Hydrol. Earth Syst. Sci. Discuss., 12, C4285–C4296, 2015
www.hydrol-earth-syst-sci-discuss.net/12/C4285/2015/


Hydrology and Earth System Sciences
Open Access
Discussions

# *Interactive comment on* "Technical note: Analytical solution for the mean drawdown of steady state pumping tests in two-dimensional isotropic heterogeneous aquifers" *by* A. Zech and S. Attinger

**A. Zech and S. Attinger**

alraune.zech@ufz.de

Received and published: 15 October 2015

First, we want to thank the referee for his fruitful comments, which helped to improve the manuscript a lot. We appreciate that he thinks our work is worthy of publication. To our understanding, the major point of the referee is to put more efforts to the analysis of pumping tests in individual heterogeneous aquifer realizations. Additionally, some technical aspects for improving the manuscript are provided. We are going to cite the corresponding comments (in italics) and respond accordingly. Major changes planned



in the manuscript will be explicitly provided. Minor changes will not be given in detail.

Comments and Responses

*This version of the paper is mostly focused on assessing the accuracy of the estimates obtained when applying the solution over the ensemble. It is just a personal opinion, but I do not find this part of the document particularly interesting as (i) effective flow parameters have been extensively studied by lots of previous works (e.g. Sanchez-Vila et al. [2006] and references therein) and (ii) the estimation of variance and correlation length from the ensemble is a nice exercise but has no real applicability.*

We agree with the reviewer, that the application of the solution to single realizations is more interesting with regard to the interpretation of field pumping test. The part of the paper, where the solution is tested against the ensemble mean, is rather of technical nature and therefore kept short. The corresponding section 3.2 in the manuscript aims to confirm the appropriateness of the Radial Coarse Graining (RCG) approach for interpreting pumping test in heterogeneous media by showing the agreement with well known effective parameters for well flow. It can be understood as the numerical proof of the hypothesis taken in the derivation of the RCG approach. We feel, that this is necessary, specifically with regard to the comment of the other referee S.P. Neuman. It is further aimed to show how the stochastic parameters of the log-normal distributed media can be directly estimated from drawdown data without going a detour on effective or equivalent transmissivity descriptions or using type curves.

*As I said, I think that the real added value of this work is when it is applied to single realizations. Thus, I think that the examples presented in the document are not really exhaustive. For instance, the solution is tested only over a few realizations of set A (Table 1), which has a relatively small variance. What would happen with more challenging realizations (e.g. set C/D or even E/F)? Also, from the two selected realizations we observe some obvious (but still interesting) effect; i.e. when the contrast* *of transmissivity between the near and far field is modest, almost no information can be inferred whereas when this contrast increases, the accuracy of the estimation also increases. I think that this need to be analyzed in a more rigorous way for instance by using (individually) the whole set of realizations. Scatter plots of $T_{\mathsf{well}}/T_{\mathsf{G}}$ vs. $\hat{\ell}/\ell$ would help to get insight into the range of applicability of the solution and its dependence on the contrasts of transmissivity.*

As recommended by the reviewer, we expanded the analysis of single realizations, in particular for highly heterogeneous media (Ensemble D with $\sigma^2 = 2.25$ and Ensemble E with $\sigma^2 = 4$). Inspired by the analysis of virtual pumping test campaigns as done e.g. by *Neuman* [2004], *Copty and Findikakis* [2004], *Firmani* [2006], we developed and tested a sampling strategy. Additional pumping test simulations were conducted to test the feasibility of the effective well flow method for interpreting a series of steady state pumping tests within a single aquifer. The procedure as well as results are presented in the following section. We aim to add this analysis (as presented below) as a separate section to the manuscript. Accordingly, the introduction and the conclusion section of the manuscript will require minor adaptions.

The referee further suggests to present results for the whole set of realizations, e.g. by scatter plots. We see the point, that effects observed in single realizations are difficult to interpret with respect to the entire ensemble. We aimed to give credit to that point by presenting a boxplot of the estimation results for $100$ realizations in Figure 4. However, the plot might not provide as much information as we wanted it to. We tested the proposed scatter plot, but they are difficult to interpret and do not provide additional information. Instead we tested histogram plots for the estimation results of the entire ensemble of $N = 5000$ realizations, which are presented in Figure 1. The plotted results support the discussion in section 3.3 (p. 6934, l14ff) and allow to give some additional interpretations. Therefore, we suggest to substitute Figure 4 of the manuscript by the histogram plots in Figure 1. Minor adaption are planned for the text in section 3.3.



*I have also two minor comments:*

- *a differential operator is missing in eq.(2) and*
- *I think that set H of Table 1 is never used.*

The manuscript will be modified according to the comments: The differential operator will be added in Eq. (2) and set H will be removed from Table 1.

The following section is meant as a draft version for an additional section in the manuscript. The focus is on individual aquifer analysis through a pumping test sampling strategy. The interpretation of the drawdown results with $h^{\mathsf{efw}}(r)$ aim to show the appropriateness of the RCG approach for pumping test analysis.

Application Example: Single Aquifer Analysis

Pumping test campaigns in the field often include the performance of multiple pumping tests within one aquifer. Drawdown measurements from these multiple tests and locations can be used to gain representative parameter values of the underlying heterogeneous transmissivity field. With multiple pumping test locations, the sampled area increases and the effect of local heterogeneity at the well reduces. In the following, it is shown, how mean $T_{\mathsf{G}}$, variance $\sigma^2$ and the correlation length $\ell$ of an individual transmissivity fields can be inferred making use of a multiple pumping sampling strategy in combination with $h_{\mathsf{efw}}(r)$.

Sampling Strategy

The sampling strategy is constructed as pumping test campaign in a virtual aquifer with heterogeneous transmissivity. A series of steady state pumping test is performed independently at $n$ different well locations. For each test, drawdowns are measured at all $n$ wells and at $m$ additional observation wells. A similar sampling strategy to infer the aquifer statistics from drawdown measurements have been pursued by e.g. *Neuman* [2004], *Copty and Findikakis0 [2004*, *Firmani* [2006].

The sampling strategy used includes $n = 8$ pumping wells and $m = 4$ observation wells. The specific location of all wells are indicated in Figures 2 and 3. All $8$ pumping wells are located within a distance of $18$ m. The observation wells are located at larger distances and in all four direction. The well locations were designed to gain numerous drawdown measurements in the vicinity of each pumping well to allow a good estimation of $T_{\text{well}}$ (or $\sigma^2_{\text{local}}$ respectively) and $\ell$. The additional observation wells should provide head observations in the far field to gain a representative value for $T_{\text{G}}$. The chosen locations of all wells do not interfere with the refinement of the numerical grid at the pumping well (section 3.1).

Each of the $8$ pumping tests is analyzed with the adaption version of $h_{\text{efw}}(r)$ (Sect. 2.3) individual pumping test analysis. $\hat{T}_{\text{G}}$, $\hat{T}_{\text{well}}$, and $\hat{\ell}$ for all tests are inferred by minimizing the residual between the analytical solution and the measurements at the $12$ observation wells. Additionally, parameter estimates are inferred by analyzing the drawdown measurements of all tests jointly.

Aquifer Analysis

The sampling strategy was applied to fields of all ensembles A-G (Table 1). Results are presented for two fields: D1 out of Ensemble D ($\sigma^2 = 2.25$, $\ell = 20$ m) and E1 out of Ensemble E ($\sigma^2 = 4.0$, $\ell = 10$ m). Each field was generated according to the theoretical values defined for the particular ensemble and afterwards analyzed geostatistically to determine the sample values. The fields D1 and E1 are visualized in Figures 2 and 3. The drawdown measurements for all $8$ pumping tests at both fields are given in Figures 4 and 5. The inverse estimates as well as theoretical input and sampling



values for the statistical parameters are summarized in Table 1.

Analyzing the data from all $8$ pumping tests at field D1 jointly yields very close estimates of all three parameters $\hat{T}_{\text{G}}$, $\hat{T}_{\text{well}}$ (corresponding to $\hat{\sigma}^2 = 2.255$), and $\hat{\ell}$ to the theoretical and sampled values. The geometric mean was estimated similarly for all of the $8$ individual pumping tests. In contrast, the local transmissivities at the well varied within a range of one order of magnitude. This behavior was expected, since $\hat{T}_{\text{well}}$ represents the local transmissivity value at the pumping well. The wide range of estimates is a results of the high variance of the transmissivity field. The estimates of the correlation length differed between the individual tests within a reasonable range of a few meters. The only exception is the estimate for pumping at PW5; $\hat{\ell}$ for this specific pumping test is highly uncertain due to the coincidence of the values of $\hat{T}_{\text{well}}$ and $\hat{T}_{\text{G}}$, similar to the realizations in Fig. 3b, as discussed in section 3.3. However, the mean value over the individual tests as well as the estimate from the joint analysis of all measurements give reliable estimates for the correlation length.

The analysis of the sampling strategy at field E1 yields similar results as for D1. The geometric mean values differed little among the $8$ individual pumping tests and for the joint analysis. The value is double the value as the theoretical one, but close to the sampled geometric mean (Table 1). The local transmissivities $\hat{T}_{\text{well}}$ again varied within one order of magnitude, reflecting the high variance of the field. The mean and jointly estimated values are higher than theoretical one, which is in correspondence to the difference in the geometric mean. The values are in the range of the sampled value and calculating the variance from $\hat{T}_{\text{G}}$ and $\hat{T}_{\text{well}}$, results in $\hat{\sigma}^2 = 3.18$, which is close to the theoretical value. The estimates of the correlation length deviate in a reasonable range of a few meters, which reflects the impact of the location of the pumping well with regard to the shape of the correlation structure around the well.

Finally, the analysis shows that representative values of the statistical parameters can be determined by performing pumping test at multiple locations of an individual transmissivity field. It was shown, that $h_{\text{efw}}(r)$ is feasible to interpret steady state pumping

**Table 1.** Parameter estimates of geometric mean transmissivity $\hat{T}_\mathsf{G}$ [$10^{-4}$ m$^2$/s], local transmissivity at the well $\hat{T}_\mathsf{well}$ [$10^{-4}$ m$^2$/s] and correlation length $\hat{\ell}$ [m] for the $8$ pumping tests at fields D1 (from Ensemble D, $\sigma^2 = 2.25$) and E1 (from Ensemble E, $\sigma^2 = 4.0$). Additionally, the theoretical and the sampled values ($T_\mathsf{well} \equiv T_\mathsf{H}$) are given.

| | D1 | | | E1 | | |
|---|---|---|---|---|---|---|
| | $\hat{T}_\mathsf{G}$ | $\hat{T}_\mathsf{well}$ | $\hat{\ell}$ | $\hat{T}_\mathsf{G}$ | $\hat{T}_\mathsf{well}$ | $\hat{\ell}$ |
| PW0 | 1.025 | 0.434 | 29.51 | 1.945 | 0.313 | 9.56 |
| PW1 | 1.023 | 0.362 | 27.23 | 2.202 | 0.445 | 11.55 |
| PW2 | 1.076 | 0.220 | 23.68 | 2.093 | 0.437 | 10.03 |
| PW3 | 0.898 | 1.057 | 9.51 | 2.052 | 0.520 | 15.34 |
| PW4 | 1.001 | 0.147 | 20.53 | 2.174 | 1.847 | 12.30 |
| PW5 | 0.889 | 1.071 | 5.33 | 1.980 | 1.117 | 5.43 |
| PW6 | 1.038 | 0.177 | 20.39 | 1.840 | 0.148 | 8.78 |
| PW7 | 0.901 | 1.700 | 16.48 | 1.969 | 0.476 | 17.04 |
| Mean of 8 | 0.981 | 0.646 | 19.08 | 2.032 | 0.663 | 9.90 |
| Jointly | 1.013 | 0.328 | 22.38 | 2.010 | 0.409 | 9.97 |
| Theory | 1.0 | 0.325 | 20.0 | 1.0 | 0.135 | 10.0 |
| Sampled | 0.985 | 0.333 | 23.43 | 1.999 | 0.491 | 12.66 |

tests in highly heterogeneous fields.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 12, 6921, 2015.



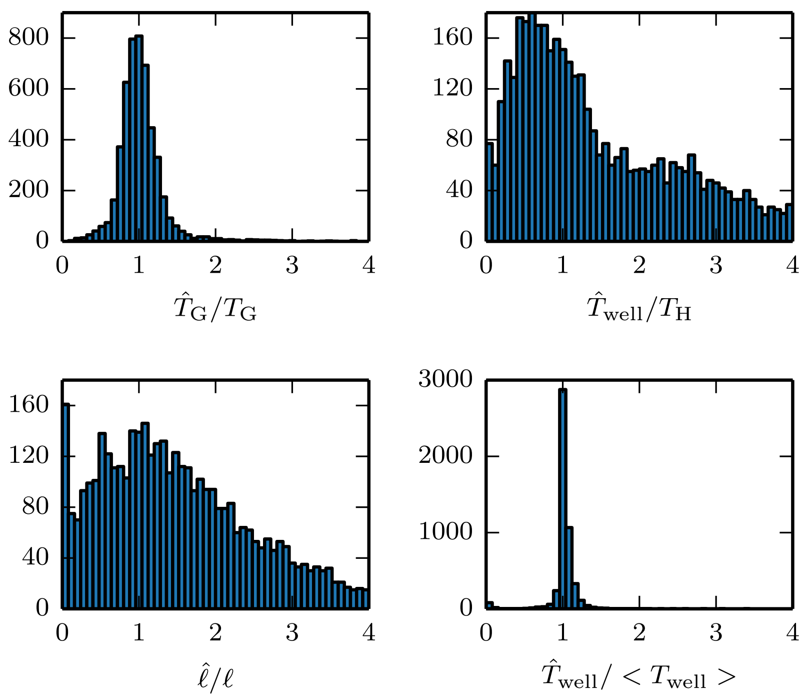
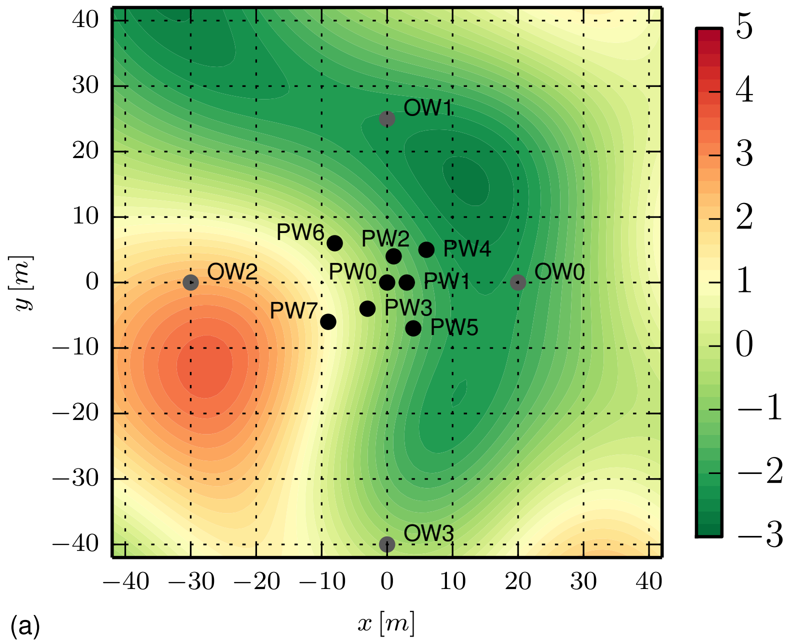


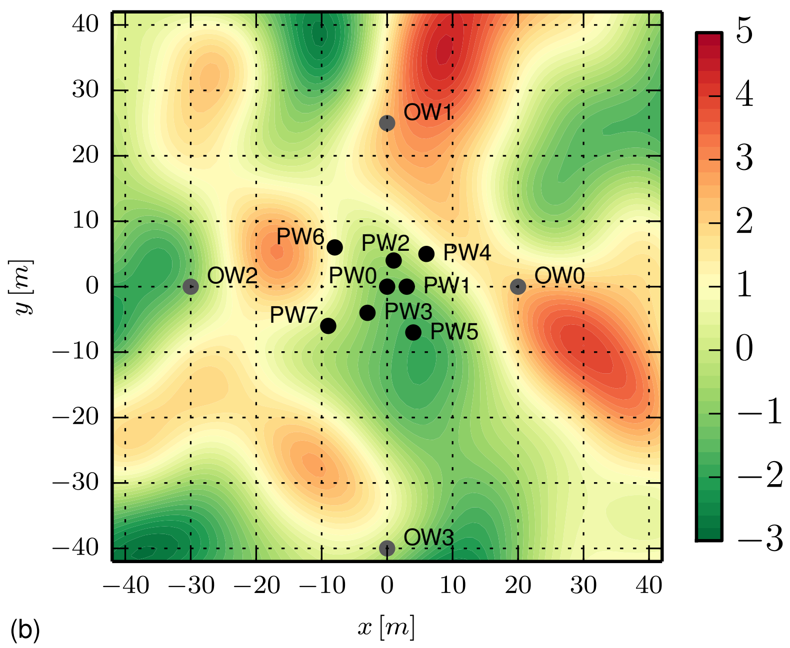

**Fig. 1.** Histogram on the best fit estimates divided by the theoretical input values and the sampled transmissivity at the pumping well for the 5000 realizations of Ensemble A.




(a)

**Fig. 2.** Spatial distribution of log-transmissivity for field D1. Locations of the 8 pumping wells and the 4 observation wells marked in black and gray.






(b)

**Fig. 3.** Spatial distribution of log-transmissivity for field E1. Locations of the 8 pumping wells and the 4 observation wells marked in black and gray.

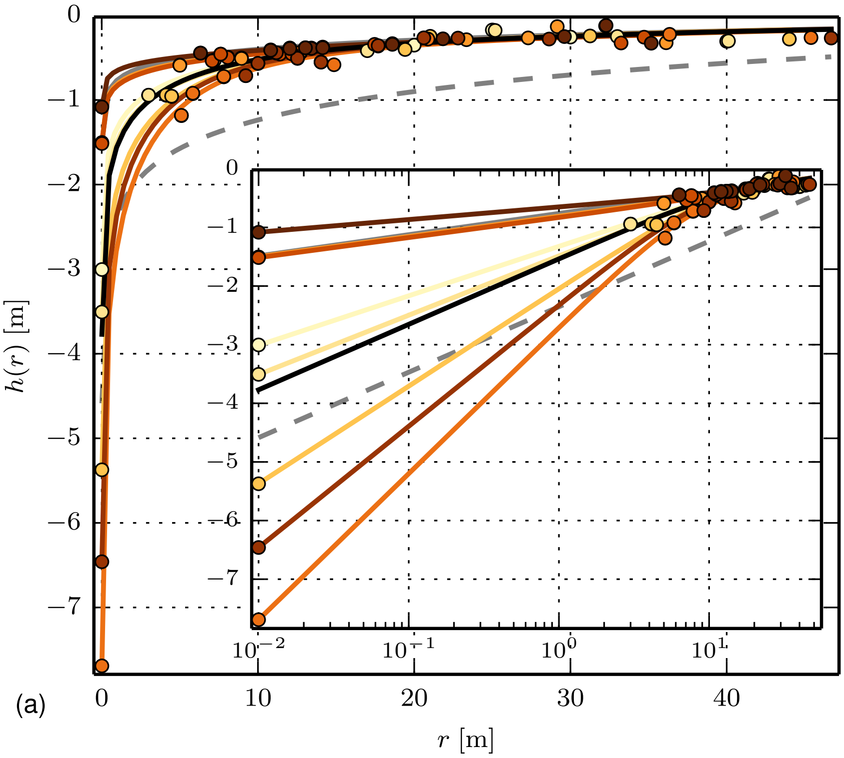



**Fig. 4.** Simulated measurements (dots) and fitted efw-solution (lines) for the 8 pumping tests (colours) within field D1. The black line denotes best fit for joint interpretation.



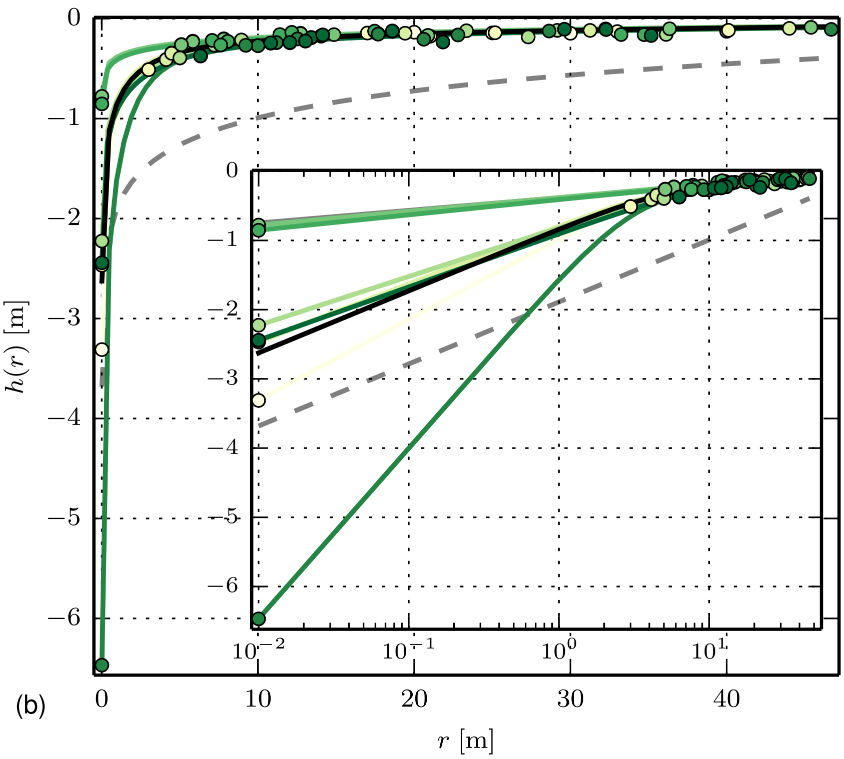



**Fig. 5.** Simulated measurements (dots) and fitted efw-solution (lines) for the 8 pumping tests (colours) within field E1. The black line denotes best fit for joint interpretation.