The mathematical manipulations and procedures are as follows.

By taking Laplace inverse transform on $Q_i(\xi_l, n, s)$, we have

$$q_i(\xi_l, n, T) = L^{-1}[Q_i(\xi_l, n, s)]$$

$$= L^{-1}\left[\sum_{k=0}^{k=i-2} \frac{\beta_{i-k-1} \prod_{j_1=0}^{j_1=k} \sigma_{i-j_1}}{\prod_{j_2=0}^{j_2=k+1} \left(s + \alpha_{i-j_2}\right)} F_{i-k-1}(s)\right]$$

$$= \sum_{k=0}^{k=i-2} \beta_{i-k-1} \prod_{j_1=0}^{j_1=k} \sigma_{i-j_1} L^{-1}\left[\frac{1}{\prod_{j_2=0}^{j_2=k+1} \left(s + \alpha_{i-j_2}\right)} F_{i-k-1}(s)\right] \tag{1}$$

Expressing $\dfrac{1}{\prod_{j_2=0}^{j_2=k+1} \left(s + \alpha_{i-j_2}\right)}$ **as the summation of partial fractions and applying the inverse Laplace transform formula, one gets**

$$L^{-1}\left[\frac{1}{\prod_{j_2=0}^{j_2=k+1} \left(s + \alpha_{i-j_2}\right)}\right] = L^{-1}\left[\sum_{j_2=0}^{j_2=k+1} \frac{1}{\prod_{j_3=i-k-1,\, j_3 \neq i-j_2}^{j_3=i} (\alpha_{j_3} - \alpha_{i-j_2})\left(s + \alpha_{i-j_2}\right)}\right]$$

$$= \sum_{j_2=0}^{j_2=k+1} \frac{e^{-\alpha_{i-j_1} T}}{\prod_{j_3=i-k-1,\, j_3 \neq i-j_1}^{j_3=i} (\alpha_{j_3} - \alpha_{i-j_1})} \tag{2}$$

The inverse Laplace transform of $F_{i-k-1}(s)$ **is** $f_{i-k-1}(T)$**.**

The Laplace inverse transform of $\dfrac{1}{\prod_{j_2=0}^{j_2=k+1} \left(s + \alpha_{i-j_2}\right)} F_{i-k-1}(s)$ **in Eq. (1) can be achieved using the convolution integral equation as**

$$L^{-1}\left[\frac{1}{\prod_{j_2=0}^{j_2=k+1}\left(s+\alpha_{i-j_2}\right)}F_{i-k-1}(s)\right]=\sum_{j_2=0}^{j_2=k+1}\frac{e^{-\alpha_{i-j_1}T}\int_0^T e^{\alpha_{i-j_1}\tau}f_{i-k-1}(\tau)d\tau}{\prod_{j_3=i-k-1,\, j_3\neq i-j_2}^{j_3=i}\left(\alpha_{j_3}-\alpha_{i-j_2}\right)} \tag{3}$$

Putting Eq. (3) into Eq. (1), we can obtain the Eq. (34) in our manuscript as the following form:

$$q_i(\xi_l,n,T)=\sum_{k=0}^{k=i-2}\beta_{i-k-1}\prod_{j_1=0}^{j_1=k}\sigma_{i-j_1}\sum_{j_2=0}^{j_2=k+1}\frac{e^{-\alpha_{i-j_1}T}\int_0^T e^{\alpha_{i-j_1}\tau}f_{i-k-1}(\tau)d\tau}{\prod_{j_3=i-k-1,\, j_3\neq i-j_2}^{j_3=i}\left(\alpha_{j_3}-\alpha_{i-j_2}\right)} \tag{4}$$