Responses to comments by Reviewer #2

Dear Reviewer #2,

Thank you very much for your constructive comments and recommendations, which have been used to improve the quality of our manuscript. Below are our point-by-point responses to each of your comments.

Major concerns:

1. **This manuscript applies an analytical framework to determine the mean water level profile in the Yangtze estuary. The manuscript overlaps significantly with a previous publication by the same authors; therefore, it is unclear to me what is new.**

Our reply: We realise that we have not clearly spelled out the innovation of our paper, which is not just an application of a model to a case study, but an analysis that provides new analytical tools to assess the influence of river discharge on water levels in estuaries. In particular, the equations (16)—(18) are new to the analytical method and have not been published before (see also Figure 9 in the manuscript).

Indeed, the proposed analytical model for hydrodynamics has been detailed in Cai et al. (2014a). However, the current work represents a further development of the analytical model to understand the mechanism of backwater effect due to tide-river interaction and its resulted mean water level profile in estuaries with substantial fresh water discharge (taking the Yangtze estuary as an example), which is not completely understood yet. For the first time, we used a fully analytical approach to quantify the contributions made by different components (tide, river, and tide-river interaction) to the residual water level. The method is subsequently used to estimate the frequency of extreme high water along the estuary, which is particularly useful for water management and flood control.

The highlights of this paper can be summarized as follows:

1. Interplay between tide and river is described using an analytical model.
2. Contributions from tide-river dynamics to rise of mean water level are quantified.
3. Response of mean water level to tide and river discharge is explored.
4. Extreme high water level frequency distribution is analytically determined.
5. The proposed analytical approach is a useful tool for water management.

In the revised paper, we shall clearly clarify the innovation of this paper by including a new paragraph in the Introduction part:

"The current work is not just an application of a model to a case study, but an analysis that provides new analytical tools to assess the influence of fresh water discharge on water levels in estuaries. For the first time, we used a fully analytical approach to quantify the contributions made by different components (tide, river, and tide-river interaction) to the residual water level, which sheds new light on how backwaters are generated as a result of tide-river interaction. The method is subsequently used to estimate the frequency of extreme high water along the estuary, which is particularly useful for water management and flood control."

2. **In the abstract the authors state that the influence of river flow, tides, and their interaction to the mean water level is not completely understood but fail at showing how the analytical approach yields new insight about this problem. The authors should show the approach is generic across various tidal rivers, or should provide new understanding on the dynamics of river-tide interaction in the Yangtze case.**
   The abstract is not very informative. The opening sentence is suggestive of a problem that is not worth studying. Midway the abstract, the authors speak of a method but it is

unclear how this is made possible in practice; the same applies to the extreme frequency analysis.

Our reply: We agree with your comments! In the revised paper, we shall completely revise the abstract to clarify the innovation and the main results of this paper. The revised abstract is as follows:

"The mean water level in estuaries rises in landward direction due to a combination of the density gradient, the tidal asymmetry, and the backwater effect. This phenomenon is more prominent under an increase of the fresh water discharge, which strongly intensifies both the tidal asymmetry and the backwater effect. However, the interactions between tide and river flow and their individual contributions to the rise of the mean water level along the estuary are not yet completely understood. In this study, we adopt an analytical approach to describe the tidal wave propagation under the influence of fresh water discharge, where the analytical solutions are obtained by solving a set of four implicit equations for the tidal damping, the velocity amplitude, the wave celerity and the phase lag. The analytical model is used to quantify the contributions made by tide, river, and tide-river interaction to the water level slope along the estuary, which sheds new light on the generation of backwater due to tide-river interaction. Analytical model results show that in the tide-dominated region the mean water level is mainly controlled by the tide-river interaction, while it is primarily determined by the river flow in the river-dominated region. The effect of the tide alone is most important in the transitional zone, where the ratio of velocity amplitude to river flow velocity approaches unity. Subsequently, the method is applied to the Yangtze estuary under a wide range of river discharge conditions where the influence of both tidal amplitude and fresh water discharge on the longitudinal variation of the mean tidal water level is explored. Finally, we demonstrate that, in combination with extreme value theory (e.g., Generalized extreme-value theory), the method can be used to predict the frequency of extreme water levels relevant for water management and flood control."

Other concerns:

1. **Beginning of section 2.2, this sentence is not very clear, is this fact or simply expectation?**

Our reply: It is a fact and we shall include some references to clarify this point. The sentence will be updated as follows:

In a tidal river, we usually observe that the tidally averaged water level rises in landward direction (e.g., Kukulka and Jay, 2003; Buschman et al., 2009; Sassi and Hoitink, 2013; Guo et al., 2015).

2. The manuscript should include an appendix explaining the parameters employed throughout. For instance, how can the dimensionless river discharge (phi) be obtained? What is the range of values phi takes on?

Our reply: Thank you for your suggestion and we agree with it. The definition of the dimensionless parameter φ is given in Table 1 of the manuscript and it ranges between 0 to infinity. In the revised paper, we shall include an appendix to summarize the parameters we used in the analytical model.

3. In section 2.2, should show the behavior of p0, p1, p2, and p3 as a function of phi. That way is easier to compare to Godin's work (by the way, the friction term in Godin's approach is not linear with velocity).

Our reply: The coefficients p_1, p_2 and p_3 quantify the contributions made by linear, quadratic and cubic frictional interaction, respectively. In Figure R1 (see below), it appears that the value of p_0 is small with respect to the values of the other coefficients. We observe that the values of p_1 and p_2 increase with increasing φ until a maximum value is reached, after which p_1 converges to 0 while p_2 converges to $-\pi$. The value of p_3 is decreased with φ and it reduces to 0 for $\varphi<1$. For $\varphi \geq 1$,

$p_0=p_1=p_3=0$ and $p_2=-\pi$, so that the friction term (8) becomes $F=U^2/\left(K^2\overline{h}^{4/3}\right)$. If $\varphi=0$ (or $Q=0$), $p_0=p_2=0$, $p_1=16/15$ and $p_3=32/15$, so that equation (8) reduces to:

$$F=\frac{16}{15\pi}\frac{\upsilon^2}{K^2\overline{h}^{4/3}}\left[\frac{U}{\upsilon}+2\left(\frac{U}{\upsilon}\right)^3\right].$$

It was shown by Godin (1991,1999) that the quadratic velocity $U|U|$ can also be approximated by using only the first and third terms of the dimensionless velocity scaled by the maximum velocity. The Godin’s approximation does perform well in the downstream part of the estuary, where the current is bi-directional. However, the approximation does not convergence to U^2 in the upstream part of the estuary, where the river flow is dominant over tidal flow (see equation A1 in the manuscript). As a result, we would prefer to use Dronkers’ approximation to the friction term, which provides a consistent description for the whole estuary.

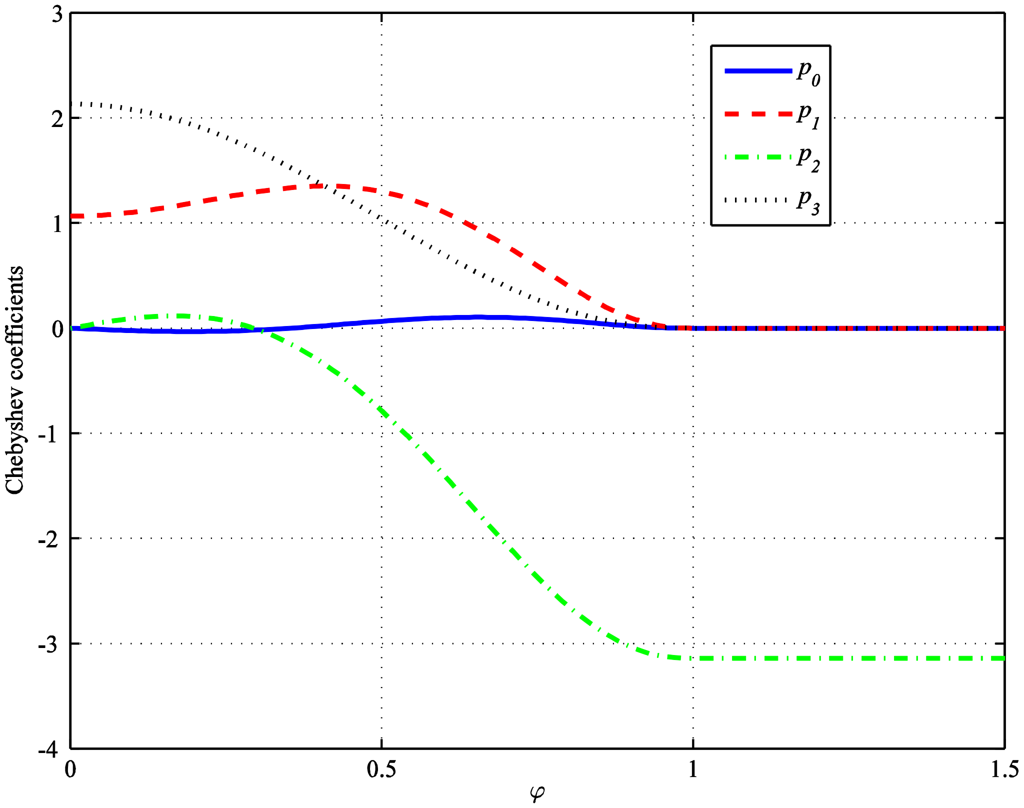

Figure R1. Variation of the Chebyshev coefficients p_i (i=0, 1, 2, 3) as a function of the dimensionless river discharge number φ.

4. **Section 2.3 should state how to arrive at the equation (i.e. by means of neglecting certain terms in the momentum equation).**

Our reply: Indeed, we shall clarify the assumptions we made to derive equation (15) with regard to the mean water level gradient $\partial\bar{z}/\partial x$. In the revised paper, we shall revise the first sentence in Section 2.3 as follows:

“Based on the assumptions of a negligible density effect and a periodic variation of velocity, the integral of the momentum equation over a tidal period yields the mean water level gradient with respect to distance (see also Vignoli et al., 2003 and Cai et al. 2014b):

$$\frac{\partial \bar{z}}{\partial x} = -\overline{F} = -\overline{\frac{1}{K^2 \bar{h}^{4/3} \pi}\left(p_0 \upsilon^2 + p_1 \upsilon U + p_2 U^2 + p_3 U^3 / \upsilon \right)}$$

where $\bar{z}$ is the mean water level or residual water level (the overbar denotes the tidally averaged value)."

5. **The approach in equation 19 is prone to error as the uncertainty in any of the terms will propagate upstream.**

Our reply: Actually, we have tested equation 19 by comparing the analytical and numerical results and the good agreement indicates that equation 19 can be used to estimate the mean water level profile along the estuary axis. For details, the reviewer can refer to Section 5 of Cai et al. (2014a).

6. **Section 3.3, it is unclear what the hydrodynamic model is and how calibration is performed. Also, should provide more details about the data used in the paper. Are these gauges vertically referenced? How do you obtain the tidal amplitudes? What is the zig-zag on the amplitudes in Fig. 6 and 8?**

Our reply: We have detailed our hydrodynamics model in Sections 2.2—2.4. It is fully analytical, although we calculate the tidal amplitudes by simple explicit integration of the analytically determined tidal damping. In the revised paper, we shall clarify the procedure to obtain the analytical solutions for the whole estuary. Basically, we adopt a multi-reach approach that divides the whole estuary into sub-reaches in order to account for the longitudinal variation of depth and bottom friction. For given topography, tidal amplitude at the estuary mouth η_0 and fresh water discharge at the upstream boundary, it is possible to compute the main tidal dynamics by solving a set of four implicit equations (3)—(6) for tidal damping, velocity amplitude, wave celerity and phase lag. Based on the computed amplification number δ, the unknown tidal amplitude η_1 at a distance Δx (such as 1 km) inland can be determined by a simple explicit integration:

$$\eta_1 = \eta_0 + \frac{\mathrm{d}\eta}{\mathrm{d}x}\Delta x = \eta_0 + \frac{\eta_0 \omega \delta}{c_0}\Delta x.$$

Based on the computed η_1 and the geometric feature (e.g., depth) of the next reach, the main tidal dynamics δ, μ, λ, and ε can be obtained by solving the set of equations (3)—(6). Such a process can be repeated by moving the origin of axis for each reach, leading to the solutions for the entire estuary. In principle, the proposed method is valid for an arbitrary bed profile, even with strong longitudinal gradient of bed elevation.

Since the geometry is defined by the exponential function describing the cross-sectional area, calibration is done merely on the channel roughness. The Manning-Strickler friction coefficient K=1/n is determined by comparing the analytically computed tidal amplitudes with observed data.

We shall clarify the data we used in this paper. In particular, the observed water levels at different gauging stations have been corrected and referenced to mean sea level of Huanghai 1985 datum. We determined the tidal amplitude by averaging the flood tidal amplitude and the ebb tidal amplitude. We observe that the Yangtze estuary has an irregular semi-diurnal tide character, suggesting two tidal cycles within a day. The zigzag line has to do with the fact that the tidal amplitude is very different between the two tidal cycles within a day.

7. **Figure 7 is unclear with regards to what is being plotted there. If it contains station data for different stations you should then indicate which stations are there, and display the data with different symbols or colors.**

Our reply: We agree with this comment. The revised figure is shown as follows.

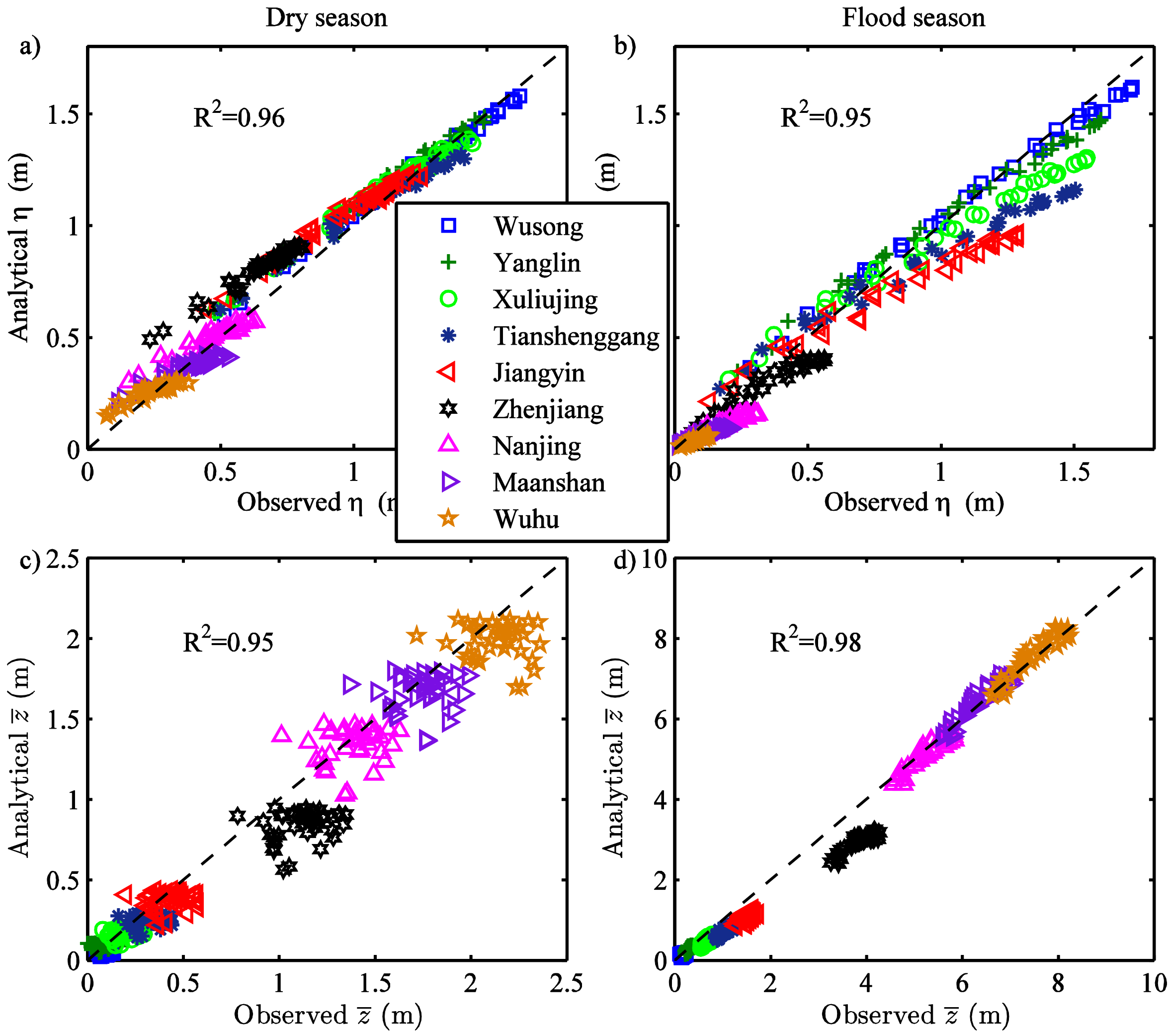

Figure R2. Comparison between analytically computed tidal amplitude η (a, b) and residual water level $\bar{z}$ (c, d) and measurements in the Yangtze estuary during 6 February 2012–26 February 2012 (a, c, representing the dry season) and during 10 August 2012–26 August 2012 (b, d, representing the flood season).

8. **Section 3.4, since the friction term is non-linear, in general, it is not true that average friction over varying tidal amplitudes equals the friction at the average tidal amplitude; Figure 9 suggests so.**

Our reply: You are right! Although we linearized the friction term by using Dronkers' Chebyshev polynomials approach, the friction term is still nonlinear. In the analytical approach, the decomposition of the friction term allows us to quantify the contributions made by different components (tide, river flow and tide-river interaction) to the mean water level.

9. **Section 3.5 has little to do with the goal set by the paper. The extreme analysis will be very much dependent on the details of the tide upstream, which is dependent on high frequency harmonics. Since the present approach ignores asymmetries and such other phenomena, its applicability, and particularly the accuracy, cannot be warranted.**

Our reply: We do not agree with this comment because the proposed approach does account for first-order tidal asymmetry induced by the interaction between predominant M_2 and fresh water discharge. For instance, in our analytical model we assume that the velocities at HW and LW are given by:

$$V_{HW} = \upsilon \sin(\varepsilon) - \frac{Q}{\bar{A}}, \quad V_{LW} = -\upsilon \sin(\varepsilon) - \frac{Q}{\bar{A}}.$$

But the reviewer is right that we did not account for the tidal asymmetry caused by the interaction between different tidal constituents (e.g., M_2 and M_4). However, since we aim to reproduce the first-order hydrodynamics with regard to tide-river interaction, this is not a critical limitation. As a result, the frequency analysis of extreme high water is useful for water management and flood control.

In the revised paper, we shall clearly clarify that the proposed analytical model does account for the tidal asymmetry induced by the fresh water discharge.

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