

Reviewer #2

General comments

Ensemble Kalman filter is used here to jointly infer hydraulic conductivities and recharge by assimilating head data. The authors evaluated effect of the prior model on success of the method and concluded that a correct prior model is critical, which is consistent with previous research. My main concern includes two items in the “specific comments”: item 4 and 8, in which the former is related to the mathematical model of EnKF and the latter the example to illustrate the compensation between hydraulic conductivity and recharge. Please revise the manuscript accordingly.

We would like to thank the reviewer for the thorough review. All comments are addressed below.

Specific comments

1. Page 5568 line 20-25: The authors cited work by Hendricks Franssen et al. (2004) who jointly estimated hydraulic conductivity and groundwater recharge. What is the difference between this work and the one in the submission? What is the improvement here? Can the authors comment it?

A major difference is that Hendricks-Franssen et al (2004) considered only spatially (blockwise) uniform recharge while the current manuscript considers spatially random fields also for the recharge. A new sentence will be added to the manuscript in order to clarify this point “The authors [*here meaning Hendricks Franssen et al. (2004)*] considered the problem of a well-capture zone, in which they estimated hydraulic conductivity as continuously varying spatial field, whereas recharge was parameterized by zones with uniform values.” Another difference is the choice of the estimation method (as already in the manuscript) and the actual purpose of the investigation.

2. Page 5572 line 1-5: Recharge rate, as a boundary condition, is determined based on such as precipitation, infiltration and geographic conditions (rivers and pumping wells). However the recharge here seems more likely the specific flux at the interface of blocks (discretized for numerical simulation) according to the authors “recharge depends on the gradient of the original transmissivity field”. Please clarify the meaning of “recharge” that is inferred from head data in this study.

Recharge in the context of this work is a flux into each cell across the top boundary; potentially variable in both space and time. In the citation above, the reviewer misses that the cited text talks about **apparent** recharge, which is a mathematical construct. Seen from the flow model it is also a flux, in the same way as the real recharge, but its value is calculated as a function of the (erroneous) conductivity field. This is the essence of Section 2.

3. Page 5572 on Fig.1: What are the initial and boundary conditions (except the recharge at the center) of this example?

Boundary conditions (west 50m, east 8m, north and south no flow) will be added to the figure in the revised manuscript. As this is a steady-state problem, the question about initial condition is not applicable.

4. Section 3.1 Kalman filter on page 5573 through 5574: Vector X_t consists of two elements, heads h_t and parameters (recharge and log-conductivity) while the head h_t can be simulated as $f_t(h_{t-1}, X_t)$, that is $h_t = f_t(h_{t-1}, X_t)$. If this is correct there would be mistake in the objective function $W(x)$ (equation 6) since the X_t contains head vector h_t that is considered again separately in the second term $(f_t(h_{t-1}, X_t) - Y_t)^T R^{-1}(f_t(h_{t-1}, X_t) - Y_t)$. Either the head vectors can be excluded from the vector X_t or the second term of the objective function should be removed. The corresponding comments and following equations should be revised accordingly, i.e., equations 7-13.

This is a bit of a misunderstanding. The term $f_t(h_{t-1}, X_t)$ does not denote the simulated head, but the observation operator. Hence, $f_t(h_{t-1}, X_t)$ represents the simulated observations and the full term is the difference between the observed heads and the simulated ones. The two terms of the objective function are the prior (first term) and the likelihood of observations (second term), hence a standard objective function formulation. In the revised manuscript we will consider reformulating/renaming the terms to avoid any misunderstanding.

5. Page 5575 equation 11-13: The series of equations are used to calculate the covariance between parameters and/or simulations. The denominator should be $n-1$ rather than n , that is, $1/(n-1)$ instead of $1/n$ in these equations.

The reviewer is right; this error will be corrected

6. Page 5577 line 6-7: What do the authors mean by “combined patterns of hydraulic conductivity and recharge”?

We meant: *[the prior knowledge determines what]* “patterns of conductivity and patterns of recharge that can be jointly inferred by the scheme”. The sentence will be reformulated in the revised manuscript.

7. Page 5577 line 20: Here it says “the conductivity and recharge fields are uncorrelated”. On the contrary the authors stated earlier that “the apparent recharge depends on the gradient of the original transmissivity field” (page 5572 line 1-2), which indicates a close correlation. Please clarify this.

We disagree that the two statements contradict each other. The first (page 5572) concerns the mathematical calculation of the **apparent** recharge that can be used to replace a false conductivity field with. The second (page 5577) on the other hand concerns the setup of the virtual experiment and simply states that the model input fields are generated without any correlation between the two fields. The **apparent** recharge is dependent on the false conductivity field, but this has no relation to the virtual experiment input data. Please also see the answer to Comment 2 above.

8. Page 5581 line 16-20: “it is always possible to compensate a missing or wrong conductivity with a recharge, and this is also clearly seen in the last column of Fig.5: the estimated recharge shows remarkable similarity with the reference conductivity field.” In Fig.5 the estimated recharge does show similarity with the reference conductivity field because the wrong recharge prior is sampled using the true conductivity field model NOT because they can compensate each other. They are two different things in my opinion. The authors need find another example to illustrate the compensation effect between conductivity and recharge derived in Section 2.

We see the reviewer's point. The text was not well formulated in this section. The compensation effect does not cause the estimated fields, which are enforced by the erroneously sampled prior parameter distributions (as rightly pointed out by the reviewer). However, if the data were uniquely informative and no compensation was possible, the effect of the erroneously sampled prior would have disappeared after 300 days of data assimilation and the estimated fields would have altered into structurally (more) correct fields! Because the head data are not unique, compensation between conductivity and recharge is possible, and wrong prior assumptions prevail in data-assimilation and parameter estimation practically forever. The combination of wrong conductivity and recharge fields reproduce the head observations quite well, and the filter algorithm sees no need to change the (erroneous) fields.

We see that the choice of wording was suboptimal and in the revised manuscript we will make it clear that what we see is that the compensation effect sustains the erroneous fields. We believe, however, that the example is illustrative and assess that adding another experiment would unnecessarily complicate the manuscript.

9. Page 5595 Fig.2: The last plot in Fig.2 shows the spatial distribution of recharge over the domain. The recharge is time-varying with a seasonal trend (page 5577 line 13 as well as shown in the "river stage" plot of Fig.2). So the question is which time does this recharge plot show? Also please add title for the X axis in the plot of "river stage" (it should be time I guess).

The actual recharge is calculated by multiplying the spatially uniform, temporal trend parameter (at the right day) with the shown (relative) recharge field. The spatial field shown is for the case that temporal trend parameter has the value of unity. The sentence "The actual recharge is calculated by multiplying the trend parameter at any given time with the shown recharge field" will be added to the revised manuscript to make this setup clearer.

Technical corrections

1. Page 5570 line 3: "...was worse then..." should be "...was worse than..."
Corrected.
2. Page 5570 line 5-10, page 5578 line 4, page 5581 line 2 and 16: The authors mentioned "Section 2", "Sect.2", "Section 3", "Sect.4" and "Sect.5". It would be better to keep consistent.
Corrected.
3. Page 5576 line 9-10: "...parameter that it is primarily required..." remove "it"
Corrected.
4. Page 5577 line 19: ". it should be..." use capital letter in "it"
Corrected.
5. Page 5579 line 7-8: "We have also conducted successful assimilations also estimating the trend parameter." Too many "also"
Corrected.
6. Page 5580 line 20: "smaller errors in predicting heads then the...". Correct "then" to "than"
Corrected.

7. Page 5581 line 28: "...it would to be difficult to..." modify this sentence.

Sentence is altered and split in two.

8. Page 5582 line 4-6: revise this sentence.

Sentence revised and shortened.