Author's Response

First of all, I would like to deeply appreciate taking precious time for carefully examining my script, suggesting improvements and providing interesting references. I apologize for taking long time to respond.

I checked the literatures and papers about asymmetry once more. As anonymous referree# 1 commented, I agree that it is similar to the definition of Joe, H. (2014) in the sense that it compares the asymmetry along both diagonals of bivariate copula.

<Comments from Anonymous Referre#1 : suggestion for asymmetry notation>
>This reviewer would like A1 and A2 to also be expressed in direct terms of integration of the copula formula or its density. For example, a Joe (2014) definition for the primary diagonal is: \int\int_{[0,1]} (v-u)C(u,v) du dv from which a secondary asymmetry definition (not identified by Joe) can result \int\int_{[0,1]} (v+u-1)C(u,v) du dv - (1/2) Can the authors of the paper expand the definitions of A1 and A2 beyond the "expectation" notation?

<Author's Response>
"expectation" notation was conventionally used, but as anonymous referee #1 commented, it seems sensible to define in integration form beyond the expectation notation as follow:

Equation (9)

$$A_1(k) = E\left[(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) + (U_{t+k} - 0.5)\right)\right]$$
$$= \int_0^1 \int_0^1 (u - 0.5)(v - 0.5)(u + v - 1)\, c(u,v)\, du dv$$

Equation (10)

$$A_2(k) = E\left[-(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) - (U_{t+k} - 0.5)\right)\right]$$
$$= \int_0^1 \int_0^1 -(u - 0.5)(v - 0.5)(u - v)\, c(u,v)\, du dv$$

In this notation, then, the terms (u+v-1) and (u-v) appears, which is clearly related to the definition of Joe(2014). In general, it seems there are other possible ways to define and apply the asymmetry as it was mentioned by anonymous referee#1with some examples such as L-comoment (bivariate L-moment, bivariate L-skew) as follow:

<Anonymous referee#1 : L-comoment, L-Skew> … But more importantly, the very recent "break through" of L-comoment (bivariate L-moment, bivariate L-skew) definition (Brahimi et al. [2015]) directly in terms of a copula. L-coskew (bivariate skew) …

<Author's Response>

I had shortly read the paper and checked the theory of L-comoment or L-coskew. As far as I understand, it’s the extension of L-moment to multivariate case in which moments are expressed as linear combination of order statistics. The advantage is that it is applicable without the assumption of second order moment, thus suitable for heavy-tail distribution. I found this technique is sophisticated and interesting options for advanced investigation of asymmetric property of distributions including skewness in the context of hydrology and earth system sciences.

<Author’s Response : additional description about asymmetry and hydrograph>
I admit that the explanation about the relation between hydrograph and asymmetry was not good enough in my manuscript, which might have caused confusion. Now, I want to modify figure3 and add another sketch to it (please see figures_eqautions_revised.pdf in supplement), hoping this can answer to some questions such as :

<Anonymous referee#1 >9164, line 24 "... A1 ... asymmetry can be related to temporal distribution of precipitation" (what scale of time?)

<Anonymous referee#1 >“is there a connection between A1/A2 and storm water hydrographs (e.g. unit hydrographs)?“

<Anonymous referee#2>“3) Section 3. I would give more practical explanation about Copula asymmetry. It is not fully clear.“

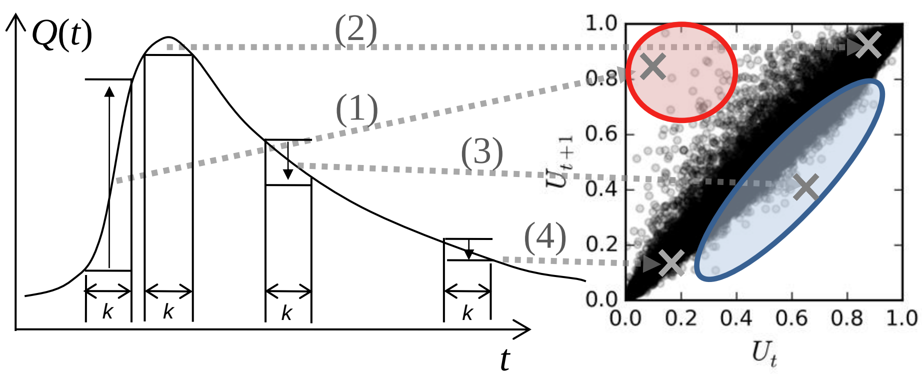

Figure 1 Sketch of the transformation from sample hydrograph (left) to empirical copula (right): Scatterplot of ranks are calculated from two values separated by time lag $k = 1$[days] in a discharge time series of Andernach where $\textit{rank correlation} = 0.9870$, $A_1(k=1) = -0.0002398$ and $A_2(k=1) = -0.00011037$. The possible combinations of high and low values, which has large impacts on asymmetry, are numbered (1) low to high, (2) high to high (3) high to low (4) low to low. Negative contribution to asymmetry2 with red circle, positive contribution with blue circle.

This figure illustrates where each pairs of values on hydrograph can be plotted on empirical copula. For example, asymmetry2 will be smaller, if there are more plots on upper left corner, which is related to curves of hydrograph.

It is also assumed from this figure that asymmetry 1 could be related to frequency and duration of rainfall:

- for small temporal scale (lag k is less than a few days), if a pair of values is like the transformation case (2), then asymmetry1 gets bigger according to the definition of asymmetry1
- For the bigger k (more than 30 days) asymmetry1 would not reflect the shape of unit hydrograph, but it would capture the asymmetric property of inter seasonal or annual cycle.

But as anonymous referee#1 commented, it is merely the assumption and this study is rather trial which is surely not enough to prove them. I also don't doubt that a great suite of similar study should be conducted to conclude something.

I would put this figure and explanation after 9164 Line 11, instead, delete the equations about de-seasonalization and shorten the explanation as anonymous referee#2 mentioned as follow:

<Anonymous referee#2> 6) Section 3.1 page 9165-9166. The de-seasonalization approach is well known (Grimaldi, S. Linear parametric models applied to daily hydrological series (2004) Journal of Hydrologic Engineering, 9 (5), pp. 383-391), maybe you can remove the equations in order to make easier the text.

In general, I agree to rest of the comments, corrections and suggestions. I would carefully modify the description about this and conclusion so that the notes and comments from the referees will be reflected.