Reply to 2nd Comment of Referee #2

We thank reviewer #2 for his/her valuable and constructive comments. Our point-by-point responses to the comments are shown below. Each point first shows our previous response, then reviewer's comment on the response, and finally our present response to reviewer's comment.

1. "Analysis of three-dimensional groundwater flow toward a radial collector well in a finite-extent unconfined aquifer"

   Comment: The word Approximate should be added as the first word in the title.

   Response: The title has been changed as suggested.

2. "The simplification from Eq. (7) to Eq. (8) was first proposed by Boulton (1954) and later used to develop analytical solutions by, for example, Neuman (1972), Zhan and Zlotnik (2002), and Yeh et al. (2010). The simplification has been validated by agreement on drawdown measured by a field pumping test and predicted by Neuman (1972) solution based on Eq. (8) (e.g., Goldscheider and Drew, 2007, p. 88). We inserted the following sentence right below Eq. (8):

   "Goldscheider and Drew (2007) revealed that pumping drawdown predicted by Neuman (1972) analytical solution based on Eq. (8) agrees well with that obtained in a field pumping test.""

   Comment: I am aware that this approximation is not uncommon, but radial collector wells are often used for pumping large quantities of water. The approximation breaks down when draw-downs become too large. The issue is that the authors fail to make this point clear, and to explain what the limitations are of their approach. In the case considered in the paper, the release from storage as a result of drawing down the water table will be larger than release from elastic storage and is therefore important. The approximation replacing (7) by (8) implies that the release from storage is entirely accounted for by the vertical component of flow, neglecting the horizontal components. This approximation breaks down when the water table slopes more than a certain amount. I suggest that the authors verify in the results section that the gradients of the water table are indeed within acceptable limits.

   Response: Thanks for the comment. The simplification from Eq. (7) to (8) is hold under two conditions. The first of the conditions is that the decline of the water table $|h|$ is smaller than 10% of initial aquifer thickness H (i.e., $|h|/H \leq 0.1$) (Nyholm et al., 2002; Yeh et al., 2010). The second is that the sum of the horizontal hydraulic gradients of $|\partial h/\partial x|$ and $|\partial h/\partial y|$ at the water table is smaller than 0.01 (Yeh et al., 2010). Nyholm et al. (2002) achieved agreement on drawdown measured in a field pumping test and predicted by MODFLOW which models flow in the study site as confined behavior because of $|h|/H \leq 0.1$ in the pumping well. In addition, Yeh et al. (2010) also achieved agreement on the hydraulic head predicted by their analytical solution based on Eq. (8), their finite-difference solution based on Eq. (7) with $\partial h/\partial y = 0$ (referring to Eq. (7a)), and Teo et al. (2003) solution derived by applying the perturbation technique to deal with Eq. (7a) when $|h|/H = 0.1$ and $\partial h/\partial x = 0.01$ (i.e., $\alpha = 0.1$ and $\partial\phi/\partial x = 0.01$ at $x = 0$, respectively, in Fig. 5(a) in Yeh et al., 2010). We therefore conclude from the result that the second-order terms of the horizontal hydraulic gradient is negligible when $|\partial h/\partial x| + |\partial h/\partial y| \leq 0.01$. With the abovementioned results, the associated paragraph is rewritten as:

"The free surface equation describing the water table decline is written as

$$K_x\left(\frac{\partial h}{\partial x}\right)^2 + K_y\left(\frac{\partial h}{\partial y}\right)^2 + K_z\left(\frac{\partial h}{\partial z}\right)^2 - K_z\frac{\partial h}{\partial z} = S_y\frac{\partial h}{\partial t} \quad \text{at} \quad z = h \tag{7}$$

Neuman (1972) indicated that the effect of the second-order terms in Eq. (7) is generally ignorable to develop analytical solutions. Eq. (7) is thus linearized by neglecting the quadratic terms, and the position of the water table is fixed at the initial condition (i.e., z = 0). The result is written as

$$K_z\frac{\partial h}{\partial z} = -S_y\frac{\partial h}{\partial t} \quad \text{at} \quad z = 0 \tag{8}$$

Notice that Eq. (8) is applicable if the conditions $|h|/H \leq 0.1$ and $|\partial h/\partial x| + |\partial h/\partial y| \leq 0.01$ are satisfied. These two conditions had been studied and verified by simulations in Nyholm et al. (2002), Goldscheider and Drew (2007) and Yeh et al. (2010). Nyholm et al. (2002) achieved agreement on drawdown measured in a field pumping test and predicted by MODFLOW which models flow in the study site as confined behavior because of $|h|/H \leq 0.1$ in the pumping well. Goldscheider and Drew (2007) revealed that pumping drawdown predicted by Neuman (1972) analytical solution based on Eq. (8) agrees well with that obtained in a field pumping test. In addition, Yeh et al. (2010) also achieved agreement on the hydraulic head predicted by their analytical solution based on Eq. (8), their finite difference solution based on Eq. (7) with $\partial h/\partial y = 0$ (referring to Eq. (7a)), and Teo et al. (2003) solution derived by applying the perturbation technique to deal with Eq. (7a) when $|h|/H = 0.1$ and $\partial h/\partial x = 0.01$ (i.e., $\alpha = 0.1$ and $|\partial\phi/\partial x| = 0.01$ at $x = 0$ in Fig. 5(a) in Yeh et al., 2010)." (lines 190-208 of the revise manuscript)

3. "We also inserted the following sentence to indicate the governing equation (i.e., Eq. (1)) is based on a concept proposed by Terzaghi: "The first term on the RHS of Eq. (1) depicts aquifer storage release based on the concept of effective stress proposed by Terzaghi (see, for example, Bear, 1979, p.84; Charbeneau, 2000, p.57)."
Comment: The governing equation is not based upon the concept of effective stress, but rather on the approximation that the total vertical stress is not changing. This is a good assumption for most groundwater

flow problems. The concept of effective stress is not, in itself, sufficient to obtain the equation used here from Biot's equations.

Response: Thanks for the comment. The phrase “which is valid under the assumption of constant total stress” is added after the sentence.

4. “A stream of partial penetration can be considered as fully penetrating if the distance between the stream and well is larger than 1.5 times the aquifer thickness (Todd and Mays, 2005).”

   Comment: I disagree with this statement. The flow pattern at distances of about 1.5 times the aquifer thickness indeed reduces to flow that is uniform over the vertical (de Saint Venant's principle). A stream, or well, with given discharge is therefore indeed indistinguishable at such distances. However, the boundary condition along a partially penetrating stream certainly affects the discharge the stream captures and replacing a head boundary condition over limited depth by one over the full depth will have an impact, not on the flow pattern at distance, but on the discharges computed. If the authors apply the constant head boundary condition over the full vertical, they must state this clearly, and, if they do this, a resistance between stream and aquifer needs to be added to obtain the proper flow rates.

   Response: Thanks for the comment. The sentence has been removed. The present model uses the Robin boundary conditions (Eqs. (5) and (6) given below) to describe the fluxes across low-permeability streambeds of two parallel streams at $y = 0$ and w_y. The SDR solution of the present model can therefore predict stream filtration rate subject to the effect of streambed resistance.

$$K_y \frac{\partial h}{\partial y} - \frac{K_1}{b_1} h = 0 \quad \text{at} \quad y = 0 \tag{5}$$

   and

$$K_y \frac{\partial h}{\partial y} + \frac{K_2}{b_2} h = 0 \quad \text{at} \quad y = w_y \tag{6}$$

   where h is the hydraulic head at stream-aquifer interfaces, K_y is the aquifer hydraulic conductivity in the y direction normal to two parallel streams, K and b are the hydraulic conductivity and thickness of the streambeds, respectively, and subscripts 1 and 2 indicate streambeds at $y = 0$ and w_y, respectively.

5. “The flux across the well screen is assumed to be uniform along each of the laterals.”

   Comment: I am not sure how it is possible in practice to maintain the flow rate constant along the lengths of the radii of the well. If this is an approximation of an actual well of constant head radii, then it should be remembered, and stated, that the head varies along the legs in the solution. It would be of more practical value to break the legs up onto segments, and solve a system of equations to fix the heads at the centers of each segment to some prescribed value.

Response: The assumption of uniform flux along the laterals of a radial collector well (RCW) is valid when the laterals are not long. We added following sentence to illustrate the validity of the assumption.

"The assumption is valid for a short lateral within 150 m verified by agreement for drawdown observed in field experiments and that predicted by existing analytical solutions based on the uniform-flux assumption (Huang et al., 2011; 2012)."

Regarding each lateral represented by several segments, it will be suitable for a long lateral of RCW to consider total head loss due to friction in the lateral but this is not our present concern. Two motives have combined to make us write this paper. One is to develop the present solution of the head and stream filtration/depletion rate (SDR) induced by RCW in finite-extent unconfined aquifers. The advantage of the present solution is that its calculation only relies on Newton's method and avoids laborious calculations involved in existing solutions. For example, Huang et al. (2012) solution should resort to numerical integration of a triple integral in predicting the hydraulic head and a quintuple integral in predicting SDR. The integrand is expressed in terms of an infinite series expanded by the roots of nonlinear equations. The integration variables are associated with those roots. The other is to provide insights or new findings accounting for flow and SDR induced by RCW, given below:

(1) We quantify a region where groundwater flow is three-dimensional. Beyond this region, the flow is horizontal and can be described by existing solutions neglecting the vertical flow (e.g., Mohamed and Rushton, 2006; Haitjema et al., 2010). Please refer to the 1st conclusion in the manuscript of Huang et al. (2015, p.7526).

(2) We provide a condition under which flow in aquifers is unidirectional and perpendicular to a horizontal well. Under the condition, existing models neglecting the flow component along the well give accurate head predictions (e.g., Anderson, 2000; Anderson, 2003; Kompani-Zare et al., 2005). Please refer to the 2nd conclusion in the manuscript of Huang et al. (2015, p.7526).

(3) We present a criterion describing that the effect of the vertical flow in aquifers on SDR can be ignored. A variety of existing solutions neglecting the vertical flow can predict accurate SDR only when the criterion is met (e.g., Glover and Balmer, 1954; Hantush, 1965; Hunt, 1999; Butler et al., 2001; Sun and Zhan, 2007; Zlotnik, 2014). Please refer to the 7th conclusion in the manuscript of Huang et al. (2015, p.7527).

(4) We find that SDR for unconfined aquifers depends on the depth of installing RCW, but for confined aquifers does not at all. Please refer to the 8th conclusion in the manuscript of Huang et al. (2015, p.7527).

6. "Please refer to 1st response for the fact that the present solution is applicable to unconfined flow."

Comment: I maintain that it should be made clear that this approximation is valid only under limited conditions, where drawdowns do not exceed some maximum.

Response: Please refer to 2nd response for the condition under which the approximation is valid.

References

Anderson, E. I.: The method of images for leaky boundaries, Adv. Water Resour., 23, 461−474, doi:10.1016/S0309-1708(99)00044-5, 2000.

Anderson, E. I.: An analytical solution representing groundwater-surface water interaction, Water Resour. Res., 39(3), 1071, doi:10.1029/2002WR001536, 2003.

Bear, J.: Hydraulics of Groundwater, McGraw-Hill, New York, 84, 1979.

Butler, J. J., Zlotnik, B. A., Tsou, M. S.: Drawdown and stream depletion produced by pumping in the vicinity of a partially penetrating stream, Ground Water, 39(5), 651 – 659, 2001.

Charbeneau, R. J.: Groundwater Hydraulics and Pollutant Transport, Prentice-Hall, NJ, 57, 2000.

Glover, R. E., Balmer, G. G.: River depletion resulting from pumping a well near a river, Trans. Am. Geophys. Union 35 (3), 468 – 470, 1954.

Goldscheider, N., and Drew, D.: Methods in karst hydrology, Taylor & Francis Group, London, UK, 88, 2007.

Haitjema, H., Kuzin, S., Kelson, V., and Abrams, D.: Modeling flow into horizontal wells in a Dupuit-Forchheimer model, Ground Water, 48(6), 878−883, doi:10.1111/j.1745-6584.2010.00694.x, 2010.

Hantush, M.S.: Wells near streams with semi-pervious beds, J. Geophys. Res., 70 (12), 2829 – 2838, 1965.

Huang, C.-S., Chen, J.-J., and Yeh, H.-D.: Analysis of three-dimensional groundwater flow toward a radial collector well in a finite-extent unconfined aquifer, Hydrol. Earth Syst. Sci. Discuss, 12, 7503–7540, doi:10.5194/hessd-12-7503-2015, 2015.

Huang, C. S., Chen, Y. L., and Yeh, H. D.: A general analytical solution for flow to a single horizontal well by Fourier and Laplace transforms, Adv. Water Resour., 34(5), 640−648, doi:10.1016/j.advwatres.2011.02.015, 2011.

Huang, C. S., Tsou, P. R., and Yeh, H. D.: An analytical solution for a radial collector well near a stream with a low-permeability streambed, J. Hydrol., 446, 48−58, doi:10.1016/j.jhydrol.2012.04.028, 2012.

Hunt, B.: Unsteady stream depletion from ground water pumping, Ground Water, 37(1), 98 – 102, 1999.

Kompani-Zare, M., Zhan, H., and Samani, N.: Analytical study of capture zone of a horizontal well in a confined aquifer, J. Hydrol., 307, 48−59, doi:10.1016/j.jhydrol.2004.09.021, 2005.

Mohamed, A. and Rushton, K.: Horizontal wells in shallow aquifers: Field experiment and numerical model, J. Hydrol., 329(1−2), 98−109, doi:10.1016/j.jhydrol.2006.02.006, 2006.

Nyholm, T., Christensen, S., and Rasmussen, K. R.: Flow depletion in a small stream caused by ground water abstraction from wells, Ground Water, 40(4), 425–437, 2002

Sun, D., Zhan, H.: Pumping induced depletion from two streams, Adv. Water Resour., 30(4), 1016–1026, 2007.

Teo, H. T., Jeng, D. S., Seymour, B. R., Barry, D. A., and Li, L.: A new analytical solution for water table fluctuations in coastal aquifers with sloping beaches, Adv. Water Resour., 26(12), 1239 – 1247, doi:10.1016/j.advwatres.2003.08.004, 2003.

Yeh, H. D., Huang, C. S., Chang, Y. C., and Jeng, D. S.: An analytical solution for tidal fluctuations in unconfined aquifers with a vertical beach, Water Resour. Res., 46, W10535, doi:10.1029/2009WR008746, 2010.

Zlotnik, V.: Analytical Methods for Assessment of Land-Use Change Effects on Stream Runoff, J. Hydrol. Eng., 06014009, doi:10.1061/(ASCE)HE.1943-5584.0001084, 2014.