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Examination for robustness of parametric estimators for flood statistics in the context of extraordinary extreme events

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Abstract

We compare several estimators, which are commonly used in hydrology, for the parameters of the distribution of flood series, like the Maximum-Likelihood estimator or L-Moments, with the robust estimators Trimmed L-Moments and Minimum Distances. Our objective is estimation of the 99 %- or 99.9 %-quantile of an underlying Gumbel or Generalized Extreme Value distribution (GEV), where we modify the generated random variables such that extraordinary extreme events occur. The results for a two- or three-parametric fitting are compared and the robustness of the estimators to the occurrence of extraordinary extreme events is investigated by statistical measures.

When extraordinary extreme events are known to appear in the sample, the Trimmed L-Moments are a recommendable choice for a robust estimation. They even perform rather well, if there are no such events.

1 Introduction

In hydrology the statistical distribution of annual maximum discharges is commonly modelled by the three parametric Generalized Extreme Value (GEV) distribution or its special case, the two parametric Gumbel distribution (Hosking et al., 1985a; N.E.R.C., 1975). This approach is based on the Fisher–Tippet-Theorem which says that the maximum of independent, identically distributed random variables (properly normalized) converges in distribution to an extreme value distribution (Fisher and Tippett, 1928).

Parameter estimators for distribution functions used in the hydrological context, especially the GEV, are often compared concerning their efficiency or biasedness in small and large samples (Hosking et al., 1985b). It is also known, which of the popular estimators have smaller variances when estimating the probability of certain values or quantiles (cf. Hosking, 1990; Madsen et al., 1997). Typically, these estimators are the Maximum-Likelihood estimator, the Method of Moments or the L-Moments (cf. Prescott and Walden, 1980; Hosking et al., 1985b), with the comparison depending on the sample size. It turns out that the L-Moment estimators are preferable to the Maximum Likelihood estimation for small sample sizes in terms of efficiency. The aforementioned works concentrate on the (un)biasdness and efficiency of these estimators under ideal conditions. However, these estimators are all known to be non-robust. That is, in the presence of atypical observations or misspecification of the underlying model these estimators tend to over- or underestimate.

In hydrological time series of maxima, for example annual floods, sometimes extraordinary extreme events occur. These are in fact events with very low exceedance probability, which deviate markedly from the other observations in the sample (Fischer and Schumann, 2015). An example of such an event is the 2002 flood in the eastern part of Germany. In Fig. 1 the Gauge Nossen at the river Freiberger Mulde in the east of Germany is shown.

This occurrence of extreme events and the related problems in estimation are a known problem and the property of robustness gets more and more into focus (cf. Garavaglia et al., 2010; Guerrero et al., 2013; Fischer and Schumann, 2015). Using non-robust estimators can lead to highly variable results in specification of distribution functions which are applied to estimate design floods, for example the 99 %-quantile. The occurrence of an extraordinary event increases the estimated quantile significantly, although the underlying distribution does not change.

An alternative worth considering in this case are robust estimators, with robustness meaning robustness of the parameter estimation against extraordinary values or misspecification of the underlying model. That is, the presence of single extraordinary events in a time series does not influence the estimation. A measure for this is for example the influence curve or its empirical equivalent, the sensitivity curve (Hampel et al., 1986).

In this article we want to investigate the behaviour of estimators, which are commonly used in practice, in the situation, where extraordinary and rare extreme events occur. We want to figure out, if the estimators are robust to small modifications of the GEV distribution. At the same time we consider estimators, which are up to now not in common use, but supposed to be robust. These are the Trimmed L-Moments, which use a trimming of the extreme values in the sample, and a Minimum Distance estimator. As a comparison, fitting a two- and a three-parametric distribution is investigated. In hydrology it is not determined, how many parameters should be used. The third parameter, characterizing the shape and therefore the skewness, gives more flexibility in estimation, especially when the data do not seem to be symmetric. However, its estimation also leads to an additional uncertainty. The DWA (2012) (Deutsche Gesellschaft für Wasserwirtschaft, Abwasser und Abfall e.V.; German Association for Water, Wastewater and Waste) recommends the usage of a two- or three-parametric distribution, depending on the length of the data sample. For small sample sizes ($n \leq 50$) application of a two-parametric distribution is recommended, taking into account the low efficiency in the estimation.

Therefore, we decide to compare both possibilities, two- and three-parametric fits, to guarantee a fair evaluation for small and large sample sizes.

In our study we choose the two parametric Gumbel distribution with distribution function

$$F(x) = \exp\left(-\exp\left(\frac{x-\mu}{\sigma}\right)\right) \quad (1)$$

and as three parametric GEV distribution with distribution function

$$G(x) = \exp\left(-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right), \quad (2)$$

for $1+\xi(x-\mu)/\sigma > 0$, where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ is the scale parameter and $\xi \in \mathbb{R}$ is the shape parameter. The Gumbel distribution corresponds to the special case $\xi = 0$.

In Sect. 2 we specify the setting for our simulations and give explicit description of the used estimators. The results are evaluated in Sect. 3 and finally a conclusion and an outlook is given.

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2 Simulation

To test several estimators with regard to their robustness and efficiency we use simulated flood series. For the simulations we first analyse Gumbel-distributed data with location parameter $\mu = 100$ and scale parameter $\sigma = 10$, and in a second step GEV-distributed data with the same location and scale parameter and shape $\xi_1 = 0.1$ (GEV1). These are representative choices fitting such distributions to maximum discharges (cf. Madsen et al., 1997). For comparison we also consider a larger shape parameter $\xi_2 = 0.2$ in the model, to which we refer as GEV2.

As a justification for the choice of these parameters we fitted the GEV distribution via L-Moments (described later on) to 33 series of annual maximum discharges of gauges in different river basins in Thuringia and Saxony in Germany. The histogram of estimated shape parameters of the GEV (rounded to one decimal figure) can be found in Fig. 2. We can see some very large values for ξ, which indicate a significant deviation from the Gumbel distribution.

In the simulations for each of the three distribution functions mentioned above two scenarios are considered. The independent, identically Gumbel respectively GEV distributed random variables are modified in one of the following ways:

1. No modification: independent identically distributed random variables.

2. We include extraordinary extreme values in these time series, which equal the 99.9 %-quantile of the underlying distribution. For this, randomly chosen 2 % of the data (rounded up) are replaced by the value of the 99.9 %-quantile.

First of all, we fit both the Gumbel and the GEV distribution, respectively, to compare fits by two and three parametric distributions. This is done by calculating the 99 %- and the 99.9 %-quantiles of the fitted distributions and considering the bias and the root mean squared error (RMSE) for the corresponding quantiles of the assumed true distribution. In both scenarios this true distribution is the one without modification, which has the following quantiles:

$$Q_{0.99;\mathrm{Gumb}} = 146.0,\ Q_{0.999;\mathrm{Gumb}} = 169.1 \quad (3)$$
$$Q_{0.99;\mathrm{GEV1}} = 158.4,\ Q_{0.999;\mathrm{GEV1}} = 199.5 \quad (4)$$
$$Q_{0.99;\mathrm{GEV2}} = 175.5,\ Q_{0.999;\mathrm{GEV2}} = 249.0 \quad (5)$$

In the simulation we consider 1000 repetitions for each of different sample lengths equal to $n = 30, 50, 100, 200$. Annual series with a length of more than 100 years are very rare in hydrology and therefore an upper length of 200 seems to be sufficient.

We compare the following five different estimators.

1. Maximum-Likelihood estimator (ML): The Maximum Likelihood estimator is among the most popular estimators. It is frequently used because of its high efficiency, consistency and asymptotical unbiasedness. However, it is not robust against outliers or model misspecification (Dupuis and Field, 1980). The calculation of the ML estimator turns out to be rather difficult and especially for three parameter distributions like the GEV it needs to be done numerically.

2. L-Moment estimator: The probability weighted moments (PWM) were developed by Greenwood et al. (1979) to express parameters of easily invertible distributions by moments. As an advancement of the standard and the probability weighted moments Hosking (1990) suggested the so called Linear-Moments (L-Moments), which are estimated by a linear combination of order statistics (that is L-statistics). The resulting estimator offers the advantages of being similar to and for small samples sometimes even more efficient than the Maximum-Likelihood estimator and also more robust than ordinary Moment estimators (Hosking, 1990). They exist in situations, where the classical moments do not exist and represent in contrast to the PWM the characteristic values of a sample such as mean or standard deviation straightforwardly. Therefore, they are used more frequently than

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the PWM but deliver the same results. The rth L-Moment

$$\lambda_r = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \mathbb{E}X_{(r-k:r)} \tag{6}$$

is estimated by

$$l_r = \frac{1}{\binom{n}{r}} \sum_{1\leq i_1<\ldots<i_r\leq n} r^{-1}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} x_{(i_{r-k}:n)}, \tag{7}$$

where $x_{(i:n)}$ is the ith value of the order statistics of the sample x_i, $i = 1,\ldots,n$ respectively X_i, and $r = 1,\ldots,n$. Often also an expression via the PWM is used.

3. Trimmed L-Moment estimator with symmetric (1,1) and asymmetric (0,1) trimming (TL(1,1)/TL(0,1))

   The TL-Moments are a generalization of the L-Moments suggested by Elamir and Seheult (2003). The general formula for the rth TL-Moment is

$$\lambda_r^{(t_1,t_2)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} \mathbb{E}\left(X_{(r+t_1-k:r+t_1+t_2)}\right), \tag{8}$$

where $t_1, t_2 \in \mathbb{N}$ is the degree of trimming of the lower and upper parts of the order statistic. By choosing $t_1 = t_2 = 0$ one gets the classical L-Moments. The trimming makes the TL-Moments more robust against outliers than the L-Moments. An unbiased estimator for the TL-Moments for the sample $x_1,\ldots,x_n$ is (Hosking, 2007)

$$l_r^{(t_1,t_2)} = \frac{1}{r\binom{n}{r+t_1+t_2}} \sum_{j=t_1+1}^{n-t_2}\sum_{k=0}^{r-1}(-1)^k \binom{j-1}{r+t_1-1-k-1}\binom{n-j}{t_2+k} x_{(j:n)}. \tag{9}$$

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For comparison, the estimator of the first L-Moment obtained from four data points $x_{(1:4)} \leq x_{(2:4)} \leq x_{(3:4)} \leq x_{(4:4)}$ is

$$l_1 = \frac{1}{4}\left(x_{(1:4)} + x_{(2:4)} + x_{(3:4)} + x_{(4:4)}\right), \tag{10}$$

whereas for the first TL(1,1)-Moment we have

$$l_1(1,1) = \frac{1}{2}\left(x_{2:4} + x_{3:4}\right), \tag{11}$$

since due to the trimming the highest and the lowest values of the sample are omitted. The question of the trimming degree is crucial and of course other trimming is possible. In our experiments, simulations with a higher trimming (0,2) did not show more robustness but lower efficiency and therefore they are not considered here. For the special case of the Gumbel- and GEV distribution one can find approximative expressions of the parameter estimations, see Elamir and Seheult (2003) and Lilienthal (2013):

For the parameters of the GEV distribution the TL(0,1)-Moment estimators are

$$z = \frac{10}{9}\left(\frac{1}{2 + l_3^{(0,1)}/l_2^{(0,1)}}\right) - \frac{2\log(2) - \log(3)}{3\log(3) - 2\log(4)} \tag{12}$$

$$\hat{\xi}_{\mathrm{TL}(0,1)} = 8.567394 \cdot z - 0.675969 \cdot z^2 \tag{13}$$

$$\hat{\sigma}_{\mathrm{TL}(0,1)} = \frac{2}{3} l_2^{(0,1)} \frac{1}{\Gamma\left(\hat{\xi}_{\mathrm{TL}(0,1)}\right)} \left(\left(\frac{1}{3}\right)^{\hat{\xi}_{\mathrm{TL}(0,1)}} - 2\left(\frac{1}{2}\right)^{\hat{\xi}_{\mathrm{TL}(0,1)}} + 1\right)^{-1} \tag{14}$$

$$\hat{\mu}_{\mathrm{TL}(0,1)} = l_1^{(0,1)} - \frac{\hat{\sigma}_{\mathrm{TL}(0,1)}}{\hat{\xi}_{\mathrm{TL}(0,1)}} - \hat{\sigma}_{\mathrm{TL}(0,1)} \Gamma\left(\hat{\xi}_{\mathrm{TL}(0,1)}\right) \left(\left(\frac{1}{2}\right)^{\hat{\xi}_{\mathrm{TL}(0,1)}} - 2\right) \tag{15}$$

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and for the Gumbel distribution

$$\hat{\sigma}_{\mathrm{TL}(0,1)} = \frac{l_2^{(0,1)}}{0.431} \tag{16}$$

$$\hat{\mu}_{\mathrm{TL}(0,1)} = l_1^{(0,1)} + 0.116\hat{\sigma}_{\mathrm{TL}(0,1)}. \tag{17}$$

For the symmetric trimming the TL(1,1)-Moment estimators are

$$z = \frac{9}{20}\left(\frac{l_3^{(1,1)}}{l_2^{(1,1)}}\right) - \frac{\log(3) - 2\log(4) + \log(5)}{\log(2) - 2\log(3) + \log(4)} \tag{18}$$

$$\hat{\xi}_{\mathrm{TL}(1,1)} = 25.31711 \cdot z - 91.5507 \cdot z^2 + 110.0626 \cdot z^3 - 46.5518 \cdot z^4 \tag{19}$$

$$\hat{\sigma}_{\mathrm{TL}(1,1)} = l_2^{(1,1)} \frac{1}{\Gamma(\hat{\xi}_{\mathrm{TL}(1,1)})} \frac{1}{3\left(\frac{1}{2}\right)^{\hat{\xi}_{\mathrm{TL}(1,1)}} - 6\left(\frac{1}{3}\right)^{\hat{\xi}_{\mathrm{TL}(1,1)}} + 3\left(\frac{1}{4}\right)^{\hat{\xi}_{\mathrm{TL}(1,1)}}} \tag{20}$$

$$\hat{\mu}_{\mathrm{TL}(1,1)} = l_1^{(1,1)} - \frac{\hat{\sigma}_{\mathrm{TL}(1,1)}}{\hat{\xi}_{\mathrm{TL}(1,1)}} - \hat{\sigma}_{\mathrm{TL}(1,1)}\Gamma(\hat{\xi}_{\mathrm{TL}(1,1)})\left(-3\left(\frac{1}{2}\right)^{\hat{\xi}_{\mathrm{TL}(1,1)}} - 2\left(\frac{1}{3}\right)^{\hat{\xi}_{\mathrm{TL}(1,1)}}\right) \tag{21}$$

for the GEV distribution and

$$\hat{\sigma}_{\mathrm{TL}(1,1)} = \frac{l_2^{(1,1)}}{0.353} \tag{22}$$

$$\hat{\mu}_{\mathrm{TL}(0,1)} = l_1^{(0,1)} - 0.459\hat{\sigma}_{\mathrm{TL}(0,1)} \tag{23}$$

for the Gumbel distribution.

4. Minimum Distance estimator: The Minimum Distance estimator for the parameter vector $\boldsymbol{\theta}$ using the Cramer-von-Mises distance is given by

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta} \int_{-\infty}^{\infty} (F_n(x) - G_\theta(x))^2 \mathrm{d}x, \tag{24}$$

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see Dietrich and Hüsler (1996). Here, F_n is the empirical distribution function of the sample and G_θ the distribution function with parameter (vector) $\boldsymbol{\theta}$ to be fitted. In our case we need to minimize (for the GEV distribution)

$$\int_{-\infty}^{\infty} \left(F_n(x) - \exp\left(- \left(1 + \xi \left(\frac{x-\mu}{\sigma} \right) \right)^{-\frac{1}{\xi}} \right) \right)^2 \mathrm{d}x \tag{25}$$

or (for the Gumbel distribution)

$$\int_{-\infty}^{\infty} \left(F_n(x) - \exp\left(- \exp\left(\frac{x-\mu}{\sigma} \right) \right) \right)^2 \mathrm{d}x, \tag{26}$$

which is done numerically.

Remark: the third classical non-robust estimator, the standard sample moments, is excluded here, since the moment estimators do not exist for a shape parameter $\xi > 1/3$, and therefore do not seem to be suitable in this hydrological context, since there can be samples with a larger shape parameter as underlined by Fig. 2.

For the simulations we used the statistical software R (R-Project, 2013) with the related packages distrMod (Kohl and Ruckdeschel, 2010), fExtremes (Wuertz et al., 2013), lmomco (Asquith, 2013), RobExtremes (Ruckdeschel et al., 2012) and VGAM (Yee, 2010). The results can be found in Tables 1–6.

2.1 Evaluation for data without disturbances

In Tables 1, 3 and 5 we can find the results for bias and root mean squared error (RMSE) for the fitting to independent, identically distributed random variables, following a Gumbel, GEV1- or GEV2-distribution.

For i.i.d. Gumbel data (Table 1) we see that a Gumbel-Fitting with Maximum Likelihood leads to the lowest RMSE for all sample sizes n and both quantiles considered here. Concerning the RMSE, TL(0,1) estimation is second best, followed by TL(1,1) and L-Moments, which do not differ much. The Minimum Distance estimator performs worst, only in the case of $n = 100$ it is better than classical L-Moments. The bias of all Gumbel-based fittings is very small, even for small sample sizes. Fitting a GEV-distribution to Gumbel-distributed data with non-robust estimators (ML and L-Moments) roughly doubles the RMSE. Robust estimation (TL(0,1), TL(1,1), and MD) worsens the results even more as both the RMSE and the bias become larger for all n and both quantiles. Robust estimators aim at reducing biases which are due to using only approximately valid models, but it seems that in this case estimation error increases when fitting an unnecessary third parameter by one of these robust estimators.

In case of i.i.d. GEV1-distributed data with a shape parameter of $\xi = 0.1$ (Table 3), fitting a Gumbel distribution causes a substantial negative bias, which dominates the RMSE. Again, the L-Moments give the best results, but the differences are not large, neither to the robust TL(1,1)-Moments. It is striking that the RMSE is nevertheless much smaller for a Gumbel-Fitting than for a GEV-Fitting if the sample size is small, and about equal for large sample sizes. This is due to the much smaller variability of the two-parametric fits. A non-robust GEV-Fitting causes a large positive bias only for the 99.9 %-quantile, but nevertheless the RMSE is large for both quantiles and all sample sizes due to the large variability. The results for the ML and the L-Moments are very similar for large sample sizes ($n = 200$), though the L-Moments deliver results with less deviation from the assumption in small samples. A robust fitting of a GEV-distribution leads, in comparison to the same estimators with a Gumbel-fitting, to a positive bias and and higher variability and therefore to larger RMSEs for all sample sizes and both quantiles.

When the shape parameter is further increased to $\xi = 0.2$ (GEV2) (Table 5) the previous remarks remain qualitatively valid. Fitting a Gumbel distribution to such data both bias and RMSE is nearly twice (2.5 times for the 99.9 %-quantile) the one as before. Very striking are the results of the L-Moment estimates for small sample sizes. They produce a large positive bias resulting in a large RMSE, which do not fit to the results

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for the other sample sizes. In case of at most 50 observations, fitting a Gumbel distribution with TL(1,1)-Moments seems best, closely followed by ML, MD and TL(0,1). In case of at least 100 observations, GEV fitting becomes worthwhile with L-Moments and ML performing best, followed by TL(1,1)- and TL(0,1)-Moments.

2.2 Evaluation in the presence of extraordinary extreme events

In Tables 2, 4 and 6 data from scenario 2 containing extraordinary extreme events are considered.

For Gumbel-distributed data and a Gumbel-Fitting we see that the smallest values for the RMSE are given for the TL(0,1)-Moments followed by the MD-estimator, which even has a smaller bias. For both estimators the RMSE is not much higher than in the case of data without disturbances. The non-robust estimators are substantially more biased, with the L-Moments behaving worst. The RMSE of the ML-estimator is comparable to that of the TL(1,1)-Moments, though ML has larger bias. If a GEV-Fitting is used, the bias increases rapidly and therefore also the RMSE is large.

Somewhat surprisingly, the results are similar to this when the data follow a GEV-distribution with $\xi = 0.1$. The occurrence of extraordinary extreme events apparently reduces the negative bias and the RMSE of the estimations based on a Gumbel fit in this situation. Fitting a Gumbel distribution by L-Moments works best for all sample sizes, followed by ML, TL(1,1)- and TL(0,1)-Moments. For the 99.9 %-quantile the difference becomes even larger. So the robust estimators have a higher RMSE and are no longer better than the non-robust ones, where the ML-estimator behaves best overall. Note that the results for a GEV-Fitting are much worse. If the value of the shape parameter is increased, the RMSE and bias results increase by the factor 2.

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3 Conclusions and outlook

This work investigates the applicability and advantages or disadvantages of some robust estimators in the context of hydrological flood estimation.

Concerning the non-robust estimators the results of Hosking et al. (1985b) are confirmed. For small sample sizes the L-Moments in the GEV-Fitting have smaller RMSE than the ML-estimator. Nevertheless, it becomes obvious that the size of the shape parameter plays a crucial role. The larger the shape parameter the more differ the RMSEs of these two estimators. Fewer observations are needed to make the ML superior to L-moments when just two parameters need to be estimated as in the case of a Gumbel distribution.

When extraordinary or rare extreme events occur in our data, the robust Minimum Distance estimator or the Trimmed L-Moments offer a smaller bias and RMSE for the higher quantile, but they have the disadvantage of having larger RMSE compared to the classical estimators ML and L-Moments, when no such extraordinary extreme events occur. This is the common lack of efficiency of many robust estimators, particularly in small samples. Among the robust estimators considered here, the TL(1,1)-Moments are preferable, since they have the smallest RMSE when extraordinary extreme events occur and not too large RMSE when there are no such events – especially for estimation of the higher quantile. Of course also other trimming is possible and could be investigated further.

Based on the results the Trimmed L-Moments with symmetric trimming seem to be a recommendable choice when extraordinary events occur in the sample. The do not have a tendency of overestimating the quantile, even if the return period of the event is much larger than the sample length. Additionally they seem to be rather efficient, which is inherited from the ordinary L-Moments. The Minimum Distance estimator considered here is apparently not efficient enough for the small sample sizes given in hydrology. If there are no obvious extraordinary events in the sample, L-Moments are recommended having the largest efficiency for the small sample sizes occurring in hydrology.

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Further hydrological phenomena can occur. Therefore, we also investigated another scenario, representing an uncertainty in the measurement of the data. We wanted to model the situation, when a rating curve does not consider the overflowing of river beds (too small discharge values are assumed) or backwater in river increases the water level (too high discharges are assumed). This was done by cutting off the 20 % highest data of the simulated distribution and replacing them by data representing an error between 0 and 30 %. For this scenario all estimators failed and were not able to cope with the misspecification of the model. Therefore, detailed results are omitted.

In the recommendations made above the question arises, how to detect extraordinary events. It is not always clear, which event is extraordinary and which is just very high. A possibility to make this choice could be the sensitivity curve. For non-robust estimators one could use this tool to examine the influence of a single event to the estimation and therefore decide, whether to use a robust estimator or not. This is beyond the scope of this work and a question of further research.

Additionally, the results give the impression that the choice of the number of parameters is crucial and should depend on the sample size and on the value of the shape parameter. The recommendations of the DWA (2012) are confirmed for the scenarios considered here but we have by far not covered all relevant cases where a two-parametric distribution function might be preferred. Since this is an important question in the estimation of flood quantiles it deserves further investigation.

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Table 1. Estimation of the 99 %- and the 99.9 %-quantile for independent, identically Gumbel (100, 10)-distributed random variables with sample size 30, 50, 100 and 200.

	Gumbel-Fitting				GEV-Fitting			
$n = 30$	99 %-quantile		99.9 %-quantile		99 %-quantile		99.9 %-quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−1.14	7.39	−1.78	10.7	0.797	18.4	9.62	77.3
L-Moments	−0.398	8.08	−0.609	11.7	0.136	14.0	4.58	35.5
TL(1,1)-Moments	−0.114	8.27	−0.139	12.1	4.14	22.3	21.2	74.8
TL(0,1)-Moments	0.418	7.77	0.626	11.2	9.86	36.9	37.7	124
MD	−0.816	8.67	−1.28	12.7	6.47	32.7	36.4	168
	Gumbel-Fitting				GEV-Fititng			
$n = 50$	99 %-quantile		99.9 %-quantile		99 %-quantile		99.9 %-quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−0.295	5.61	−0.52	8.07	−0.9	11.1	1.00	26.8
L-Moments	0.078	6.45	0.108	9.37	0.593	10.6	3.27	25.0
TL(1,1)-Moments	−0.176	6.47	0.283	9.54	3.08	16.4	13.3	48.2
TL(0,1)-Moments	0.155	5.99	0.243	8.66	3.46	21.4	13.4	57.1
MD	−0.362	6.60	−0.58	9.67	3.54	20.8	17.7	75.5
	Gumbel-Fitting				GEV-Fitting			
$n = 100$	99 %-quantile		99.9 %-quantile		99 %-quantile		99.9 %-quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−0.24	4.01	−0.39	5.76	0.355	8.11	2.39	19.2
L-Moments	0.101	4.71	0.103	6.86	0.23	7.78	1.83	18.3
TL(1,1)-Moments	0.256	4.71	0.38	6.97	0.755	10.2	4.19	26.8
TL(0,1)-Moments	0.117	4.32	0.159	6.22	1.206	14.1	5.14	33.8
MD	−0.01	4.48	−0.03	6.55	1.37	12.7	6.59	34.6
	Gumbel-Fitting				GEV-Fitting			
$n = 200$	99 %-quantile		99.9 %-quantile		99 %-quantile		99.9 %-quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−0.192	2.95	−0.31	4.24	−0.07	5.24	0.373	11.7
L-Moments	−0.061	3.24	−0.09	4.72	−0.08	5.16	0.476	11.8
TL(1,1)-Moments	0.029	3.32	0.022	4.86	0.747	7.42	2.95	18.3
TL(0,1)-Moments	0.042	3.03	0.044	4.38	1.49	9.73	4.51	21.8
MD	−0.035	3.29	−0.06	4.81	0.565	8.57	2.3	21.5

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Table 2. Estimation of the 99 %- and the 99.9 %-quantile for independent, identically Gumbel (100, 10)-distributed random variables with sample size 30, 50, 100 and 200 and simulated extreme events.

	Gumbel-Fitting				GEV-Fitting			
$n = 30$	99 %-quantile		99.9 %-quantile		99 %-quantile		99.9 %-quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	6.29	9.79	8.90	14.0	27.7	34.8	81.7	123
L-Moments	11.8	14.2	17.2	20.7	26.7	29.3	74.1	84.6
TL(1,1)-Moments	4.67	10.7	6.84	15.7	18.9	34.9	66.4	135
TL(0,1)-Moments	3.04	8.83	4.32	12.7	15.4	31.1	47.8	99.1
MD	2.80	9.75	3.96	14.3	25.1	53.2	103	315
	Gumbel-Fitting				GEV-Fitting			
$n = 50$	99 %-quantile		99.9 %-quantile		99 %-quantile		99.9 %-quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	3.81	6.89	5.37	9.84	16.2	19.9	42.7	57.0
L-Moments	6.81	8.90	10.1	13.0	15.7	18.6	41.1	50.5
TL(1,1)-Moments	3.06	7.73	4.46	11.3	10.2	22.2	31.7	70.0
TL(0,1)-Moments	2.35	6.85	3.34	9.84	8.60	20.5	25.0	58.4
MD	1.70	7.32	2.39	10.7	12.3	29.9	43.9	117
	Gumbel-Fitting				GEV-Fitting			
$n = 100$	99 %-quantile		99.9 %-quantile		99 %-quantile		99.9 %-quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	3.87	5.60	5.50	8.01	14.2	16.0	35.0	40.7
L-Moments	6.85	8.01	10.1	11.8	15.2	16.7	38.1	42.9
TL(1,1)-Moments	3.25	6.09	4.78	8.95	12.0	17.8	32.5	50.8
TL(0,1)-Moments	2.30	4.92	3.26	7.10	8.50	15.3	21.7	40.4
MD	2.14	5.33	3.05	7.75	9.54	18.3	26.6	52.5
	Gumbel-Fitting				GEV-Fitting			
$n = 200$	99 %-quantile		99.9 %-quantile		99 %-quantile		99.9 %-quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	4.04	4.96	5.78	7.09	13.9	14.8	33.0	35.7
L-Moments	6.92	7.60	10.2	11.1	14.9	15.6	36.7	39.1
TL(1,1)-Moments	3.37	4.81	4.95	7.10	11.5	14.5	29.0	38.3
TL(0,1)-Moments	2.23	3.87	3.14	5.55	8.73	12.5	20.8	31.3
MD	2.12	4.01	3.04	5.82	7.63	12.8	18.7	32.9

Table 3. Estimation of the 99 %- and the 99.9 %-quantile for independent, identically GEV (0.1, 100, 10)-distributed random variables with sample size 30, 50, 100 and 200.

	Gumbel-Fitting				GEV-Fitting			
n = 30	99 %-quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−10.6	13.8	−28.1	29.2	2.68	27.3	21.9	103
L-Moments	−7.08	13.3	−22.7	28.1	0.292	21.3	8.34	65.9
TL(1,1)-Moments	−8.74	13.3	−25.0	29.0	5.63	33.3	37.2	138
TL(0,1)-Moments	−11.0	14.0	−28.6	31.1	7.11	34.5	36.9	133
MD	−11.2	14.9	−28.7	32.1	11.6	58.3	86.5	550
	Gumbel-Fitting				GEV-Fitting			
n = 50	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−10.7	12.6	−28.1	29.7	0.939	18.2	8.94	56.0
L-Moments	−6.64	11.0	−22.0	25.5	−0.05	17.5	4.75	51.6
TL(1,1)-Moments	−9.37	12.3	−26.0	28.4	3.92	23.3	20.9	83.6
TL(0,1)-Moments	−10.9	12.9	−28.5	30.1	2.97	23.1	16.1	79.2
MD	−11.0	12.1	−28.4	29.3	6.93	34.1	39.8	167
	Gumbel-Fitting				GEV-Fitting			
n = 100	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−9.98	10.5	−27.1	27.5	0.718	12.1	4.67	33.9
L-Moments	−6.95	8.14	−22.5	23.3	−0.17	11.8	2.13	32.8
TL(1,1)-Moments	−9.19	9.93	−25.7	26.3	2.01	15.6	10.2	47.2
TL(0,1)-Moments	−10.9	11.9	−28.4	29.2	2.17	15.1	9.41	44.2
MD	−11.0	12.1	−28.4	29.3	1.84	17.9	11.1	56.7
	Gumbel-Fitting				GEV-Fitting			
n = 200	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−9.98	10.5	−27.1	27.5	0.167	8.23	1.77	21.9
L-Moments	−6.95	8.14	−22.5	23.3	0.301	8.36	1.90	22.4
TL(1,1)-Moments	−9.19	9.93	−25.7	26.3	1.06	10.3	4.83	29.9
TL(0,1)-Moments	10.2	11.6	−28.8	29.1	1.19	10.6	4.89	30.2
MD	−10.7	11.3	−28.0	28.5	2.07	12.3	8.33	36.3

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Table 4. Estimation of the 99 %- and the 99.9 %-quantile for independent, identically GEV (0.1, 100, 10)-distributed random variables with sample size 30, 50, 100 and 200 and simulated extreme events.

$n = 30$	Gumbel-Fitting				GEV-Fitting			
	99 %-quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−0.363	8.75	−13.3	18.2	46.5	60.4	172	283
L-Moments	11.0	14.9	4.02	15.1	43.8	47.4	150	168
TL(1,1)-Moments	−3.08	12.0	−16.7	23.8	31.7	56.5	135	284
TL(0,1)-Moments	−7.09	11.5	−22.9	26.3	27.0	51.6	105	221
MD	−6.92	13.1	−22.6	27.8	38.4	95.3	216	917
$n = 50$	Gumbel-Fitting				GEV-Fitting			
	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−4.12	8.19	−18.7	21.3	25.1	31.7	79.4	110
L-Moments	3.85	9.03	−6.57	13.6	27.1	30.4	85.6	98.5
TL(1,1)-Moments	−5.33	10.1	−20.0	23.6	17.3	32.6	61.9	123
TL(0,1)-Moments	−8.75	11.3	−25.3	27.3	15.3	33.0	53.2	117
MD	−8.68	11.7	−25.1	27.5	18.7	41.8	75.9	186
$n = 100$	Gumbel-Fitting				GEV-Fitting			
	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−3.98	6.28	−18.4	19.7	23.3	26.3	68.8	80.4
L-Moments	3.69	6.95	−6.80	10.9	26.2	28.4	81.3	90.1
TL(1,1)-Moments	−4.79	7.53	−19.2	21.0	18.2	26.8	59.1	92.2
TL(0,1)-Moments	−8.73	9.97	−25.3	26.2	14.7	23.6	44.3	74.0
MD	−8.46	10.2	−24.7	26.0	13.8	27.3	47.0	96.3
$n = 200$	Gumbel-Fitting				GEV-Fitting			
	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−3.71	5.10	−18.0	18.7	21.6	23.1	60.9	66.2
L-Moments	3.84	5.63	−6.59	8.92	25.1	26.2	75.6	79.6
TL(1,1)-Moments	−5.08	6.49	−19.6	20.5	18.8	23.2	58.0	74.1
TL(0,1)-Moments	−8.18	8.93	−24.5	25.0	14.5	19.7	41.4	58.2
MD	−8.37	9.29	−24.6	25.3	11.8	19.2	35.3	60.3

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Table 5. Estimation of the 99 %- and the 99.9 %-quantile for independent, identically GEV (0.2, 100, 10)-distributed random variables with sample size 30, 50, 100 and 200.

	Gumbel-Fitting				GEV-Fitting			
$n = 30$	99 %-quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−23.6	26.4	−71.8	73.8	10.5	68.1	101	1233
L-Moments	73.0	78.0	314	343	0.435	33.7	18.8	145
TL(1,1)-Moments	−21.7	25.2	−68.5	70.9	5.94	45.7	53.5	226
TL(0,1)-Moments	−26.9	28.7	−76.5	77.8	7.56	44.7	51.6	205
MD	−25.7	27.9	−74.7	76.4	23.7	82.8	172	607
	Gumbel-Fitting				GEV-Fitting			
$n = 50$	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−23.7	25.3	−72.0	73.1	2.50	28.3	22.0	111
L-Moments	43.5	49.0	173	178	0.009	24.5	9.69	93.5
TL(1,1)-Moments	−21.8	23.9	−68.7	70.0	5.03	32.7	33.9	132
TL(0,1)-Moments	−26.6	27.7	−76.0	76.8	5.69	34.7	34.74	141
MD	−26.2	27.5	−75.3	76.2	11.0	48.2	69.3	248
	Gumbel-Fitting				GEV-Fitting			
$n = 100$	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−23.3	24.1	−71.3	71.8	1.98	19.2	11.2	63.8
L-Moments	−17.2	18.9	−62.0	63.1	−0.65	17.9	2.84	60.5
TL(1,1)-Moments	−22.2	23.1	−69.1	69.8	2.74	21.8	16.5	78.5
TL(0,1)-Moments	−26.6	27.2	−76.1	76.5	2.23	22.9	14.5	79.8
MD	−25.6	26.3	−74.3	74.8	5.27	28.8	29.2	115
	Gumbel-Fitting				GEV-Fitting			
$n = 200$	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−23.2	23.6	−71.1	71.5	0.874	12.8	5.67	41.9
L-Moments	−17.1	18.1	−61.9	62.6	0.11	13.3	2.98	44.1
TL(1,1)-Moments	−22.3	22.7	−69.3	69.7	1.92	16.1	11.1	57.6
TL(0,1)-Moments	−26.6	26.9	−76.0	76.2	1.69	15.2	8.49	50.8
MD	−25.8	26.2	−74.7	74.9	2.05	18.5	11.4	65.8

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Table 6. Estimation of the 99 %- and the 99.9 %-quantile for independent, identically GEV (0.2, 100, 10)-distributed random variables with sample size 30, 50, 100 and 200 and simulated extreme events.

	Gumbel-Fitting				GEV-Fitting			
$n = 30$	99 %-quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−7.16	13.6	−47.9	50.7	84.6	122	436	1254
L-Moments	11.4	17.6	−19.8	27.9	73.0	78.0	314	343
TL(1,1)-Moments	−14.8	20.1	−58.2	61.6	51.9	95.3	292	646
TL(0,1)-Moments	−21.6	24.3	−68.9	70.7	43.4	79.5	201	419
MD	−22.0	25.1	−69.1	71.4	63.3	146	439	1504
	Gumbel-Fitting				GEV-Fitting			
$n = 50$	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−13.6	16.3	−57.2	58.6	40.6	53.8	160	239
L-Moments	0.530	11.5	−35.9	39.5	43.5	49.0	173	201
TL(1,1)-Moments	−17.5	20.2	−62.3	64.0	28.5	53.6	131	257
TL(0,1)-Moments	−23.8	25.1	−72.0	72.9	22.6	47.4	95.1	206
MD	−23.4	24.9	−71.1	72.3	28.9	65.1	145	347
	Gumbel-Fitting				GEV-Fitting			
$n = 100$	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−13.7	15.2	−57.3	58.1	36.1	42.1	131	160
L-Moments	0.290	7.97	−36.2	38.0	42.6	45.6	162	176
TL(1,1)-Moments	−16.9	18.3	−61.3	62.1	32.6	45.6	132.2	197
TL(0,1)-Moments	−22.9	23.6	−70.7	71.2	24.4	37.7	91.3	149
MD	−23.8	24.6	−71.8	72.4	20.8	39.3	84.2	165
	Gumbel-Fitting				GEV-Fitting			
$n = 200$	99 %- quantile		99.9 %- quantile		99 %- quantile		99.9 %- quantile	
Estimator	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
ML	−13.6	14.3	−57.2	57.5	34.4	37.5	119	133
L-Moments	−0.489	5.93	−37.3	38.3	41.9	43.4	157	164
TL(1,1)-Moments	−16.2	17.0	−60.3	60.7	31.8	38.1	120	151
TL(0,1)-Moments	−23.0	23.3	−70.8	71.1	23.4	30.3	81.0	109
MD	−23.6	24.1	−71.5	71.8	16.9	28.1	61.0	105

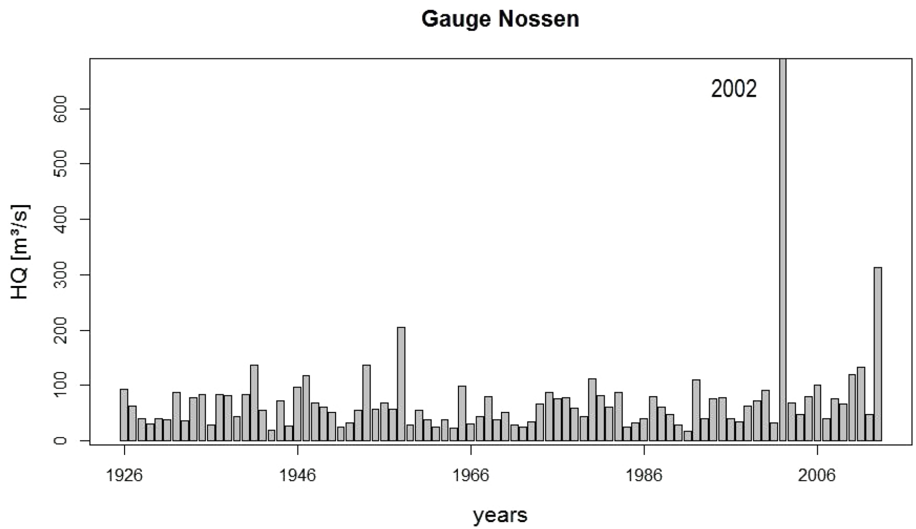

Figure 1. Annual Maxima at the gauge Nossen/Freiberger Mulde.

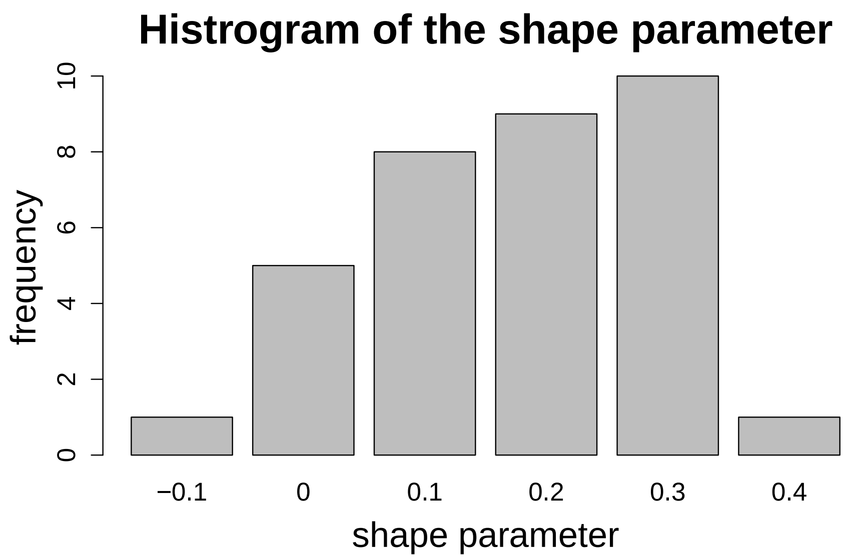

Figure 2. Histogram of the estimated shape parameter for annual maxima of three river basins.

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