Interactive comment on "Investigation of hydrological time series using copulas for detecting catchment characteristics and anthropogenic impacts" by

T. Sugimoto, A. Bárdossy, G. S. S. Pegram and J. Cullmann

General remarks of authors

First of all, we deeply appreciate the care and effort taken by the reviewers in examining this paper.

We checked the literature and the papers concerning asymmetry once more. We agree with the comments of anonymous referee #1 that our definition of asymmetry is similar to the definition of Joe H. (2014) in the sense that it compares the asymmetry along both diagonals of a bivariate copula. Although advanced modeling with new asymmetry function is intriguing, it is beyond the scope of this study.

It seems that the explanation about the relation between asymmetry and hydrograph was not clear, leading to questions and comments from both referees. An effort has been made to substitute more comprehensive figures and text for a better explanation of the material.

The main modifications and improvements in the manuscript are

- Expansion of asymmetry definition from expectation notation to integration notation
- Further comprehensive illustration for the relation between hydrograph and asymmetry.
- In Figure 6 (Asymmetry and Catchment), x-coordinate is log scaled.
- In Section 3.1 (deseasonalization of data) redundant equations and explanations are deleted
- The mistakes in the equations and English have been corrected

Point-to-Point Response to Anonymous Referee #1 – *referee comments in italics*

Nice study that appears to be the first dealing with select asymmetrical properties and interpretations of copula models in a context of daily streamflow statistics for which asymmetry is known to exists. The asymmetry is related to the generalized hydrograph shape. Much of the authoritative text literature (e.g. Nelsen, 2006; Joe, 2014; Durante and Sempi, 2015) do not comprehensively tackle the asymmetry problem of a copula.

Nelsen (2006) is basically devoid of "skewness" (asymmetry) computations— understandably so. Joe (2014, p.66) discusses skewness of a copula and the orientation of the skewness appears conceptually similar (not necessarily numerically equal) to the A1 definition (primary diagonal) of eq. 9. A unique contribution by the paper is the A2 definition (secondary diagonal) of eq. 10. This reviewer has seen many bivariate plots of hydrologic phenomena (such as daily streamflow) and notes the secondary diagonal asymmetry. This asymmetry means a fair share of copula families seen in the literature arguably are in applicable because they have symmetry on the secondary diagonal. This reviewer would like A1 and A2 to also be expressed in direct terms of integration of the copula formula or its density. For example, a Joe (2014) definition for the primary diagonal is: \int\int_{[0,1]} (v-u)C(u,v) du dv from which a secondary asymmetry definition (not identified by Joe) can result \int\int_{[0,1]} (v+u-1)C(u,v) du dv - (1/2) Can the authors of the paper expand the definitions of A1 and A2 beyond the "expectation" notation?

Author's Response (Definition of Asymmetry1 and Asymmetry2)**:**

The "expectation notation" was conventionally used in this research, so there is no reason not to express the equation in integration form beyond the expectation notation as follows:

$$\begin{aligned} A_1(k) &= E\left[(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) + (U_{t+k} - 0.5)\right)\right] \\ &= \int_0^1 \int_0^1 (u_t - 0.5)(u_{t+k} - 0.5)(u_t + u_{t+k} - 1)\, c(u_t, u_{t+k})\, du_t du_{t+k} \end{aligned} \tag{9}$$

$$\begin{aligned} A_2(k) &= E\left[-(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) - (U_{t+k} - 0.5)\right)\right] \\ &= \int_0^1 \int_0^1 -(u_t - 0.5)(u_{t+k} - 0.5)(u_t - u_{t+k})\, c(u_t, u_{t+k})\, du_t du_{t+k} \end{aligned} \tag{10}$$

It seems sensible, because the terms such as $(u_t + u_{t+k} - 1)$ and $(u_t - u_{t+k})$ appear in this notation, which is comparable to the asymmetry definition by Joe (2014) and anonymous referee #1 :

Asymmetry1 : $\int \int (v - u) c(u, v) du dv$

Asymmetry2 : $\int \int (v + u - 1) c(u, v) du dv - 0.5$

In general, there seem other ways to define and apply the asymmetry. L-comoments (L-coskew) suggested by anonymous referee#1 can be one of them.

*Have the authors considered the **L-comoments** (Serfling and Xiao, 2007)? But more importantly, the very recent "break through" of L-comoment (bivariate L-moment, bivariate L-skew) definition (Brahimi et al. [2015]) directly in terms of a copula. L-coskew (bivariate skew) \delta^{[12]}_{3;\mathbf{C}} = \int\int_{[0,1]} (60v^2 - 60v + 12) * C(u,v) du dv - (1/2) \delta^{[21]}_{3;\mathbf{C}} = \int\int_{[0,1]} (60u^2 - 60u + 12) * C(u,v) du dv - (1/2)*

Author's Response (Suggestion for using L-comoments and L-coskew)**:**

L-comoments or L-coskew (Serfling and Xiao, 2007) were not really known to our group. So, we quickly checked the theory in the papers and summarize the main features below.

- L-moments are defined as linear combinations of order statistics.
- The advantage of using order statistics is that, it is not necessary to assume the existence of second order statistics or the statistics of higher order. This can be suitable for heavy-tail distributions.
- L-comoments or L-coskew are extensions of L-moments to the multivariate case.

These functions are theoretically interesting and can be regarded as an advanced definition of copula asymmetry.

The authors generally think that the use of such sophisticated functions enable us to tackle with problems of hydrology and earth system sciences in different ways. For example, the application of such functions for asymmetry1 might be interesting, although their application is beyond the main focus in this research.

These integrals can readily by numerically approximated or integrated by Monte Carlo methods enhanced by low-discrepancy sequence methods. Some final thoughts. A similar study as this does not really appear to have been done. Whereas, this review generally thinks that the physical interpretations of the watershed

U_t

and climatology are mechanism producing asymmetry, care is suggested to avoid over interpretations until a great suite of similar studies can be conducted. For example, 9164, line 24 "... A1 ... asymmetry can be related to temporal distribution of precipitation" (what scale of time?) or "... A2 ... more related to catchment and rainfall characteristics ... or ... interseasonal characteristics of climate".

These are deeply important properties and suggest that copulas are an avenue forward in watershed/climate stochastic modeling. Intuition seems to be correct, but expansion of the authors' thoughts and statements to interpretation of A1 and A2 or other skewness measures or bivariate moment (L-moment) would be informative.

Also, given that we know typical storm water hydrographs are asymmetrical and are inherently formed by a cascade of processes (e.g. water parcel survival from input to output — Markov of sorts), is there a connection between A1/A2 and storm water hydrographs (e.g. unit hydrographs)?

Author's Response (Relation between asymmetry and hydrograph):

We note that anonymous referee#1 gave some positive comments but also the warning about the necessity of careful thought and expressions for asymmetry. It seems important, because this can influence the decent usage of copulas and its fruitful results in the future.

In retrospect, our explanation about the relation between hydrograph and asymmetry seemed to be not good enough in this manuscript, which raised several questions or remarks.

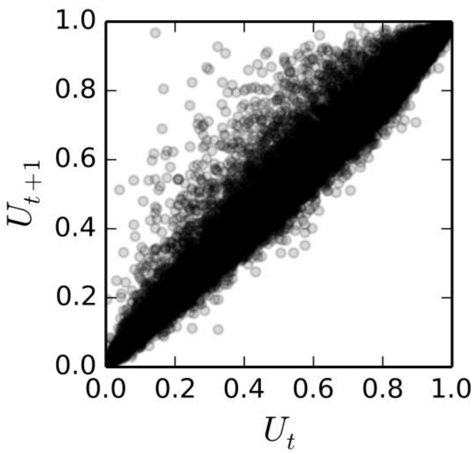
from interactive comment of anonymous referee #1:

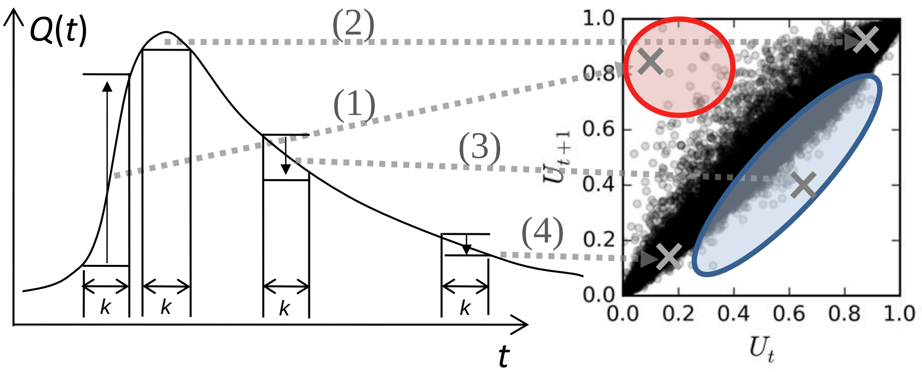
- *... is there a connection between A1/A2 and storm water hydrographs (e.g. unit hydrographs)?*
- *A1 ... asymmetry can be related to temporal distribution of precipitation" (what scale of time?)*
- *expansion of the authors' thoughts and statements to interpretation of A1 and A2*

from interactive comment of anonymous referee #2:

- *Section 3. I would give more practical explanation about Copula asymmetry. It is not fully clear.*

In order to answer these questions, Figure3 has been modified as shown below





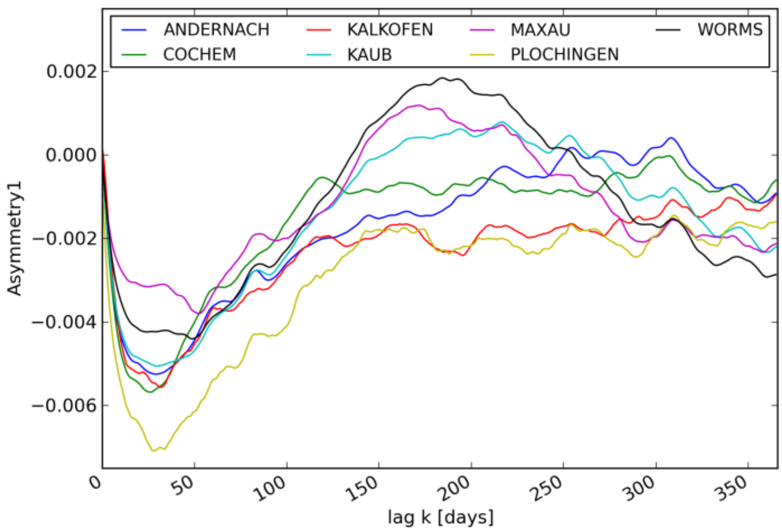
Previous version of figure 3 (top) and New version of figure 3 (bottom)
Sketch of the transformation of the values from sample hydrograph (left) to the points on scatterplot of ranks (right): empirical copula calculated from two values separated by time lag $k=1$ [days] in a discharge time series of Andernach where $rank\,correlation = 0.9870$, $A_1(k=1) = -0.0002398$ and $A_2(k=1) = -0.00011037$. The possible combinations of high and low values, which has large impacts on asymmetry, are numbered: (1) low to high, (2) high to high, (3) high to low, (4) low to low. Negative contribution to asymmetry2 is drawn with red circle and positive contribution with blue oval.

This figure illustrates where each pair of values on hydrograph can be plotted on empirical copula. For example, it can be seen there are more points in upper left corner, which demonstrates how the shape of hydrograph can be related to the asymmetry of these empirical copulas. This figure and additional explanation will replace the current figure3 and explanation.

A1 ... asymmetry can be related to temporal distribution of precipitation" (what scale of time?)

Author's Response (Further explanation about asymmetry1):

The asymmetry1 would change depending on the lag k [days] similar to the case of asymmetry 2 (please see the figure below) but based on different reasons. The answer to the question is that the asymmetry1 is significantly small (-0.002 ~ -0.006) for small time scale (lag k = 1~100 [days]) . This is important because this asymmetry can be potentially related to the precipitation of the region. Some basic investigation for asymmetry1 was conducted in the original study (Sugimoto, T., 2014. Copula based stochastic analysis of discharge time series. PhD Thesis. Nr. 232. University of Stuttgart, Germany). It is copied below, but finally not included for the organization of this paper.



In this sense, no concrete conclusion or over interpretation should be given, but it still may make sense to mention the possible mechanism behind it so that it can be the hint for the possible future research works.

or other skewness measures or bivariate moment (L-moment) would be informative.

Hopefully, the new Figure3 and some additional explanation about asymmetry1 will carry the message, so the following sentences were slightly corrected:

(original text at 9164 Line 25 in discussion paper)
This asymmetry can be related to the intrinsic temporal distribution of precipitation.

(improved text)
This implies that the intrinsic temporal distribution of precipitation can be investigated based on this asymmetry, possibly with advanced asymmetry functions such as bivariate moments based on L-moments (Brahimi et al., 2015).

(original text at 9165 Line 2 in discussion paper)
This asymmetry can be related to the characteristics of the runoff and catchment.

(improved text)
This asymmetry can be related to the shape of the hydrograph, therefore the characteristics of the runoff and catchment.

References

Brahimi, Chebana, and Necir (2015) Copula representation of bivariate L-moments: A new estimation method for multi-parameter two-dimensional copula models, Statistics, 49(3)[497–521].

Durante and Sempi (2015) Principles of copula theory, CRC Press.
Nelsen, RB (2006) An introduction to copulas, Springer.
Joe, H. (2014) Dependence modeling with copulas, CRC Press.
Serfling and Xiao (2007) A contribution to multivariate L-moments: L-comoment matrices, Journal of Multivariate Analysis, 98[1765-1781].

9160, Lines 25 and 30: There is confusion in the technical writing aspect of mentioning ARIMA and then evidently switching conceptually to "Fourier analysis". This review suggests that a proof reading would resolve potential confusion.

Author's Response (technical proof reading about ARIMA and Fourier Analysis)

We checked again the literature (Huang et al., 1998) . For the Fourier Analysis, the system must be periodic or stationary and EMD methods have been developed to overcome the restriction. ARIMA is designed originally for stationary process, assuming the no change of the background system. In this sense ARIMA and Fourier analysis is related, but maybe the technical description was not clear, so the text at 9160 Line1 in discussion paper was improved.

Huang, N.E., Shen, Z., Long, S.R., Wu, M.C., Shih, H.H., Zheng, Q., Yen, N.-C., Tung, C.C., Liu, H.H., 1998. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proc. R. Soc. A Math. Phys. Eng. Sci. doi:10.1098/rspa.1998.0193

9162, Line 9: "this statistics" —> "these statistics"

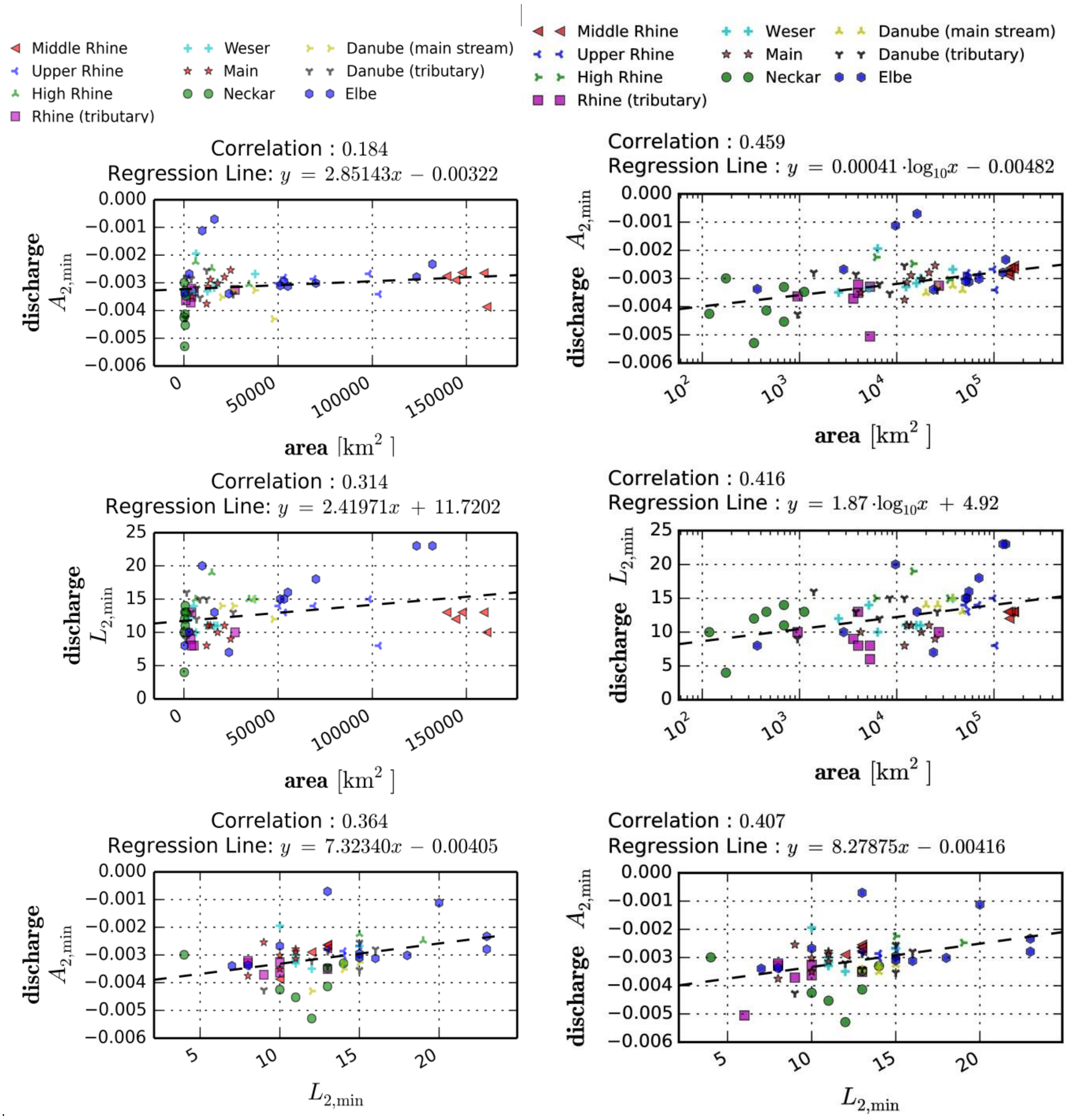
Author's Response:
Thank you very much for pointing out the mistakes. This will be corrected in the revised version of manuscript.

9168, Line 14: missing minus sign in definition of A2(k,t)?

Author's Response:
Yes, this is again a mistake. We thank you for pointing out this error.

Figure 6: Shouldn't the horizontal axis be cast in logarithms?

Author's Response:
For the figure 6 (figure6 old), the same result was plotted on the graph with log-scaled x-coordinate (figure6 new). The correlation and regression line were also calculated based on the log-scaled catchment area. ($x' = \log_{10} x$). Now, it is more clear that there are linear relationships between area and asymmetry measures (A2min, L2min). Thank you very much for pointing this out.



Old version of Figure6 (left) and new version of Figure6 (right):
Relation between Asymmetry and catchment characteristics: minimum of asymmetry2 of discharge and catchment area (top), lag at minimum of asymmetry2 of discharge and catchment area (middle), minimum of asymmetry2 of discharge and lag at minimum of asymmetry2 of discharge (bottom)

Point-to-Point Response to Anonymous Referee #2 – *referee comments in italics*

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The manuscript provides an interesting set of tools based on copula function for investigating discharge time series dynamic.

The topic is particularly interesting since it is in line with the recent and innovative use of copula. Up today copula was applied mainly to perform multivariate frequency analysis while it is potentially useful for detecting and interpreting observed data. This paper is a clear example. The manuscript is easy and pleasant to read, <u>however it includes many analyses and methods that</u>, maybe, it could be worth to split it in two papers.

In the following minor and major concerns are listed.

1) In the abstract API acronym should be defined.

Author's Response:

Thank you for pointing out this. It will be corrected.

2) In the Introduction line 20-22. If the aim is to investigate on the catchment status and the anthropogenic impact, I do not think it is obvious that the solution is to analyze the discharge time series, the reader could expect to see the analysis of the crosscorrelation between rainfall and runoff time series.

Author's Response:

we agree that cross correlation between rainfall and runoff can be the first choice. There are several studies about them, but in our opinion, not enough to explain the causality. The corresponding expressions in abstract will be reconsidered.

3) Section 3. I would give more practical explanation about Copula asymmetry. It is not fully clear.

Author's Response:

Please see "Author's Response (Relation between asymmetry and hydrograph)" in the previous section in this document

4) Section 3.1 line 15. "and instead of "und" 5) Section 3.1 line 25. related "to" temporal distribution

Author's Response:

Thank you very much for pointing out the mistakes

6) Section 3.1 page 9165-9166. The de-seasonalization approach is well known (Grimaldi, S. Linear parametric models applied to daily hydrological series (2004) Journal of Hydrologic Engineering, 9 (5), pp. 383-391), maybe you can remove the equations in order to make easier the text.

Author's Response:

Thank you for pointing out this. Section3.1 , the several equation and redundant explanation were removed, instead reference to the study of Grimaldi (Grimaldi, 2004) was added.

7) Section 3.1 pag 9166. I am not surprised to have a residual periodicity since you have removed the annual one. Maybe a weekly periodicity could be still detected.

Author's Response:
Yes, the weekly periodicity might still exist. The important argument here is that the asymmetry remains after certain normalizing treatment of original. This asymmetry is now more reasonable to explain catchment characteristics, because the influence of annual cycle is eliminated. (Not that asymmetry itself is different from month to month. In this sense, the seasonality cannot be fully removed).

8) Section 4.1.In general this section is very interesting. I would suggest to better explain if the distance D is based on empirical copula and why this is important; and ***the uncertainty*** *of the estimated distance. Maybe these notions are already included in the text but it should be better clarified.*

Author's Response:

I would suggest to better explain if the distance D is based on empirical copula and *why this is important*

Yes, it is based on empirical copula. This study started with the analyzing the asymmetry of empirical copula. After that distance D was examined as an extension to it. It is not necessarily important to use empirical copula, but seems sensible to use it for the purpose of this study.

and the uncertainty of the estimated distance.

There seem two aspects about uncertainty:

1. Uncertainty of Model

From the definition, copula variance can be related to the model uncertainty; how much the natural system is varying. This can be related to the potential calibration difficulty of hydrological model or any parameter estimation of global circulation model.

(the following text is added at 9179 line 15 in discussion paper, original text)
This asymmetry can be related to the intrinsic temporal distribution of precipitation.

The copula based measures introduced in this study can be related to the potential model uncertainty, that is, how much the natural system is varying.

2. Uncertainty of the statistic

Estimating uncertainty of copula distances might be interesting, but seems complicated. It is possible to calculate copula distances for 77 discharge data from different gauging stations, but these data from the same river or same regions should be interrelated and not independent. Thus, it seems not to be simple to estimate the uncertainty of copula distances, therefore this matter is not really discussed in this paper. Copula distances are just calculated for the independent stationary Gaussian processes in order to provide some impression.

These arguments are not clear in the manuscript, so some correction has been done so that these are clearer.

Investigation of hydrological time series using copulas for detecting catchment characteristics and anthropogenic impacts

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Abstract. Global climate change can have impacts on characteristics of rainfall-runoff events and subsequently on the hydrological regime. Meanwhile, the catchment itself changes due to anthropogenic influences. However, it is not easy to prove the link between the hydrology and the forcings. In this context, it might~~can~~ be meaningful to detect the temporal changes of catchments independent from climate change by investigating existing long term discharge records. For this purpose, a new stochastic system based on copulas for time series analysis is introduced. While widely used time series models are based on linear combinations of correlations assuming a Gaussian behavior of variables, a statistical tool like the copula has the advantage to scrutinize the dependence structure of the data in the uniform domain independent of the marginal.

Two measures in the copula domain are introduced herein:

1. Copula asymmetry is defined for copulas and calculated for discharges; this measure describes the non symmetric property of the dependence structure and differs from one catchment to another due to the intrinsic nature of both runoff and catchment.

2. Copula distance is defined as Cramér-von Mises type distance calculated between two copula densities of different time scales. This measure describes the variability and

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interdependency of dependence structures similar to variance and covariance, which can assist in identifying the catchment changes.

These measures are calculated for 100 years of daily discharges for the Rhine rivers and tributaries. Comparing the results of copula asymmetry and copula distance between an Antecedent Precipitation Index (API) and simulated discharge time series by a hydrological model we ~~can~~ show the interesting signals of systematic modifications along the Rhine rivers in the last 30 years.

Keywords : Catchment discharge characteristics, Copula stochastic analysis, API, Model uncertainty

1. Introduction

In order to understand the water cycle behavior of a region, it is important to determine its characteristics, but this is difficult to achieve due to the diversity of the system response at different time and space scales. In particular, temporal variability makes parameter estimation difficult and the assessment of model uncertainty essential. As a part of the endeavor to understand~~grasp~~ the hydrological system, the objective of this research, assessing the anthropogenic impacts on the catchment characteristic independent of the climate change, is therefore important, yet hard to accomplish.

The first possible approach is to statistically test the existence or change of trend in hydrological time series which can be related to climate changes or anthropogenic impacts. Mann-Kendall's Test was performed to confirm the existence of a trend in the annual discharge, precipitation and sediment loads, then ~~and discussed~~ the human intervention and climate impacts based on the available information of the catchments were discussed (Wu et al., 2012). Pettitt's Method (Pettitt, 1979) can be used to detect the time point of trend alternation and analyze the impacts based on a double mass curve (Gao et al., 2012) or a hydrological model

(Karlsson et al., 2014). These non-parametric methods for detecting the signal seem, however, not capable enough of explaining when and how much the system had changed, thus making it still difficult to relate the change due to~~to~~ human activities.

On the other hand, runoff events are initiated by precipitation then modified by the state and physical features of the catchment. This implies that the integrated information of catchment status might be retrieved by analyzing the discharge time series itself. Focusing on this property, the attempts can be made for capturing the temporal dependence structure of runoff by time series models. The classical time series model, autoregressive integrated moving average (ARIMA), is designed to describe a stationary stochastic process based on the temporal correlation structure of Gaussian random variables (Box and Jenkins, 1976). However, the stationarity of the data is not guaranteed in reality, thus a number of alternative approaches have been suggested. While the application of Fourier analysis is basically for stationary process, the analysis using e~~E~~mpirical mode decomposition (Huang et al., 1998) ~~is~~ overcomes the restriction of stationarity ~~a method designed to overcome the drawbacks of Fourier analysis~~ by allowing the frequency and local variance of a time series to vary within a component and to separate the signals adaptively by scale. Autoregressive Conditional Heteroskedasticity (ARCH) models l~~o~~ose the assumption of stationarity to a certain extent so that variance is not constant, however models the variance in a similar way to ARIMA. Although ~~the~~ inventions and efforts to overcome the limitation of stationarity have been~~are~~ made, it seems still inadequate to model dynamic changes of hydrological processes with these time series models.

Alternatively there is a statistical concept, the copula~~s~~, which has advantages to model the multivariate dependence independently from marginals and recently adopted in the field of hydrology. A Copula (Sklar, 1959) is a multivariate probability distribution designed to flexibly model dependence structure in the uniform (quantile) domain. The use of copulas in hydrology can be found for the assessment of extreme events by considering flooding as a joint behavior of peak and volume (De Michele and Salvadori, 2003). Copulas have been applied to describe the spatio-temporal uncertainty of precipitation (Bárdossy and

Pegram, 2009) or the inhomogeneity of groundwater parameters (Bárdossy and Li, 2008). Asymmetry of dependence in a time series can be tested in the framework of a finite state Markov chain's transition probability matrix (Sharifdoost et al., 2009). Dissimilarity measures can be defined by means of a copula modelling the correlation structure of pairs of discharge time series in order to identify the similarity of catchments with the purpose of transferring catchment properties from one to the other (Samaniego et al., 2010). We aim at utilizing copulas as an alternative to classical time series models and an efficient tool for time series analysis to overcome these hydrological challenges.

The main interest of this study is to~~ precisely~~ assess the human intervention and climate change impacts on hydrological regime for the strategy of future development in the region. For achieving this goal, 7 daily discharge gauging stations in South-West Germany (Figure 1), which have 100 years daily discharge records, were chosen and extensively analyzed. The gauging stations Andernach, Kaub, Worms and Maxau are located in the main stream of the Rhine, while Kalkofen, Cochem and Plochingen are located on tributaries. For further analysis, daily precipitation and temperature records in the Baden-Württemberg state of Germany for the last 50 years were obtained from the German Weather Service. Also, 77 discharge records obtained from the Global Runoff Date Centre in Germany were utilized.

The following are the novel~~What follows is the new~~ aspects introduced in this study: (1) The catchment characteristics are~~ is~~ defined based on copulas and estimated from discharge data. Also the changes of catchment characteristics are investigated by tracing the temporal change of these~~is~~ statistics. (2) A method to model systematic changes of dependence structure with the help of copulas is suggested, then its variability and interrelationship with~~of~~ the time series are examined. (3) Anthropogenic impacts are assessed by the discharge - precipitation relation using API and a hydrological model with copula based measures.

This article is divided into five sections. After the introduction, the basic methodology for applying copulas to discharge time series is introduced in the second section. Thirdly, the measures of asymmetry in copulas are defined and estimated for the discharges of the river Rhine and other catchments. The determination of the temporal change of the asymmetry of the copulas is treated in the third section as well. In the fourth section two topics are treated: (i) the analysis based on copula distances for the observed discharges and (ii) the comparison of observed discharge with API (Antecedent Precipitation Index) time series and simulated discharge time series with a hydrological model. The conclusion is given in the fifth section.

2. Methodology

In this section, the application of copulas to time series is articulated after a brief introduction~~ of copulas~~. The very basics about copulas are presented here ;~~and~~ further information can be obtained from (Joe, 1997) or (Nelsen, 2006).

2.1 Basic Methodology

In probability theory and statistics, a copula is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform.

$$C:[0,1]^n \to [0,1] \tag{1}$$

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$$C\left(\mathbf{u}^{(i)}\right) = u_i \qquad if\ u^{(i)} = (1,\ldots,1,u_i,1,\ldots 1) \tag{2}$$

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Any multivariate distribution can be described by a copula and its marginal distributions as was proven by Sklar’s theorem (Sklar, 1959):

$$F(\mathbf{x}) = C\left(F_{X_1}(x_1),\ldots,F_{X_n}(x_n)\right) \tag{3}$$

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where $F_{\mathbf{X}_i}(x_i)$ represents the i-th marginal distribution of a multivariate random variable $\mathbf{X}$. The copula density can be derived by taking partial derivatives of the copula:

$$c(u_1,\ldots,u_n) = \frac{\partial^n C(u_1,\ldots,u_n)}{\partial u_1 \ldots \partial u_n} \tag{4}$$

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The advantage of using copulas is that the marginal is detached from the multivariate distribution and the dependence structure can be examined in the uniform compact domain for different types of data.

2.2 Basic Hypothesis of Temporal Copulas

For the application of copulas to time series analysis, a stochastic system should be presumed to be similar to the case of spatial copulas (Bárdossy and Li, 2008): the random variable at time t is described as $Z(t)$ and in general there may exist non-Gaussian dependency among the elements of $Z(t)$. Then stationarity is defined for each subset of times $t_1,\ldots,t_n \subset N$ and time lag k such that $\{t_1+k\ldots,t_n+k\} \subset N$ and for each set of possible values $z_1,\ldots,z_n$:

$$\begin{aligned} &P\left(Z(t_1)<z_1,\ldots,Z(t_n)<z_n\right)= \\ &P\left(Z(t_1+k)<z_1,\ldots,Z(t_n+k)<z_n\right) \end{aligned} \tag{5}$$

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For the given random function $Z(t)$, a set $S(k)$ containing pairs of ranked values is defined as a function of time lag k as follows:

$$S(k)=\left\{\left(F_Z\left(z(t)\right)\right),\left(F_Z\left(z(t+k)\right)\right)\right\} \tag{6}$$

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Thus, a 2-dimentional autocopula for stochastic time series is a function of time lag k for the set $S(k)$ similar to the case of ~~a~~spatial copula (Bárdossy and Li, 2008):

$$\mathbf{C}_t\left(k,u_1,u_2\right)=P\left[F_z\left(Z(t)\right)<u_1,F_z\left(Z(t+k)\right)<u_2\right] \tag{7}$$

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where $(u_1,u_2)\in S(k)$. Thus, a 2-dimentional empirical copula density can be constructed based on conditional empirical frequencies on a regular $g\times g$ grid and kernel density smoothing (Bárdossy, 2006):

$$\begin{aligned} &c^*\left(\frac{2i-1}{2g},\frac{2j-1}{2g}\right)=\frac{g^2}{\left|S(k)\right|} \\ &\cdot\left|\left\{(u_1,u_2)\in S(k);\frac{i-1}{g}<u_1<\frac{i}{g}\ and\ \frac{j-1}{g}<u_2<\frac{j}{g}\right\}\right| \end{aligned} \tag{8}$$

Field Code Changed

where $\left|S(k)\right|$ denotes the cardinality (the number of elements in a set) of set $S(k)$.

3. Copula Asymmetry in Discharge Time Series

High and low values might have different dependences in general. Measuring the asymmetry of copulas could reveal substantial aspects of time series data, which are not illuminated in the Gaussian approach. Statistics defined on copula shape and calculated from observed discharge time series we believe to be a new idea. Asymmetry functions are defined on 2-dimensional copulas as a function of time lag k (Li, 2010):

Asymmetry 1 and Asymmetry2 are ~~is~~ defined as:

$$A_1(k) = E\left[(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) + (U_{t+k} - 0.5)\right)\right] = \int_0^1\int_0^1 (u_t - 0.5)(u_{t+k} - 0.5)(u_t + u_{t+k} - 1)\, c(u_t, u_{t+k})\, du_t du_{t+k} \quad (9)$$

$$A_2(k) = E\left[-(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) - (U_{t+k} - 0.5)\right)\right] = \int_0^1\int_0^1 -(u_t - 0.5)(u_{t+k} - 0.5)(u_t - u_{t+k})\, c(u_t, u_{t+k})\, du_t du_{t+k} \quad (10)$$

$$\cancel{A_1(k) = E\left[(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) + (U_{t+k} - 0.5)\right)\right]}$$

~~(9)~~

~~Asymmetry 2 is defined as:~~

$$\cancel{A_2(k) = E\left[-(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) - (U_{t+k} - 0.5)\right)\right]}$$

~~(10)~~

where $u_t = F_Z(z(t))$, $u_{t+k} = F_Z(z(t+k))$. Figure 2 shows an idealization of the ~~two~~ asymmetries between a pair of variables $U(t)$ and $U(t+k)$, showing that the tails of the distributions have a large impact on each type of asymmetry. The measure of asymmetry ~~(i)~~ compares the dependency between low and high values and ~~(ii)~~ quantifies how much it is not symmetric. For example, in a 2-dimensional copula, $A_1(k)$ is positive if the probability density is higher in the upper right corner than in the lower left corner. On the contrary ~~the other hand~~, $A_1(k)$ is negative if the probability density is higher in the lower left corner ~~than in the upper right~~. $A_2(k)$ is the asymmetry for the other diagonal of a 2-dimensional copula.

Figure 3 shows the scatterplot of ranked values of a discharge time series with time lag k = 1 as a sample of an empirical autocopula and its relation with storm hydrographs. This figure displays (i) where each pair of values on a hydrograph can be plotted on an empirical copula, demonstrating that (ii) the dependence structure is not symmetric especially for $A_2(k)$.

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This illustration provides the insight that asymmetry can be related to the shape of a unit hydrograph as well as the notion that asymmetry might be used for advanced modeling of hydrological time series.

~~Figure 3 shows the scatterplot of ranked values of a discharge time series with time lag $k = 1$ as a sample of an empirical autocopula, demonstrating the structure is not symmetric especially for $A_2(k)$.~~

3.1 Asymmetry and catchment characteristics

Asymmetries can be considered as statistics calculated from the observed discharge time series and ~~leads to have~~ an important assumption can be made: ‘assymetry2 is related to catchment characteristics’. This idea will be ~~intensively~~ discussed ~~u~~and demonstrated in this section. Figure 5 (upper left) shows parts of the hydrographs of 7 gauging stations in southwest Germany.

First, an important ~~and obvious~~ natural property of discharge seen in this figure is that the durations of high flow and low flow periods are~~is~~ not symmetric: Flood events, which are initiated by rainfall or snowmelt, do not continue for a long time because the duration of runoff to rivers is comparatively short. On the other hand, discharge keeps decreasing and stays low for no rain periods. This means that, if two consecutive values in a time series are chosen for small time lag k, these two values are likely to be less correlated for high values but more correlated for low values, which leads to negative value of $A_1(k)$. This implies that the intrinsic temporal distribution of precipitation can be investigated based on this asymmetry, possibly with advanced asymmetry functions such as bivariate moments based on L-moments (Brahimi et al., 2015).

~~This asymmetry can be related to the intrinsic temporal distribution of precipitation.~~

Second, the rates of increase and decrease of discharge are~~is~~ not symmetrical: Soon after the rainfall, the river flow rises sharply. Once the rain stops and peak discharge is observed, then the water level starts to decrease, typically more slowly on the recession than the rising limb of the hydrograph, which leads to negative values of $A_2(k)$ for small time lags k. This asymmetry can be related to the shape of the

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The change of $A_2(k)$ with time lag k [days] is now discussed. The point is that these statistics for small time lags k can be more related to the catchment and rainfall characteristics of the region, while asymmetry for larger time lags k can capture the inter-seasonal characteristic of the climate in the region.

In order to reduce such seasonal impacts on the analysis of hydrological time series, deseasonalization measures can be applied, for example, for daily stream flow (Grimaldi, 2004). Adopting this method, all the time seires are normalized in this study.

~~In order to reduce this seasonal impact, normalization was adopted for the time series similar to z-score in the following way. F~~First, ~~the annual cycle of~~ the mean μ_i on the i-th calender day is calculated as the expectation of the random variable X_i on the i-th calender day. Then, the annual cycle of the mean μ_i^* is calculated as a smoothed version of μ_i by linearly weighting the neighboring values along i and summing them up. The smoothed annual cycle of standard deviations σ_i^* (Figure 4 left) can be obtained in the same way. Then the normalized time series is defined by dividing the original time seires Z(t) by σ_i^* after subtracting μ_i^* as follows

$$Z_{norm}(t) = \frac{Z(t) - \mu^*_t}{\sigma_t^*} \quad (11)$$

~~and smoothed by linear weighting~~

$$\mu_{t|365} = E\left[X_{t|365}\right]$$
$$\mu^*_{t|365} = \frac{1}{2N}\sum_{i=0}^{N/2}\left(\frac{1}{2} - \frac{i}{N}\right)\left(\mu_{t+i|365} + \mu_{t-i|365}\right) \quad \cancel{(11)}$$

where $t\,|\,365$ is $t(\mathrm{mod}\,365)$ and represents calendar day at time t [day]. ~~X_i denotes the random variable of discharge, μ_i denotes mean and μ_i^* denotes mean after smoothing on calendar day i respectively. After~~

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~~subtraction of the annual mean from the original time series $Z(t)$, the annual cycle of standard deviation is defined.~~

$$\sigma_{t|365} = E\left[\sqrt{\left(X_{t|365} - \mu_{t|365}\right)^2}\right]$$

$$\sigma^*_{t|365} = \frac{1}{2N}\sum_{i=0}^{N/2}\left(\frac{1}{2} - \frac{i}{N}\right)\left(\sigma_{t+i|365} + \sigma_{t-i|365}\right) \qquad (12)$$

~~Figure 4 shows the annual cycles after smoothing described by equations (11) and (12). By subtracting the annual mean cycle and dividing by annual standard deviation cycle, the normalized time series is defined.~~

$$Z_{norm}(t) = \left(\frac{Z(t) - \mu^*_{t|365}}{\sqrt{\sigma^*_{t|365}}}\right) \qquad (13)$$

Figure 5 (upper right) shows parts of normalized discharge time series from the 7 gauging stations. It should be noted that the process still appears to be non-Gaussian after this transformation and the seasonality for small time lags k might not have been fully eliminated. Figure 5 (bottom left and bottom right) shows the variation of asymmetry functions for 7 discharge time series corresponding to time lag k~~-~~, similar to ~~the~~ correlograms, in addition to the confidence interval of Gaussian process.

The confidence intervals in the figures are gained by calculating $A_2(k)$ for 100 realizations of stationary Gaussian process which are fitted to the observed discharge of Andernach. The result shows that the process is clearly different from Gaussian and the influence of asymmetry is significantly large.

It can be seen that the variation of $A_2(k)$ of discharge without normalization (Figure 5 bottom left) has a larger impact of seasonality for bigger k ($k > 40$), while its impacts are mitigated after the normalization (Figure 5 bottom right). Furthermore, as a consequence of normalization, a sharp drop down of $A_2(k)$ for small time lags k emerged which might be regarded as a catchment indicator. Therefore, the selected/critical properties for small time lags k is formulated by (i) taking the minimum value of $A_2(k)$ for the time lag $k < 50$ and (ii) the lag k at the minimum of asymmetry2:

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$$A_{2,\min} = \min_{k<50} A_2(k) \tag{12}$$

$$L_{2,\min} = \min_{0<k<50} \left\{ k; A_2(k) = A_{2,\min} \right\} \tag{13}$$

The question is whether they are really related to catchment characteristics. Now, these statistics estimated for 77 discharge data recorded at the gauging stations in Germany are compared with the catchment area as one of the simplest possible indicators of the catchment as shown in Figure 6.~~:~~ $A_{2,\min}$ - *area* (Figure 6 top) and ~~shows a more clear linear relation than~~ $L_{2,\min}$ - area (Figure 6 middle) ~~are~~ both show~~ing~~ a linear relationship with~~to~~ the log-scaled x-axis of catchment area, with positive correlation. There seems also to be a ~~the~~ linear relation between~~while the dispersions~~ $A_{2,\min}$ and $L_{2,\min}$ ~~-~~as a consequence~~s~~ of the above relationships in. ~~for the smaller catchments are big for both cases. The correlation between $A_{2,\min}$ and $L_{2,\min}$ (~~Figure 6 bottom~~) is slightly positive~~.

This demonstrates that the information extracted from discharge is related to the basic information of its catchment to a certain extent. Since the principal objective is to assess anthropogenic impacts, the idea introduced now is to use this measure for evaluating the catchment change by calculating chronological changes of $A_{2,\min}$.

3.2 Time Series Analysis with Asymmetry

Temporal change of asymmetry $A_2(k,t)$ is defined on the set representing a moving time window of size *w*.

$$S^*(k,t) = \left\{ (F_Z(z(a))), (F_Z(z(a+k))); t - \frac{w}{2} < a < t + \frac{w}{2} \right\} \tag{14}$$

$$\begin{aligned} A_2(k,t) &= E\left[-(U_t - 0.5)(U_{t+k} - 0.5)\left((U_t - 0.5) - (U_{t+k} - 0.5)\right)\right] \\ &= \int_0^1 \int_0^1 -(u_t - 0.5)(u_{t+k} - 0.5)(u_t - u_{t+k})c(u_t, u_{t+k})\,du_t\,du_{t+k} \end{aligned}$$

(15)

where $u_t \in U_t, u_{t+k} \in U_{t+k}$, $\left(u_t, u_{t+k}\right) \in S^*\left(k,t\right)$ .Then the minimum of asymmetry2 and lag k at the minimum of asymmetry2 at time t are given by

$$A_{2,\min}\left(t\right) = \min_{k<30} A_2\left(k,t\right) \tag{16}$$

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$$L_{2,\min}\left(t\right) = \min_{0<k<30}\left\{k; A_2\left(k,t\right) = A_{2,\min}\left(t\right)\right\} \tag{17}$$

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Figure 7 shows the temporal changes of $A_{2,\min}\left(t\right)$ with window size $w = 3000$ [days] for 7 gauging stations in southwest Germany in addition to the confidence interval calculated for 100 times independently generated Gaussian process.

The comparison of $A_{2,\min}\left(t\right)$ from observed discharges with $A_{2,\min}\left(t\right)$ from a Gaussian process exhibits (i) the influence of asymmetry in discharge is significantly large as ~~it~~ was seen in Figure 5, ~~.~~ (ii) The fluctuations of $A_{2,\min}\left(t\right)$ of 7 observed discharge time series appear to be bigger than the one calculated for a realization of a Gaussian process and ~~.~~ (iii) $A_{2,\min}\left(t\right)$ of these 7 discharge records shows a similar trend: there are big drop-downs around 1945 and after 1980 for all the discharges.

However, it cannot be ascertained whether this is caused by the simultaneous change of the catchments, the long term meteorological behavior in the region or just randomness in the stationary process. To overcome this, temporal behavior of discharge and temperature were first checked by calculating the mean, the standard deviation and the minimum in a time window centered on~~at~~ time t defined by

$$\begin{aligned} Mean\left(t\right) &= \frac{1}{w}\int_{t-w/2}^{t+w/2} z\left(a\right)da \\ Std\left(t\right) &= \sqrt{Var\left(t\right)} = \frac{1}{w}\left(\int_{t-w/2}^{t+w/2}\left(z\left(a\right) - E\left[Z\left(t\right)\right]\right)^2 da\right)^{\frac{1}{2}} \\ Min\left(t\right) &= \min\left\{Z\left(a\right); t - \frac{w}{2} < a < t + \frac{w}{2}\right\} \end{aligned} \tag{18}$$

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where w is the size of time window. Figure 8 shows the moving average and moving standard deviation of discharge records with windows size $w = 3000$ [days], but it is hard to say whether the behavior around 1945 and after 1980 is unusual. Figure 9 shows mean and minimum of temperature in the time window of~~with~~ size 365 [days] which correspond to annual mean and minimum. Roughly speaking, there are certain cold periods around 1940, 1955 and 1985, which might influence the snow accumulation and melting in the region, but the relation with asymmetry2 is rather obscure.

What seems to be a useful outcome from the above exploratory analysis is that (i) the behavior of asymmetry2 is different from catchment to catchment showing a statistical relation with the catchment area and (ii) temporal behaviors of asymmetry2 of 7 discharges time series are dependent on each other, which implies the existence of a background mechanism common to the region.

4. Analysis of hydrological time series with Copula Distance

As an alternative to copula asymmetry, which emphasizes the behavior in~~on~~ the corners of copulas, copula distance is here suggested so that the characteristic behavior can be captured in the entire domain of the copula. Calculating this for each time step for different time series and comparing them hopefully exhibits the changes of dependence structure and therefore the catchment change.

4.1 Introduction of Copula Distance

The basic idea behind the copula distance is to apply the Cramér-von Mises type distance

$$D=\int_0^1\int_0^1\left(C^*\left(u_1,u_2\right)-C\left(u_1,u_2\right)\right)^2 du_1du_2 \qquad (19)$$

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~~,~~ which by design measures the goodness of fit between two distribution functions~~,~~ to two copulas. This type of distance was tested to measure the difference between empirical and theoretical copulas in the bootstrap framework for the evaluation of spatial dependence of ground water quality (Bárdossy, 2006). For the analysis of time series data, it still needs to be carefully thought out how (and which) copulas should be chosen.

4.1.1 Introduction of Copula Distance to single time series

In order to apply the concept of copula distance to time series, the adoption of two copulas in different time scales is considered. An empirical copula can be obtained from an entire time series which contains the averaged information of all the time points (*global copula*). Another empirical copula can be obtained for a certain time window of width *w* centered at time step t (*local copula*). In order to make the concept clear, two sets containing pairs of ranked values with different time scales are specified.

$$S_{global}(k) = \left\{ \left(F_Z\left(z(t)\right)\right), \left(F_Z\left(z(t+k)\right)\right); t_1 < t < t_n \right\} \tag{20}$$

$$S_{local}(k,t) = \left\{ \left(F_Z(z(a))\right), \left(F_Z(z(a+k))\right); t - \frac{w}{2} < a < t + \frac{w}{2} \right\} \tag{21}$$

$S_{local}(k,t)$ can be interpreted as a moving time window where the reference time t is set to the middle of the window of size *w*, while $S_{global}(k)$ represents a set of the entire time series. *Global copula* and *local copula* are the empirical autocopula densities defined on these sets based on Equation (8)~~(8)(8)~~, there denoted by $c^*_{global}(\mathbf{u})$ and $c^*_{local}(\mathbf{u}, t, w)$ respectively for the n-dimensional case. In this analysis, 3000 [days] for the time window *w* and a 3-dimensional copula separated with 1 day gap between each variable are employed. This means that

$$\mathbf{u} = (u_0, u_1, u_2) \tag{22}$$

where $u_0 = F_z\left(Z(t)\right), u_1 = F_z\left(Z(t+1)\right), u_2 = F_z\left(Z(t+2)\right)$, then the deviation of local copula from global copula is defined by

$$\Delta c(\mathbf{u}, t) = c^*_{local}(\mathbf{u}, t) - c^*_{global}(\mathbf{u}) \tag{23}$$

For the first approach, the comparison of dependence structures between entire and local time series is done for detecting unusual dependence structures. To this end, *copula distance type1* is defined by taking the copula distance between global and local copulas at each time step t

$$\begin{aligned} D_1(c,t) &= \int_0^1 \ldots \int_0^1 \left(c^*_{global}(\mathbf{u}) - c^*_{local}(\mathbf{u}, t)\right)^2 du_1 \ldots du_n \\ &= \int_0^1 \ldots \int_0^1 \Delta c(\mathbf{u}, t)^2 du_1 \ldots du_n \end{aligned} \tag{24}$$

Second, *copula distance type 2* is introduced for indicating the point at which the structure of copulas starts to change. For this method, the distance between two local copulas is calculated at two instants:~~from the 2 time intervals~~

$$D_2(c,t) = \int_0^1 \dots \int_0^1 \left(c^*_{local}\left(\mathbf{u}, t - \frac{w}{2}\right) - c^*_{local}\left(\mathbf{u}, t + \frac{w}{2}\right) \right)^2 du_1 \dots du_n \qquad (25)$$

Note that reference time is set to the middle of both time windows and shifted for *w/2* [days] from each other where the size of the time windows is *w*. Therefore, there is no overlapping part between the two time intervals of these two local copulas. For the comparison, the moving variance is introduced as follows:

$$\begin{aligned} E\left[Z(t)\right] &= \frac{1}{w}\int_{t-w/2}^{t+w/2} z(a)\,da \\ Var(t) &= \frac{1}{w}\int_{t-w/2}^{t+w/2} \left(z(a) - E\left[Z(t)\right]\right)^2 da \end{aligned} \qquad (26)$$

Figure 10 shows the result of $D_1(t), D_2(t)$ and $Var(t)$ in the moving time window for the normalized discharge time series between 1940 to 2000 at 4 gauging stations located in the main stream of the Rhine (Andernach, Maxau) and its two different tributaries (Cochem, Plochingen) in addition to the 90 % confidence intervals calculated for the Gaussian process fitted to the discharge data of Andernach.

First of all, the values of ~~these 2~~ $D_1(t)$ and $D_2(t)$ ~~measures~~ at Cochem and Plochingen are bigger and more fluctuating than in general. The reason could be that their catchments and discharges are smaller, thus more sensitive to changes. Second, it can be said that the dependence structure is not homogeneous over the time period, but the local copula clearly deviates from the global copula for certain time periods. For example, the value of $D_1(t)$ is remarkably big around 1947, 1982 and 2000 for all the 4 discharge records (indicated~~pointed~~ by white arrows). $D_2(t)$ is also big around 1977 for all the data. The~~is~~ signal of $D_2(t)$ implies that a simultaneous change of runoff behavior occurred in this region in~~at~~ 1977, which can be related to the high value of $D_1(t)$ at 1982. $Var(t)$ is also changing, but a~~the~~ direct relation with $D_1(t)$

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and $D_2(t)$ is hard to recognize. Also the confidence interval of the Gaussian process is clearly smaller than the observed one. This indicates the copula distances of the stationary process are small while the nature process is non-stationary and its dependence structure is more varying.

For copula distance type1, the global copula can be considered as an average state of the copula, while the local copula can be regarded as a realization of a possible state of a copula at time step *t*. This concept can be comparable to variance and leads to a new measure, *copula variance*, which is the summation of copula distances between global and local copula over the time.

$$Var_{cop}(c) = \frac{1}{t_n - t_1} \int_{t_1}^{t_n} D_1(c,t)\,dt \tag{27}$$

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Table 1~~Table 1Table 1~~ shows the variance and copula variance calculated for the 4 discharge data. The result demonstrates that copula variance of the time series can be higher, even if the conventional variance is lower for example in case of Maxau.

4.1.2 Copula Distance for two time series

In the previous section, copula variance was defined as a measure of the variability characteristic of the copula itself. Here, it is determined~~examined~~ whether covariance can be defined for two copula densities c_1 and c_2 from two time series as *copula distance type3*, which shows whether the variability characteristic of copulas is related to each other. The measure introduced is :

$$D_3(c_1, c_2, t) = \int_0^1 \ldots \int_0^1 \Delta c_1(\mathbf{u}, t)\, \Delta c_2(\mathbf{u}, t)\, du_1 \ldots du_n \tag{28}$$

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where

$$\begin{aligned} \Delta c_1(\mathbf{u}, t) &= c^*_{1,local}(\mathbf{u}, t) - c^*_{1,global}(\mathbf{u}) \\ \Delta c_2(\mathbf{u}, t) &= c^*_{2,local}(\mathbf{u}, t) - c^*_{2,global}(\mathbf{u}) \end{aligned} \tag{29}$$

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By its definition, the value of $D_3(t)$ can be related to $D_1(t)$ because $D_3(t)$ compares the deviation of local copulas from global copulas in a similar way to $D_1(t)$ in Equation (26). In order to reduce the influence of

$D_1(t)$ on $D_3(t)$, *copula distance type4* is introduced as a normalized measure bounded between -1 and 1 analogous to correlation.

$$D_4(c_1, c_2, t) = \frac{D_3(c_1, c_2, t)}{\sqrt{D_1(c_1, t)} \cdot \sqrt{D_1(c_2, t)}} \tag{30}$$

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where $|D_4(c_1, c_2, t)| \leq 1$. For comparison, covariance and correlation in a moving window are introduced for two random variables $Z_1(t)$ and $Z_2(t)$ as follows:

$$Cov(t) = \int_{t-w/2}^{t+w/2} \left(z_1(a) - E\left[Z_1(t)\right]\right)\left(z_2(a) - E\left[Z_2(t)\right]\right) da \tag{31}$$

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$$Corr(t) = \frac{Cov(t)}{\sqrt{Var(Z_1(t))} \cdot \sqrt{Var(Z_2(t))}} \tag{32}$$

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Figure 11 shows the copula distance between two time series $D_3(t)$ and $D_4(t)$ in addition to the covariance and correlation in a moving time window.

First, it can be said that the behavior of covariance and correlation in a moving window are different from $D_3(t)$ and $D_4(t)$. This implies these two copula based statistics exhibit different properties of the time series from ordinary statistics. Second, $D_3(t)$ shows high values around 1947, 1982 and 2000, which is similar~~ame~~ to the case of $D_1(t)$ in Figure 10. This indicates that unusual states of copulas in 4 discharge time series can be related to each other. Third, $D_4(t)$ is in general high except for the period around 1970 and 1990. This means, the temporal behavior of dependence structures for these 4 discharges are actually similar except for these periods even if $D_1(t)$ and $D_3(t)$ are small.

Copula covariance and copula correlation can be defined similar to copula variance in order to quantify the overall behavior of two time series.

$$Cov_{cop}(c_1, c_2) = \frac{1}{t_2 - t_1} \int_{t_2}^{t_1} D_3(t)\, dt \tag{33}$$

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$$Corr_{cop}(c_1, c_2) = \frac{Cov_{cop}(c_1, c_2)}{\sqrt{Var_{cop}(c_1)} \cdot \sqrt{Var_{cop}(c_2)}} \tag{34}$$

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where $\left|Corr_{cop}(c_1, c_2)\right| \leq 1$ and its derivation can be found in appendix A. In Table 2~~Table 2Table 2~~, these copula based statistics are compared with ordinary statistics. For example, Cochem and Plochingen are located remotely in different tributaries, thus covariance and correlation are lower than the others, but copula covariance and copula correlation are not the lowest.

The measures using copula distance are different from the conventional statistics. This behavior can be explained by the fact that the autocopula has more substantial information about temporal dependence structure than the autocorrelation. Using these measures might enable us to take advantage of a different way of seeing the dependence between time series.

What is new in the analysis of this section is that (i) measures based on copula distance show the different properties of time series in comparison to conventional statistics and (ii) there are significant signals of copula distances for certain time periods in common to all the discharge data.

4.2 Copula based Stochastic Analysis with API and a Hydrological Model

The difficulty of analyzing discharge time series in order to detect catchment change is that it is not clear whether the temporal change of stochastic information is caused by catchment change or merely by random behavior of precipitation. To gain an understanding of this process, we attempted to eliminate the influence of precipitation using, first, an Antecedent Precipitation Index (API) (~~antecedent precipitation index~~) for comparison with discharge , second, using a hydrological model with the parameter sets calibrated and fixed for the entire simulation time period.

4.2.1 Copula Distance Analysis with API

An API ~~(Antecedent Precipitation Index)~~ time series, which is generated from observed precipitation time series and behaves similarly to discharge, is used instead of precipitation.

$$API(t+1) = \alpha API(t) + P(t+1) \tag{35}$$

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where $P(t)$ is daily precipitation [mm/day], $API(t)$ is time series of API [mm/day] and α = *0.85* was chosen. The assumption for this method is that the API time series has the stochastic information purely originated from the precipitation, while observed discharge is ~~supposed to be~~ influenced by both catchment~~s~~ and precipitation~~s~~ characteristics. If the stochastic information derived from these two data sets is the same, this indicates that the stochastic turbulence is originating from precipitation; otherwise the change is from the catchment.

For this investigation, precipitation data was carefully chosen for 4 regions (northwest, northeast, southwest and central) of Baden-Württemberg (Germany) so that they have several almost continuous daily records between 1935 and 2005. Figure 12 shows the locations of measuring stations. The precipitation time series were aggregated into one for each region by taking their daily average, then 4 API time series were~~as~~ calculated in total by Equation (35)~~(35)(35)~~. Figure 13 shows the resulting~~ of~~ copula distances $D_1(t)$, $D_2(t)$ and moving average $Var(t)$ for API time series with the 90% confidence intervals of the Gaussian process. Figure 14 shows the result of copula distances $D_3(t)$, $D_4(t)$ and moving covariance and correlation for API time series.

What can be recognized first from~~in this~~ Figure 13 is that the magnitudes of $D_1(t)$ and $D_2(t)$ are smaller than the case of discharge. This is considered to be~~happen as~~ a result of aggregation of precipitation time series and adoption of API, but some signals can be still identified: $D_1(t)$ around 1947 and 2000 is high, but not as high~~much~~ for 1982. The signal of $D_2(t)$ which was detected around 1977 in Figure 11 does not seem to exist for API. This is~~can be~~ even more clear for $D_3(t)$ in Figure 14 in that there is no common change of the dependence structure around 1982 in API time series. This is interesting due to the following implications: (i) the noises of $D_1(t)$ in Figure 13~~Figure 13Figure 13~~ were reduced and signals in common were amplified (ii) the unusual state of the copula around 1982 is not caused ~~not~~ by precipitation, but could be caused by the catchment change.

For further verification, copula distance type3 and type4 between discharge and API time series were calculated as shown in Figure 15. This result also shows there is no clear relation between API and discharge time series around 1982.

4.2.2 Copula based analysis with a hydrological model

~~API time series were calculated by spatially aggregating several daily precipitations records in each region of Baden-Württemberg state.~~ In this section, simulated discharges time series are generated by a conceptual hydrological model, HBV (Bergström 1976 ; Bergström, Singh, and others 1995) ,which takes daily precipitation~~s~~ and temperature~~s~~ records as input and simulates discharges for smaller catchments as an example ~~more robust sample~~ of discharge, to compare with observed discharge in order to check if differences might occur due to the method.

Thus the idea behind this methodology is similar to the case of API: a~~A~~ hydrological model with the parameters fixed for the entire time period represents the catchment not influenced by anthropogenic impacts. Then, the discharges simulated by this model should not depend~~reflect~~ on~~ the~~ catchment change, while observed discharge is assumed to be influenced by both catchment and precipitation.

For the study area, the Upper Neckar Catchment was chosen as shown~~drawn~~ in Figure 12. One parameter set needed for this model constitutes of 13 parameters which are calibrated based on the Nash–Sutcliffe model efficiency coefficient using the simulated annealing algorithm for the period between 1960 and 2000. Then, 30 parameter sets are independently calibrated in total and, subsequently, 30 simulated discharges time series are generated to compare with one observed discharge.

Figure 16 shows the result of copula based analysis calculated for single time series $(D_1(t), D_2(t), A_{2,\min}(t))$. It can be seen that $A_{2,\min}(t)$ in Figure 16 (top) that (i) fluctuations of $A_{2,\min}(t)$ of observed and simulated discharge are locally identical. This implies that the short term behavior of $A_{2,\min}(t)$ is originated from the temporal behavior of precipitation but (ii) there exists a change of trend around 1976: $A_{2,\min}(t)$ of observed discharge is slightly bigger than simulated before 1976, while

$A_{2,\min}(t)$ of observed discharge clearly undershoot the simulated ones of after 1976. This change of trend was also seen in the previous analyses ($D_2(t)$ in Figure 10). Furthermore, $D_1(t)$ in Figure 16 (middle) is high before 1976 which indicates the state of the copula is different from the rest, while the result of simulated discharges does not show such tendency. $D_2(t)$ in Figure 16 (bottom) indicates the change of dependence structure happened around 1970 and 1977. These results using the HBV model indicate the change of the dependence structure detected using copulas around 1976 is not caused by the random behavior of precipitation, but by the behavior of the catchment itself.

The fact and the notion obtained in this section is that (i) both results from API and HBV based on copula measures indicate that the catchment changed around 1976 and (ii), by comparing the simulated discharge with observed discharge, the origin of the change of stochastically information can be assessed.

Conclusion

In this paper the application of copulas for hydrological time series data is newly explored for the detection of catchment characteristics and their temporal changes.

1. A Copula based measure, asymmetry, was defined and newly applied for the identification of catchment characteristics. Indeed, it was~~is~~ presumed~~presented~~ that asymmetry2 is~~can be~~ related to the runoff characteristics.

2. The relation between the minimum of asymmetry2 and catchment characteristics was tested for 77 discharge records. Asymmetry2 has a certain relation especially with the size of ~~bigger~~ catchments and this strengthens the notion that asymmetry2 can be used as a statistic to explain the catchment state.

3. Temporal change of asymmetry2 was calculated as an index of the catchment state and demonstrated it keeps changing coincidentally with time. However, it is difficult to explain the causality, at least, by long term behavior of discharge and temperature time series.

4. A method based on copula distance was examined for the investigation of temporal behavior of hydrological time series. This measure can detect the time period where dependence structure is unusual and its interdependency. Clear signals were detected that the dependence structure is unusual for a certain time period and the signal was not found by investigating the time series with variance, covariance or correlation.

5. API time series were generated for each region in the Baden-Württemberg state and simulated discharge time series were generated using the HBV model for the Upper Neckar Catchment. These are the data not influenced by ~~the~~ catchment change, thus compared with observed discharge to assess the anthropogenic impacts. The results showed that there was a signal detected only in the observed discharge around 1982, but not in the API or simulated time series, which implies the anthropogenic impacts on the catchment. Also it was shown in the results of copula asymmetry that ~~the difference of $A_{2,\min}(t)$ between observed and simulated discharge was not constant, but~~ the trend clearly changed around 1976.

The results of copula based analysis of hydrological time series seem to support the assumption that the catchment had started to change around 1976 and stayed unusual until 1990. These changes could correspond to the construction of flood retention basins started around 1982 (Lammersen et al., 2002) and ecological flooding strategy, which let small floods to happen for the rehabilitation of ecological systems in the floodplain, introduced in the Upper Rhine since 1989 (Siepe, 2006).

Copulas can be seen as an alternative method to analyze ~~the~~ hydrological time series data by focusing on the dependence structure, but further exploratory applications and theoretical developments are expected. The copula based measures introduced in this study can be related to the potential model uncertainty, that is, how much the natural system is varying. Empirical autocopula analysis is a more data driven approach which retains more information than the copulas estimated with parametric methods, but it is also numerically demanding. The effective way to analyze time series and build up a time series model based on copulas can be further explored.

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Appendix A

Suppose that a random variable at time t is denoted as $X(t)$ and $c_X(\mathbf{u},t)$ is an autocopula obtained from $X(t)$. Assuming $c_{X,mean}(\mathbf{u})$ as an average state of $c_X(\mathbf{u},t)$, deviation of copula $\Delta c_X(\mathbf{u},t)$ at time t is defined by

$$\Delta c_X(\mathbf{u},t) = c_X(\mathbf{u},t) - c_{X,mean}(\mathbf{u}) \tag{A1}$$

For the empirical case, $c_X(\mathbf{u},t)$ and $c_{X,mean}(\mathbf{u})$ can be regarded as local copula and global copula respectively similar to Equation (29)(29)~~(29)~~. Since global and local copula are empirical copula density as defined in equation (8)(8)~~(8)~~, $\Delta c_X(\mathbf{u},t)$ can be regarded as a vector of values on finite number of grids:

$$\mathbf{\Delta c}_X(t) = \left(\Delta c_{X,1}(t), \Delta c_{X,2}(t), \ldots, \Delta c_{X,i}(t), \ldots, \Delta c_{X,N}(t)\right) \tag{A2}$$

where $\Delta c_{X,i}(t)$ denotes the value of copula density at i-th grid and N is the number of grids. From Cauchy-Schwarz inequality

$$\left\|\mathbf{\Delta c}_X(t)\right\| \left\|\mathbf{\Delta c}_Y(t)\right\| \geq \left|\left\langle \mathbf{\Delta c}_X(t), \mathbf{\Delta c}_Y(t)\right\rangle\right|^2 \tag{A3}$$

where $\left\|\mathbf{\Delta c}_X(t)\right\|$ is norm and $\left\langle \mathbf{\Delta c}_X(t), \mathbf{\Delta c}_Y(t)\right\rangle$ is inner product of vector $\mathbf{\Delta c}_X(t)$ and $\mathbf{\Delta c}_Y(t)$ .Then

$$\begin{aligned} \left\|\mathbf{\Delta c}_X(t)\right\| &= \sum_{i=1}^{N} \Delta c_{X,i}(t)^2 \\ &= \int_0^1 \ldots \int_0^1 \left(\Delta c_X(\mathbf{u},t)\right)^2 du_1 \ldots du_n = D_1(c_X, t) \end{aligned} \tag{A4}$$

$$\begin{aligned} &\left|\left\langle \mathbf{\Delta c}_X(t), \mathbf{\Delta c}_Y(t)\right\rangle\right|^2 \\ &= \left\langle \mathbf{\Delta c}_X(t), \mathbf{\Delta c}_Y(t)\right\rangle = \sum_{i=1}^{N} \Delta c_{X,i}(t) \cdot \Delta c_{Y,i}(t) \\ &= \int_0^1 \cdots \int_0^1 \Delta c_X(\mathbf{u},t) \Delta c_Y(\mathbf{u},t) du_1 \ldots du_n = D_3(c_X, c_Y, t) \end{aligned} \tag{A5}$$

$$\frac{\left|\left\langle \mathbf{\Delta c}_X(t), \mathbf{\Delta c}_Y(t)\right\rangle\right|^2}{\left\|\mathbf{\Delta c}_X(t)\right\| \left\|\mathbf{\Delta c}_Y(t)\right\|} = \frac{D_3(c_X, c_Y, t)^2}{D_1(c_X, t) \cdot D_1(c_Y, t)} = D_4(c_X, c_Y, t)^2 \leq 1 \tag{A6}$$

Therefore $\left|D_4\left(c_X, c_Y, t\right)\right| \leq 1$ in Equation (30)~~(30)~~~~(30)~~. Above inequality is valid for certain time point t and summing up (A6) for all the time steps t leads to

$$\sum_{t=1}^{T}\left(\left\|\Delta\mathbf{c}_X(t)\right\| \cdot \left\|\Delta\mathbf{c}_Y(t)\right\|\right) \geq \sum_{t=1}^{T}\left|\left\langle \Delta\mathbf{c}_X(t), \Delta\mathbf{c}_Y(t)\right\rangle\right|^2 \tag{A7}$$

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where T is the number of time steps. $\left\|\Delta\mathbf{c}_X(t)\right\|$ is a norm and can be denoted for simplicity as $x_t = \left\|\Delta\mathbf{c}_X(t)\right\|$. Then

$$\sum_{t=1}^{T}\left(\left\|\Delta\mathbf{c}_X(t)\right\| \cdot \left\|\Delta\mathbf{c}_Y(t)\right\|\right) = \left\langle \mathbf{x}, \mathbf{y}\right\rangle \tag{A8}$$

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where $\mathbf{x} = \left(x_1, x_2, \ldots x_T\right), \mathbf{y} = \left(y_1, y_2, \ldots y_T\right)$ for $t = 1 \ldots T$. Again from Cauchy-Schwarz inequality

$$\left|\left\langle \mathbf{x}, \mathbf{y}\right\rangle\right|^2 \leq \|\mathbf{x}\| \cdot \|\mathbf{y}\| \tag{A9}$$

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where

$$\begin{aligned}\|\mathbf{x}\| \cdot \|\mathbf{y}\| &= \sum_{t=1}^{T} x_t^{\,2} \cdot \sum_{t=1}^{T} y_t^{\,2} = \sum_{t=1}^{T}\left\|\Delta\mathbf{c}_X(t)\right\|^2 \cdot \sum_{t=1}^{T}\left\|\Delta\mathbf{c}_Y(t)\right\|^2 \\ &= \sum_{t=1}^{T} D_1\left(c_X, t\right)^2 \cdot \sum_{t=1}^{T} D_1\left(c_Y, t\right)^2 = T^2 \cdot Var_{cop}\left(c_X\right) \cdot Var_{cop}\left(c_Y\right)\end{aligned} \tag{A10}$$

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$$\begin{aligned}\left\langle \mathbf{x}, \mathbf{y}\right\rangle &= \sum_{t=1}^{T}\left(x_t \cdot y_t\right) = \sum_{t=1}^{T}\left(\left\|\Delta\mathbf{c}_X(t)\right\| \cdot \left\|\Delta\mathbf{c}_Y(t)\right\|\right) \geq \sum_{t=1}^{T}\left|\left\langle \Delta\mathbf{c}_X(t), \Delta\mathbf{c}_Y(t)\right\rangle\right|^2 \\ &= \sum_{t=1}^{T} D_{3,XY}(t) = T \cdot Cov_{cop}\left(c_X, c_Y\right)\end{aligned} \tag{A11}$$

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Then $\left|\left\langle \mathbf{x}, \mathbf{y}\right\rangle\right|^2 \leq \|\mathbf{x}\| \cdot \|\mathbf{y}\|$ indicates

$$\begin{aligned}&\left|Cov_{cop}\left(c_X, c_Y\right)\right|^2 \leq Var_{cop}\left(c_X\right) \cdot Var_{cop}\left(c_Y\right) \\ &\left|Cor_{cop}\right| = \frac{Cov_{cop}\left(c_X, c_Y\right)}{\sqrt{Var_{cop}\left(c_X\right)} \cdot \sqrt{Var_{cop}\left(c_Y\right)}} \leq 1\end{aligned} \tag{A12}$$

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Acknowledgment

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Table 1 Variance and copula variance calculated for 4 discharge time series

	ANDE	COCH	MAXA	PLOC
Var	1.79	2.24	1.75	2.72
Var_{cop} [×10 $^{-5}$]	3.01	1.64	5.39	1.27

Table 2 Covariance, correlation, copula covariance and copula correlation between 4 discharge data (AN:Andernach, CO:Cochem, MA:Maxau, PL:Plochingen)

	AN-CO	AN-MA	AN-PL	CO-MA	CO-PL	MA-PL
Cov	1.68	1.60	1.33	1.38	1.31	1.41
Cor	0.84	0.90	0.60	0.70	0.53	0.64
Cov_{cop} [×10 $^{-6}$]	4.90	3.40	3.39	7.16	9.90	5.47
Cor_{cop}	0.60	0.77	0.46	0.71	0.60	0.59

Table 3 Variance and copula variance calculated for API time series of 4 regions in the Baden-Württemberg state of Germany

	C	SW	NW	NE
Var	1.70	1.66	1.72	1.78
Var_{cop} [×10 $^{-6}$]	3.00	4.02	3.35	3.21

Table 4 Covariance, correlation, copula covariance and copula correlation between API time series from 4 regions in the Baden-Württemberg state of Germany

	C-SW	C-NW	C-NE	SW-NW	SW-NE	NW-NE
Cov	1.35	1.33	1.44	1.25	1.41	1.42
Cor	0.80	0.77	0.84	0.74	0.84	0.83
Cov_{cop} [×10 $^{-7}$]	1.46	1.16	8.94	4.42	1.11	8.80
Cor_{cop}	0.36	0.29	0.29	0.09	0.26	0.24

Figure Captions

Figure 1 Locations of 7 discharge gauging stations in the Upper Rhine Region

Figure 2 Visualization of the functions which displays the contribution of a realization of $\left(U_t, U_{t+k}\right)$ to *assymetry1* (left) and *asymmetry2* (right)

Figure 3 Sketch of the transformation from sample hydrograph (left) to empirical copula (right): Scatterplot of ranks are calculated from two values separated by time lag $k=1$[days] in a discharge time series of Andernach where $\textit{rank correlation} = 0.9870$, $A_1\left(k=1\right) = -0.0002398$ and $A_2\left(k=1\right) = -0.00011037$. The possible combinations of high and low values, which has large impacts on asymmetry, are numbered (1) low to high, (2) high to high (3) high to low (4) low to low. Negative contribution to asymmetry2 is drawn with red circle, positive contribution with blue circle.

Figure 4 Annual cycle of mean discharge after smoothing (left) and annual cycle of standard deviation after smoothing (right)

Figure 5 Discharge time series between 1950 and 1955 before applying normalization (upper left) and after applying normalization (upper right). The variation of asymmetry2 function calculated for entire time series before applying normalization (bottom left) and after applying normalization (bottom right) with 90% confidence intervals (grey) calculated for 100 realizations of Gaussian process (dashed line is $A_2\left(k\right)$ calculated for one of the realization of Gaussian process).

Figure 6 Relation between Asymmetry and catchment characteristics: minimum of asymmetry2 of discharge and catchment area (top), lag at minimum of asymmetry2 of discharge and catchment area (middle), minimum of asymmetry2 of discharge and lag at minimum of asymmetry2 of discharge (bottom)

Figure 7 Temporal change of minimum of asymmetry2 for 7 discharge records and confidence intervals calculated from the Gaussian process (90% confidence interval with grey color and 60% confidence interval with dark grey color) and one of its realizations (dashed line)

Figure 8 Moving average and standard deviation of the 7 daily discharge records for the window size *w* = 3000

Figure 9 Annual minimum and mean of aggregated daily temperature in the Baden-Württemberg state of Germany

Figure 10 Copula distances of discharge time series in moving time window: moving variance (top), distance type1 (middle) and distance type2 (bottom) with 80% confidence interval of Gaussian process and one of its realization (dashed line)

Figure 11 Copula distances of discharge time series in moving time window: moving covariance (top), moving correlation (second), distance type3 (third) and distance type4 (bottom)

Figure 12 Locations of the precipitation gauge stations within the Baden-Württemberg (Germany) indicated by coloured circles. Upper Neckar catchment is drawn with green area and the location of gauging station is drawn with a square

Figure 13 Copula distances of API time series in moving time window: moving variance (top), copula distance type1 (middle) and copula distance type2 (bottom) where ‘C’ denotes central, ‘SW’ denotes southwest, ‘NW’ denotes northwest and ‘NE’ denotes northeast part of Baden-Württemberg State of Germany respectively with 80% confidence interval of Gaussian process and one of its realization (dashed line).

Figure 14 Copula distances of API time series in moving time window: moving covariance (top), moving correlation (second), distance type3 (third) and distance type4 (bottom)

Figure 15 Copula distance type3 (top) and type4 (bottom) between 4 discharge and 1 API time series which is aggregated for all the daily precipitations depicted in Figure 12

Figure 16 Copula asymmetry and copula distances for 30 simulated and one observed discharge time series at Plochingen between 1965 and 2000: minimum of asymmetry2 for the time lag $k = 2$ [days] (top), copula distance type1 (middle), copula distance type2 (bottom)

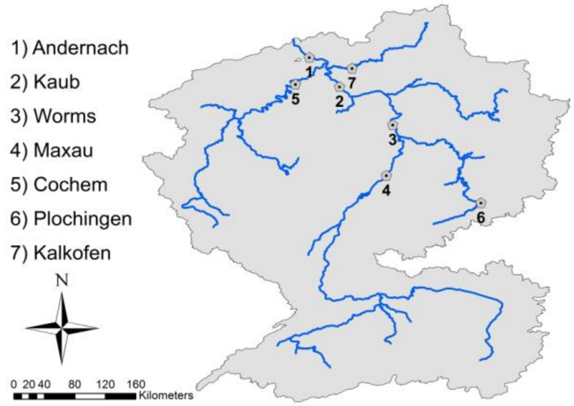

Figure 1 Locations of 7 discharge gauging stations in the Upper Rhine Region

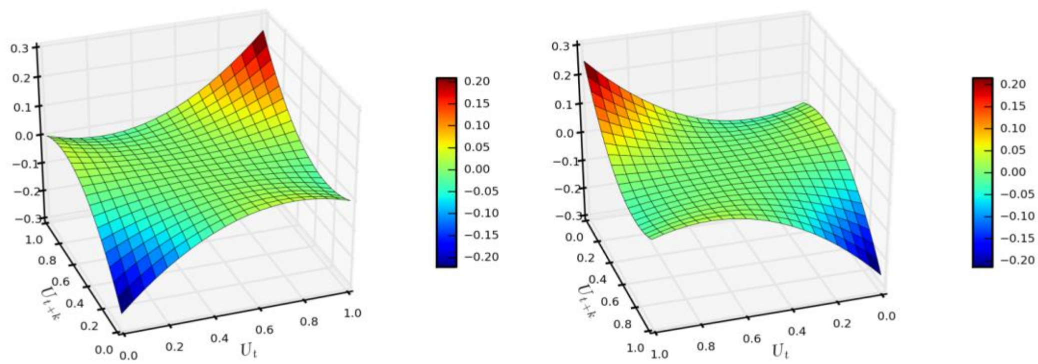

Figure 2 Visualization of the functions which displays the contribution of a realization of $\left(U_t, U_{t+k}\right)$ to *assymetry1* (left) and *asymmetry2* (right)

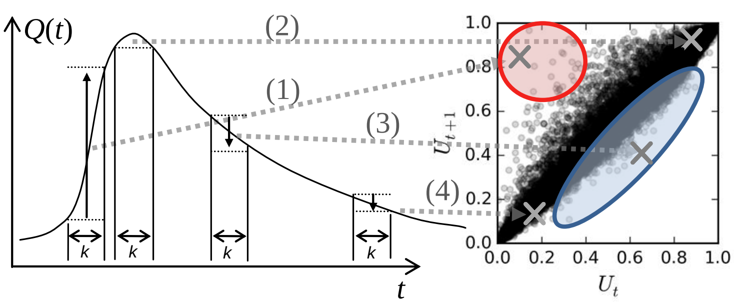

Figure 3 Sketch of the transformation of the values from sample hydrograph (left) to the points on scatterplot of ranks (right): empirical copula calculated from two values separated by time lag $k=1$ [days] in a discharge time series of Andernach where $rank\,correlation = 0.9870$, $A_1(k=1) = -0.0002398$ and $A_2(k=1) = -0.00011037$. The possible combinations of high and low values, which has large impacts on asymmetry, are numbered: (1) low to high, (2) high to high, (3) high to low, (4) low to low. Negative contribution to asymmetry2 is drawn with red circle and positive contribution with blue oval.

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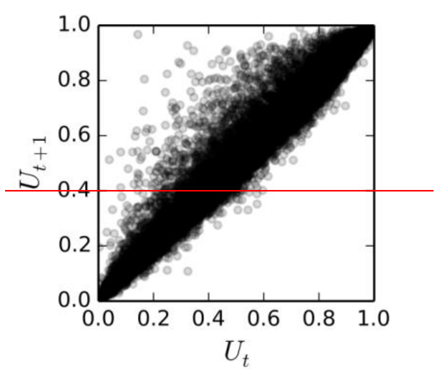
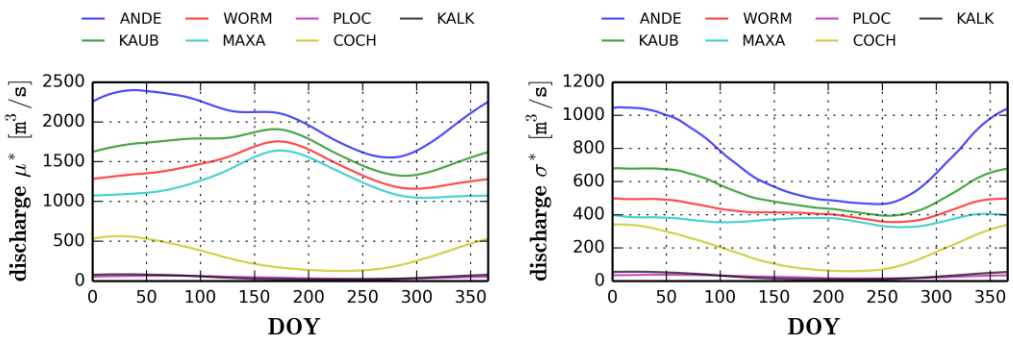


~~Figure 3 Scatterplot of ranks calculated from two values separated by time lag~~ $\cancel{k=1}$ ~~[days] in a discharge time series of Andernach where~~ $\cancel{rank\,correlation = 0.9870}$, $\cancel{A_1(k=1) = -0.0002398}$ ~~and~~ $\cancel{A_2(k=1) = -0.00011037}$



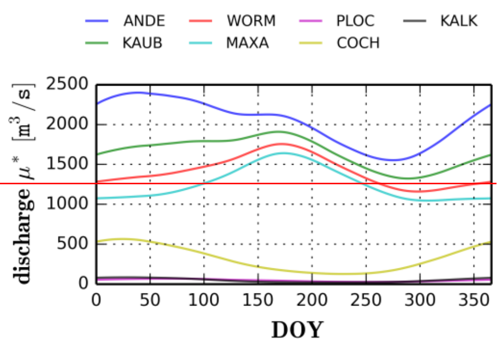
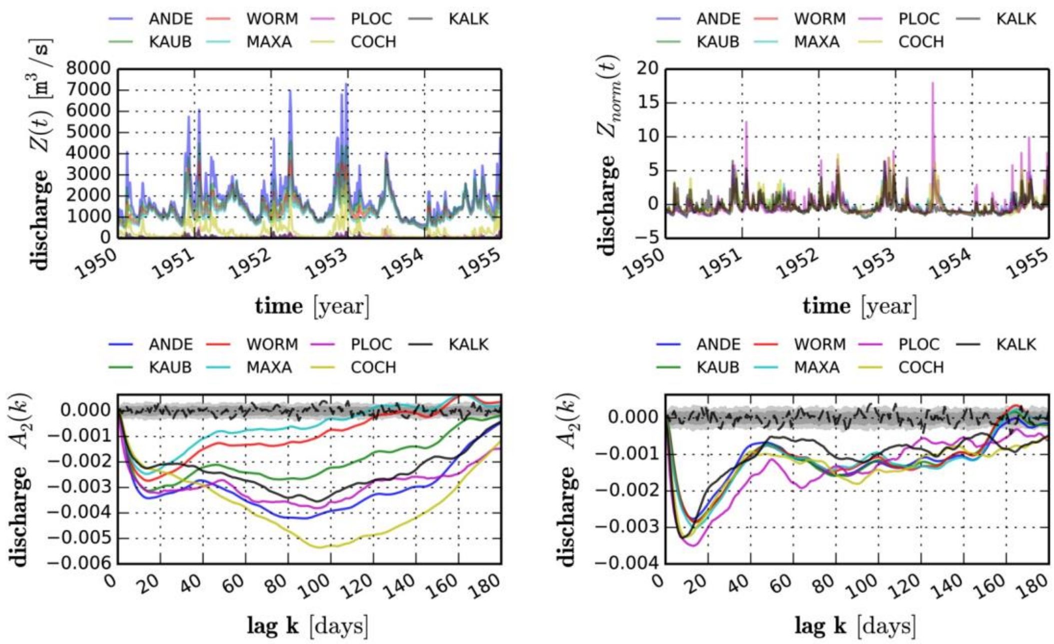


Figure 4 Annual cycle of mean discharge after smoothing (left) and annual cycle of standard deviation after smoothing (right)



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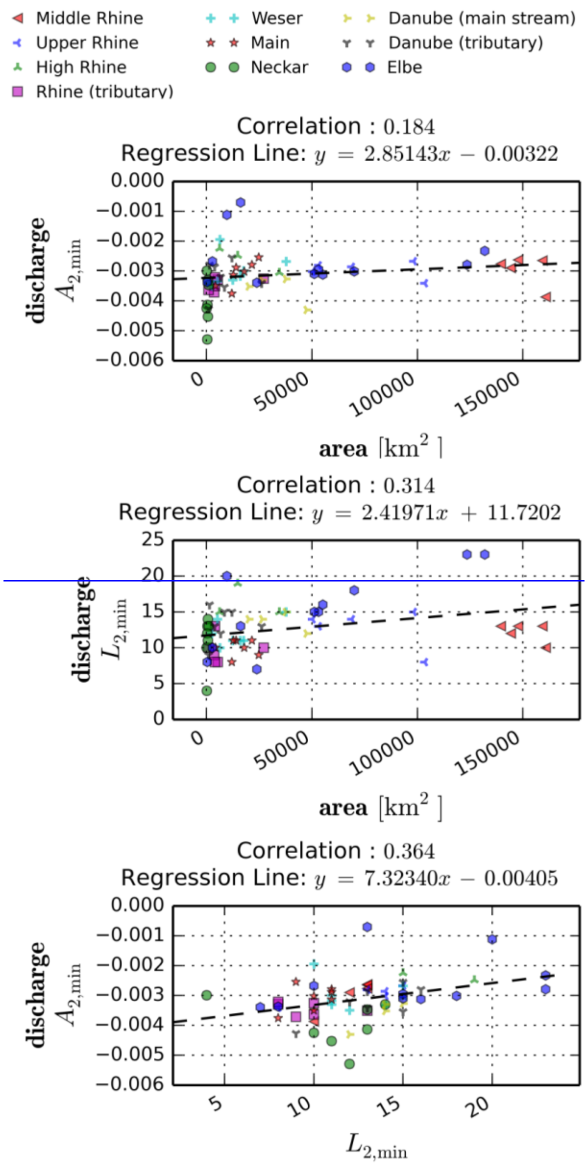
Middle Rhine
Upper Rhine
High Rhine
Rhine (tributary)
Weser
Main
Neckar
Danube (main stream)
Danube (tributary)
Elbe
Correlation : 0.184
Regression Line: $y = 2.85143x - 0.00322$
discharge $A_{2,\min}$
area [km^2]
Correlation : 0.314
Regression Line: $y = 2.41971x + 11.7202$
discharge $L_{2,\min}$
area [km^2]
Correlation : 0.364
Regression Line: $y = 7.32340x - 0.00405$
discharge $A_{2,\min}$
$L_{2,\min}$

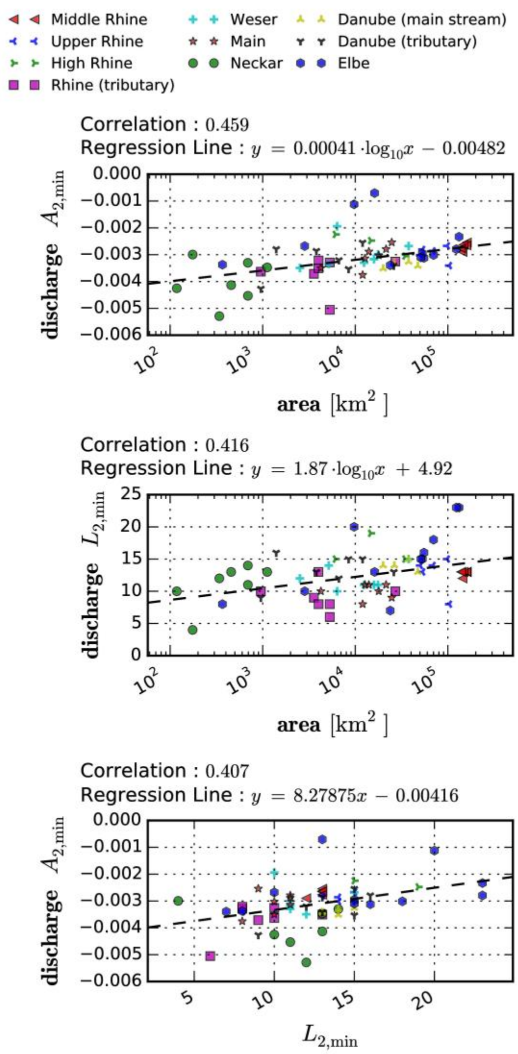

Figure 6 Relation between Asymmetry and catchment characteristics: minimum of asymmetry2 of discharge and catchment area (top), lag at minimum of asymmetry2 of discharge and catchment area (middle), minimum of asymmetry2 of discharge and lag at minimum of asymmetry2 of discharge (bottom)

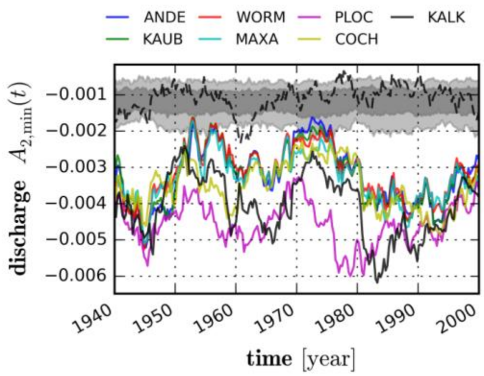

Figure 7 Temporal change of minimum of asymmetry2 for 7 discharge records and confidence intervals calculated from the Gaussian process (90% confidence interval with grey color and 60% confidence interval with dark grey color) and one of its realizations (dashed line)

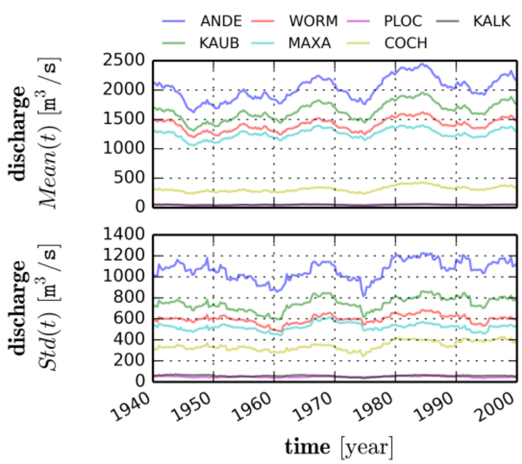

Figure 8 Moving average and standard deviation of the 7 daily discharge records for the window size w = 3000

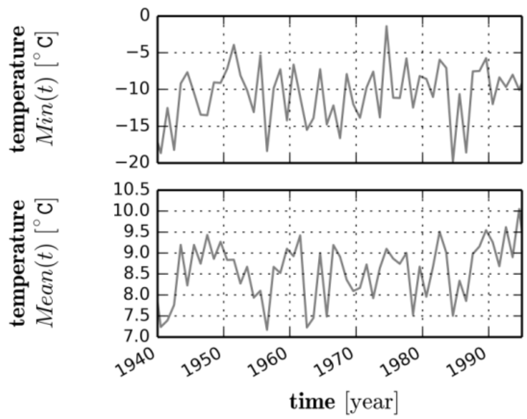

Figure 9 Annual minimum and mean of aggregated daily temperature in the Baden-Württemberg state of Germany

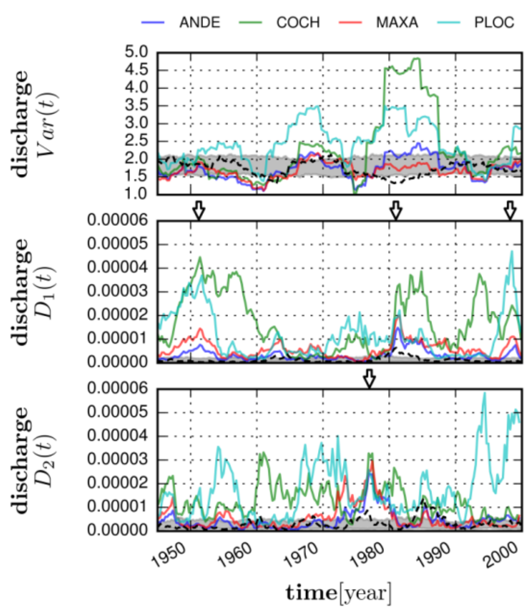

Figure 10 Copula distances of discharge time series in moving time window: moving variance (top), distance type1 (middle) and distance type2 (bottom) with 80% confidence interval of Gaussian process and one of its realization (dashed line)

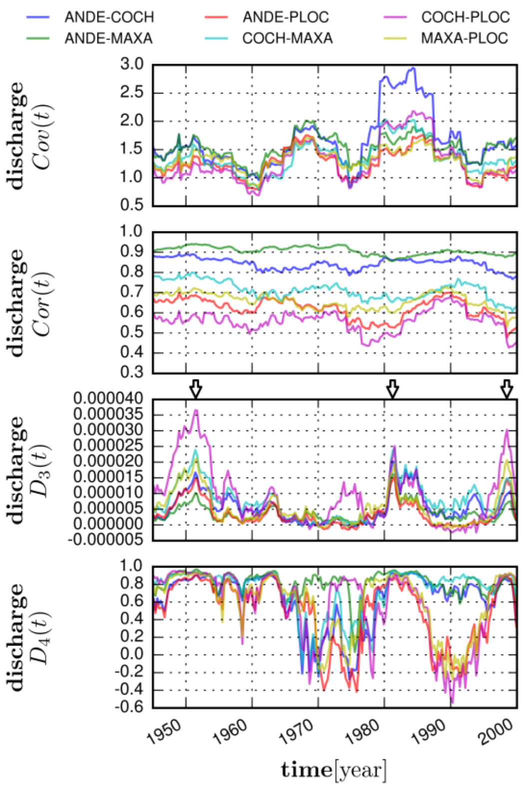

Figure 11 Copula distances of discharge time series in moving time window: moving covariance (top), moving correlation (second), distance type3 (third) and distance type4 (bottom)

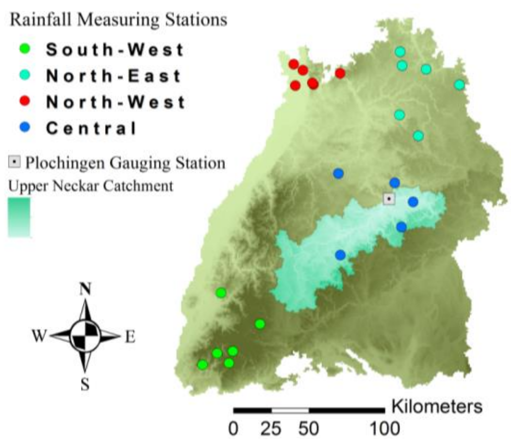

Figure 12 Locations of the precipitation gauge stations within the Baden-Württemberg (Germany) indicated by coloured circles. Upper Neckar catchment is drawn with green area and the location of gauging station is drawn with a square

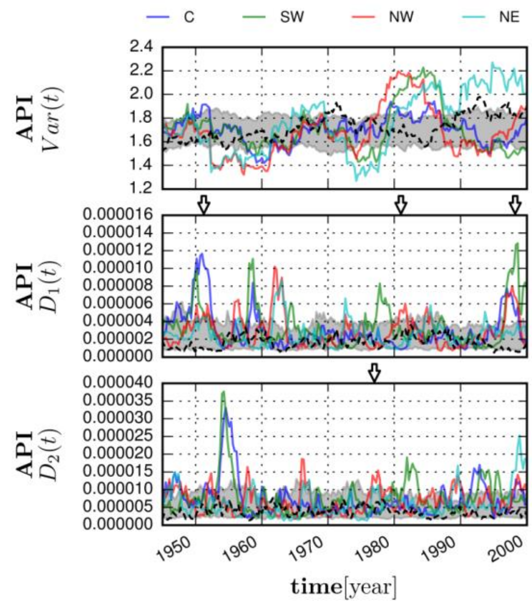

Figure 13 Copula distances of API time series in moving time window: moving variance (top), copula distance type1 (middle) and copula distance type2 (bottom) where ‘C’ denotes central, ‘SW’ denotes southwest, ‘NW’ denotes northwest and ‘NE’ denotes northeast part of Baden-Württemberg State of Germany respectively with 80% confidence interval of Gaussian process and one of its realization (dashed line).

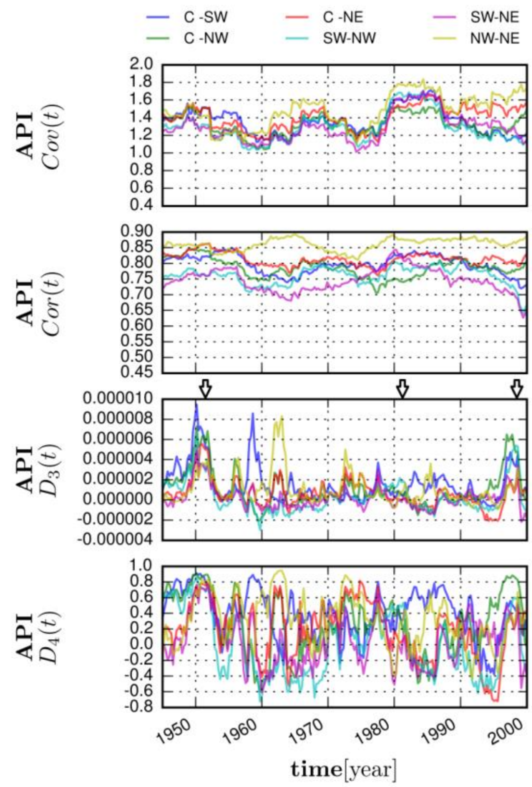

Figure 14 Copula distances of API time series in moving time window: moving covariance (top), moving correlation (second), distance type3 (third) and distance type4 (bottom)

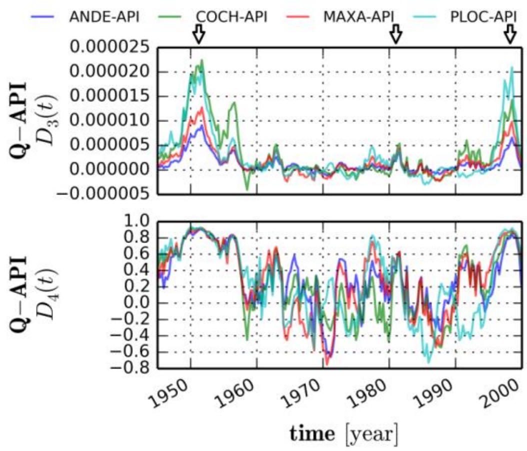

Figure 15 Copula distance type3 (top) and type4 (bottom) between 4 discharge and 1 API time series which is aggregated for all the daily precipitations depicted in Figure 12~~Figure 12Figure 12~~

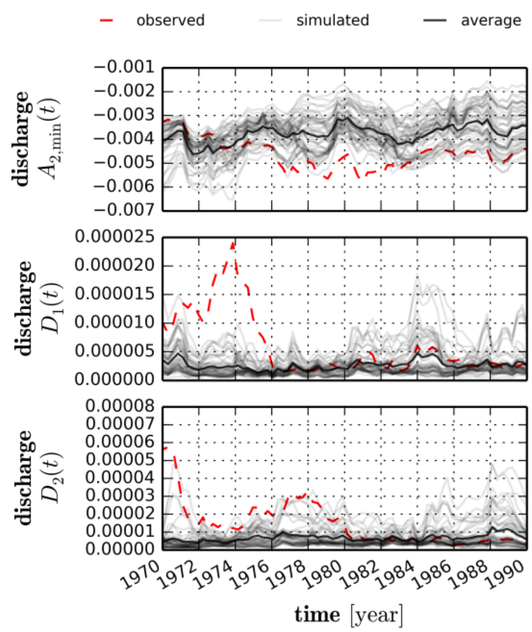

Figure 16 Copula asymmetry and copula distances for 30 simulated and one observed discharge time series at Plochingen between 1965 and 2000: minimum of asymmetry2 for the time lag $k = 2$ [days] (top), copula distance type1 (middle), copula distance type2 (bottom)