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Hydrology and Earth System Sciences
Discussions

Interactive comment on "Investigation of hydrological time series using copulas for detecting catchment characteristics and anthropogenic impacts" *by* T. Sugimoto et al.

Anonymous Referee #1

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Nice study that appears to be the first dealing with select asymmetrical properties and interpretations of copula models in a context of daily streamflow statistics for which asymmetry is known to exists. The asymmetry is related to the generalized hydrograph shape. Much of the authoritative text literature (e.g. Nelsen, 2006; Joe, 2014; Durante and Sempi, 2015) do not comprehensively tackle the asymmetry problem of a copula.

Nelsen (2006) is basically devoid of "skewness" (asymmetry) computations—understandably so. Joe (2014, p.66) discusses skewness of a copula and the orientation of the skewness appears conceptually similar (not necessarily numerically equal) to the A1 definition (primary diagonal) of eq. 9. A unique contribution by the paper is

the A2 definition (secondary diagonal) of eq. 10. This reviewer has seen many bivariate plots of hydrologic phenomena (such as daily streamflow) and notes the secondary diagonal asymmetry. This asymmetry means a fair share of copula families seen in the literature arguably are in applicable because they have symmetry on the secondary diagonal.

This reviewer would like A1 and A2 to also be expressed in direct terms of integration of the copula formula or its density. For example, a Joe (2014) definition for the primary diagonal is: 6\int\int_{[0,1]} (v-u)C(u,v) du dv from which a secondary asymmetry definition (not identified by Joe) can result 6\int\int_{[0,1]} (v+u-1)C(u,v) du dv - (1/2) Can the authors of the paper expand the definitions of A1 and A2 beyond the "expectation" notation?

Have the authors considered the L-comoments (Serfling and Xiao, 2007)? But more importantly, the very recent "break through" of L-comoment (bivariate L-moment, bivariate L-skew) definition (Brahimi et al. [2015]) directly in terms of a copula. L-coskew (bivariate skew) \deltaˆ{[12]}_{3;\mathbf{C}} = \int\int_{[0,1]} (60vˆ2 - 60v + 12) * C(u,v) du dv - (1/2) \deltaˆ{[21]}_{3;\mathbf{C}} = \int\int_{[0,1]} (60uˆ2 - 60u + 12) * C(u,v) du dv - (1/2)

These integrals can readily by numerically approximated or integrated by Monte Carlo methods enhanced by low-discrepancy sequence methods.

Some final thoughts. A similar study as this does not really appear to have been done. Whereas, this review generally thinks that the physical interpretations of the watershed and climatology are mechanism producing asymmetry, care is suggested to avoid over interpretations until a great suite of similar studies can be conducted. For example, 9164, line 24 "... A1 ... asymmetry can be related to temporal distribution of precipitation" (what scale of time?) or "... A2 ... more related to catchment and rainfall characteristics ... or ... interseasonal characteristics of climate". These are deeply important properties and suggest that copulas are an avenue forward in wa-

tershed/climate stochastic modeling. Intuition seems to be correct, but expansion of the authors' thoughts and statements to interpretation of A1 and A2 or other skewness measures or bivariate moment (L-moment) would be informative.

Also, given that we know typical storm water hydrographs are asymmetrical and are inherently formed by a cascade of processes (e.g. water parcel survival from input to output — Markov of sorts), is there a connection between A1/A2 and storm water hydrographs (e.g. unit hydrographs)?

References

Brahimi, Chebana, and Necir (2015) Copula representation of bivariate L-moments: A new estimation method for multi-parameter two-dimensional copula models, Statistics, 49(3)[497–521].

Durante and Sempi (2015) Principles of copula theory, CRC Press.

Nelsen, RB (2006) An introduction to copulas, Springer.

Joe, H. (2014) Dependence modeling with copulas, CRC Press.

Serfling and Xiao (2007) A contribution to multivariate L-moments: L-comoment matrices, Journal of Multivariate Analysis, 98[1765-1781].

———————— 9160, Lines 25 and 30: There is confusion in the technical writing aspect of mentioning ARIMA and then evidently switching conceptually to "Fourier analysis". This review suggests that a proof reading would resolve potential confusion.

9162, Line 9: "this statistics" —> "these statistics"

9168, Line 14: missing minus sign in definition of A2(k,t)?

Figure 6: Shouldn't the horizontal axis be cast in logarithms?

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 12, 9157, 2015.