# Interactive comment on “Technical Note: Approximate solution of transient drawdown for constant-flux pumping at a partially penetrating well in a radial two-zone confined aquifer” by C.-S. Huang et al.

**Anonymous Referee #1**
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**General Comments**

C.-S. Huang, S.-Y. Yang, H.-D. Yeh present in the technical note a newly developed approximate solution for the drawdown of a pumping test at a partially penetrating well in a radial two-zone confined aquifer under constant-flux pumping conditions. The analytical solution for steady state and the approximate solution for transient pumping test is something new and interesting to the hydrological community. In general the publication is well-written. The readability could be improved by language check by a native speaker and restructuring several subsections, as described later. Figures and tables are in a good shape, minor improvements are suggested later on.

Response: Thanks for the comment. The manuscript has been revised on the basis of the comments below and edited by a colleague who is good at English writing.

**Specific Comments**

*Abstract*

Clarify in the abstract the type of pumping test solution you derive: Is it for homogeneous media (or heterogeneous media)? It is limited to 2D or valid for 3D aquifer description?

Response: To address the problem, we added the statement: “This study develops a new approximate solution for the problem based on a mathematical model describing steady-state radial and vertical flows in a two-zone aquifer. Hydraulic parameters in these two zones can be different but are assumed homogeneous in each zone.” (lines 22 – 25).

*Introduction*

What are potential applications of the derived approximate solution?

Response: Two sentences shown below are added in the revised manuscript to state its potential applications:

"The transient solution is in term of simple series with advantages of fast convergence, simplicity, and good accuracy from practical viewpoint. It can be used as a convenient tool to estimate temporal and spatial drawdown distributions for the constant-flux pumping and explore physical insight into the flow behavior affected by hydrogeological properties and aquifer configuration." (lines 115 – 119)

*Mathematical Model*

Specify the aim of the section at the beginning (p 2745, line 22).

Response: We added a sentence: "This section introduces a new mathematical model for steady-state flow due to the CFP at a finite-radius partially penetrating well in a radial two-zone confined aquifer." (lines 125 – 126).

The ordering of the content of the section could be improved: first specify the process of interest (pumping test), including boundary conditions, assumptions and characteristics (e.g. points mentioned in Table 1) first in words, than refer to figure and than in equations.

Response: The section is rewritten as suggested. The new one is listed below:

"This section introduces a new mathematical model for steady-state flow due to the CFP at a finite-radius partially penetrating well in a radial two-zone confined aquifer. The symbols representing variables and parameters for the model are listed in Table 2. The hydraulic parameters in the two zones are different but in each zone are assumed homogeneous. The outer boundary is considered to be under the Dirichlet condition of $\bar{s}_2 = 0$ at $\bar{r} = \bar{R}$. The top and bottom confining beds are under the no-flow conditions of $\partial \bar{s}_i / \partial \bar{z} = 0$ where $i \in (1, 2)$. The effect of wellbore storage on aquifer drawdown is assumed ignorable. Note that this effect diminishes when $t > 2.5 \times 10^2 r_c^2 / T_2$ mentioned in Papadopulos and Cooper (1967). In addition, Yeh and Chang (2013) also mentioned that this effect can be neglected for a well with $r_c \leq 0.25$ m. A schematic diagram for the CFP problem is illustrated in Figure 1.

The governing equations describing steady-state dimensionless drawdown distributions in the skin and formation zones are expressed, respectively, as

$$\frac{\partial^2 \bar{s}_1}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{s}_1}{\partial \bar{r}} + \alpha_1 \frac{\partial^2 \bar{s}_1}{\partial \bar{z}^2} = 0 \quad \text{for} \quad 1 \leq \bar{r} \leq \bar{r}_s \tag{1}$$

and

$$\frac{\partial^2 \bar{s}_2}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{s}_2}{\partial \bar{r}} + \alpha_2 \frac{\partial^2 \bar{s}_2}{\partial \bar{z}^2} = 0 \quad \text{for} \quad \bar{r}_s \le \bar{r} \le \bar{R} \tag{2}$$

where $\alpha_1$ and $\alpha_2$ reflect the effect of aquifer anisotropy on dimensionless aquifer drawdown. The inner boundary designated at the rim of the wellbore is under the Neumann condition as

$$\frac{\partial \bar{s}_1}{\partial \bar{r}} = -\frac{\gamma}{\phi}\left(U(\bar{z}-\bar{z}_1) - U(\bar{z}-\bar{z}_2)\right) \quad \text{at} \quad \bar{r} = 1 \quad \text{and} \quad 0 \le \bar{z} \le 1 \tag{3}$$

where $U(\cdot)$ is the unit step function. Equation (3) indicates that the flux is uniformly distributed over the screen. Two continuity conditions required at $\bar{r} = \bar{r}_s$ are

$$\bar{s}_1 = \bar{s}_2 \quad \text{at} \quad \bar{r} = \bar{r}_s \tag{4}$$

and

$$\frac{\partial \bar{s}_1}{\partial \bar{r}} = \gamma \frac{\partial \bar{s}_2}{\partial \bar{r}} \quad \text{at} \quad \bar{r} = \bar{r}_s \qquad (5)”$$

(lines 125 – 148)

A table containing the symbols of variables and parameters would improve the readability of the work significantly. Refer to that table in caption of fig. 1 and in the text.

Response: Thanks for the comment. We added Table 2 in which the symbols are defined. Two sentences given below related to the table are added in the revised manuscript. “The symbols representing variables and parameters for the model are listed in Table 2.” (lines 126 – 127) and “The symbols of the variables are defined in Table 2” (in the caption of Figure 1).

*Steady-State Solution*

Is the derived solution for steady state already published before? If not, specify that these are new results. If yes give a reference.

Response: To our knowledge, the steady-state solution has not been published elsewhere, which is stated in the sentence of “A new solution derived by the application of the finite Fourier cosine transform to the model can be written as” (lines 150 – 151).

*Approximate Solution*

The ordering of the content of the section could be improved: First state the aim of the approach (why), than the idea of the approach (as given in line 9-11, p 2749), than how it is done (line 20, p 2748 – line 6 p 2749) and than the result (line 17, p 2748). Finally elaborate in more detail on the way how R(t) was found (line 15, p. 2748): how was the trail and error procedure performed, what was the tested range of parameters, to what was the approximated solution compared to and how?

Response: Thanks for the comment. The section is rewritten on the basis of the comment. The new one is shown below:

"The inverse Laplace transform to Chiu et al. (2007) semi-analytical solution of drawdown leads to a time-domain result for the CFP in a two-zone aquifer system; however, the resultant solution involves laborious calculations. We therefore develop an approximate transient solution of drawdown for the CFP problem. The idea originated from the concept of a time-dependent diffusion layer for the solution of the diffusion equation in the field of electrochemistry (Fang et al., 2009). The approximate transient solution is obtained by replacing the $\bar{R}$ in the steady-state solution (i.e., Eqs. (6) – (15)) with a dimensionless time-dependent radius of influence $\bar{R}(\bar{t})$. The result is in terms of dimensionless time denoted as

$$s_1(\bar{r},\bar{z},\bar{t}) = \ln(\bar{R}(\bar{t})/\bar{r}_s) + \gamma \ln(\bar{r}_s/\bar{r}) + \frac{2\gamma}{\bar{z}_2 - \bar{z}_1}\sum_{n=1}^{\infty} F_1(\bar{r},n,\bar{t})\cos(n\pi\bar{z}) \quad \text{for} \quad 1 \le \bar{r} \le \bar{r}_s \qquad (16)$$

$$s_2(\bar{r},\bar{z},\bar{t}) = \ln(\bar{R}(\bar{t})/\bar{r}) + \frac{2\gamma}{\bar{r}_s(\bar{z}_2 - \bar{z}_1)}\sum_{n=1}^{\infty} F_2(\bar{r},n,\bar{t})\cos(n\pi\bar{z}) \quad \text{for} \quad \bar{r}_s \le \bar{r} \le \bar{R} \qquad (17)$$

and

$$\bar{R}(\bar{t}) = 1 + \sqrt{\pi\bar{t}/1.4} \qquad (18)$$

where $F_1(\bar{r},n,\bar{t})$ and $F_2(\bar{r},n,\bar{t})$ obtained from Eqs. (8) and (9), respectively, with coefficients $\psi$, $\zeta$, $\xi$, and $G(\mu,c)$ defined in Eqs. (10) – (13), respectively, are functions of dimensionless time due to substitution of Eq. (18). The time-dependent radius of influence $\bar{R}(\bar{t})$ was first assumed as $\bar{R}(\bar{t}) = 1 + \sqrt{\pi\bar{t}/c}$ where $c$ is a constant. By trial and error, we found that the drawdowns predicted by the approximate solution and Chiu et

al. (2007) Laplace-domain solution with the Crump method agree well when *c* approaches 1.4. Detailed discussion is shown in section 3.1. Notice that Eq. (18) is similar to an equation given in Yang et al. (2014, Eq. (25)) but has a different coefficient value."
(lines 169 – 189)

*Special Case*

Give a link to the relation of the special case solution to previously derived results as given in the introduction. (Similar to the sentence in line 6, p. 2750 for the special case in 2.5.)

Response: To our knowledge, the special case of the present solution (i.e., eq. (19) and (20) in the revised manuscript) has never been published before.

*Accuracy of approximate Solution*

Specify the meaning of the parameters (e.g. line 12, p 2750 state what gamma is, etc.) for easier readability. Give a reason for the choice of parameters, e.g. the point in time t in Figure 2a. Did you test all choices of parameters? What are the ranges of tested parameters? For which choice of parameters did the solutions not match? I recommend to start a new paragraph in line 16, p 2750. The same questions concerning the choice and tested range of parameters as for Fig. 2a apply for Fig. 2b.

Response: To clarify the problem, a new text describing the choice of parameter values for plots in Figure 2 is added in the revised manuscript and also given below:

"On the basis of the comparison of predictions from the approximate solution and Chiu et al. (2007) Laplace-domain solution, we have concluded that the accuracy of the present solution depends only on dimensionless time $\bar{t}$ and radial distance $\bar{r}$ and does not relate to other dimensionless parameters and space variable. Consider representative parameters and variables as follows: $\bar{z} = 0.5$, $\bar{r}_s = 5$, $\bar{z}_1 = 0.4$, $\bar{z}_2 = 0.6$, $\alpha_1 = \alpha_2 = 10^{-7}$, and $\gamma = 0.1$ for positive skins, 1 for no skin and 10 for negative skins." (lines 208 – 213)

Which discrepancies you mean in line 20, p 2750? I do not understand the message of the last sentence, especially what do you mean with " time during which the radius of influence arrives"?

Response: To clarify the problem, the associated sentence is rewritten as "The discrepancy in dimensionless drawdown at the early period of $0 \le \bar{t} \le 600$ can be attributed to the absence of the time derivative term in both Eqs. (1) and (2)." (lines 220 – 222)

The phrase "time during which the radius of influence arrives" has been deleted. In addition, the last sentence is rewritten as "It seems reasonable to conclude that the approximate transient solution gives good predicted drawdown in an observation well over the entire pumping period except at early time when the dynamic radius of influence reaches the well (i.e., $\bar{t} \cong 1.4(\bar{r}-1)^2/\pi$ derived by substituting $\bar{R}(\bar{t}) = \bar{r}$ into Eq. (18) and rearranging the result)." (lines 223 – 226)

*Vertical Flow*

Specify the meaning of the parameters (e.g. line 5, p 2751 alpha, etc.) to improve the readability. What is b in line 12, p 2751?

Response: Those parameters influence the vertical flow near the partially penetrating well. The original sentence is rewritten as "The vertical flow induced by well partial penetration is strongly dependent on both dimensionless lumped parameters $\alpha_1 \bar{r}^2$ and $\alpha_2 \bar{r}^2$ (i.e., $K_{z1} r^2 /(K_{r1} b^2)$ and $K_{z2} r^2 /(K_{r2} b^2)$, respectively)." (lines 228 – 230). The symbol $b$ is aquifer thickness defined in Table 2 of the revised manuscript.

Give a reason for the choice of parameters (line 7, p 2751).

Response: Arbitrary choice of the parameter values won't affect the conclusion that the vertical flow induced by well partial penetration is ignorable when $\alpha_1 \bar{r}^2 \ge 1$ and $\alpha_2 \bar{r}^2 \ge 1$.

Are the results shown in Fig. 3 representative for other choices of parameters of r, z and gamma?

Response: Yes, they are.

*Concluding remarks*

I do not understand what is meant with " during which the time-dependent radius of influence just touches." (line 24, p 2751) State in words (not in formulas) what you mean in line 1-4, p 2752. The conclusion should be understandable without searching for the meaning of the parameters.

Response: The phrase “during which the time-dependent radius of influence just touches” has been deleted. The conclusion is rewritten as:

“The analysis of the temporal drawdowns predicted by Eqs. (17) and (20) indicates that the vertical flow due to a partially penetrating well prevails under the conditions of thick aquifers, vicinity to the well, and/or small conductivity ratios (i.e., $\alpha_1 \bar{r}^2 < 1$ or $\alpha_2 \bar{r}^2 < 1$ ). Accordingly, conventional models neglecting the vertical flow will underestimate drawdown under those conditions.” (lines 248 – 252)

*Figures and Tables*

Table 1: Avoid the abbreviation CFP in the caption.

Response: It is replaced by the constant-flux pumping.

Figure 1: Is the abbreviation CFT a typo or was it introduced before? Recommendation of not using abbreviation in caption in general. Variables and parameters used in the Figure are not explained in the caption. This would be OK, if it is given a link to a Table, where they are listed separately.

Response: We appreciate reviewer’s eye for detail. The CFT is replaced by the constant-flux pumping.

Figure 2: The different lines in the plots are difficult to distinguish; probably use thicker lines and marker in combination with lines. Give a link to the equation in the text for the "approximate solution“.

Response: Figure 2 is redrawn and also shown below.

Figure 3: The different lines in the plots are difficult to distinguish; probably use thicker lines. The color and line scheme appears somewhat arbitrary, this could be improved. List the choice of parameters (as done in caption of Fig. 2).

Response: Figure 3 is redrawn and its caption is rewritten. They are shown at the end of this response.

**Technical Corrections**

Language could be improvements by native speaker.

Response: The manuscript has been edited by a colleague who is good at English writing.

The usage of the abbreviation CFP is OK, but I would recommend to avoid it in the abstract and figure/table captions.

Response: The CFP in the abstract and figure/table captions are replaced by the constant-flux pumping.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 12, 2741, 2015.

**Table 2.** Summary of symbols used in the text and their definitions

| Symbols | Definitions |
|---|---|
| ($s_1$, $s_2$) | Drawdowns in skin and formation zones, respectively |
| $r$ | Radial distance from the center of the well |
| $r_s$ | Radius of skin zone |
| $R$ | Radius of cylinder aquifer domain or the radius of influence |
| ($r_w$, $r_c$) | Outer and inner radiuses of well, respectively |
| $z$ | Elevation from the aquifer bottom |
| ($z_1$, $z_2$) | Lower and upper elevations of well screen, respectively |
| $t$ | Time since pumping |
| $b$ | Aquifer thickness |
| $Q$ | Pumping rate of well |
| ($K_{r1}$, $K_{r2}$) | Radial hydraulic conductivities of skin and formation zones, respectively |
| ($K_{v1}$, $K_{v2}$) | Vertical hydraulic conductivities of skin and formation zones, respectively |
| $S_{s2}$ | Specific storage of formation zone |
| ($T_1$, $T_2$) | Transmissivities of skin and formation zones, respectively |
| ($\bar{s}_1$, $\bar{s}_2$) | ($2\pi T_2 s_1/Q$, $2\pi T_2 s_2/Q$) |
| $\bar{t}$ | $K_{r2}t/(S_{s2}r_w^2)$ |
| ($\bar{r}$, $\bar{r}_s$, $\bar{R}$) | ($r/r_w$, $r_s/r_w$, $R/r_w$) |
| ($\bar{z}$, $\bar{z}_1$, $\bar{z}_2$) | ($z/b$, $z_1/b$, $z_2/b$) |
| ($\phi$, $\gamma$) | ($\bar{z}_2 - \bar{z}_1$, $K_{r2}/K_{r1}$) |
| ($\alpha_1$, $\alpha_2$) | ($K_{z1}r_w^2/(K_{r1}b^2)$, $K_{z2}r_w^2/(K_{r2}b^2)$) |

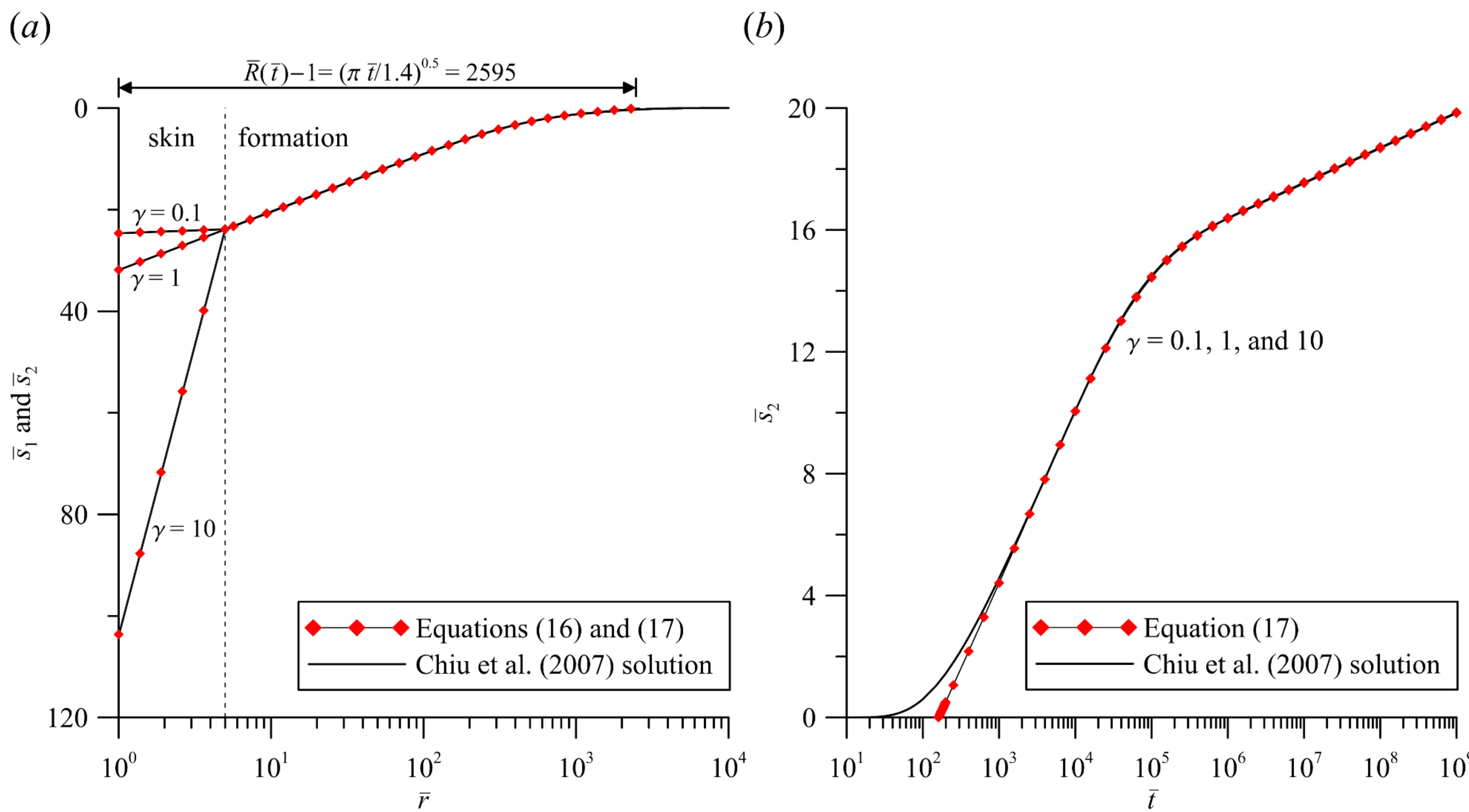


**Figure 2.** Predicted drawdowns by Chiu et al. (2007) solution and the approximate solution, Eqs. (16) and (17), with $\gamma$ = 0.1, 1, and 10 for (a) spatial distributions at $\bar{t}=3\times10^6$ and (b) temporal distributions at $\bar{r}$ = 20 with $\bar{z}$ = 0.5, $\bar{r}_s$ = 5, $\bar{z}_1$ = 0.4, $\bar{z}_2$ = 0.6, and $\alpha_1 = \alpha_2 = 10^{-7}$

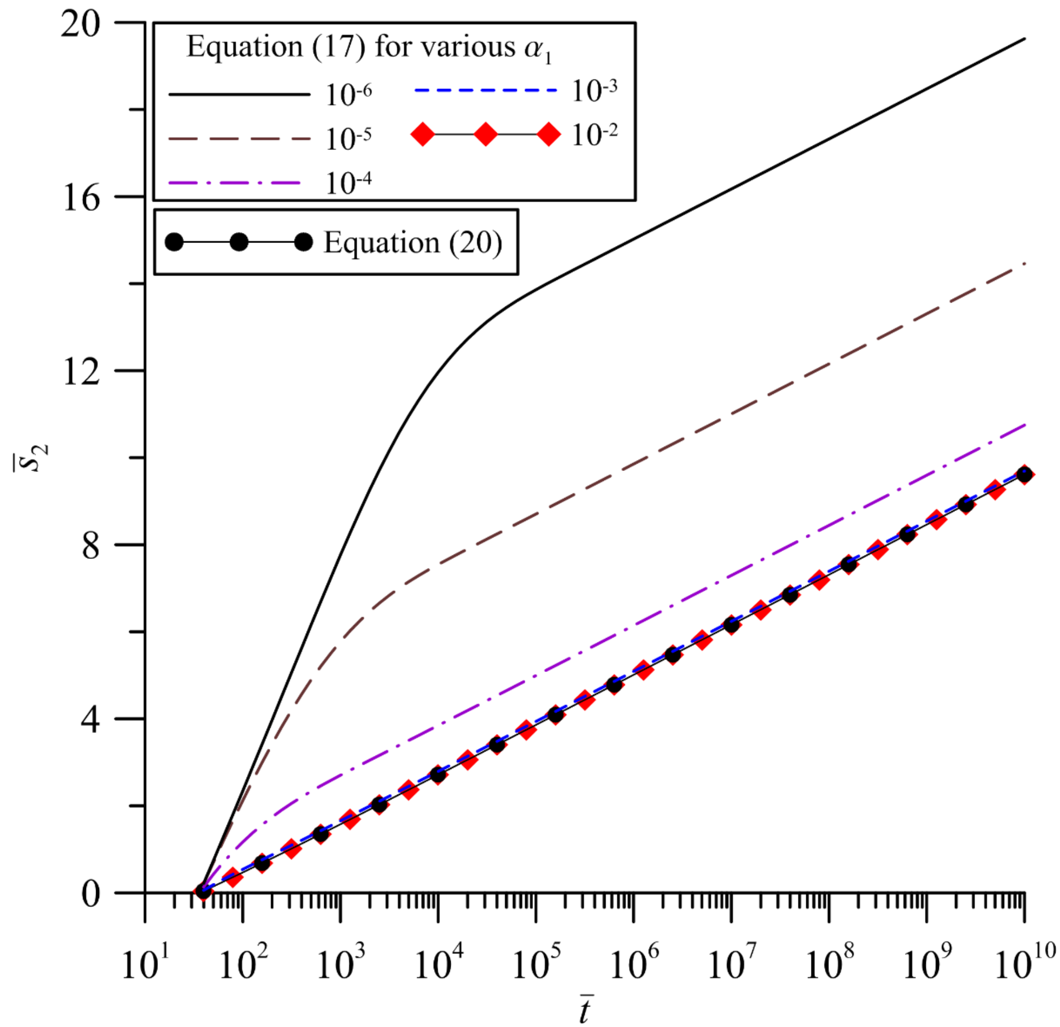


**Figure 3.** Temporal drawdown distributions predicted by the approximate solution, Eq. (17), with $\bar{r} = 10$, $\bar{z} = 0.5$, $\bar{z}_1 = 0.4$, $\bar{z}_2 = 0.6$, $\bar{r}_s = 5$, $\gamma = 0.1$ and various values of $\alpha_1$ with $\alpha_1 = \alpha_2$