Interactive comment on "Technical Note: Approximate solution of transient drawdown for constant-flux pumping at a partially penetrating well in a radial two-zone confined aquifer" by C.-S. Huang et al.

Anonymous Referee #2
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General Comments

C.-S. Huang, S.-Y. Yang and H.-D. Yeh present in the technical note a new approximate solution for the drawdown induced by a constant rate pumping test in a radial two-zone confined aquifer. By considering partial penetration of the pumping well, their approximate solution for the transient drawdown is new and interesting to the hydrologic community. The publication is well-written. A more detailed description of the mathematical model and results in the discussion section could improve readability and understanding, as described later. Tables and figures are clear and comprehensible, minor improvements are suggested for captions.

Response: Thanks for the comment. The manuscript has been revised on the basis of the comments below.

Specific Comments

Abstract

Please clarify under which assumptions your solution is valid: 2D or 3D aquifer, heterogeneous or homogeneous media?

Response: To address the problem, we added the statement: "This study develops a new approximate solution for the problem based on a mathematical model describing steady-state radial and vertical flows in a two-zone aquifer. Hydraulic parameters in these two zones can be different but are assumed homogeneous in each zone." (lines 22 – 25).

Introduction

The introduction might concentrate on studies directly related to the presented work, i.e. solutions which consider partial penetration or two-zone aquifers, respectively. A comprehensive overview on available solutions for constant rate pumping tests is already given in table 1.

Response: Thanks.

What are potential applications for the presented solution?

Response: Two sentences shown below are added in the revised manuscript to state its potential applications:

"The transient solution is in term of simple series with advantages of fast convergence, simplicity, and good accuracy from practical viewpoint. It can be used as a convenient tool to estimate temporal and spatial drawdown distributions for the constant-flux pumping and explore physical insight into the flow behavior affected by hydrogeological properties and aquifer configuration." (lines 115 – 119)

Mathematical Model

Please state the aim of the section a the beginning and explain model configurations and assumptions (2D or 3D, homogeneous or heterogeneous media, boundary conditions).

Response: Thanks for the comment. We added a new paragraph given below to state the aim and describe model configurations and assumptions.

"This section introduces a new mathematical model for steady-state flow due to the CFP at a finite-radius partially penetrating well in a radial two-zone confined aquifer. The symbols representing variables and parameters for the model are listed in Table 2. The hydraulic parameters in the two zones are different but in each zone are assumed homogeneous. The outer boundary is considered to be under the Dirichlet condition of $\bar{s}_2 = 0$ at $\bar{r} = \bar{R}$. The top and bottom confining beds are under the no-flow conditions of $\partial \bar{s}_i / \partial \bar{z} = 0$ where $i \in (1, 2)$. The effect of wellbore storage on aquifer drawdown is assumed ignorable. Note that this effect diminishes when $t > 2.5 \times 10^2 r_c^2 / T_2$ mentioned in Papadopulos and Cooper (1967). In addition, Yeh and Chang (2013) also mentioned that this effect can be neglected for a well with $r_c \leq 0.25$ m. A schematic diagram for the CFP problem is illustrated in Figure 1." (lines 125 – 134).

Figure 1 shows the observation well to be screened over the entire aquifer. Is this a prerequisite for your solution or is it also valid for partially penetrating observation wells?

Response: The present solution is applicable to a fully or partially penetrating observation well.

Refer to the meaning of α in the text.

Response: We added a new text shown below to describe it.

"The analysis of the temporal drawdowns predicted by Eqs. (17) and (20) indicates that the vertical flow due to a partially penetrating well prevails under the conditions of thick aquifers, vicinity to the well, and/or small conductivity ratio (i.e., $\alpha_1 \bar{r}^2 < 1$ or $\alpha_2 \bar{r}^2 < 1$)." (lines 248 – 251)

α includes the vertical hydraulic conductivity, K_z. Are you assuming the aquifer and the skin zone to be anisotropic? Please elaborate this in more detail.

Response: Yes, we added the phrase "α_1 and α_2 reflect the effect of aquifer anisotropy on dimensionless aquifer drawdown" in lines 140 – 141.

Approximate Solution

Please elaborate in more detail on the procedure how equation 18 was found to allow for reproducibility. What was the range of tested parameters. How is the accuracy of equation 18 for early, intermediate and late pumping times?

Response: The dynamic radius of influence, $\bar{R}(\bar{t}) = 1 + \sqrt{\pi \bar{t} / 1.4}$, defined in the equation is applicable to any value of time t, well radius r_w, radial hydraulic conductivity K_{r2}, and specific storage S_{s2}. In addition, we added a new text in lines 184 – 187, also shown below, to explain how to obtain the equation:

"The time-dependent radius of influence $\bar{R}(\bar{t})$ was first assumed as $\bar{R}(\bar{t}) = 1 + \sqrt{\pi \bar{t} / c}$ where c is a constant. By trial and error, we found that the drawdowns predicted by the approximate solution and Chiu et al. (2007) Laplace-domain solution with the Crump method agree well when c approaches 1.4."

Why is the coefficient different to the one obtained by Yang et al. (2014)?

Response: The coefficient equals unity found by Yang et al. (2014) for constant-head pumping tests and 1.4 found in this work for constant-flux pumping tests.

Accuracy of approximate Solution

Please specify "the approximate solution" in line 10, p 2750 by giving the equations you refer to.

Response: The equation numbers are provided as suggested.

Presented results generally assume that $\alpha_1 = \alpha_2$. What does this assumption imply?

Response: The assumption $\alpha_1 = \alpha_2$ (i.e., $K_{z1} r_w^2 /(K_{r1} b^2) = K_{z2} r_w^2 /(K_{r2} b^2)$) means that the conductivity ratios for both formation and skin zones are the same because of constant well radius r_w and aquifer thickness b.

Figures 2a and 2b show examples of the solution at specific values of dimensionless time and distance. Were these values chosen randomly or by some criterion? Did you test other choices?

Response: Yes, we did. The agreement on dimensionless drawdown in the figures stands for any chosen values of dimensionless parameters and variables.

I assume that by "discrepancy" in line 20, p 2750, you mean the deviation of your solution from Chiu et al. (2007) during early pumping times. As I understand, equation 18 should compensate for neglecting the temporal derivative in equations (2) and (3). Might the deviation between the two solutions possibly come from the definition of $\bar{R}(\bar{t})$ in equation 18? This could be answered by a more detailed description on the trail and error procedure regarding $\bar{R}(\bar{t})$ in section 2.3.

Response: Yes, you are right. The present solution was developed using equation 18 to compensate the neglect of the temporal derivative term in the transient groundwater flow equation. To clarify the problem, the associated sentence is rewritten as "The discrepancy in dimensionless drawdown at the early period of $0 \le \bar{t} \le 600$ can be attributed to the absence of the time derivative term in both Eqs. (1) and (2)." (lines 220 – 222)

The phrase "except at early time during which the radius of influence arrives" (line 2, p 2751) is somehow unclear to me.

Response: The phrase "time during which the radius of influence arrives" has been deleted. The associated sentence is rewritten as "It seems reasonable to conclude that the approximate transient solution gives good predicted drawdown in an observation well over the entire pumping period except at early time when the dynamic radius of influence reaches the well (i.e., $\bar{t} \cong 1.4(\bar{r}-1)^2/\pi$ derived by substituting $\bar{R}(\bar{t}) = \bar{r}$ into Eq. (18) and rearranging the result)." (lines 223 – 226)

Vertical Flow

What does the assumption $\alpha_1 = \alpha_2$ imply? Did you test for $\alpha_1 \neq \alpha_2$?

Response: Yes, we did. The assumption $\alpha_1 = \alpha_2$ indicates that the conductivity ratios for the formation zone and skin zone are the same.

How/why did you choose the set of parameters ($\bar{r}$, $\bar{z}$,...)? Did you test for other choices?

Response: Yes, we did. The criterion that the vertical flow effect on the aquifer drawdown vanishes is valid (or applicable) for any parameter values. We take a representative one of them for example.

In line 10, p 2751, "vertical flow vanishes": I guess the effect of vertical flow on the drawdown at an observation vanishes, but vertical flow itself does continue.

Response: Thanks for the comment. The original sentence is rewritten as "We may, therefore, reasonably conclude that the vertical flow effect on the aquifer drawdown at an observation well vanishes when $\alpha_1 \bar{r}^2 \geq 1$ and $\alpha_2 \bar{r}^2 \geq 1$, i.e., b is small, r is large, and/or the values of K_{z1}/K_{r1} and K_{z2}/K_{r2} are large." (lines 235 – 237)

You state a criterion for the presented model by which vertical flow can be neglected. Do comparable criteria exist for other models (e.g. models mentioned in the introduction)? If so, please relate to those. Such criteria could also be mentioned in the introduction.

Response: To our knowledge, our work is the first to provide such a criterion.

Concluding remarks

Line 23, p 2751: The meaning of the phrase "during which the time-dependent radius of influence just touches" is somehow unclear to me. Please consider revising.

Response: The phrase “during which the time-dependent radius of influence just touches” has been deleted. The associated sentence is rewritten as:

“The comparison with the Chiu et al. (2007) solution reveals that the approximate solution gives accurate temporal drawdown distributions in an observation well over the entire pumping period except at early time when the dynamic radius of influence reaches the well (i.e., $\bar{t} \cong 1.4(\bar{r}-1)^2/\pi$ derived by substituting $\bar{R}(\bar{t}) = \bar{r}$ into Eq. (18) and rearranging the result).” (lines 244 – 248)

Figures

Figure 1: Specify abbreviations.

Response: The abbreviation of “CFT” in the figure caption is replaced by “the constant-flux pumping”.

Figure 2: Specify the approximate solution by referring to equations.

Response: Figure 2 is redrawn as suggested.

Figure 3: List chosen parameters (r, z, ...) in the caption, as done for figure 2.

Response: The caption is rewritten as suggested.

Technical Corrections

Figure 1: Is CFT a typo? If so, please change to CFP.

Response: We appreciate reviewer’s eye for detail. It has been changed to “constant-flux pumping”.

Please consider revising the language by a native speaker.

Response: The manuscript has been edited by a colleague who is good at English writing.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 12, 2741, 2015.

Table 2. Summary of symbols used in the text and their definitions

Symbols	Definitions
(s_1, s_2)	Drawdowns in skin and formation zones, respectively
r	Radial distance from the center of the well
r_s	Radius of skin zone
R	Radius of cylinder aquifer domain or the radius of influence
(r_w, r_c)	Outer and inner radiuses of well, respectively
z	Elevation from the aquifer bottom
(z_1, z_2)	Lower and upper elevations of well screen, respectively
t	Time since pumping
b	Aquifer thickness
Q	Pumping rate of well
(K_{r1}, K_{r2})	Radial hydraulic conductivities of skin and formation zones, respectively
(K_{v1}, K_{v2})	Vertical hydraulic conductivities of skin and formation zones, respectively
S_{s2}	Specific storage of formation zone
(T_1, T_2)	Transmissivities of skin and formation zones, respectively
$(\bar{s}_1, \bar{s}_2)$	$(2\pi T_2 s_1/Q, 2\pi T_2 s_2/Q)$
$\bar{t}$	$K_{r2}\, t/(S_{s2} r_w^2)$
$(\bar{r}, \bar{r}_s, \bar{R})$	$(r/r_w, r_s/r_w, R/r_w)$
$(\bar{z}, \bar{z}_1, \bar{z}_2)$	$(z/b, z_1/b, z_2/b)$
(ϕ, γ)	$(\bar{z}_2 - \bar{z}_1, K_{r2}/K_{r1})$
(α_1, α_2)	$(K_{z1} r_w^2/(K_{r1} b^2), K_{z2} r_w^2/(K_{r2} b^2))$

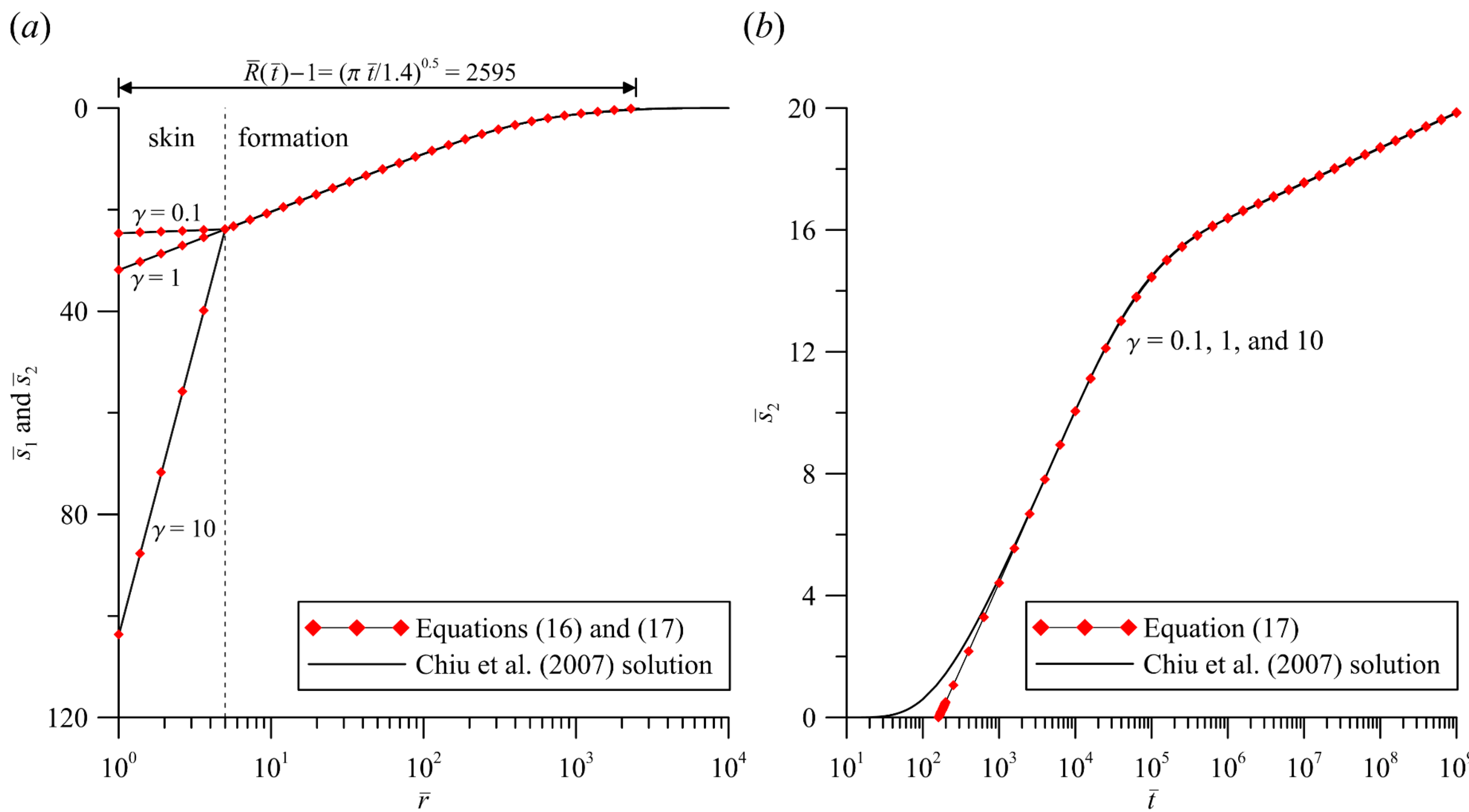

Figure 2. Predicted drawdowns by Chiu et al. (2007) solution and the approximate solution, Eqs. (16) and (17), with $\gamma = 0.1$, 1, and 10 for (a) spatial distributions at $\bar{t} = 3\times10^6$ and (b) temporal distributions at $\bar{r} = 20$ with $\bar{z} = 0.5$, $\bar{r}_s = 5$, $\bar{z}_1 = 0.4$, $\bar{z}_2 = 0.6$, and $\alpha_1 = \alpha_2 = 10^{-7}$