We would like to thank all referees for their feedback and generally positive remarks, which helped us to improve the manuscript. Below is a point by point reply to all comment. The referee's comment is in italic and our response in normal font. The page and line numbers we mention in the response refer to the 'track and trace' version of the revised manuscript.

Reply to Referee 1

General remarks: The manuscript presents a derivation of the gradients driving evapotranspiration and runoff based on the maximum power principle and the Budyko curve. It is an interesting concept, to use the Budyko curve not as an evaluation criterion, but instead as an additional constraint in the optimisation procedure, and I think this is in principle suitable for publication. The manuscript, however, could be much more clear on the goals of the optimisation. In particular, it should be made clear that the authors do not really predict "the Budyko curve", but rather the shape of the gradient functions and the value of conductances. The Budyko curve is used as a constraint. I suggest that several parts are extended (see below) to make the study comprehensible also for readers who are not familiar with the maximum power principle.

We do agree that the goals of the study have to be rephrased. In the revised version we did this in line 74-89, stating that we "define a model which, under constant forcing, leads to a point on the asymptotes of the Budyko curve when flow conductances are optimized by maximizing power. The model is comparable to the one proposed by Porada et al. (2011), but with different relations between relative wetness of the subsurface store and driving gradients. We derived the gradients driving evaporation and runoff in an inverse manner, with both the asymptotes of the Budyko curve and the maximum power principle as constraints. Subsequently, we added dynamics in forcing or in actual evaporation (similar to Westhoff et al., 2014) to move away from these asymptotes to more realistic values of the aridity and evaporation index, without calibrating any parameter. Finally, these sensitivities were compared to observations."

Detailed comments:

p7822,l9 "...the asymptotes closely." - please add a short sentence why you did that.

Our aim was to start with a curve expressing the asymptotes of the Budyko curve and deviate from these curves by only adding dynamics in forcing or evaporation. In the original manuscript we used the formulation in Eq. (9), which, with a large n, follows the asymptotes closely. So the parameter n was only introduced because it is in one of the available mathematical expressions of the Budyko curve. In the revised version we use the expression of Wang and Tang [2014] which follows the asymptotes exactly (see Eq. 9 in the revised manuscript). Because we now use a different expression for the (asymptotes of the) Budyko curve, we do not refer explicitly to parameter n anymore.

p7822,l12 I guess it should be "sensitivity OF the model TO dry spells..."

We have rephrased this part of the abstract.

p7822,l15 This should be more specific, the Budyko curve itself is not "derived" here, it is prescribed in Eq. 9

We rephrased this with "Thus by constraining the – with the maximum power principle optimized – model with the asymptotes of the Budyko curve we were able to derive more realistic values of the aridity and evaporation index without any parameter calibration." (L11-13).

p7823,l23 "...coincidence." - could you please add one or two recent references for the debate?

We now refer to Dewar (2009).

p7826,l6 Please use "t" instead of "T" for time.

In the HESS guidelines, a capital T is used for time, so we will leave it this way as well.

p7827,l21 There should be an additional remark here that, consequently, the model ignores the influence of radiation on evapotranspiration. This is important, since some established approaches (e.g. equilibrium evapotranspiration) assume the opposite.

Thank you. We have added this remark (line 167-168)

p7828,l5 This is quite difficult to comprehend: Power is defined as flux times gradient (Eq. 3), hence I assume the authors are looking for a function "G" so that dP/dk=0 etc. The way this is described here sounds like the expression for power could take on any form. This is not correct, it is the expression for G that is assumed to be flexible, which I am ok with, and resulting from the form of G, the corresponding power is maximised. The authors should make this part more clear, maybe rearrange the equations 11-13.

We are indeed looking for a function G(h) so that $dP/dk_e = 0$, but we do that in a backward analysis, meaning that we start with the power function. We first chose a function for power depending on k_e with the constraint that this function has a maximum and that it is always above zero. Once this function is chosen, the gradients are fixed (as a function of k_e!) and will always lead to a maximum in power corresponding to a point on the (asymptotes of the) Budyko curve (assuming constant forcing). Only in the forward model we make a link between $G(k_e)$ and G(h).

p7830,l11 I would rather say, Gr is a linear function of h, that fits better to Eq. 19
In our opinion this is a matter of taste. We choose to leave the expression as it is, because it is h which is being scaled between zero and unity.

p7831,l21 The authors should shortly explain here why they did not include the parameter n in the optimisation. This would have been a real step to "move away from empiricism".
The parameter n was only introduced because, when infinitely large, lead to a point on the asymptotes of the Budyko curve. However, in the revised manuscript we use a different formula describing the asymptotes, so the parameter n is obsolete now. See also our response to the previous comment (about p7822,l9).

p7831,l22 I would like to know why the authors did not additionally use a very small value of n for the initial curve and started from there. Is it only possible for the slope of the curve to decrease and, if so, why?
The answer is indeed that when dynamics are introduced, the slopes of the curves decrease. This is because when these dynamics are introduced, the optimum k_{e*} values always tends to increase [which is consistent with the results of Westhoff et al. 2014] and therefore the aridity index as well. Also, starting from the other extreme of n = 0 results in a $k_{e*} = 0$ in Eq. (10) and (14) and subsequently $G_e(k_e)$ (Eq. 13) and power are zero.

p7834,l17 I am not sure if using a large value for n really corresponds to an "uncalibrated" Budyko model. As the authors state, a large n reflects the asymptotes of the Budyko curve, and therefore corresponds to the energetic and mass constraints of the Budyko model. An uncalibrated version of the Budyko curve would, in my opinion, rather be associated with an unknown value of n, treating n as a free parameter.
As said earlier in this response, we aimed to start from the asymptotes of the Budyko curve and only deviate from this by adding dynamics in boundary conditions. Because the asymptotes are the extremes of the Budyko curve, we do not consider it arbitrary, although we can understand the confusion: The confusion probably arises because we used an expression for the Budyko curve which only follows the asymptotes exactly for n goes to infinity. Therefore it seems that any lower value for n seems arbitrary (and thus can be seen as a calibration parameter). In the revised manuscript we tried to avoid this confusion by using a different expression to describe the asymptotes, in which no 'arbitrary' parameter is present.

Reply to Referee 2

I read this paper with a great interest. Considering Budyko curve as an optimized result is an inspiring idea. The paper is well-written and technically sound. I see great contribution of this paper to HESS. This paper is almost ready for publication except for minor corrections that authors already expressed to implement.

I have little comment to make on specific details as this paper is of high standard. Rather, I would like to make a general inquiry. As described in introduction, there have been previous attempts to investigate Budyko curve from optimization framework such as maximum entropy production. In this paper, authors used maximum power approach. It is curious how the maximum power principle works and how it is compared with other principles. Well this is beyond the scope of this paper. However, it would be very nice if authors can comment on this somewhere in the manuscript. Or it can be a subject of future study.

Again, I find this paper is of high standard. I have been picky reviewer in many occasions and it is my great pleasure to encounter such a high quality manuscript. Thank you.

Thank you for this very positive comment. To shed light on the difference between maximum power principle and maximum entropy production we can say the following: For the example of two heat reservoirs, power is given as the heat flux times the normalized temperature difference, which follows directly from the first and second laws of thermodynamics (as explained in the paper). In hydrological settings, power is often generated by water fluxes and is determined by the product of the mass flux and the potential difference $P = \partial M/\partial t(\mu_{high} - \mu_{low})$. It seems that several authors simply divided this equation by the absolute temperature and called it entropy production. Note, that in isothermal conditions (which are often assumed in these cases) maximizing power is mathematically the same as maximizing entropy production. We have added this note in the revised manuscript on lines 115-121.

Reply to Referee 3

Westhoff et al. present an analysis based on maximum entropy production principle of the Budyko curve using a steady state mass balance model and the assumption that evaporation is at its maximum when the soil is fully saturated and the soil chemical potential is zero. I find this an interesting study, though I doubt that the content warrants a research paper. To me this looks more like a technical note, even if there is comparison to observations.

We find it positive that the referee finds this an interesting study, but we do not agree that it is rather a technical note than a research paper. Derivation of the Budyko curve from an organizing principle is much more than a technical issue. It deals with a very fundamental issue, namely whether terrestrial systems operate according to thermodynamic optimality. And although we do this in a backwards analysis, where the optimality principle is used as a constraint, the fact that even a relatively simple model forced with simplified precipitation and potential evaporation dynamics compares reasonably well with observations, hints that terrestrial systems indeed operate according to thermodynamic optimality.

Below are additional more specific and general comments.

Simplifying assumptions are central to the analysis, such as the one mentioned above and h being a linear function of Gr. Perhaps, the authors could touch on possibilities to evaluate the impact of these assumptions on the results and relax them in future studies.

We believe that the assumption that evaporation is at its maximum when the soil is saturated is a very reasonable assumption: The reason for water limitation of actual evaporation is that roots cannot extract water against the strong capillary forces. As there is no water limitation in case of absent capillary forces, actual evaporation can at best be energy limited, which is expressed by assuming actual evaporation being equal to potential evaporation when the soil chemical potential is zero. Note, that this assumption is also used in many other models such as the HBV (Lindström et al., 1997), SUPERFLEX model framework (Kavetski and Fenicia 2011) or the GR4J model (Perrin et al., 2003).

In contrast, we do agree to investigate the assumption of h being a linear function of G_r (although we believe this is a reasonable assumption, since runoff is driven by gravity). In the revised manuscript we added this sensitivity analysis as supplementary material. We showed that when h is assumed to be a quadratic function of G_r the model is insensitive compared to the linear assumption. However, we also tested h as a linear function of G_e and h as a linear function of k_e. Applying these functions resulted in completely different Budyko curves which is mainly explained by the fact that a too large part of the G_r curves had to be adapted to make sure that the $G_r \geq 0$ AND monotonously increasing with h.

The authors should state clearly right at the beginning, which parameters are known (chem. pot. atmosphere and Qin, in my understanding) and which are unknown/they are solving for.

In the revised manuscript we added this at the beginning of the method section (L133-137)

In order to arrive at eqns 13 and 21, the authors need to introduce an additional equation i.e. eqn 11. This seems arbitrary to me; please comment on that. What are the reference power and conductance?

We do agree that this is a somewhat arbitrary function: but this is the very essence of a backward analysis. We chose this function since it satisfies the constraints $P_e(k_e) > 0$ for $k_e \in (0,+\infty)$ and $\partial P_e/\partial k_e = 0$ at $k_e = k_{e^*}$. We also tested the function $P_e(k_e) = P_0 \exp-((k_e - a)/k_0)^2$, but this led to two values of k_{e^*} (we added this in a footnote on page 6), which was reason to use the formulation of Eq. 11. We introduced the reference power and conductance in the formula to get the correct units. In all calculations, we have set them to unity. This is explained in L182.

Isn't it a given that if one applies eqn (9) then finds an expression for Epot for a known Qin that the results are consistent and fit the Budyko concept? In this context, what is Ge(h)? I am somehow missing a functional relationship for Ge(h) or soil chemical potential as a function of h.*

It is indeed a given that when applying Eq. 9, we end up at the Budyko curve. This is also inherent to the backwards analysis we made, which forms the basis of finding relations between h and G_e and between h and G_r.

The gradient $G_e(h^*)$ is the gradient for evaporation corresponding to the relative wetness that lead to a point at the (asymptotes of the) Budyko curve (under constant forcing!). The more general term $G_e(h)$ is introduced because we aimed to build a forward model to test sensitivities to dynamics in boundary conditions. When introducing these dynamics, we first derived the gradients assuming a Budyko curve that follows the asymptotes closely (Eq.9, with n = 20). We will better explain this in the revised manuscript (see also our reply to Referee 1).

On behalf of all authors,

Martijn Westhoff

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Does the Budyko curve reflect a maximum power state of hydrological systems? A backward analysis

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Abstract. Almost all catchments plot within a small envelope around the Budyko curve. This apparent behaviour suggests that organizing principles may play a role in the evolution of catchments. In this paper we applied the thermodynamic principle of maximum power as the organizing principle.

In a top-down approach we derived mathematical formulations of the relation between relative wetness and gradients driving runoff and evaporation for a simple one-box model. We did this in ~~such a way~~ an inverse manner such that when the conductances are optimized with the maximum power principle, the steady state behaviour of the model leads exactly to a point on the asymptotes of the Budyko curve. Subsequently~~we derived gradients that, under constant forcing~~, ~~resulted in a Budyko curve following the asymptotesclosely. With these gradients we explored the sensitivity of dry spells and dynamics in actual evaporation.~~ we added dynamics in forcing and actual evaporations, causing the Budyko curve to deviate from the asymptotes. Despite the simplicity of the model, catchment observations compare reasonably well with the Budyko curves ~~derived with~~ subject to observed dynamics in rainfall and ~~evaporation. This indicates that the~~actual evaporation. Thus by constraining the – with the maximum power principle ~~may be used (i) to derive the Budyko curve and (ii) to move away from the empiricism in free parameters present in many Budyko functions.~~ optimized – model with the asymptotes of the Budyko curve we were able to derive more realistic values of the aridity and evaporation index without any parameter calibration. Future work should focus on better representing the boundary conditions of real catchments and eventually adding more complexity to the model.

1 Introduction

In different climates, partitioning of rainwater into evaporation and runoff is different as well. Yet, when plotting the evaporation fraction against the aridity index (ratio of potential evaporation and rainfall), almost all catchments plot in a small envelope around a single empirical curve known as the Budyko curve ~~(e.g. ?)~~ (Budyko, 1974) . The fact that almost all catchments worldwide plot within this small envelope around this curve inspired several scientists to speculate whether this is due to co-evolution of climate and terrestrial catchment characteristics (e.g. Harman and Troch, 2014). Co-evolution between climate and the terrestrial system could in turn be explained by an underlying organizing principle which determines optimum system functioning (Sivapalan et al., 2003; McDonnell et al., 2007; Schaefli et al., 2011; Thompson et al., 2011; Ehret et al., 2014; Zehe et al., 2014). As hydrological processes are essentially dissipative, we suggest that thermodynamic optimality principles are deemed to be very interesting candidates.

~~Belonging to this class of principles~~ The most popular among these are the closely related principles of maximum entropy production (Kleidon and Schymanski, 2008; Kleidon, 2009; Porada et al., 2011; Wang and Bras, 2011; del Jesus et al., 2012; Westhoff and Zehe, 2013) and maximum power (Kleidon and Renner, 2013; Kleidon et al., 2013; Westhoff et al., 2014) on the one hand – both defining the optimum configuration between competing fluxes across the system boundary – and, on the other hand, minimum energy dissipation (Rinaldo et al., 1992; Rodriguez-Iturbe et al., 1992; Hergarten et al., 2014) or maximum free energy dissipation (Zehe et al., 2010, 2013), focusing on free energy dissipation associated with changes in internal state variables as a result of boundary fluxes, i.e. soil moisture and capillary potential, and a related optimum system configuration. In this research we focus on the maximum power principle.

~~With these principles, an optimum configuration between two competing fluxes can be determined. It seems therefore potentially suitable to derive the Budyko curve from such a principle, since the Budyko curve describes the competition between runoff and evaporation. This is also the aim of this study.~~

The validity and the practical value of thermodynamic optimality principles are still debated (e.g. Dewar, 2009) and the partly promising results reported in the above listed studies might be just a matter of coincidence. There is a vital search for defining rigorous tests to assess how far thermodynamic optimality principles bears and applies. The Budyko curve appears very well suited for such a test, as it condenses relative weights of the steady state water fluxes in most catchments around the world. It is thus not astonishing that there have been several attempts to reconcile the Budyko curve with thermodynamic optimality principles. For example, Porada et al. (2011) used the maximum entropy production principle to optimize the runoff conductance and evaporation conductance of a bucket model being forced with observed rainfall and potential evaporation of the 35 largest catchments in the world. The resulting modelled fluxes were plotted in the Budyko diagram and followed the curve with a similar scatter as real world catchments.

Another very interesting approach was presented by Kleidon and Renner (2013) and Kleidon et al. (2014), using the perspective of the atmosphere. They maximized power of the vertical convective motion transporting heat and moisture upwards using the Carnot limit to constrain the sensible heat flux. This motion is driven by the temperature differences between the surface and the atmosphere, while at the same time depleting this temperature gradient, leading to a maximum in power. Additionally, evaporation at the surface and condensation in the atmosphere depletes this gradient even further at the expense of more vertical moisture transport and thus more convective motion. Their approach showed some more spreading around the Budyko curve for the same 35 catchments as used in Porada et al. (2011), but they used a simpler model that has to be forced with much less observations, namely solar radiation, precipitation and surface temperature.

Very recently, Wang et al. (2015) used the maximum entropy production principle to derive directly an expression for the Budyko curve. They started from the expression of Kleidon and Schymanski (2008) and by maximizing the entropy production of the whole system they reached the expression for the Budyko curve as formulated by Wang and Tang (2014). This is an intriguing result that partly contradicts the findings of Westhoff and Zehe (2013), whose study revealed within simulations with an HBV type conceptual model, that joint optimization of overall entropy production results in optimum conductances approaching zero.

~~In this study we used a model comparable to~~ The objective of this study is to define a model which, under constant forcing, leads to a point on the asymptotes of the ~~one proposed by Porada et al. (2011) and derived the~~ Budyko curve ~~from the maximum powerhypothesis in an inverse manner. With this backward analysis we found proper~~ when flow conductances are optimized by maximizing power. The model is comparable to the one proposed by Porada et al. (2011) , but with different relations between relative ~~saturation~~ wetness of the subsurface ~~and~~ store and driving gradients. We derived the gradients driving ~~runoff and evaporation.~~

~~This backward analysis is performed for constant forcing and evaporation. Since Westhoff et al. (2014) showed mathematically that~~ evaporation and runoff in an inverse manner, with both the asymptotes of the Budyko curve and the maximum power principle as constraints. Subsequently, we added dynamics in forcing or in actual evaporation ~~may result to different optimum conductances (and sometimes even two maxima in power) we tested sensitivities to these dynamics here as well. We expect these dynamics to influence the optimum conductance k_e^* and subsequently the whole Budyko curve.~~ (similar to Westhoff et al., 2014) to move away from these asymptotes to more realistic values of the aridity and evaporation index, without calibrating any parameter. Finally, these sensitivities were compared to observations.

2 The maximum power principle

The maximum power principle implies that a system evolves in such a way that steady state fluxes across a systems boundary produce maximum power. It is directly derived from the first and the second laws of thermodynamics, and is very well explained in Kleidon and Renner (e.g. 2013). Here we give only a short description: let us start by considering a warm and a cold reservoir, which are connected to each other. The warm reservoir is forced by a constant energy input J_{in} and the cold reservoir is cooled by a heat flux J_{out}. In steady state $J_{\text{in}} = J_{\text{out}}$ and both reservoirs have a constant temperature T_{h} and T_{c}, respectively, with $T_{\text{h}} > T_{\text{c}}$. The heat flux between the two reservoirs produces entropy, which is given by:

$$\sigma = \frac{J_{\text{out}}}{T_{\text{c}}} - \frac{J_{\text{in}}}{T_{\text{h}}}. \tag{1}$$

However, instead of transferring all incoming energy to the cold reservoir, the heat gradient can also be used to perform work (to create other forms of free energy). This means that in steady state, the incoming energy flux J_{in} equals the outgoing energy flux J_{out} plus the rate of work P (which is power) performed by the system.

For given temperatures of both reservoirs, the theoretical maximum rate of work is given by the Carnot limit:

$$P_{\text{Carnot}} = J_{\text{in}} \frac{T_{\text{h}} - T_{\text{c}}}{T_{\text{h}}}. \tag{2}$$

Power is thus given as the product of a flux (in this case J_{in}) and its driving potential difference (in this case $(T_{\text{h}} - T_{\text{c}})$ scaled by T_{h}). Since the temperature of both reservoirsis also influenced by the heat flux Now we introduce an extra flux cooling the hot reservoir as a function of its temperature $J_{\text{h,out}} = f(T_{\text{h}})$. This flux is in competition with the flux $J_{\text{h-c}}$ between both reservoirs, while both reduce the temperature gradient between the two reservoirs, there exist a trade-off between the . In Eq. (2) J_{in} should then be replaced by $J_{\text{h-c}}$, while T_{h} and T_{c} are not fixed anymore, but a function of all fluxes. In this setting, there exists a flux $J_{\text{h-c}}$, maximizing power. In the extreme cases of $J_{\text{h-c}} = 0$ and $J_{\text{h-c}} \to \infty$, power is zero, while for intermediate values power is larger than zero.

In hydrological systems, power is often generated by water fluxes and is given as the product of a mass flux and the temperature difference potential difference driving this flux. Note that several authors divided this formulation by the absolute temperature, while naming it maximum entropy production: (e.g. Kleidon and Schymanski, 2008; Porada et al., 2011; Westhoff and Zehe, 2013; Westhoff et al., 2014; Kollet, 2015) . Although, these formulations are equivalent in isothermal circumstances, the here derived maximum power principle is, in our opinion, more sound. Subsequently, a maximum in power exists.

In the remainder of this article we used specific water fluxes [L T^{-1}] and potential differences $\mu_{\text{high}} - \mu_{\text{low}}$ in meter water column [L], where the flux is given as the product of a specific conductance k [T^{-1}] and the potential difference. We recognize that, in order to come to the same units as power, these formulations should be multiplied by the water density, gravitational acceleration and a cross-sectional area, but since we are looking for a maximum, and these parameters are constant, we can leave them out. We also use the word gradient for the potential difference $\mu_{\text{high}} - \mu_{\text{low}}$, where the length scale with which the difference should be divided is incorporated in the conductance. With these formulation, power is given by

$$P = k\left(\mu_{\text{high}} - \mu_{\text{low}}\right)^2 \tag{3}$$

where k is the free parameter we optimized to find a maximum in power.

3 Mathematical framework

Here we derive the model that, when conductances are optimized with the maximum power principle, always result in a point on the asymptotes of the Budyko curve independent of the value of the given constant atmospheric inputs (here rainfall and chemical potential of the atmosphere). To reach this, proper relations between relative wetness and gradients driving runoff and evaporation were derived, which is explained in the following.

3.1 Initial model setup

Our model consists of a simple reservoir being filled by rainfall Q_{in} and drained by evaporation E_{a} and runoff Q_{r}. Using the same expressions as in Kleidon and Schymanski (2008), the steady state mass balance and corresponding fluxes are expressed by

$$Q_{\text{in}} = E_{\text{a}} + Q_{\text{r}} \tag{4}$$

$$E_{\text{a}} = k_{\text{e}}\left(\mu_{\text{s}} - \mu_{\text{atm}}\right) \tag{5}$$

$$Q_{\text{r}} = k_{\text{r}}\left(\mu_{\text{s}} - \mu_{\text{r}}\right) \tag{6}$$

where μ_{s}, μ_{r} and μ_{atm} are the chemical potential of the soil, chemical potential of the free water surface of the nearest river and chemical potential of the atmosphere, while k_{e} and k_{r} are the specific conductances of evaporation and runoff. In these expressions, μ_{s} and $\mu_{\text{s}} - \mu_{\text{r}}$ are functions of the relative saturation h in the reservoir:

$$G_{\text{e}}(h) = \mu_{\text{s}}(h) \tag{7}$$

$$G_{\text{r}}(h) = \mu_{\text{s}}(h) - \mu_{\text{r}}(h) \tag{8}$$

where $G_{\text{e}}(h)$ and $G_{\text{r}}(h)$ can have any form as long as they are strictly monotonically increasing with increasing relative saturation. For example, Porada et al. (2011) used the van Genuchten model (van Genuchten, 1980) and gravitational potential to derive the chemical potential of the soil. However, here we will derive them in such a way that, under constant forcing, we end up exactly at the Budyko curve.

3.2 Backwards analysis to determine the driving gradients

3.2.1 Optimum k_e^* matching the Budyko curve

Let us first find an optimum conductance k_e^* leading to a point on the Budyko curve. We started with the following expression for the Budyko curve (e.g. Choudhury, 1999; Yang et al., 2008, although other expression can in principle be used as well) :

$$\frac{E_\mathrm{a}}{Q_\mathrm{in}} = \frac{1}{\left(1+\frac{Q_\mathrm{in}^n}{E_\mathrm{pot}^n}\right)^{1/n}}$$

asymptotes of the Budyko curve B. An expression describing these asymptotes exactly is given by (adapted from Wang and Tang, 2014) :

$$B = \frac{E_a}{Q_{in}} = \frac{1+E_{pot}/Q_{in} - \sqrt{(E_{pot}/Q_{in}-1)^2}}{2} \tag{9}$$

with E_pot being the potential evaporation. Now we make an important assumption to define E_pot: we assume that evaporation is purely described as the product of a gradient and conductance; ignoring the influence of radiation. It is assumed to be maximum when in Eqs. (5) and (8), $\mu_\mathrm{s} = 0$, meaning that the relative wetness is 1, implying no water limitation. With this assumption, potential evaporation is given by $E_\mathrm{pot} = k_\mathrm{e}^*(-\mu_\mathrm{atm})$ (note that μ_atm is always negative). Combining this equation with Eqs. (5), (7) and (9) results in:

$$k_\mathrm{e}^* = \frac{Q_\mathrm{in}}{(G_\mathrm{e}(h^*)-\mu_\mathrm{atm})\left(1+\left[\frac{Q_\mathrm{in}}{-k_\mathrm{e}^*\mu_\mathrm{atm}}\right]^n\right)^{1/n}} \frac{Q_\mathrm{in}}{(G_\mathrm{e}(h^*)-\mu_\mathrm{atm})} B(k_\mathrm{e}^*) \tag{10}$$

where h^* is the steady state relative wetness leading to a point at the Budyko curveon the asymptotes of the Budyko curve (note that this is the relative wetness occurring when $k_\mathrm{e} = k_\mathrm{e}^*$).

3.2.2 Maximum power by evaporation

As mentioned above, k_e^* should also correspond to a maximum in power by evaporation (P_e). This means that We achieved this in a backward analysis, implying that we start with defining a function $P_\mathrm{e}(k_\mathrm{e})$ should be found which is always larger than zero for $k_\mathrm{e} \in (0,+\infty)$ and where $\partial P_\mathrm{e}/\partial k_\mathrm{e} = 0$ at $k_\mathrm{e} = k_\mathrm{e}^*$. A possible function satisfying these constraints is[1]:

$$P_\mathrm{e}(k_\mathrm{e}) = k_\mathrm{e}\frac{P_0}{k_0} e^{-\left(\frac{k_\mathrm{e}-a}{k_0}\right)^2} \tag{11}$$

[1] We have also tested a the function $P_\mathrm{e}(k_\mathrm{e}) = P_0 exp\left(-((k_\mathrm{e}-a)/k_0)^2\right)$, but this led to two non-trivial solutions for k_e^*, and is thus less convenient to use than the expression in Eq. (11)

where P_0 and k_0 are the reference power $[\mathrm{L}^2\mathrm{T}^{-1}]$ and reference conductance $[\mathrm{T}^{-1}]$, ~~respectively.~~ introduced to come to the correct units. In all computations they have been set to unity. Setting the derivative to zero for $k_\mathrm{e} = k_\mathrm{e}^*$ yields:

$$\frac{\partial P_\mathrm{e}}{\partial k_\mathrm{e}} = \left(2k_\mathrm{e}^* a - 2k_\mathrm{e}^{*2} + k_0^2\right)\frac{P_0}{k_0^3} e^{-\left(\frac{k_\mathrm{e}^*-a}{k_0}\right)^2} = 0 \tag{12}$$

$$\rightarrow a = k_\mathrm{e}^* - \frac{k_0^2}{2k_\mathrm{e}^*}$$

resulting in $P_\mathrm{e}(k_\mathrm{e}) = k_\mathrm{e} P_0/k_0 e^{-((k_\mathrm{e}-k_\mathrm{e}^*)/k_0 + k_0/(2k_\mathrm{e}^*))^2}$.

Combining this expression with Eqs. (3) and (7): $P_\mathrm{e} = k_\mathrm{e}\left(G_\mathrm{e} - \mu_\mathrm{atm}\right)^2$, G_e is expressed as:

$$G_\mathrm{e}(k_\mathrm{e}) = \pm\sqrt{\frac{P_0}{k_0} e^{-\left(\frac{k_\mathrm{e}-k_\mathrm{e}^*}{k_0} + \frac{k_0}{2k_\mathrm{e}^*}\right)^2}} + \mu_\mathrm{atm}. \tag{13}$$

Since we neglect condensation ($G_\mathrm{e}(k_\mathrm{e}) - \mu_\mathrm{atm} \geq 0$), only the positive solution remains. Inserting Eq. (13) into Eq. (10) and setting $k_\mathrm{e} = k_\mathrm{e}^*$ yields:

$$k_\mathrm{e}^* = \frac{Q_\mathrm{in}}{\sqrt{\frac{P_0}{k_0} e^{-\frac{k_0^2}{4k_\mathrm{e}^{*2}}}}\left(1 + \left[\frac{Q_\mathrm{in}}{-k_\mathrm{e}^* \mu_\mathrm{atm}}\right]^n\right)^{1/n}} \quad \frac{Q_\mathrm{in}}{\sqrt{\frac{P_0}{k_0} e^{-\frac{k_0^2}{4k_\mathrm{e}^{*2}}}}} B(k_\mathrm{e}^*) \tag{14}$$

which can be solved iteratively for k_e^*.

Combining these results with the mass balance (Eqs. 4–6) yields the following expression for runoff gradient G_r as a function of k_e:

$$G_\mathrm{r}(k_\mathrm{e}) = \frac{Q_\mathrm{in}}{k_\mathrm{r}} - \frac{k_\mathrm{e}}{k_\mathrm{r}}\sqrt{\frac{P_0}{k_0} e^{-\left(\frac{k_\mathrm{e}-k_\mathrm{e}^*}{k_0} + \frac{k_0}{2k_\mathrm{e}^*}\right)^2}}. \tag{15}$$

Note that any value of k_r does lead to a point on the Budyko curve.

3.2.3 Maximum power by runoff

Although the Budyko curve does not depend on the value of k_r, an optimum k_r^* can still be found by maximizing power by runoff. For this, the similar steps as for optimizing k_e are used, where in Eqs. (11)–(13) k_e is simply replaced by k_r, resulting in a gradient for runoff as a function of k_r:

$$G_\mathrm{r}(k_\mathrm{r}) = \sqrt{\frac{P_0}{k_0} e^{-\left(\frac{k_\mathrm{r}-k_\mathrm{r}^*}{k_0} + \frac{k_0}{2k_\mathrm{r}^*}\right)^2}} \tag{16}$$

while from the mass balance (Eqs. 4–8), k_r is given by

$$k_\mathrm{r} = \frac{Q_\mathrm{in} - [G_\mathrm{e}(h) - \mu_\mathrm{atm}]}{G_\mathrm{r}(h)}. \tag{17}$$

Combining these two equations and setting k_r to k_r^* yields:

$$k_\mathrm{r}^* = \frac{Q_\mathrm{in} - k_\mathrm{e}^*\left[G_\mathrm{e}(k_\mathrm{e}^*) - \mu_\mathrm{atm}\right]}{\sqrt{\frac{P_0}{k_0} e^{-\frac{k_0^2}{4k_\mathrm{r}^{*2}}}}} \tag{18}$$

which can also be solved iteratively for k_r^*.

3.3 Forward analysis

To apply the maximum power principle in any hydrological model, the model should run until a (quasi-)steady state is reached. Within the above presented backward analysis the steady state optimum gradients are simply found by giving k_e the value of k_e^* in Eq. (13) and $k_\mathrm{r} = k_\mathrm{r}^*$ in Eq. (15).

However, when the relative wetness h evolves over time, the gradients should be resolved as a function of the relative wetness ($G_\mathrm{e} = G_\mathrm{e}(h)$ and $G_\mathrm{r} = G_\mathrm{r}(h)$). To do this, we assumed that h is a linear function of $G_\mathrm{r}(k_\mathrm{e})$ scaled between zero and unity (for sensitivities to different initial relations between relative wetness and one of the gradients see Supplement S1):

$$G_\mathrm{r}(h) = \min\left[G_\mathrm{r}(k_\mathrm{e})\right] + \left(\max\left[G_\mathrm{r}(k_\mathrm{e})\right] - \min\left[G_\mathrm{r}(k_\mathrm{e})\right]\right) h \tag{19}$$

where the maximum in $G_\mathrm{r}(k_\mathrm{e})$ occurs when the second term on the right-hand-side of Eq. (15) is zero: $\max\left[G_\mathrm{r}(k_\mathrm{e})\right] = \frac{Q_\mathrm{in}}{k_\mathrm{r}}$ and the minimum value is derived when this second term is maximum, occurring at $k_\mathrm{e} = k_\mathrm{e}^{\max} = 1/2\left(k_\mathrm{e}^* - \frac{k_0^2}{2k_\mathrm{e}^*} + \sqrt{\left(k_\mathrm{e}^* - \frac{k_0^2}{2k_\mathrm{e}^*}\right)^2 + 4}\right)$. Inserting this into Eq. (15) yields:

$$\min\left[G_\mathrm{r}(k_\mathrm{e})\right] = \frac{Q_\mathrm{in}}{k_\mathrm{r}} - \frac{k_\mathrm{e}^{\max}}{k_\mathrm{r}}\sqrt{\frac{P_0}{k_0} e^{-\left(\frac{k_\mathrm{e}^{\max} - k_\mathrm{e}^*}{k_0} + \frac{k_0}{2k_\mathrm{e}^*}\right)^2}}. \tag{20}$$

If we now plot h vs. G_e, a unique relation between the two exists (Fig. 1).

With the gradients as functions of h, the non-steady mass balance equation is written as

$$S_\mathrm{max}\frac{\mathrm{d}h}{\mathrm{d}t} = Q_\mathrm{in} - k_\mathrm{r} G_\mathrm{r}(h) - k_\mathrm{e}\left(G_\mathrm{e}(h) - \mu_\mathrm{atm}\right) \tag{21}$$

where S_max is the maximum storage depth [L] and t is time [T]. Now, the time evolution of the relative wetness can be simulated.

4 Results and discussion from forward analysis

4.1 Constant forcing

With the known relations between relative wetness and gradients driving evaporation and runoff, the forward model was run and k_e be optimized by maximizing power. With constant forcing, each value of μ_atm resulted in a point on the asymptotes of the Budyko curve (Fig. 2a~~, a value of $n = 2$ is used~~). In Fig. 2b, the time evolution of the relative wetness and both gradients are shown for an initially saturated and an initially dry state indicating that irrespective of the initial state, the forward model evolves to a steady state.

4.2 Sensitivity to dry spells

By introducing dynamics in forcing, we expected the resulting budyko curve to deviate from the ~~initial one derived with constant forcing. It therefore matters how the initial one looks like. The parameter n in Eq. (9) is the key parameter to adapt the initial curve.~~ asymptotes.

In literature, a value of $n = 2$ (and small variations around) is often used since it gives a good fit for many catchments: In fact, n is an empirical parameter often linked to catchment properties. the deviation from the asymptotes is often done by introducing a empirical parameter (e.g. Choudhury, 1999; Wang and Tang, 2014) . To move away from this empiricismfor n, we subsequently used a much larger value in order to closely follow , we start at the asymptotes of the Budyko curve. A value of $n \longrightarrow +\infty$ follows these asymptotes exactly, but for numerical reasons we used $n = 20$ (differences with $n = 10$ are minor (not shown)). Next, we Subsequently, we added dry spells and dynamics in evaporation (e.g. when trees lose their leaves the evaporative conductance k_e goes to zero) and tested how this influenced the Budyko curve.

To test sensitivities to dry spells, simple block functions were used, with either a predefined constant input or no input at all. For longer relative lengths of the dry spell, the slope of the curves becomes smaller until a maximum of $E_\mathrm{a}/Q_\mathrm{in} = 0.98$ (Fig. 3). The reason the asymptotes do not reach unity lies in the fact that already at very short dry spells a second maximum in power evolves, while the first maximum disappears quickly with increasing dry spells. This is in line with results of Westhoff et al. (2014) while also in Zehe et al. (2013) a second optimum is present. Although interesting, we leave a better exploration of this transition zone where two maxima exist for future research.

These curves were compared with data of real catchments that have a relatively stable wet period interspersed with a regular dry period. The Mupfure catchment (Zimbabwe, Savenije, 2004) with approximately seven months without rain (Fig. S1 in the S2.1 of Supplement), plots very close to the theoretical curve with the same length of the dry spell. However, catchments from the MOPEX database (Schaake et al., 2006) with clear consistent dry spells plot still far from the respective theoretical curves. This discrepancy can be partly explained by the somewhat arbitrary way the number of dry months are determined: The MOPEX catchments are filtered to have only those catchments having at least one month with a median rainfall $< 2.5\,\mathrm{mm\,month^{-1}}$ and a coefficient of variance < 0.5 for all months with a median rainfall $> 25\,\mathrm{mm\,month^{-1}}$. The final number of dry months were determined maximizing the difference between the mean monthly precipitation of the X driest months minus the mean monthly precipitation of the $1 - X$ wettest months, where $X = 1, 2 \ldots 12$.

For example, the MOPEX catchment with a four month dry spell could also be argued to have a dry spell of seven months (Fig. S1S2.1, MOPEX ID: 11222000) and similarly, the MOPEX catchment with a five month dry spell (Fig. S1S2.1, MOPEX ID: 11210500) could also be argued to have one of six months. If these “corrections” are made, the variability within the MOPEX catchments is consistent (with longer dry spells plotting more to the right), but there is still a discrepancy of one to two months, indicating that the model should still be improved.

4.3 Sensitivity to dynamics in actual evaporation

We also tested the sensitivity of dynamics in actual evaporation by periodically turning k_e on and off, while keeping the rainfall constant. This sensitivity analysis shows that the longer actual evaporation is switched off, the smaller the slope of the Budyko curve and the smaller the maximum value of the evaporation index (Fig. 4). Comparing the different curves with real catchments, shows that data from the Ourthe catchment (Belgium) is relatively close to its respective line (its months without actual evaporation are estimated from Fig. 6.1 of Aalbers, 2015). Also the MOPEX catchments plot relatively close to their respective lines. However, the way the MOPEX catchments were filtered is somewhat arbitrary (only those having a coefficient of variance < 0.12 for monthly median rainfall and with at least one month with a monthly median maximum ambient temperature $< 0\,^\circ\mathrm{C}$ are taken into account; a month is considered to have no actual evaporation if the monthly median maximum air temperature $< 0\,^\circ\mathrm{C}$; after Devlin, 1975, Fig. S2.2 of Supplement).

At first sight the comparison with data looks better than in the case of dry spells. However, all plotted catchments have an aridity index between 0.5 and 0.71 and within this range the different curves plot also close to each other. Yet, it is still somewhat surprising that the comparison is relatively good, since the modelled lines have been created by assuming a constant atmospheric demand (μ_atm) for each run, which is different from real catchments that have a more or less sinus shape potential evaporation over the year. However, we consider it as future work to better represent the real world dynamics in the model.

5 Conclusions and outlook

The Budyko curve is an empirical proof that only a subset of all possible combinations of aridity index and evaporation index emerges in nature. It belongs to the, so-called Darwinian models (Harman and Troch, 2014), focusing on emergent behaviour of a system as a whole. Since the maximum power principle links Newtonian models with the Darwinian models, it has indeed potential to derive the Budyko curve with an, in essence, Newtonian model.

We presented a top-down approach in which we derived relations between relative wetness and chemical potentials that lead, under constant forcing, to a point on the asymptotes of the Budyko curve when the maximum power principle is applied. Subsequently sensitivities to dynamics in forcing and actual evaporation were tested.

Since the Budyko curve is an empirical curve, ~~the parameter n~~ a calibration parameter is often linked to catchment specific characteristics such as land use, soil water storage, climate seasonality or spatial scales (e.g. Milly, 1994; Choudhury, 1999; Zhang et al., 2004; Potter et al., 2005). Although correlations between characteristics and ~~n~~ the calibration parameter have been found, it remains a calibration parameter.

Here ~~, to avoid an arbitrary (calibrated) value for n we used a large n, reflecting the two asymptotes of~~ we presented a method to derive the the Budyko curve ~~, and analysed deviations from this line by introducing temporal dynamics .~~ without any calibration parameter, but sensitive to temporal dynamics in boundary conditions. Although we used simple block functions to test these sensitivities they compare reasonably well with observations. Nevertheless, improvements could be made by modelling dynamics closer to reality, or even by adding multiple parallel reservoirs to account for spatial variability within a catchment.

Even though the model represents observations reasonably well (despite its simplicity), the method used here is by no means a proof that the maximum power principle does apply for hydrological systems. This is due to the top-down derivation of the gradients in which the maximum power principle is used explicitly. In principle, the method could also be used with respect to any other optimization principle. However, the reasonable fits with observations gives floor to further explore this methodology – including the maximum power principle.

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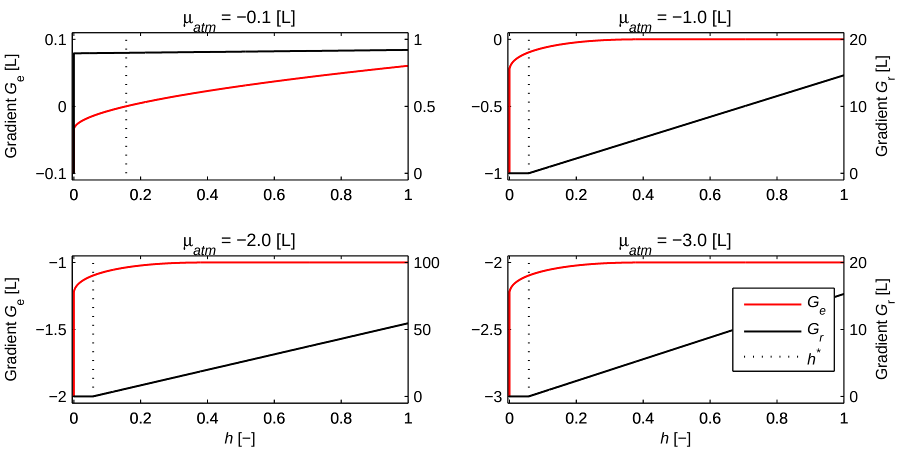

Figure 1. The gradients driving evaporation (G_e) and runoff (G_r) as a function of the relative saturation (h) for different values of μ_atm with $k_\mathrm{r} = k_\mathrm{r}^*$ and $n = 2$. At $h = 0$, the slope of the gradient G_e is vertical, while the value of G_r is set to zero to avoid runoff at zero saturation.

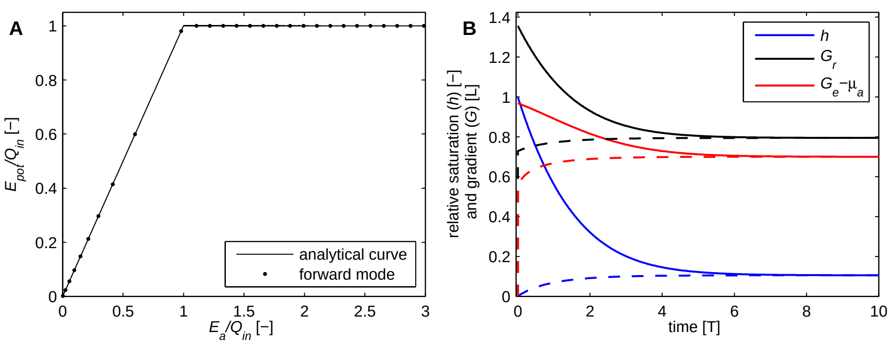

Figure 2. **(a)** Analytical Budyko curve (Eq. 9) and result from forward mode with constant forcing ~~for $n = 2$~~ and **(b)** time evolution of relative saturation and both gradients for complete initial saturation (solid lines) and initial dry state (dashed lines). ~~$\mu_\mathrm{atm} = -1$~~$\mu_\mathrm{atm} = -0.7$.

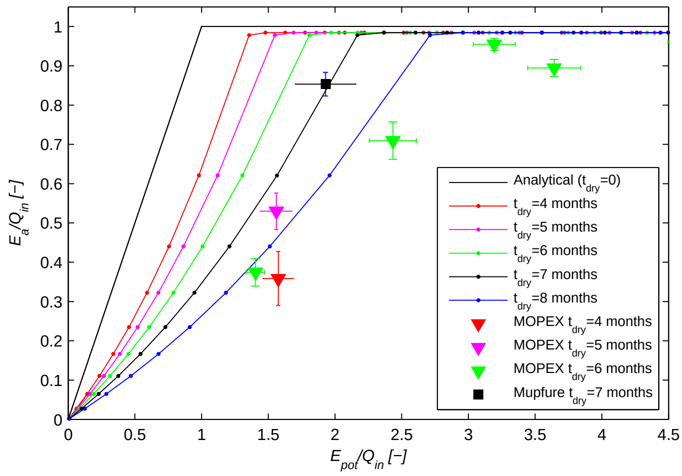

Figure 3. Sensitivity to periodic dry spells to the forward model. MOPEX catchments are filtered to have only those catchments having at least one month with a median rainfall $< 2.5\,\mathrm{mm\,month}^{-1}$ and a coefficient of variance < 0.5 for all months with a median rainfall $> 25\,\mathrm{mm\,month}^{-1}$. The final number of dry months were determined maximizing the difference between the mean monthly precipitation of the X driest months minus the mean monthly precipitation of the $1-X$ wettest months, where $X = 1, 2 \ldots 12$. Error bars indicate one standard deviation and are determined with bootstrap sampling.

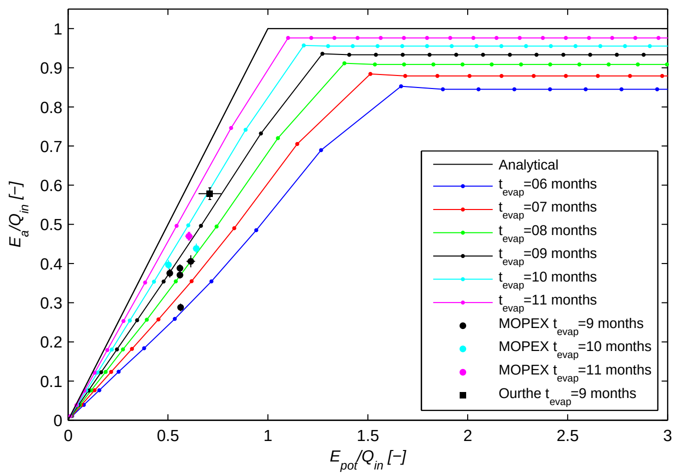

Figure 4. Sensitivity to on-off dynamics in actual evaporation to the forward model. MOPEX catchments were filtered to have only those catchments having a coefficient of variance < 0.12 for monthly median rainfall and with at least one month with a median maximum air temperature $< 0\,^{\circ}\mathrm{C}$; a month is considered to have no actual evaporation if the monthly median maximum air temperature $< 0\,^{\circ}\mathrm{C}$ (after Devlin, 1975). Error bars indicate one standard deviation and are determined with bootstrap sampling.