Referee #1: Comments and Responses

Interactive comment on “Analysis of three-dimensional groundwater flow toward a radial collector well in a finite-extent unconfined aquifer” by C.-S. Huang et al.

Anonymous Referee #1
Received and published: 31 August 2015

General Comments

This is my reivew of "Analysis of three-dimensional groudnwater flow toward a radial collector well in a finite-extent unconfined aquifer" submitted by Huang, Chen, and Yeh to HESSD. This manuscript presents a modification of one of the Latinopoulis (1985) solutions for a rectangular domain (combinations of type I, II and III boundary conditions on the lateral edges), by including the effects of a water table at the top of the aquifer. They then take the point source solution and integrate it to approximate line source solution to represent a horizontal well. I think the authors’ derivation of the line-source approximation for a finite domain may be in error, as don’t seem to have handled the boundary conditions in their transition from point source to line source rigorously (or it may not be presented clearly). The boundary condition used to represent the river (a fully penetrating type III boundary condition) is not realistic, and would not be widely useful. I have not seen any rectangular aquifers with trenches cut down to the bottom of the aquifer on one or two parallel sides.

Response: The solution based on the assumption of a fully penetrating stream is applicable to most of real-world stream depletion (or filtration) problems when the distance between a partially penetrating stream and pumping well is larger than 1.5 times the aquifer thickness (Todd and Mays, 2005).

We consider a rectangular aquifer for two reasons. One is that the present model based on two parallel streams at two sides of the aquifer can be used to solve the problems involving water right distributions from the streams (Sun and Zhan, 2007). The other is that two no-flow boundaries at the other two sides of the aquifer significantly improve calculation efficiency in stream depletion/filtration rates (SDR) by the fact that the triple series reduces to double series when deriving the present SDR solution (i.e., derivation from Eq. (50) to (52)). Conventional solutions derived based on aquifers of semi-infinite extent from a nearby stream can be considered as particular cases of the present solution if the three adjacent sides of the rectangular aquifer are far away from the pumping well. Regarding the derivation of the line-source approach, please refer to the response to Specific Comment 9.

We added following sentences in Introduction section:

“The streams fully penetrate the aquifer thickness and connect the aquifer with low-permeability streambeds. A stream of partial penetration can be considered as a fully penetrating one if the distance between the stream and the well is larger than 1.5 times the aquifer thickness (Todd and Mays, 2005). The model based on the two parallel streams can be used to determine the fraction of water filtration from two streams and solve the associated water right problem (Sun and Zhan, 2007).” (lines 133 – 138 of the revised manuscript)

Specific Comments

1. The manuscript introduction and abstract should mention the river boundary conditions are "fully penetrating". The river is assumed to penetrate the entire thickness of the aquifer (treating river as a type III boundary condition), and the aquifer is not affected by anything occuring on the other side of th aquifer.

Response: Thanks for the comment. We insert following two sentences in Abstract and Introduction sections, respectively.

“The streams with low-permeability streambeds fully penetrate the aquifer thickness.” (lines 22 – 23 of the revised manuscript)

“The streams fully penetrate the aquifer thickness and connect the aquifer with low-permeability streambeds.” (lines 133 – 134 of the revised manuscript)

2. page 7505 line 9: your proposed solution also assumes flux along the well screen is uniform; please state this.

Response: Thanks for the suggestion. We add following sentence in the last paragraph of the Introduction section: “The flux across the well screen is assumed to be uniform along each of the laterals.” (lines 132 – 133 of the revised manuscript).

3. Figure 1 does not match the problem description in the text. The boundary conditions are rotated 90 degrees. Page 7509 indicates no-flow boundary conditions at $x = 0$ and $x = W_x$, but Fig 1 shows no-flow boundary conditions at $y = 0$ and $y = W_y$.

Response: The figure has been redrawn and also shown at the end of this reply.

4. Equation 7: The references associated with the water table boundary condition (Yeh et al 2010) should be Boulton (1954), Dagan (1967), and/or Neuman (1972).

- N. S. Boulton. The drawdown of the water-table under non-steady conditions near a pumped well in an unconfined formation. Proceedings Institution of Civil Engineers, 3(4):564–579, 1954.
- G. Dagan. A method of determining the permeability and effective porosity of unconfined anisotropic aquifers. Water Resources Research, 3(4):1059–1071, 1967.
- S. P. Neuman. Theory of flow in unconfined aquifers considering delayed response of the water table. Water Resources Research, 8(4):1031–1045, 1972.

Response: The citation “Yeh et al. (2010)” has changed to “Neuman (1972)”.

5. page 7510 lines 18-19: The boundary condition is linearized by uncoupling the water table location from the head and by fixing the water table position through time. "replacing $z = h$ with $z = 0$" is only partially true. This solution (and all analytical solutions) does not modify the position of the water table and boundary condition, even though the drawdown near the well increases with time.

Response: Thanks for the comment. The sentence is rewritten as "Equation (7) is thus linearized by neglecting the second-order terms, and the position of the water table is fixed at the initial condition (i.e., $z = 0$)" (lines 193 – 194 of the revised manuscript).

6. Equation 10: give some of the key values used to non-dimensionalize the solution in the text. Do not relegate all this to Table 1. Explicitly stating the characteristic length, time, and head would be useful here. Since this is a finite domain, there are multiple ways the characteristic length could be chosen.

Response: The characteristic length is y_0 defined as a distance from stream 1 at $y = 0$ to the center of a radial collector well. With definitions of dimensionless variables and parameters, the associated paragraph is rewritten as:

"Define dimensionless variables as $\bar{h} = (K_y\, H\, h)/Q$, $\bar{t} = (K_y\, t)/(S_s\, y_0^2)$, $\bar{x} = x/y_0$, $\bar{y} = y/y_0$, $\bar{z} = z/H$, $\bar{x}_0' = x_0'/y_0$, $\bar{y}_0' = y_0'/y_0$, $\bar{z}_0' = z_0'/H$, $\bar{w}_x = w_x/y_0$ and $\bar{w}_y = w_y/y_0$ where the overline denotes a dimensionless symbol, and y_0, a distance between stream 1 and the center of the RCW, is chosen as a characteristic length. On the basis of the definitions, Eq. (1) can be written as

$$\kappa_x \frac{\partial^2 \bar{h}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{h}}{\partial \bar{y}^2} + \kappa_z \frac{\partial^2 \bar{h}}{\partial \bar{z}^2} = \frac{\partial \bar{h}}{\partial \bar{t}} + \delta(\bar{x} - \bar{x}_0')\delta(\bar{y}' - \bar{y}_0')\delta(\bar{z} + \bar{z}_0') \quad (10)$$

where $\kappa_x = K_x/K_y$ and $\kappa_z = (K_z\, y_0^2)/(K_y\, H^2)$.

Similarly, the initial and boundary conditions are expressed as

$$\bar{h} = 0 \quad \text{at} \quad \bar{t} = 0 \quad (11)$$

$$\partial \bar{h}/\partial \bar{x} = 0 \quad \text{at} \quad \bar{x} = 0 \quad (12)$$

$$\partial \bar{h}/\partial \bar{x} = 0 \quad \text{at} \quad \bar{x} = \bar{w}_x \quad (13)$$

$$\partial \bar{h}/\partial \bar{y} - \kappa_1 \bar{h} = 0 \quad \text{at} \quad \bar{y} = 0 \quad (14)$$

$$\partial \bar{h}/\partial \bar{y} + \kappa_2 \bar{h} = 0 \quad \text{at} \quad \bar{y} = \bar{w}_y \quad (15)$$

$$\frac{\partial \bar{h}}{\partial \bar{z}} = -\frac{\gamma}{\kappa_z} \frac{\partial \bar{h}}{\partial \bar{t}} \quad \text{at} \quad \bar{z} = 0 \quad (16)$$

and

$$\partial \bar{h}/\partial \bar{z} = 0 \quad \text{at} \quad \bar{z} = -1 \quad (17)$$

where $\kappa_1 = (K_1\, y_0)/(K_y\, b_1)$, $\kappa_2 = (K_2\, y_0)/(K_y\, b_2)$, and $\gamma = S_y/(S_s\, H)$." (lines 199 – 215 of the revised manuscript)

7. Add "finite" before "integral transform" when referring to the Latinopoulus solution (e.g., p7511 l18, p7512 l3, p7513 l18)

Response: The phrase "double-integral transform" has changed to "finite integral transform".

8. page 7513 line 7: what exactly is meant by "$-z_0'-$" and "$-z_0'+$"? Either explain the notation, or use clearer notation.

Response: Thanks for pointing out the problem. They are revised as "$-\bar{z}_0'^{\,-}$" and "$-\bar{z}_0'^{\,+}$", respectively. In fact, the typo was made by the staff of HESSD in the proofread version.

9. Based on what is written on page 7515 (lines 18-21) and page 7516 (lines 1-2) (and the discussion about how the current approach is much faster than other approaches), it appears the line source solution is computed after the numerical inversions for the double finite x and y transforms are computed for a single point source. A single point source solution is computed, then this is shifted and added to a new solution. It is not totally clear exactly how it is being done (this should be more explicit). The finite domain requires a totally new solution for each point source, since the distance to each of the boundary conditions is part of the solution. If the solution is just shifted and summed up, the boundary conditions will not line up – the boundary conditions will extruded over the length of the well. The authors may be doing it the right way, but they are too vague in their specification of how they do it for me to tell one way or the other.

Response: The derivation of the solution from a point sink to a line sink (representing the water extraction over the lateral of radial collector well, RCW) is under the condition that the Cartesian coordinate system and aquifer boundaries are fixed. We integrate the point sink solution along the lateral by varying the locations of the point sinks without shifting the coordinate system. For more detailed derivation, following paragraph is provided:

"The lateral of RCW is approximately represented by a line sink composed of a series of adjoining point sinks. The locations of these point sinks are expressed in terms of $(\bar{x}_0+\bar{l}\cos\theta,\ \bar{y}_0+\bar{l}\sin\theta,\ \bar{z}_0)$ where $(\bar{x}_0,\ \bar{y}_0,\ \bar{z}_0) = (x_0/y_0,\ 1,\ z_0/H)$ is the central of the lateral, and $\bar{l}$ is a variable to define different locations of the point sink. The solution of head $\bar{h}_w(\bar{x},\bar{y},\bar{z},\bar{t})$ for a lateral can therefore be derived by substituting $\bar{x}_0'=\bar{x}_0+\bar{l}\cos\theta$, $\bar{y}_0'=1+\bar{l}\sin\theta$ and $\bar{z}_0'=\bar{z}_0$ into the point-sink solution, Eq. (30), then by integrating the resultant solution to $\bar{l}$, and finally by dividing the integration result into the sum of lateral lengths. The derivation can be denoted as

$$\bar{h}_w(\bar{x},\bar{y},\bar{z},\bar{t}) = \left(\sum_{k=1}^{N}\bar{L}_k\right)^{-1}\sum_{k=1}^{N}\int_0^{\bar{L}_k}\bar{h}(\bar{x},\bar{y},\bar{z},\bar{t})\,d\bar{l} \qquad (43)$$

where $\bar{L}_k=L_k/y_0$ is the k-th dimensionless lateral length. Note that the integration variable $\bar{l}$ (i.e., $\bar{x}_0'$ and $\bar{y}_0'$) appears only in X_n and $X_{m,n}$ in Eq. (31). The integral in Eq. (43) can thus be done analytically by integrating

X_n and $X_{m,n}$ with respect to $\bar{l}$. After the integration, Eq. (43) can be expressed as

$$\bar{h}_w(\bar{x},\bar{y},\bar{z},\bar{t}) = \left(\sum_{k=1}^{N}\bar{L}_k\right)^{-1}\sum_{k=1}^{N}\begin{cases}\Phi(-\bar{z}_0,\bar{z},1) & \text{for } -\bar{z}_0\le\bar{z}\le 0\\ \Phi(\bar{z},\bar{z}_0,-1) & \text{for } -1\le\bar{z}\le-\bar{z}_0\end{cases} \quad (44)$$

where Φ is defined by Eqs. (31) – (38), and X_n and $X_{m,n}$ in Eq. (31) are replaced, respectively, by

$$\hat{X}_{n,k} = -G_k/(\beta_n \sin\theta_k) \quad (45)$$

and

$$\hat{X}_{m,n,k} = \frac{\alpha_m F_k \cos\theta_k + \beta_n G_k \sin\theta_k}{\alpha_m^2\cos^2\theta_k - \beta_n^2\sin^2\theta_k} \quad (46)$$

with

$$F_k = \sin(X\alpha_m)[\beta_n\cos(Y\beta_n) + \kappa_1\sin(Y\beta_n)] - \sin(\bar{x}_0\alpha_m)(\beta_n\cos\beta_n + \kappa_1\sin\beta_n) \quad (47)$$

$$G_k = \cos(X\alpha_m)[\kappa_1\cos(Y\beta_n) - \beta_n\sin(Y\beta_n)] - \cos(\bar{x}_0\alpha_m)(\kappa_1\cos\beta_n - \beta_n\sin\beta_n) \quad (48)$$

where $X = \bar{x}_0 + \bar{L}_k\cos\theta_k$ and $Y = 1 + \bar{L}_k\sin\theta_k$. Notice that Eq. (45) is obtained by substituting $\alpha_m = 0$ into Eq. (46). When $\theta_k = 0$ or π, Eq. (45) reduces to Eq. (49) by applying L'Hospital's rule.

$$\hat{X}_{n,k} = \bar{L}_k(\beta_n\cos\beta_n + \kappa_1\sin\beta_n) \quad (49)”$$

(lines 290 – 315 of the revised manuscript)

10. page 7504 line 24: petroleum engineering does not use radial collector wells, and this solution would be of no use to a petroleum engineer (even though they have horizontal wells). Remove this statement.

Response: It has been removed as suggested.

Technical Corrections

1. page 7507 line 4: delete "depending on situations"
2. page 7507 line 15: change "One grouped the solutions involving" to "One group involved"
3. page 7507 line 17: change "organized the" to "group included"
4. page 7508 line 7: delete "The" before "Robin boundary conditions"
5. page 7509 line 6: the $\times$ in $0 \le \times \le W_x$ is a multiplication symbol, rather than the variable *x*

Responses: Thanks, we have done the corrections.

6. Figure 1: W_x is a capital *W* in the figure, and a lowercase w everywhere in the text and Table 1.

Response: The figure is redrawn with replacing W_x and W_y by w_x and w_y, respectively. The new figure is also shown at the end of this response.

7. page 7510 lines 3-5: these two sentences seem out of place, since they refer to equations on later pages. Move this statement to the conclusions or summary section.

Response: These two sentences are arranged in Concluding Remarks and rewritten as:

"The integration can be done analytically due to the aquifer of finite extent with Eqs. (3) – (6)." (lines 530 – 531 of the revised manuscript)

"The series term of $2\sum_{m=1}^{\infty} \phi_{m,n} X_{m,n} \cos(\alpha_m \bar{x})$ in Eq. (31) of the head solution disappears when deriving the SDR solution (i.e., Eqs. (50) and (51))." (lines 533 – 534 of the revised manuscript)

8. page 7510 line 7: "permeability is usually less permeable" : remove "permeable"
9. page 7510 line 8: delete "the" before "Robin"

Responses: Thanks, They have been done as suggested.

10. page 7509 line 9: do not refer to a negative *z* coordinate as "depth". Depth is an always-positive scalar, which is the distance below the land surface.

Response: Thanks for the comment. The phrase "at depth z_0 measured from water table" has changed to "at $z = -z_0$".

11. page 7511 line 1: change "as the no-flow" to "a no-flow"
12. Equation 30: "for" should have spaces around it and should not be in italics (like equations 26 and 27)
13. page 7515 lines 2-3: "expended by" should be "expanded in"
14. Equation 42: the parentheses around κ_z and the square root should be large, to make association in the equation clearer.
15. page 7515 line 18: add commas between arguments of $\bar{h}_w$ like: $\bar{h}_w(\bar{x}, \bar{y}, \bar{z}, \bar{t})$

Responses: They have been revised according the suggestions.

16. Equation 44: remove the bar between the two options in the choice (it looks like a big fraction)

Response: The bar was inserted by the typesetter of this journal. We will remove it.

17. page 7518 line 18: in "under the confined condition" delete "under the" and "condition"

Response: Taken.

18. page 7520 line 1: add $y = 0$ before "and $y = w_y$"

Response: We would like to indicate the effect of boundaries at $\bar{x} = 0$, $\bar{x} = \bar{w}_x$ and $\bar{y} = \bar{w}_y$ on filtration from a stream at $\bar{y} = 0$. Therefore, $y = 0$ could not be added.

19. pages 7522,7523 & 7526: change "strap" to "strip" (lines 19 & 24 on 7522, line 6 on 7523, and line 22 on 7526)

Response: Done as suggested.

20. page 7527 line 20: change "no-flow" to "homogeneous Neumann" to be congruent with Dirichlet and Robin.

Response: The no-flow condition $\partial h/\partial n = 0$ is in fact a special case of the Neumann one $\partial h/\partial n = c$ with $c = 0$. The finite integral transform proposed by Latinopoulos (1985) is based on the former condition rather than the latter one.

21. Figure 2: what is the domain size assocaited with these figures? $W_x = W_y = 800$? or 20?

Response: Thanks for the comment. We consider $\bar{w}_x = \bar{w}_y = 20$ for dimensionless aquifer domain ($\bar{w}_x = W_x/y_0$ and $\bar{w}_y = W_y/y_0$). We add the phrase "$\bar{w}_x = \bar{w}_y = 20$" in the associated text. (line 412 of the revised manuscript)

22. Figure 4: change "Nirmalized" to "Normalized" or "Scaled"

Response: We appreciate reviewer's eye for detail. The typo has been corrected as "Normalized".

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 12, 7503, 2015.

References

Latinopoulos, P.: Analytical solutions for periodic well recharge in rectangular aquifers with third-kind boundary conditions, J. Hydrol., 77(1), 293−306, 1985.

Sun, D. and Zhan, H.: Pumping induced depletion from two streams, Adv. Water Resour. 30(4), 1016−1026, 2007.

Todd, D. K. and Mays, L. W.: Groundwater Hydrology, 3rd ed., John Wiley & Sons, New York, 2005.

Figure

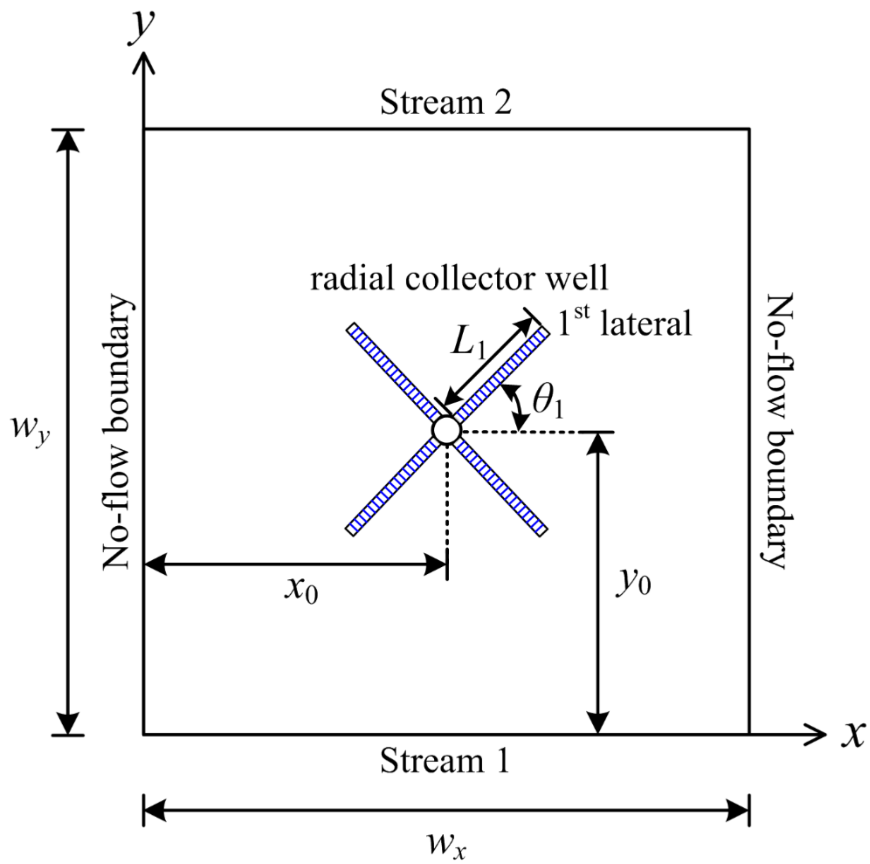

Figure 1. Schematic diagram of a radial collector well in a rectangular unconfined aquifer