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# New interpretation of the role of water balance in an extended Budyko hypothesis in arid regions

C. Du<sup>1,2</sup>, F. Sun<sup>1</sup>, J. Yu<sup>1</sup>, X. Liu<sup>1</sup>, and Y. Chen<sup>3</sup>

<sup>1</sup>Key Laboratory of Water Cycle and Related Land Surface Processes, Institute of Geographic Science and Natural Resources Research, Chinese Academy of Sciences, Beijing 100101, China

<sup>2</sup>University of Chinese Academy of Sciences, Beijing, 100049, China

<sup>3</sup>State Key Laboratory of Desert and Oasis Ecology, Xinjiang, Institute of Ecology and Geography, Chinese Academy of Sciences, Urumqi, 830011, China

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Correspondence to: [F. Sun \(sunfb@igsnr.ac.cn\)](mailto:sunfb@igsnr.ac.cn)

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## Abstract

The Budyko hypothesis (BH) is an effective approach to investigating long-term water balance at large basin scale under steady state. The assumption of steady state prevents applications of the BH to basins, which is unclosed, or with significant variations in **soil water storage**, i.e., under unsteady state, such as in extremely arid regions. In this study, we choose the Heihe River Basin (HRB) in China, an extremely arid inland basin, as the study area. We firstly use a calibrated and then validated monthly water balance model, i.e., the *abcd* model to quantitatively determine annual and monthly variations of water balance for the sub-basins and the whole catchment of the HRB and find that the role of **soil water storage** change and that of inflow from upper sub-basins in monthly water balance are significant. With the recognition of the inflow water from other regions and the **soil water storage** change as additional possible water sources to evapotranspiration in unclosed basins, we further define the equivalent precipitation ( $P_e$ ) to include local precipitation, inflow water and **soil water storage** change as the water supply in the Budyko framework. With the newly defined water supply, the Budyko curve can successfully describe the relationship between the evapotranspiration ratio and the aridity index at both annual and monthly timescales, whilst it fails when only the local precipitation being considered. Adding to that, we develop a new *Fu*-type Budyko equation with two non-dimensional parameters ( $\omega$  and  $\lambda$ ) based on the deviation of *Fu*'s equation. Over the annual time scale, the new *Fu*-type Budyko equation developed here has more or less identical performance to *Fu*'s original equation for the sub-basins and the whole catchment. However, over the monthly time scale, due to large seasonality of **soil water storage** and inflow, the new *Fu*-type Budyko equation generally performs better than *Fu*'s original equation. The new *Fu*-type Budyko equation ( $\omega$  and  $\lambda$ ) developed here enables one to apply the BH to interpret regional water balance over extremely dry environments under unsteady state (e.g., unclosed basins or sub-annual timescales).

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are significantly changing natural hydrological cycle and breaking the original water balance to form a new balance under the new hydroclimatic conditions. For example, the transferring water becomes the new water source of the basin to evapotranspiration due to the implemented inter-basin water transfer project (Bonacci and Andric, 2010).

In dry regions, croplands expand with irrigation, which increased water availability for evapotranspiration (Gordon et al., 2005). Land use/cover changes have also caused the change of runoff (Li et al., 2014). Nowadays, most of the inhabited basins have been developed or disturbed by so large-scale human activities. Therefore, lots of basins were no longer closed or natural and the relationship between annual evapotranspiration ratio and potential evapotranspiration ratio hardly meet the first condition of the BH, which presents great challenge in applying the BH in unclosed basins.

Secondly, water storage change can be assumed to be negligible at the basin scale and at long-term time scale. However, over finer temporal scales, it becomes increasingly concerned of the importance of water storage in water balance in the Budyko framework. For example, Wang et al. (2009) found that the inter-annual water storage change should be considered due to the hysteresis response of the base flow to the inter-annual precipitation change in Nebraska Sand Hills. Zhang et al. (2008) considered the impacts of soil water and groundwater storage and developed a monthly water balance model based on the BH with application in 265 catchments in Australia. Yokoo et al. (2008) highlighted the importance of soil water storage change in determining both annual and seasonal water balances. Wang (2012) evaluated changes in inter-annual water storage at 12 watersheds in Illinois using the field observation of long-term groundwater and soil water found that the impact of inter-annual water storage changes on the water supply in the BH need to be considered. Chen et al. (2013) defined the difference between rainfall and storage change as effective precipitation to develop a seasonal model for construction long-term evapotranspiration. Therefore, water storage change should be taken into account as the important part of the steady state assumption of the BH (Zhang et al., 2008).

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In summary, it has been more and more recognized that water systems are no longer natural to different extents (Sivapalan et al., 2011). Hence, it presents a **grant** challenge to apply the BH to unsteady state conditions (unclosed basins or intense water storage changes). The BH has been widely applied to mild arid basins with precipitation of 300–400 mm and aridity index of less than, for example, five, such as over northern China (Yang et al., 2007), the southwest regions of MOPEX catchments (Gentine et al., 2012; Carmona et al., 2014) and the west of Australia (Zhang et al., 2008). However, it is rare of applying the BH in extremely arid environments (say, the aridity index over five), where water systems are typically unclosed with intense human interference and irrigation. For example, rivers in the arid region of Northwestern China are typically from upper mountains with little human interference, and **flowing** through middle regions with intensive irrigation and human interferences and finally into extremely dry desert plains. To investigate it in more detail, we choose the Heihe River Basin (HRB), the second largest arid inland basin in northwestern China (mean annual aridity index = 10). Being an inland basins, the HRB consists of six sub-basins with different landscapes and climate conditions, where the upper mountainous basins are closed and natural with little human interference (long term mean annual water storage change approaches zero), the middle basins are arid and intensively irrigated plain with strong human interference (mean annual evapotranspiration is higher than the local precipitation), and the lower basin is extremely dry Gobi desert plain without any runoff flowing out (evapotranspiration is mainly the local precipitation, mean annual evapotranspiration approaches to mean annual precipitation). **We aim at (1) testing whether the BH is applicable for the unsteady state condition in extremely arid basins, (2) improving the original BH by including observed water balance and (3) extending its applicability at unclosed basin scale and annual or monthly time scales.**

## 2 Theory and method

### 2.1 Annual and monthly water balance analysis

In the original BH, the basin is a natural hydrologic unit, and the only possible water source to evapotranspiration is the local precipitation. Annual or monthly water balance equation can be written as.

$$P = ET + Q_{\text{out}} - Q_{\text{in}} + \Delta S + \Delta G \quad (1)$$

where,  $P$  is the local annual or monthly precipitation (mm);  $ET$  is the sum of soil evaporation and vegetation transpiration (mm);  $Q_{\text{out}}$  is the outflow away from a basin (mm);  $Q_{\text{in}}$  is the transferring water with other basins (mm);  $\Delta S$  is the soil water storage change (mm);  $\Delta G$  is the groundwater storage change (mm).

Because of human interference (land cover change, dams, irrigation and other withdrawals) to the hydrologic system worldwide, the water supply to evapotranspiration in a basin has changed. Local groundwater and soil water and external water transfer also become new possible water sources. However, that new non-ignorable part of available water for evapotranspiration has yet been explicitly considered in the Budyko framework in an unclosed basin. More specifically, the inflow or/and inter-basin water transfer may affect the available water for evapotranspiration largely. By considering that, here we rearrange Eq. (1) as  $P + Q_{\text{in}} - \Delta S = ET + Q_{\text{out}} + \Delta G$  the available water for evapotranspiration in Eq. (1) as

$$P_e = P + Q_{\text{in}} - \Delta S \quad (2)$$

Where, the total water supply to evapotranspiration in an unclosed basin is denoted as  $P_e$  and for simplicity,  $P_e$  hereafter is defined as the equivalent precipitation of the BH at finer time scales. If  $\Delta S$  is more than zero, it means the surplus water is stored in the vadose zone, which should be deducted from the water sources. If  $\Delta S$  is less than zero, it means soil water contributes to the evapotranspiration consumption. Note

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that the change of groundwater storage ( $\Delta G$ ) is the result of the exchange between groundwater and baseflow and is not directly interacted with evapotranspiration, so that  $\Delta G$  is not included into the defined  $P_e$  in Eq. (2). It will be discussed in the results section.

## 2.2 Budyko hypothesis model at annual and monthly scale

As discussed above, in the original Budyko framework, the water supply to land evapotranspiration is mean annual precipitation, and the energy supply to land evapotranspiration is estimated by mean annual potential evapotranspiration. The general Budyko equation can be written as:

$$\frac{ET}{P} = F\left(\frac{ET_0}{P}\right) \quad (3)$$

where,  $\frac{ET}{P}$  is the evapotranspiration ratio;  $\frac{ET_0}{P}$  is the aridity index.  $F()$  is the function to be determined. The general analytical solution to Eq. (3) over mean annual timescales is derived by Fu (1981) and is written as follows:

$$ET = ET_0 + P - (ET_0^\omega + P^\omega + C)^{1/\omega} \quad (4)$$

where,  $\omega$  is the parameter, which reflects the integrated effects of soil, vegetation and topography on **separating the local precipitation**. If the local precipitation is zero, evapotranspiration approaches to zero due to no available water,  $C$  is zero constant. Note that another form of the BH is also given by Mezentsev (1955) (later, Choudhury, 1999 and Yang et al., 2008), which is, in fact, identical to Fu's equation (Zhou et al., 2015) with the parameters linearly related ( $R^2 = 0.9997$ ) (Sun, 2007).

Water balance analysis in Sect. 2.1 concludes that the water supply in the BH under the unsteady state condition is the equivalent precipitation instead of the local precipitation. So the annual (or monthly) evapotranspiration ratio is redefined as the ratio of annual (or monthly) evapotranspiration and equivalent precipitation,

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and the annual (or monthly) aridity index is redefined as the ratio of annual (or monthly) potential evapotranspiration and equivalent precipitation. They are described as follows:

$$\frac{ET}{P_e} = \frac{ET}{P + Q_{in} - \Delta S} \quad (5)$$

$$\frac{ET_0}{P_e} = \frac{ET_0}{P + Q_{in} - \Delta S} \quad (6)$$

If the equivalent precipitation can be evaporated by enough available energy ( $ET_0/P_e \rightarrow \infty$ ), then annual (or monthly) evapotranspiration may approach annual (or monthly) precipitation ( $ET/P_e \rightarrow 1$ ). Such condition is moisture – constrained. While, if the available energy to evaporate the annual (or monthly) precipitation is limited ( $ET_0/P_e \rightarrow 0$ ), the annual (or monthly) evapotranspiration may approach annual (or monthly) potential evapotranspiration ( $ET/ET_0 \rightarrow 1$ ). Such condition is energy-constrained. Figure 1 describes partitioning of the equivalent precipitation into evapotranspiration and streamflow, which follows the BH. The Budyko equation under unsteady state assumption can be written as,

$$\frac{ET}{P_e} = F \left( \frac{ET_0}{P_e} \right) \quad (7)$$

Under the unsteady state conditions for a region, when the local precipitation in the origin  $Fu$ 's equation is zero, evapotranspiration may not be zero due to other water sources (e.g. inter-basin water transfer), so following the derivation of  $Fu$ , 1981.

**Equation (4)** can be rewritten as,

$$\frac{ET}{P_e} = 1 + \frac{ET_0}{P_e} - \left[ 1 + \left( \frac{ET_0}{P_e} \right)^\omega + \lambda \right]^{1/\omega} \quad (8)$$

where,  $\omega$  and  $\lambda$  are two fitting parameters and both non-dimensional.  $\omega$  has been widely discussed and is greater than 1 (Fu, 1981; Yang et al., 2007). By meeting the

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constraints formed by the BH, we can derive that  $\lambda \geq -1$  (see Appendix A). When  $\lambda = 0$  (Fig. 2a), Eq. (8) is the same as the *Fu*'s Equation in its original form (Fu, 1981; Zhang et al., 2004; Yang et al., 2007). For  $\lambda$  becomes positive, e.g., 1, the lower end of the Budyko curve adjusts to the right (Fig. 2b, c). And  $\lambda = 1$  (Fig. 2c, d) sets up the upper theoretical constraint of the Budyko curve (Fig. 2c, d).

### 2.3 A monthly water balance model: *abcd* model

Regional evapotranspiration and soil water cannot be measured directly and they are usually provided by using monthly water balance models. Monthly water balance models were first developed in the 1940s. From that, many models have been developed in hydrological studies, such as *T* model, *T $\alpha$*  model, *P* model, *abc* model and *abcd* model are often popular due to relatively simple structure and fewer parameters (Fernandez et al., 2000).

Among these monthly models, the *abcd* model was proposed by Thomas (1981) has been widely applied to assess regional water resources due to its explicit model structure and only four parameters, of which two parameters pertain to runoff characteristics and the other two relate to groundwater sound physical meanings.

The model was originally applied at the annual time scale and later extended to the monthly time scale (Alley, 1984). Moreover, Savenije (1997) has verified that the *abcd* model to derive expressions for the evapotranspiration ratio has better agreement with observations than Budyko-type curves. Inputs to the *abcd* model are monthly precipitation and potential evapotranspiration. Outputs include monthly runoff (direct and indirect), soil water groundwater storage and actual evapotranspiration. Therefore, this study employs the *abcd* model to provide monthly actual evapotranspiration and soil water storage.

The partitioning of monthly precipitation  $P_t$  in the model is as follows; runoff  $Q_t$  (direct and indirect), evapotranspiration  $ET_t$ , soil water storage  $S_t$ , and groundwater storage  $G_t$ . The partitioning is controlled by the magnitude of precipitation  $P_t$ , potential evapotranspiration  $ET_{0t}$ , and the initial storages in soil  $S_{t-1}$  and groundwater  $G_{t-1}$ . The

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following equation controls the partitioning:

$$Y_t(W_t) = \frac{W_t + b}{2a} - \sqrt{\left(\frac{W_t + b}{2a}\right)^2 - \frac{W_t b}{a}} \quad (9)$$

where  $Y_t$  is the sum of monthly evapotranspiration and soil water storage at the end of the month, namely evapotranspiration opportunity.  $W_t$  is the sum of monthly precipitation and initial soil moisture, named as available water. The parameter  $a$  ( $0 \sim 1$ ) means the propensity in a catchment for runoff to occur before the soil becomes saturated. The parameter  $b$  is the maximum value of  $Y_t$ . Available water partitioning between  $ET_t$  and  $S_t$  is controlled by the assumption that the loss rate of actual evaporation from soil water storage is proportional to the evapotranspiration capacity. So the soil water storage at the end of period  $t$  is written as:

$$S_t = Y_t \exp(-ET_{0t}/b) \quad (10)$$

The actual evapotranspiration at the period  $t$  is the difference between evapotranspiration opportunity and soil water storage ( $Y_t - S_t$ ). The streamflow, including direct runoff and groundwater recharge, is determined by the difference between available water and evapotranspiration opportunity ( $W_t - Y_t$ ). The parameter  $c$  separates the direct runoff  $(1 - c)(W_t - Y_t)$  and groundwater recharge  $c(W_t - Y_t)$ . Groundwater discharge  $dG_t$  as the base flow is determined by the parameter  $d$  and groundwater storage at the end of period  $t$ . The streamflow is sum of direct runoff and the base flow. For a given set of  $b$ ,  $c$  and  $d$  and initial soil water storage and groundwater storage, the allocation of monthly precipitation can be computed one by one.

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### 3 Study area and data

#### 3.1 Study area

The HRB, originating from Qilian Mountains, is the second largest inland river basin in the arid area of the northwestern China (Fig. 3). The drainage map and the basin border are extracted using a 90 m resolution digital elevation model (DEM) data from the Shuttle Radar Topography Mission (SRTM) website of the NASA (<http://srtm.csi.cgiar.org/SELECTION/inputCoord.asp>) (basin length: 820 km; total area: 143 044 km<sup>2</sup>; elevation: 870–5545 m). The HRB is in the middle of Eurasia and away from oceans, characterized with dry and windy climate, and very limited precipitation (mean annual precipitation: 126 mm yr<sup>-1</sup>) but plentiful radiation (mean annual solar radiation: 1780 MJ m<sup>-2</sup> yr<sup>-1</sup>, ~ 660 mm yr<sup>-1</sup> in the unit of evaporation).

The HRB is divided into six sub-basins according to basin characteristics, distributing along eastern and western tributaries, shown in Fig. 3. Regions I and II are upper mountainous regions with the elevation of 3000–5500 m and belong to the cold and semiarid mountainous zone dominated by shrubs and trees with mean annual temperature of less than 2 °C and annual precipitation of 200 ~ 400 mm. And these two sub-basins are the water source area to the middle and lower reaches and have little interference of human activities. Regions III and V with annual precipitation of 100–250 mm are the main irrigation zone and residential area with more than 90 % of total population of the HRB. The two sub-basins are the main water-consuming regions and largely disturbed by human activities. Regions IV and VI located at lower reaches are extremely arid and the mean annual precipitation is less than 100 mm.

#### 3.2 Data

The required data for Eq. (8) and the *abcd* model include monthly precipitation, potential evapotranspiration and runoff from those sub-basins in the HRB.

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The daily precipitation data of all stations during 1978–2012 are obtained from the year book hydrology of China including 28 rainfall stations and the China Administration of Meteorology including 19 meteorological stations (Fig. 3). The monthly precipitation of each station is calculated by summing daily precipitation. The gridded data set with 1 km resolution across the whole basin is obtained by interpolation of the site data. The monthly precipitation of the six sub-basins is obtained by the extraction from the monthly precipitation in the whole basin. Daily meteorological data of 19 stations during 1978 and 2012 are also available. Daily potential evapotranspiration is estimated in each station using the FAO Penman–Monteith equation recommended by Allen et al. (1998). The monthly  $ET_0$  at each station is the sum of the daily  $ET_0$  and then interpolated to the whole basin. Finally, annual runoff, precipitation and potential evapotranspiration are obtained by summing monthly data.

The runoff data set includes monthly runoff at 4 stations located at the inflow or outflow of the six sub-basins. The red points in Fig. 3 locate the hydrological stations. Monthly runoff data are obtained from the year book of hydrology of China and are intended for calibrate the *abcd* model. The annual runoff is obtained by summing monthly runoff. The data time series for Regions I and III are from 1978 to 2012. The same period is for Regions II and V but with the period of 1998–2006 missing. The length of data time series for Regions IV and VI is from 1988 to 2012.

The natural runoff in Regions III and IV were strongly disturbed by human activities and there is no runoff for the Regions V and VI and the whole basin. To validate the outputs of the *abcd* model for those regions, this study employs the evapotranspiration of remote sensing products from Heihe Plan Science Data Center (Wu et al., 2012) as a reference. The same data have been widely used as a reference for modeling evaluations and is supported by a State Key Research Program-Heihe Eco-hydrological Research Project of National Natural Science Foundation of China (Yan et al., 2014; Yao et al., 2014). The monthly evapotranspiration datasets (2000–2012) with 1 km spatial resolution over the HRB (<http://westdc.westgis.ac.cn>), are estimated by ETWatch model based on multi-source remote sensing data (Wu et al., 2012).

## 4 Results

### 4.1 Calibration of the *abcd* model

In extremely dry basins like the HRB, the lack of observed hydro-climatic data presents great challenge. A monthly water balance model becomes an effective tool to estimate actual evapotranspiration, change of soil water storage and change of groundwater storage. This study employs the *abcd* water balance model due to its simple and sound physical structure tested and recommended by Alley (1984) and Fernandez et al. (2000). We calibrate and validate the *abcd* model using monthly time series of precipitation, potential evapotranspiration and runoff at each of the seven regions (the six sub-basins and the whole basin) using the generalized pattern search optimization method. Nash–Sutcliffe efficiency (NSE) is used to assess the goodness of fit of the monthly water balance for the seven regions.

Figure 4 shows the results of the modeled streamflow at monthly time scale in Regions I and II. Regions I and II are the water source area of the whole basin with little interference of human activities and both keep relatively natural steady state. The NSE for the Regions I and II is for 0.92 and 0.83, respectively. The results illustrate that the simulated monthly streamflow agrees well with the observation and other modeled components can be reasonable estimates, for instance, monthly actual evapotranspiration, soil water storage change and groundwater storage change in the two sub-basins.

~~The streamflow in Regions III and IV was completely controlled by hydrological stations due to water resources allocation, so the observed monthly streamflow cannot be directly used to validate the simulations of the *abcd* model. To validate the results of actual evapotranspiration, we compare the simulated ET by the *abcd* model and remote sensing ET (Fig. 5) calculated by remote sensing data during 2000–2012. The NSE values for Regions III–VI and the whole basin are not less than 0.8, which illustrate that simulated ET have good agreement with ET by remote sensing.~~

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for the sub-basins and the whole HRB on the monthly timescales, which in turn requires new treatments in the BH as further investigated in following sections.

### 4.3 The annual Budyko curve analysis

Figure 8 (left panel) plots the original Budyko curves for the six sub-basins and the whole basin. For Regions I and II, the points of annual evapotranspiration ratio and aridity index fall in the domain of water and energy limit boundary and they can be well fitted by  $F_u$ 's equation. The relationship between water and energy in Regions I and II can be described by the original BH as expected in the section above. However, the points of evapotranspiration ratio and aridity index for other regions exceed the water limit boundary. And the results show the relationship of water and energy in Regions III–VI and the whole basin is inconsistent with the original BH. After using the equivalent precipitation instead of the local precipitation, the new  $F_u$ -type Budyko curves (Eq. (8) with  $\lambda = 0$ ) for all regions are shown in Fig. 8 (middle panel). Compared with the original Budyko curve, the new curves for Regions I and II did not behave differently, because the two basins are natural and closed. The obvious change between the improved and original Budyko curves are for the Regions III and IV. For the whole basin and Regions V and VI, the new curves fall on the upper limit of  $ET/P_e = 1$  due to no runoff flowing out. These improved Budyko curves can be fitted using  $F_u$ 's equation and the parameters are listed in Table 3. Interestingly for the annual time scale, the fitted performances of  $F_u$ 's equation and Eq. (8) are almost identical. Therefore, the new  $F_u$ -type Budyko curves (Eq. 8) with fitted values of  $\lambda$  (right panel, Fig. 8) do not show much difference from those curves with  $\lambda$  set zero.

In summary, if a basin (sub-basin) is closed, the original BH can be applicable at the annual time scale. However under unsteady state, the new  $F_u$ -type BH, instead of the original BH is more applicable to describe the annual water balance.

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#### 4.4 The monthly Budyko curves analysis

Again as expected based on the monthly water balance analysis, the points of monthly evapotranspiration ratio and aridity index exceed the water limit boundary for all the basins (Fig. 9, left panel). The value of evapotranspiration ratio can be up to 40, which means that the local precipitation in original water balance is well below the actual water supply to the evapotranspiration. The new  $F_u$ -type Budyko curves at the monthly timescale are shown in Fig. 9 on the middle panel (Eq. 8 with setting  $\lambda = 0$ ) and on the right panel (Eq. 8 with calibrated  $\lambda$ ). It is remarkable that the points of monthly evapotranspiration ratio and aridity index distribute regularly in the Budyko framework (in Fig. 9, middle panel and right panel). The improved Budyko curves with calibrated  $\lambda$  perform better than  $F_u$ 's original equation (i.e.,  $\lambda = 0$ ) by 5–10% in terms of NSF. The fitting parameter  $\lambda$  introduced in this study (Eq. 8) can add further improvement to the BH, in despite of obviously deserving further investigations.

The fitted values of the parameters in the Budyko curves for Regions I to VI are listed in the Table 4. These curves and the parameters have significantly seasonal characteristics. For example, the Budyko curves in Regions I and II can be divided to five groups (Fig. 9). The values of the integrated parameter  $\omega$  in Eq. (8) gradually decrease from the summer months to winter months. The absolute values of parameters  $\lambda$  gradually increase, which illustrates that the points in summer months are more centralized than those in winter months. Moreover, in Regions V and VI and the whole basin, all the equivalent precipitation is consumed by evapotranspiration, and therefore the ratio of evapotranspiration to the equivalent precipitation is almost one.

#### 4.5 Storage change and inflow water impact on the BH

In this study, we intended to extend the BH to the annual and sub-annual time scales by explicitly considering the soil water storage and new water source from other regions. To further investigate it, we choose Region I and Region III as typical cases in Fig. 10. In Region I, as there is no inflow into the region, we can separate the impact of soil

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water storage and groundwater storage on the BH (Fig. 10a). With subtle difference, the impacts of changes in soil water storage and groundwater storage on water balance can be almost ignored at annual scales. Region III is another extreme case where only if the role of the inflow water being considered, the BH can perform well under unsteady state (Fig. 10b).

In Fig. 11, we further adopted the approach presented by Chen et al. (2013) to examine the impacts of soil water storage, groundwater storage and inflow water on monthly water balance. We test different combinations in monthly water balance in Region III, a midstream sub-basin of the HRB (Fig. 11a–c) and found that when the equivalent precipitation includes the soil water storage change the BH performs well at the monthly scale. However, the inclusion of the groundwater storage change into the equivalent precipitation does not improve as much (Fig. 11b, c). By examining the impact of monthly inflow water on the BH in Region III (Fig. 11d, f), we find that inflow water at the monthly scale has as much impact as that at annual scale. The results presented above highlight the fact that the water supply cannot be the local precipitation only, but should have included soil water storage change and inflow water.

## 5 Conclusions

The Budyko Hypothesis (BH) is a useful approach to depicting and understanding the long term mean water balance at large basin scale under steady state condition. However, river systems worldwide have in fact been disturbed by human to different extents. That is important for extremely arid environments (say, the aridity index over five) especially in China, where water systems are typically unclosed with intense human inference and irrigation. That presents grand challenge if one is applying the BH to those regions under unsteady state e.g., unclosed or significant variation in soil water storage, or those time scales finer than a year.

To investigate it, we choose an extremely arid inland basin, the Heihe River Basin in China as the study area, which is divided into six sub-basin based on catchment

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hydrologic characteristics. We first calibrate and validate a widely used monthly water balance model, i.e., the *abcd* model. For the two upper sub-basins, the simulated monthly water balance is compared against monthly streamflow from hydrological gauges, and for the other sub-basins and the whole catchment, the simulated evapotranspiration is compared with widely used remote sensing ET products in the HRB. The *abcd* model can successfully simulate the monthly water balance and capture the inter-annual variations (NSE over 0.85). Based on that, we found that the role of soil water storage change in monthly water balance is significant but almost negligible over timescales longer than a year. And the impact of inflow from upper sub-basins is also significant and does not rely on the timescale. We concluded that the upstream basin in the HRB are almost closed basins, which meet the two steady state conditions of the BH and other sub-basins become an unclosed basin due to impact of the inflow water and human interference.

With the recognition that the inflow water from other regions and the water storage change are both new possible water sources to evapotranspiration in unclosed basins, we define the equivalent precipitation ( $P_e$ ) including the local precipitation, inflow water and water storage change as the water supply, instead of just the local precipitation, in the Budyko framework. (The evapotranspiration ratio and the aridity index are also redefined using the equivalent precipitation.) In addition to the new definition of the water supply, we develop a new *Fu*-type Budyko equation with two non-dimensional parameters ( $\omega$  and  $\lambda$ ) based on the deviation by Professor Baopu Fu, i.e., *Fu*'s equation to consider the effect of the change of soil water storage and the inflow water on the water and energy constraints. Over the annual time scale, the new *Fu*-type Budyko equation developed here has more or less identical performance to the *Fu*'s equation for the sub-basins and the whole catchment. However, for the monthly time scale, the new *Fu*-type Budyko equation performs better than *Fu*'s original equation when the ratio of evapotranspiration to equivalent precipitation less than one, and performs the same when the evapotranspiration ratio is very close to one. The new *Fu*-type Budyko equation ( $\omega$  and  $\lambda$ ) developed in this study enables one to apply the BH to interpret

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regional water balance over extremely dry environments under unsteady state (e.g., unclosed basins or sub-annual timescales).

## Appendix A

For an unclosed basin or region, the water supply to evapotranspiration is defined as equivalent precipitation ( $P_e = P + Q_{in} - \Delta S$ ). Evapotranspiration ratio:  $\varepsilon = ET/P_e$  and aridity index:  $\phi = ET_0/P_e$ . The Budyko equation is written the same as Eq. (8)

$$\varepsilon = 1 + \phi - (1 + \phi^\omega + \lambda)^{1/\omega} \quad (A1)$$

According to the constrained boundary of the BH, Eq. (1) evapotranspiration ratio is less than or equal to aridity index, namely  $\varepsilon \leq \phi$ , and Eq. (1) the evapotranspiration ratio is no more than 1, i.e.,  $\varepsilon \leq 1$ ,

With  $\varepsilon \leq \phi$ , we can have,

$$1 + \phi - (1 + \phi^\omega + \lambda)^{1/\omega} \leq \phi \quad (A2)$$

Therefore,

$$\phi^\omega + \lambda \geq 0 \quad (A3)$$

where  $\phi \geq 0$  and  $\omega > 1$ .

For the other constraint,  $\varepsilon \leq 1$  we can derive,

$$1 + \lambda \geq 0. \quad (A4)$$

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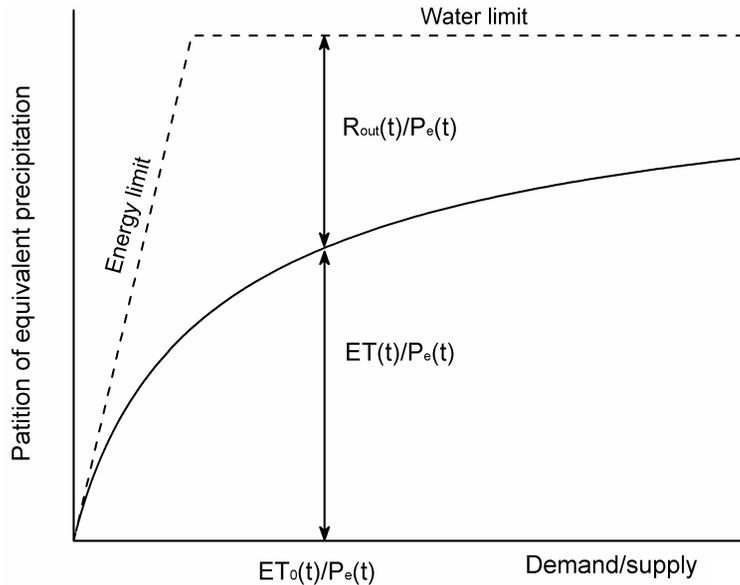
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**Figure 1.** A schematic diagram of the BH under the unsteady state condition.

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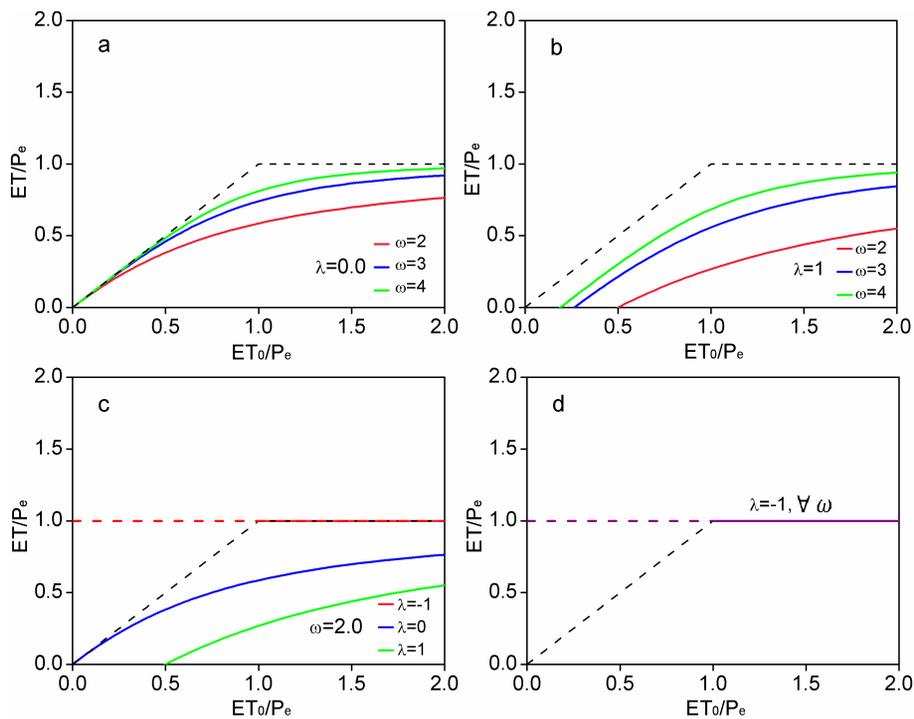
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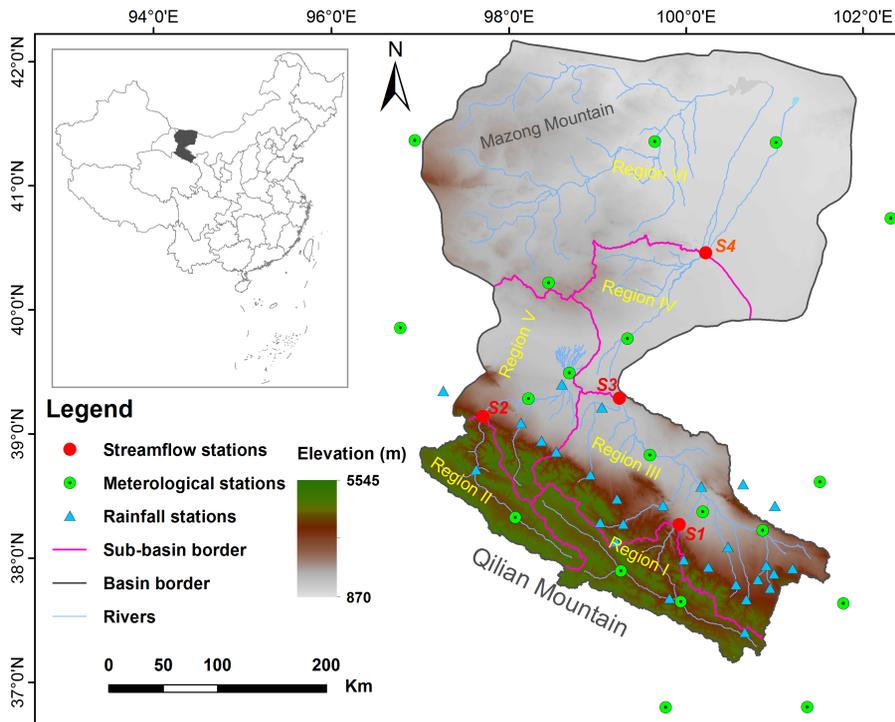
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**Figure 2.** The Budyko curves in Eq. (8) with different combinations of parameters  $\omega$  and  $\lambda$ .



**Figure 3.** Location of study area and the distribution of hydrological stations and meteorological stations.

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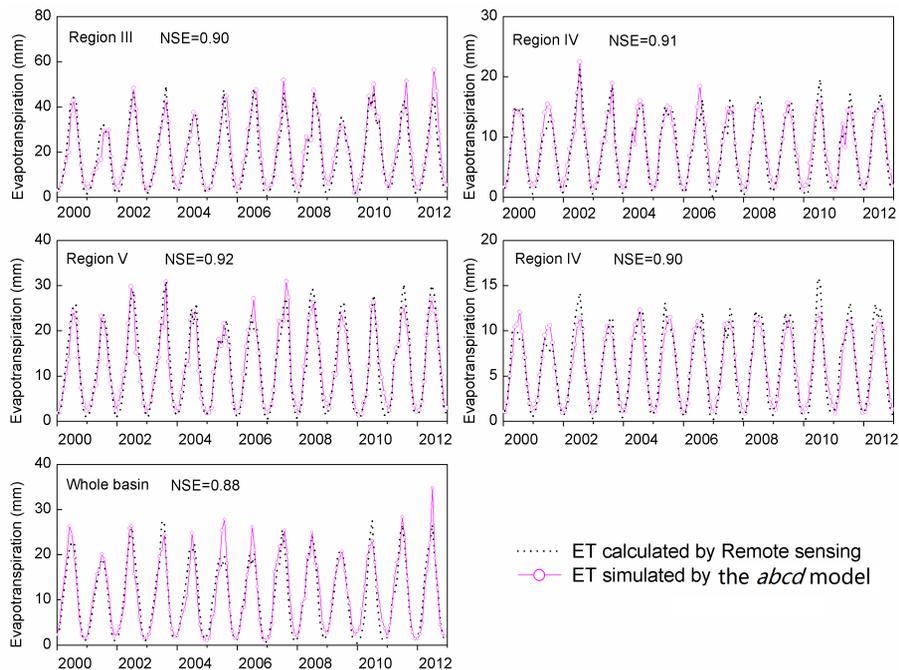
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**Figure 5.** Comparison between ET simulated by the *abcd* model and ET calculated by remote sensing data for Regions III–VI and the whole basin during 2000–2012. “WBM” denotes the *abcd* water balance model.

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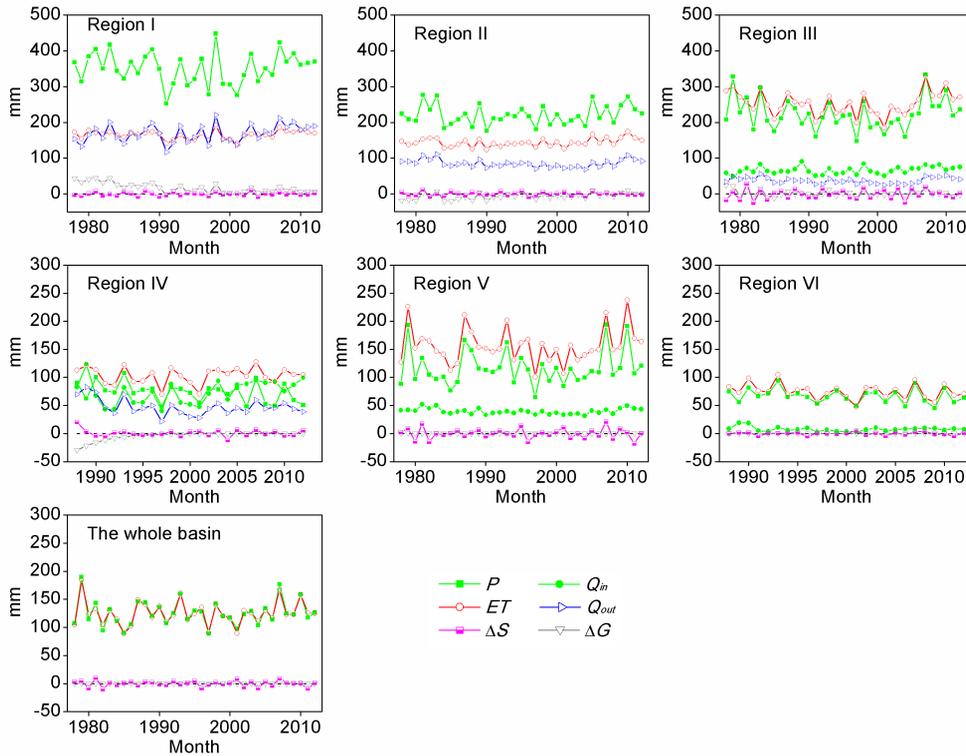
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**Figure 6.** Variation of annual water balance for all the regions simulated using the *abcd* model.

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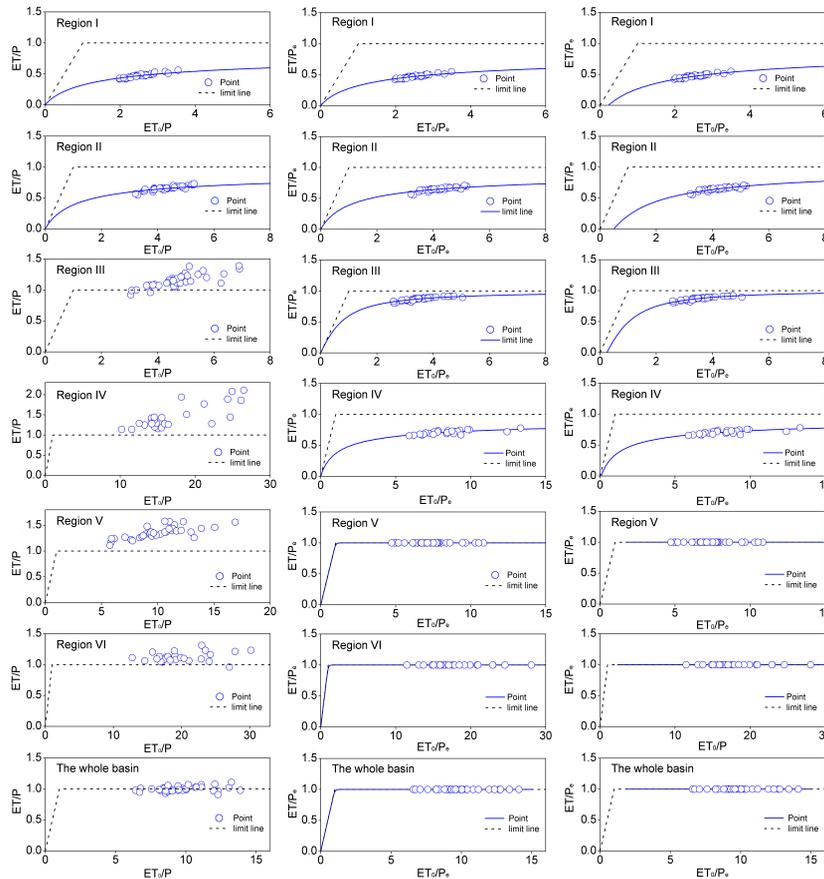
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**Figure 8.** Comparison of the original Budyko curves (left panel) and the new  $F_u$ -type Budyko curves (middle panel, with  $\lambda = 0$ ) and the new  $F_u$ -type Budyko curves (right panel, with  $\lambda > 0$ ) for Regions I–VI and the whole basin at the annual time scale.

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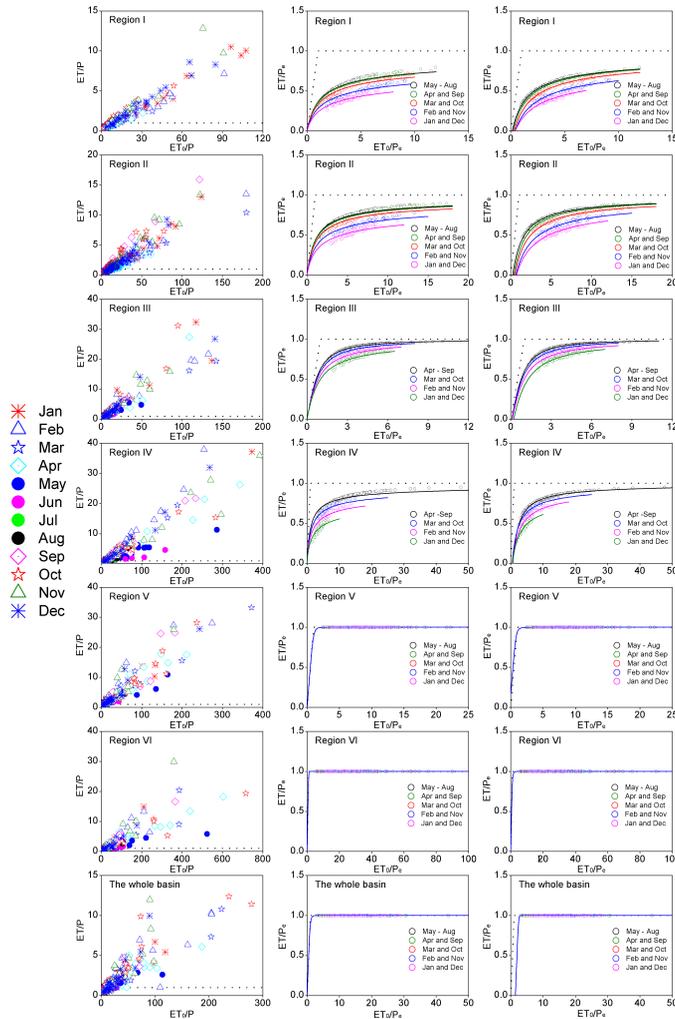
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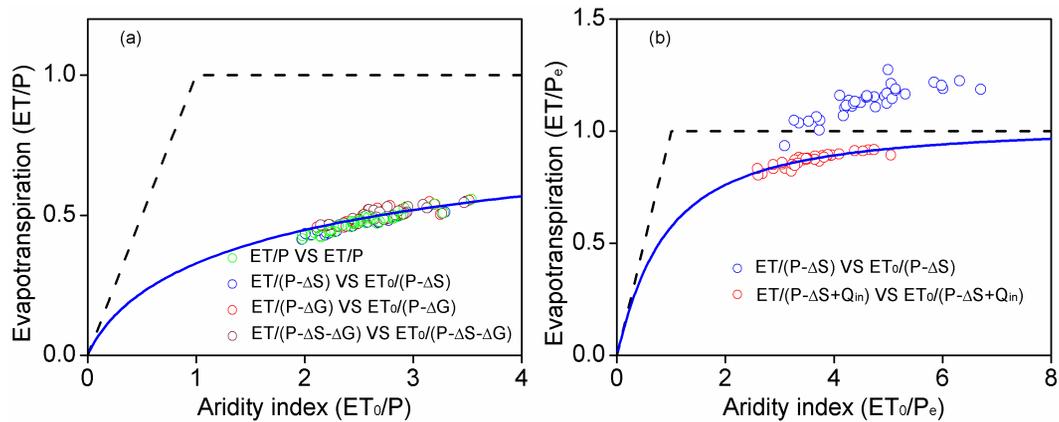
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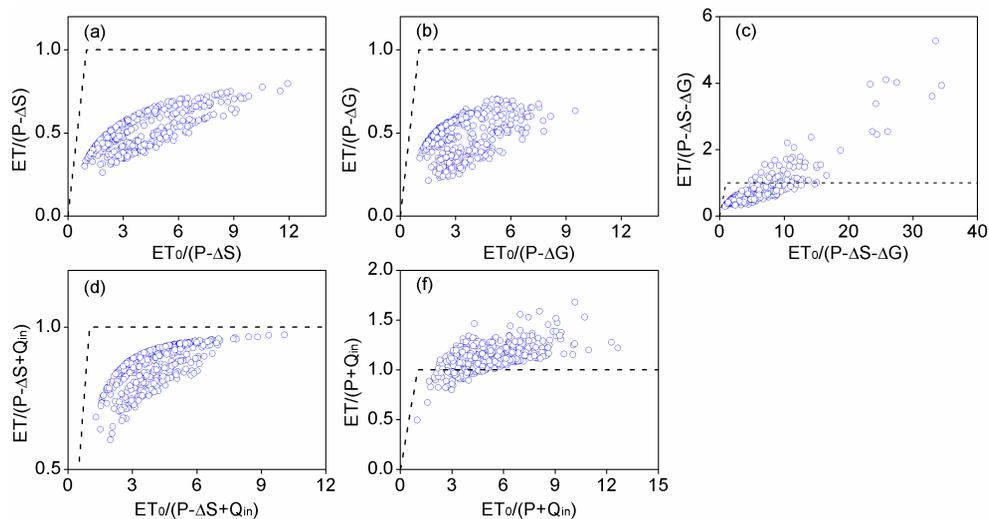


**Figure 10.** Different presentations of annual water balance for **(a)** Region I and **(b)** Region III.

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**Figure 11.** Five presentations of monthly water balance for Region III considering different combinations in the water supply to evapotranspiration.

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