

A two-parameter Budyko function to represent conditions under which evapotranspiration exceeds precipitation

by P. Greve, L. Gudmundsson, B. Orlowsky, and S. I. Seneviratne

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Referee comment

Referee #2:

The authors have improved the manuscript and have implemented most of the changes suggested by this reviewer in the past two reviews.

I still have a few minor comments:

- equation 6 is still wrong. It is OK to assume this for simplicity but this should be made explicit. (Note that in my previous review I didn't use partial derivative and its conditional value because we cannot use latex in the review and not because I didn't understand equation 6 ;). As I was describing before we know that equation 6 is wrong from the complementary relationship (see work of Brutsaert), indeed plotting E_p vs ET gives a near linear slope for all E_p/ET values so even for your $y=0$!! You should acknowledge this. (And this is fine but you need to mention it).

We thank the reviewer for his evaluation of the manuscript. However, we still think that equation 6 is not wrong for several reasons. The underlying assumption of equations 3-6 is that $P-E-Q=0$ (which is maybe also a legitimate subject for debate, but lets just assume for now that this is valid at long-term mean annual time scales). Considering this, equation 6 is not a matter of available energy but of available water. And if $y=0$ and therefore $P=E$ and $Q=0$ there is simply no water left in the system to further increase E . In this case a change in E_p can not have an effect on E since there is no water left in the system. And in case E_p is smaller than P , equation 5 comes into play. Further, $P=E$ and $Q=0$ describes a very extreme case and the limit of what is possible given $P-E-Q=0$ (therefore characterizing the supply limit). We of course acknowledge all work on the complementary relationship, but it is our assessment that it does not apply to conditions where $P=E$ and $Q=0$ on mean annual time scales.

However, we would like to emphasize again that equations 3-6 are not our invention, but are based on Fu, 1981 and Zhang et al., 2004 (which is cited over 250 times). Our work applies and extends this very widely-used formulation of the Budyko framework. We therefore think that the discussion of this well-established theory in the context of the complementary relationship is outside the scope of our study. If the reviewer thinks equations 3-6 are wrong, this will not just question a substantial amount of scientific work on the Budyko framework and in particular Fu's equation of the last 10 years, but it would also question the whole Budyko hypothesis since it implies that the concept of the water supply limit is flawed. The discussion of such fundamental issues should be done in a separate paper.

We are sorry if our answer does not satisfy the reviewer, but we still don't see the necessity to add a discussion on this topic, since in our assessment this issue is also not critical for the main outcome of the study.

- I still have concerns regarding both the seasonality and steady-state assumption, which are somewhat incoherent... Maybe add some more discussion and limitations in the conclusion on this.

We added a few sentences on limitations of the framework to the conclusions.

Specific comments:

- reformulate line 6 on the budyko curve limitation: statement is too strong

We now removed "major downside" and just state that the framework is limited to steady-state conditions.

- line 11 space is missing between instantaneous and precipitation

Changed.

- line 29: You should also mention F_u 's equation and the omega parameter which gives some rational for the scatter

We added an additional sentence to this part.

- line 68 remove rigorous

Removed

- line 75: remove "transpired and" (again there are many definitions, it is different whether it is pan evaporation or lake water surface...)

Removed

- discuss equation 6

See above.

- line 234: you should mention the limits of your model: you need to characterize it regionally and it can be evolving, it is also based on a quasi steady-state assumption (which might not be true - e.g. monsoonal climate)

We added a few lines on this to the conclusions. Thanks

Referee #4:

I have been asked to evaluate the above manuscript for final publication in HESS in view of the interactive discussion.

Overall I am quite positive about the achievements of the manuscript. The first major achievement is the derivation of the new seasonal time scale Budyko framework from quite general assumptions in line of the Fu 1981 derivation. The second achievement is that this framework allows to analyze the seasonal water balance as a function of the aridity index within a general Budyko framework. While these achievements by itself should warrant publication in HESS, however, the authors did not satisfactorily address all critical comments of the reviewers. I came up with similar concerns when reading the revised version of the manuscript as the reviewers (whose comments I read afterwards). As the achievements are substantial, I believe it is sufficient to reformulate critical passages and add one or two paragraphs to the discussion. Therefore, I recommend acceptance, after implementation of the following critical points:

We sincerely thank Reviewer 4 for his thoughtful and positive evaluation of the manuscript.

a) the framework is not predictive – if I understand correctly, the critical new parameter which is defined by $y_0 = E - P / E_p$ depends on E itself – therefore it can not be determined a priori. The Budyko function or the Fu curve with the commonly accepted value for $w = 2.6$ allows to predict E from P and E_p alone. Therefore the framework allows to analyse the seasonal course of E in hindcast as shown in Figures 7-9. As I noted earlier this is in itself quite an achievement (see Fig. 9). This point needs to be stated clearly and at central position within the manuscript – abstract – section interpreting the new parameter - conclusions.

The new parameter is not directly defined by $y_0 = (E-P)/E_p$, but is defined as the minimum value of $(E-P)/E_p$ at a certain location and within a certain (long) time period. It determines the maximum amount of water besides P that is available to E and defines the new (rotated) supply limit. It is thus fixed within a considered time period and at a particular location. This means, y_0 can technically be estimated a priori, which, however, does not mean that the estimation of y_0 is trivial (like the estimation of w is also not trivial). We further clarified this now in Sec. 3 and added a few sentences on this to the conclusions.

Additionally, we already avoided to use the word “predict” throughout the previous version of the manuscript (except for introducing the original framework), in order to prevent false expectations of readers.

b) In the abstract it is being argued that runoff assessment is critical. However, only E is being determined by the framework. Please show how runoff can be determined within the framework – it might be obvious but a worked example (in Michael Roderick style) for the two examples shown in Fig. 9 would be very beneficial and would clearly help the reader how to make use of the proposed framework.

Thanks. We added a short paragraph on this to the conclusions.

c) Discuss what makes the proposed framework different from the ones cited in the introduction. This point was already remarked by the first reviewer but was not implemented.

We now added a discussion on this issue to Sec. 6. Note that in light of this comment we also removed some of the references in the introduction, since the research question of these references is somewhat similar, but more pointing towards intra- and interannual rainfall variability.

d) The second reviewer asked if y_0 is being determined for each month. In Figure 9 and 7 it seems that k and y_0 are constant. Please make this point clear.

We now clarified in Sec. 5 that k and y_0 are fixed and not determined for every individual month.

Minor comments

Figure 8b) (Original Budyko) → from the text it should be Fu 1981 curve.

Adjust the Short Summary

Changed. Thanks!

A two-parameter Budyko function to represent conditions under which evapotranspiration exceeds precipitation

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Abstract. A comprehensive assessment of the partitioning of precipitation (P) into evapotranspiration (E) and runoff (Q) is of major importance for a wide range of socio-economic sectors. For climatological averages, the Budyko framework provides a simple first order relationship to estimate water availability represented by the ratio E/P as a function of the aridity index (E_p/P , with E_p denoting potential evaporation). However, ~~a major downside of~~ the Budyko framework is ~~its limitation to steady state~~ limited to steady-state conditions, being a result of assuming negligible storage change in the land water balance. Processes leading to changes in the terrestrial water storage at any spatial and/or temporal scale are hence not represented. Here we propose an analytically derived modification of the Budyko framework including a new parameter explicitly representing additional water available to evapotranspiration besides instantaneous precipitation. The modified framework is comprehensively analyzed, showing that the additional parameter leads to a rotation of the original water supply limit. We further evaluate the new formulation in an example application at mean seasonal time scales, showing that the extended framework is able to represent conditions in which monthly to annual evapotranspiration exceeds monthly to annual precipitation.

15 1 Introduction

The Budyko framework serves as a tool to ~~predict~~ estimate mean annual water availability as a function of aridity. It is widely-used and well-established within the hydrological community, both due to its simplicity and long history, combining experience from over a century of hydrological research. Budyko (1956, 1974) derived a formulation of the function based on findings of Schreiber (1904) and Ol'Dekop (1911), but also several other formulations have been postulated, which, how-

ever, are numerically very similar (Schreiber, 1904; Ol'Dekop, 1911; Turc, 1955; Mezentsev, 1955; Pike, 1964; Fu, 1981; Choudhury, 1999; Zhang et al., 2001, 2004; Porporato et al., 2004; Yang et al., 2008; Donohue et al., 2012; Wang and Tang, 2014; Zhou et al., 2015b). Many of these formulations are empirically derived and only few are analytically determined from simple phe-
 25 nomenological assumptions (Fu, 1981; Milly, 1994; Porporato et al., 2004; Zhang et al., 2004; Yang et al., 2007; Zhou et al., 2015b). Numerous studies further assess controls determining the observed systematic scatter within the Budyko space. This scatter is, however, inherent, being also justified by the existence of free parameters within analytically-derived formulations of the Budyko curve (Fu, 1981; Choudhury, 1999; Zhang et al., 2004; Yang et al., 2007). A variety of catchment and cli-
 30 mate characteristics, such as e.g. vegetation (Zhang et al., 2001; Donohue et al., 2007; Williams et al., 2012; Li et al., 2013; Zhou et al., 2015a), seasonality characteristics (Milly, 1994; Potter et al., 2005; Gentine et al., 2012; Chen et al., 2013; Berghuijs et al., 2014), soil properties (Porporato et al., 2004; Shao et al., 2012; Donohue et al., 2012), and topographic controls (Shao et al., 2012; Xu et al., 2013) have been proposed to exert a certain influence on the scatter within the Budyko space. Also
 35 more complex approaches to combine various controls (Milly, 1994; Gentine et al., 2012; Donohue et al., 2012; Xu et al., 2013) have been considered. Nonetheless, until present no conclusive statement on controls determining the scatter within the Budyko space has been made. In a recent assessment, Greve et al. (2015) further suggested a probabilistic Budyko framework by assuming that the combined influence of all possible controls is actually nondeterministic and follows a prob-
 40 ability distribution instead.

In this study we make use of the formulation introduced by Fu (1981) and Zhang et al. (2004). They derived a functional form between E/P and $\Phi = E_p/P$ at mean annual catchment scales analytically from simple physical assumptions,

$$\frac{E}{P} = 1 + \Phi - (1 + (\Phi)^\omega)^{\frac{1}{\omega}}, \quad (1)$$

45 where ω is a free model parameter. The original formulation introduced by Budyko (1956, 1974) is best represented by setting $\omega = 2.6$ (Zhang et al., 2004). The obtained function is subject to two physical constraints constituting both the water demand and supply limits. The water demand limit represents E being limited by E_p , whereas the water supply limit determines E to be limited by P (see Fig. 1). To maintain the supply limit, steady-state conditions are required. Therefore, the storage
 50 term (dS/dt) in the land water balance equation at catchment scales

$$\frac{dS}{dt} = P - E - Q \quad (2)$$

is assumed to be zero, which is generally a valid assumption at mean annual scales. It is further important to note that groundwater flow is not included in equation 2 and neglected throughout

the following analysis. However, the assumption of negligible storage changes constitutes a major
55 limitation to the original Budyko framework. As a consequence, the framework is not valid under
conditions in which additional storage water besides instantaneous P is available to E and $E > P$.
We note here that by instantaneous P (from here on just referred to as P) we mean all P within the
considered time interval. Conditions under which the framework is not valid can occur e.g. at sub-
60 annual or inter-annual time scales due to changes in terrestrial water storage terms such as soil mois-
(Jaramillo and Destouni, 2014), human interventions (Milly et al., 2008) or phase changes of water
within the system or supplied through precipitation (Jaramillo and Destouni, 2014; Berghuijs et al.,
2014). Also long-term changes in soil moisture may happen, e.g. under transient climate change
(Wang, 2005; Orłowsky and Seneviratne, 2013). Only few assessments addressed this limitation and
65 provided further insights on how the Budyko hypothesis could be extended to conditions under which
 E exceeds P (Milly, 1993; Potter and Zhang, 2007; Zhang et al., 2008; Zarnado et al., 2012; Chen et al., 2013) (Zhang et al., 2008;
Nonetheless, so far a theoretical ~~rigorous~~ incorporation of conditions in which $E > P$ into the
Budyko framework is missing. Here we aim to address this issue by analytically deriving a new,
modified Budyko formulation from basic phenomenological assumptions by using the approach of
70 Fu (1981) and Zhang et al. (2004).

2 Deriving a modified formulation

2.1 Preliminary Assumptions

In the following we will make use of the concept of potential evapotranspiration, which provides an
estimate of the amount of water that would be ~~transpired and~~ evaporated under conditions of a well-
75 watered surface. Fu (1981) and Zhang et al. (2004) suggested that for a given potential evaporation,
the rate of change in evapotranspiration as a function of the rate of change in precipitation ($\partial E / \partial P$)
increases with residual potential evaporation ($E_p - E$) and decreases with precipitation. Similar
assumptions were made regarding the rate of change in evapotranspiration as a function of the rate
of change in potential evaporation ($\partial E / \partial E_p$) by considering residual precipitation ($P - E$). Hence,
80 both ratios can be written as

$$\frac{\partial E}{\partial P} = f(x) \tag{3a}$$

$$\frac{\partial E}{\partial E_p} = g(y) \tag{3b}$$

with

$$x = \frac{E_p - E}{P} \quad (4a)$$

$$85 \quad y = \frac{P - E}{E_p} \quad (4b)$$

Considering E_p being a natural constraint of E , it follows that

$$\left. \frac{\partial E}{\partial P} \right|_{x=0} = 0. \quad (5)$$

The original approach of Fu (1981) further assumes that P is a natural constraint of E , constituting the following boundary condition

$$90 \quad \left. \frac{\partial E}{\partial E_p} \right|_{y=0} = 0. \quad (6)$$

The coupled boundary conditions 5 and 6 mathematically represent the supply and demand limit of the Budyko framework (see Fig. 1). Considering the definitions of x and y given by equation 4, $x = 0$ yields that $E = E_p$ and $y = 0$ yields $E = P$. Equation 5 thus states that conditional on $x = 0$, i.e. $E = E_p$, no further change in E occurs no matter how P changes, since E is already limited
 95 by E_p (constituting the demand limit). Equation 6 states that conditional on $y = 0$, i.e. $E = P$, no further change in E occurs no matter how E_p changes, since E is already limited by P (constituting the supply limit). In case $x \neq 0$ or $y \neq 0$, the gradients $\partial E/\partial P$ or $\partial E/\partial E_p$ are not (necessarily) zero.

The boundary condition 6 further requires steady-state conditions and is consequently considered
 100 to be valid at mean annual catchment scales (such that $P - E \geq 0$) only. However, as mentioned in the introduction, a wealth of possible mechanisms and processes can induce conditions in which E exceeds P . In such cases, E_p remains the only constraint of E . Consequently, since we explicitly aim to account for conditions of $E \geq P$, the value $y = (P - E)/E_p$ (see equation 4) is not necessarily positive, but larger than -1 since we assume that $E \leq E_p$. The minimum value of y , denoted as
 105 y_{min} , thus lies within the interval between -1 and 0 and depends on the additional amount of water being available for E besides water supplied by P . For convenience we define $y_0 = -y_{min}$ (and thus $y_0 \in [0, 1]$). As a consequence the boundary condition 6 is then redefined as

$$\left. \frac{\partial E}{\partial E_p} \right|_{-y_0} = 0. \quad (7)$$

2.2 Solution

110 Solving the system of the differential equations 3a,b using boundary condition 5 and the new condition 7 yields the following solution (details are provided in Appendix A):

$$E = E_p + P - ((1 - y_0)^{\kappa-1} E_p^\kappa + P^\kappa)^{\frac{1}{\kappa}} \quad (8)$$

with κ being a free model parameter. It follows

$$\frac{E}{P} = F(\Phi, \kappa, y_0) = 1 + \Phi - (1 + (1 - y_0)^{\kappa-1} (\Phi)^\kappa)^{\frac{1}{\kappa}}. \quad (9)$$

115 Similar to the traditional Budyko approach a free model parameter (named κ to avoid confusion with the traditional ω) is obtained. The second parameter y_0 , as introduced in the previous section, is directly related to the new boundary condition. Hence, in contrast to κ , which is a mathematical constant, y_0 has a physical interpretation as it accounts for additional water (i.e. storage water). However, similar to the ω -parameter in Fu's equation, κ can be interpreted as an integrator of the
120 variety of catchment-properties factors other than the aridity index that influence the partitioning of P into Q and E.

3 Characteristics of the modified framework

The newly derived formulation given (equation 9) is similar to the classical solution (equation 1), but includes y_0 as a new parameter. Assuming e.g. For different values of y_0 and $\kappa = 2.6$ (corresponding
125 to the best fit to the original Budyko function with $\omega = 2.6$ in Fu's equation) and example-values of y_0 -values, Fig. 2 shows a set of curves providing insights on the basic characteristics of the modified equation.

In case $y_0 = 0$ (being the original boundary condition), the obtained curve corresponds to the steady-state framework of Fu (1981) and Zhang et al. (2004). This shows that both model formulations are consistently transferable. If $y_0 > 0$, the supply limit is systematically exceeded. The exceedance of the supply limit increases with increasing y_0 . If $y_0 = 1$, the curve follows the demand limit. All curves are further continuous and strictly increasing.

Taking a closer look at the underlying boundary conditions and definitions (see section 2.1) reveals that y_0 explicitly accounts for the maximum amount of additional water (besides water supplied
135 through P) available for at a certain location and within a certain time period that is available to E . Since y_{min} is defined to be the minimum of $y = (P - E)/E_p$, the quantity $y_0 = -y_{min}$ physically represents the maximum fraction of E relative to E_p , which is not originating from P . A larger fraction consequently results in higher y_0 -values and thus in a stronger exceedance of the original supply limit. Further details on y_0 is-are provided in section 4.

140 The sensitivity partial derivative $\partial F(\Phi, \kappa, y_0)/\partial \Phi$ under varying κ and for three preselected values of y_0 is illustrated in Fig. 3. The sensitivity $\partial F(\Phi, \kappa, y_0)/\partial \Phi$ for different values of y_0 and κ shows the effect of the parameter choice on changes in E/P relative to changes in Φ . In general, the sensitivity is largest for small Φ (humid conditions), due to the fact that changes in E/P basically follow the demand limit (resulting in a sensitivity close to 1) regardless of parameter set (κ, y_0) . For
 145 different parameter settings, the sensitivity generally decreases with increasing Φ . For small values of y_0 (close to zero), sensitivity becomes smallest with increasing Φ , since small values of y_0 indicate conditions similar to the classical solution (equation 1). Further, the smallest sensitivity is reached for large values of κ . Large values of y_0 (close to 1) indicate conditions mainly constrained by the demand limit, thus implying a sensitivity close to 1.

150 A similar analysis is performed for varying values of κ under three preselected levels of y_0 (see Fig. 4). For $y_0 = 0$ (steady-state conditions), the sensitivity $\partial F/\partial \Phi$ is under humid conditions ($\Phi < 1$) rather large, since changes in E/P are mainly constrained by demand limit. This especially applies for large values of κ . Under more arid conditions ($\Phi > 1$), the Budyko curve slowly converges towards the (horizontal) supply limit, resulting in a near-zero sensitivity. For $y_0 = 0.2$,
 155 denoting conditions relatively similar to steady-state conditions, the decrease in sensitivity with increasing Φ is weaker, whereas for $y_0 = 0.8$, denoting conditions where E is mainly constraint by the demand limit, sensitivity is large for large κ -values and decreases rather slowly with increasing Φ .

4 Interpreting the new parameter y_0

The new parameter y_0 is, in contrast to κ , physically well defined. The combination of equation 4b
 160 and 7 shows that y_0 is explicitly related to the amount of additional water (besides water supplied through P), which is available to E . If we rewrite equation 4b with respect to y_0

$$y_0 = -y_{min} = -\left(\frac{P-E}{E_p}\right)_{min} \leq -\frac{P_{min}-E_{max}}{E_p}, \quad \text{if } P_{min}-E_{max} < 0, \quad (10)$$

where P_{min} and E_{max} are chosen in order to minimize y_{min} for a given E_p , we obtain a linear equation in terms of aridity index

$$165 \left(\frac{E}{P}\right)_{max} = y_0 \left(\frac{E_p}{P_{min}}\right) + 1, \quad (11)$$

which constitutes the mathematical interpretation of y_0 within the modified framework. That is, that y_0 determines the maximum slope of the upper limit, against which the obtained curve from equation 9 asymptotically converges to if $\kappa \rightarrow \infty$ (see Fig. 5). Physically, y_0 determines the maximum E/P that is reached in relation to Φ within a certain time period and spatial domain. It thus
 170 represents an estimate of the maximum amount of additional water that contributes to E and originates from other sources than P . Technically speaking, y_0 determines the slope of the upper limit

such that all possible pairs $(\Phi, E/P)$ are just below the line $y_0\Phi + 1$. It is further important to note that for mean annual conditions ($P - E \geq 0$), $y_0 = 0$ is considered, which results in a zero slope and thus determines the original supply limit of 1. Please also note, that this approach is not valid if

175 $P_{min} = 0$.

However, the actual slope m of the upper limit is smaller than y_0 , but directly related to both y_0 and κ as follows (see Appendix B for more information)

$$m = 1 - (1 - y_0)^{1 - \frac{1}{\kappa}}. \quad (12)$$

The relative difference between the maximum slope y_0 and the actual slope m of the upper limit
180 (being the ratio of y_0/m) is thus determined following the relationship

$$\frac{y_0}{m} = (1 - y_0)^{1/\kappa}. \quad (13)$$

The ratio y_0/m as a function of both y_0 and κ is illustrated in Fig. 6. For small κ and large y_0 (close to 1), the difference between the actual slope m and the maximum slope y_0 is large, whereas for large κ the actual slope m converges towards y_0 . However, in any case, y_0 determines the maximum
185 overshoot allowed with respect to the original supply limit at $y_0 = 0$.

5 Example application: Seasonal carryover effects in terrestrial water storage

At monthly time scales, changes in terrestrial water storage (due to changes in water storage components such as soil moisture, snow or groundwater) potentially play an important role in E and Q and are by no means negligible. Such changes can provide a significant source of additional water
190 that is (besides P) available to E . Here we analyse the climatological mean seasonal cycle of E/P by using gridded, monthly data estimates of P , E and E_p . This allows us to evaluate the capability of the obtained framework (given by equation 9) to represent additional water sources at such time scales.

We employ the following, well-established, gridded data products: (i) the Global Precipitation
195 Climatology Project (GPCP) P dataset (Adler et al., 2003), (ii) an E_p estimate (Sheffield et al., 2006, 2012) based on the Penman-Monteith E_p algorithm (Monteith, 1965; Sheffield et al., 2012) with the stomatal conductance set to zero and aerodynamic resistance defined after (Maidment, 1992), and (iii) the LandFlux-Eval E dataset (Mueller et al., 2013). All data is bilinearly interpolated to a unified 1° -grid and the mean seasonal cycle for the 1990-2000 period is calculated at gridpoint-scale. Please
200 note that the combination of datasets used here is arbitrary and only used to illustrate the capability of the newly developed framework to represent the climatological mean annual cycle of E/P .

We estimate the parameter set (κ, y_0) from equation 9 by minimizing the residual sum of squares (see Fig. 7). This means that at every gridpoint 12 monthly climatologies of E/P (representing the mean seasonal cycle of E/P) are used to determine ~~a one~~ specific parameter set (for all months).

205 To evaluate the modified framework, the derived parameter sets at each gridpoint are used in equation 9 to compute mean seasonal cycles of E/P . The correlation between the computed and the observed seasonal cycle is shown in Fig. 8a. The correlations are relatively large in most regions. Largest correlations (>0.9) are found in most mid to high latitude and tropical areas, clearly showing the capability of the modified formulation to represent the seasonal cycle in E/P . Correlations are
210 generally somewhat lower in drier regions, especially in parts of Africa and Central Asia, probably occurring due to more complex seasonal patterns in E/P and phenology, which is not considered here. Using instead Fu's original equation (or setting $y_0 = 0$) to estimate the mean seasonal cycle of E/P shows overall lower correlations, especially in semi-arid regions (Fig. 8b).

Taking a closer look at the mean seasonal cycle for example gridpoints in (i) Central Europe
215 (humid climate) and (ii) Africa (semi-arid climate) clearly shows the improvement gained through the use of the modified formulation (Fig. 9). In Central Europe, additional water is available in the early summer months due to e.g. depletion of soil moisture or snow melt, resulting in values of E/P exceeding the original supply limit. The modified formulation has the ability to represent this exceedance, whereas the original formulation is naturally bounded to 1. This is even more evident
220 for the example grid point in Africa, showing a large overshoot of the original supply limit under dry season conditions.

6 Conclusions

In conclusion we present an extension to the Budyko framework that explicitly accounts for conditions under which E is also driven by other water sources than P (i.e. changes in water storage).
225 The original Budyko framework is limited to mean annual catchment scales that constitute P and E_p to be natural constraints of E . Here we assume that the boundary condition constituted by E_p remains overall valid, whereas the boundary condition constituted by P is also subject to additional water stemming from other sources. Such additional water could e.g. originate from changes in the terrestrial water storage, landscape changes and human interventions.

230 In order to account for such additional water, we modified the set of equations underlying the derivation of Fu's equation (Fu, 1981; Zhang et al., 2004) and obtained a similar formulation including an additional parameter. The additional parameter is physically well defined and technically rotates the original supply limit upwards. Similar to the original Budyko framework, the derived two-parameter Budyko model represents the influence of first-order controls (namely P and E_p) on water
235 availability. The integrated influence of second-order controls (like e.g. vegetation, topography, etc.) is, comparable to Fu's equation, represented by the first parameter. Analyzing such controls in Fu's

formula was subject to numerous studies, but no conclusive assessment was conducted until present. Assessing the combined influence of climatic and catchment controls is hence clearly beyond the scope of this study. However, the additional second parameter of the modified formulation y_0 does
 240 have a clear physical interpretation as it represents a measure of the maximum amount of additional water being (besides P) available to E at a certain location and within a particular time period.

The Besides this study, a limited number of previous studies assessed the Budyko hypothesis under conditions of E exceeding P , especially at seasonal time scales. In a top-down approach, Zhang et al. (2008) was showing that the Budyko model has to be extended in order to model the
 245 water balance on shorter than mean annual time scales. Their extended Budyko model (which was also based on Fu (1981)) was showing good performance in modeling monthly Q , but includes four additional parameters that require extensive calibration. Chen et al. (2013) further introduced an approach (referring to Wang (2012)) that is based on replacing P by effective precipitation, which is the difference between P and soil water storage change. This allows to extend the framework to
 250 seasonal time scales, but requires explicit knowledge of changes in the soil water storage. In our approach we, however, provide an analytical derivation of an extension to Fu's equation that is able to account for conditions under which E exceeds P by including only one additional parameter. However, the framework is also subject to some limitations. The estimation of the parameter y_0 is, similar to the estimation of the ω in Fu's equation (Fu, 1981), nontrivial and the parameter
 255 apparently varies in space and potentially also in time, therefore questioning steady-state assumptions. The framework is further not capable to directly estimate Q . Since in contrast to the original Budyko framework changes in terrestrial water storage are not negligible, the runoff ratio Q/P can not be assessed through $1 - E/P$. Hence, explicit knowledge of changes in the terrestrial water storage is required, therefore aggravating assessments of Q .

260 The new framework was validated for the special case of average seasonal changes in water storage by using monthly climatologies of global, gridded standard estimates of P , E and E_p . The computed gridpoint-specific seasonal cycle of E/P using the modified framework did adequately represent mean seasonal storage changes for many parts of the world. However, the application of the modified framework is by no means limited to this case and could be extended to a variety of
 265 climatic conditions under which additional water besides P is available to E .

Appendix A: Complete Solution

Equations 3, 5 and 7 form a system of differential equations. A necessary condition to solve this system is

$$\frac{\partial f(x)}{\partial E_p} + \frac{\partial f(x)}{\partial E} g(y) = \frac{\partial g(y)}{\partial P} + \frac{\partial g(y)}{\partial E} f(x) \quad (\text{A1})$$

270 Combining equation A1 with equation 4 yields

$$\frac{\partial f(x)}{\partial E_p} = \frac{\partial f(x)}{\partial E_p} \frac{\partial x}{\partial x} = \frac{1}{P} \left(1 - \frac{\partial E}{\partial E_p}\right) \frac{\partial f(x)}{\partial x} = \frac{1}{P} (1 - g(y)) \frac{\partial f(x)}{\partial x} \quad (\text{A2a})$$

$$\frac{\partial f(x)}{\partial E} = \frac{\partial f(x)}{\partial E} \frac{\partial x}{\partial x} = \frac{1}{P} \left(\frac{\partial E_p}{\partial E} - 1\right) \frac{\partial f(x)}{\partial x} = \frac{1}{P} \left(\frac{1}{g(y)} - 1\right) \frac{\partial f(x)}{\partial x} \quad (\text{A2b})$$

$$\frac{\partial g(y)}{\partial P} = \frac{\partial g(y)}{\partial P} \frac{\partial y}{\partial y} = \frac{1}{E_p} \left(1 - \frac{\partial E}{\partial P}\right) \frac{\partial g(y)}{\partial y} = \frac{1}{E_p} (1 - f(x)) \frac{\partial g(y)}{\partial y} \quad (\text{A2c})$$

$$\frac{\partial g(y)}{\partial E} = \frac{\partial g(y)}{\partial E} \frac{\partial y}{\partial y} = \frac{1}{E_p} \left(\frac{\partial P}{\partial E} - 1\right) \frac{\partial g(y)}{\partial y} = \frac{1}{E_p} \left(\frac{1}{f(x)} - 1\right) \frac{\partial g(y)}{\partial y} \quad (\text{A2d})$$

275 Substituting the factors in equation A1 with those given in equations A2 gives:

$$\begin{aligned} \frac{\partial f(x)}{\partial x} \left((1 - g(y)) + \left(\frac{1}{g(y)} - 1\right) g(y) \right) &= \frac{P}{E_p} \frac{\partial g(y)}{\partial y} \left((1 - f(x)) + \left(\frac{1}{f(x)} - 1\right) f(x) \right) \\ (1 - g(y)) \frac{\partial f(x)}{\partial x} &= \frac{P}{E_p} (1 - f(x)) \frac{\partial g(y)}{\partial y} \end{aligned} \quad (\text{A3})$$

Expanding P/E_p yields under consideration of equations 4

$$280 \quad \frac{P}{E_p} = \frac{\frac{E_p + P - E}{E_p}}{\frac{E_p + P - E}{P}} = \frac{1 + \frac{P - E}{E_p}}{1 + \frac{E_p - E}{P}} = \frac{1 + y}{1 + x} \quad (\text{A4})$$

From equation A3 and equation A4 follows

$$\begin{aligned} (1 - g(y)) \frac{\partial f(x)}{\partial x} &= \frac{1 + y}{1 + x} (1 - f(x)) \frac{\partial g(y)}{\partial y} \\ \frac{1 + x}{1 - f(x)} \frac{\partial f(x)}{\partial x} &= \frac{1 + y}{1 - g(y)} \frac{\partial g(y)}{\partial y} \end{aligned} \quad (\text{A5})$$

285 where each side is a function of x or y only. Assuming the result of each side is α it follows

$$\frac{1 + x}{1 - f(x)} \frac{\partial f(x)}{\partial x} = \alpha \quad (\text{A6a})$$

$$\frac{1 + y}{1 - g(y)} \frac{\partial g(y)}{\partial y} = \alpha \quad (\text{A6b})$$

Integrating equation A6a under consideration of the boundary condition given by equation 5 leads to the following expression for $f(x)$

$$\begin{aligned}
290 \quad & \int_0^x \frac{1}{1-f(t)} \frac{\partial f(t)}{\partial t} dt = \alpha \int_0^x \frac{1}{1-t} dt \\
& [-\ln(1-f(t))]_0^x = \alpha [\ln(1+t)]_0^x \\
& \ln(1-f(x)) = -\alpha \ln(1+x) \\
& 1-f(x) = (1+x)^{-\alpha} \\
295 \quad & f(x) = 1 - (1+x)^{-\alpha} \tag{A7}
\end{aligned}$$

Integrating equation A6b is different from the traditional solution given in Zhang et al. (2004), as we are using the new boundary condition given by equation 7

$$\begin{aligned}
& \int_{-y_0}^y \frac{1}{1-g(t)} \frac{\partial g(t)}{\partial t} dt = \alpha \int_{-y_0}^y \frac{1}{1-t} dt \\
& [-\ln(1-g(t))]_{-y_0}^y = \alpha [\ln(1+t)]_{-y_0}^y \\
300 \quad & \ln(1-g(y)) - \ln(1-g(-y_0)) = \alpha (\ln(1-y_0) - \ln(1+y)) \\
& \ln(1-g(y)) = \alpha \ln \left(\frac{1-y_0}{1+y} \right) \\
& 1-g(y) = \left(\frac{1-y_0}{1+y} \right)^\alpha \\
& g(y) = 1 - \left(\frac{1-y_0}{1+y} \right)^\alpha \tag{A8}
\end{aligned}$$

305 Considering the expansion from equation A4 finally gives

$$\partial E / \partial P = 1 - (1+x)^{-\alpha} = 1 - \left(\frac{P}{E_p + P - E} \right)^\alpha \tag{A9}$$

$$\partial E / \partial E_0 = 1 - (1-y_0)^\alpha (1+y)^{-\alpha} = 1 - (1-y_0)^\alpha \left(\frac{E_0}{E_0 + P - E} \right)^\alpha \tag{A10}$$

In the next step, equation A9 is integrated over P . As equation A9 is identical to those in Zhang et al. (2004), we follow their substitution approach. It follows

$$310 \quad E = E_0 + P - (k + P^{\alpha+1})^{\frac{1}{\alpha+1}} \tag{A11}$$

where k is a function of E_0 only. Differentiate equation A11 with respect to E_0 gives an estimate of $\partial E/\partial E_0$, which used with equation A10 determines k

$$\frac{\partial E}{\partial E_0} = 1 - \frac{1}{\alpha + 1} (k + P^{\alpha+1})^{-\frac{\alpha}{\alpha+1}} \frac{\partial k}{\partial E_0} = 1 - (1 - y_0)^\alpha \left(\frac{E_0}{E_0 + P - E} \right)^\alpha \quad (\text{A12})$$

This leads under consideration of equation A11 to the following expression

$$\begin{aligned} 315 \quad \frac{\partial k}{\partial E_0} &= (\alpha + 1)(1 - y_0)^\alpha \left(\frac{E_0}{E_0 + P - E} \right)^\alpha (k + P^{\alpha+1})^{\frac{\alpha}{\alpha+1}} \\ &= (\alpha + 1)(1 - y_0)^\alpha \left(\frac{E_0}{E_0 + P - (E_0 + P - (k + P^{\alpha+1})^{\frac{1}{\alpha+1}})} \right)^\alpha (k + P^{\alpha+1})^{\frac{\alpha}{\alpha+1}} \\ &= (\alpha + 1)(1 - y_0)^\alpha E_0^\alpha \\ k &= (\alpha + 1)(1 - y_0)^\alpha \int E_0^\alpha dE_0 \\ 320 \quad k &= (1 - y_0)^\alpha E_0^{\alpha+1} + C \end{aligned} \quad (\text{A13})$$

with C being an integration constant. Substituting equation A13 back into equation A11, one obtains the following expression

$$E = E_0 + P - ((1 - y_0)^\alpha E_0^{\alpha+1} + C + P^{\alpha+1})^{\frac{1}{\alpha+1}} \quad (\text{A14})$$

and as $\lim_{P \rightarrow 0} E = 0$ follows $C = 0$. Substituting $\kappa = \alpha + 1$ finally gives

$$325 \quad E = E_p + P - ((1 - y_0)^{\kappa-1} E_p^\kappa + P^\kappa)^{\frac{1}{\kappa}} \quad (\text{A15})$$

and further provides by writing $\Phi = E_p/P$

$$\frac{E}{P} = 1 + \Phi - (1 + (1 - y_0)^{\kappa-1} (\Phi)^\kappa)^{\frac{1}{\kappa}} \quad (\text{A16})$$

$$F\left(\frac{E}{E_p}, \kappa, y_0\right) = \frac{E}{E_p} = 1 + \frac{P}{E_p} - \left((1 - y_0)^{\kappa-1} + \left(\frac{P}{E_p}\right)^\kappa \right)^{\frac{1}{\kappa}} \quad (\text{A17})$$

Appendix B: Solution of the actual slope

330 The actual slope m of the upper limit against which the obtained Budyko curve is converging to is smaller than y_0 . We introduced equation 12 to calculate m and in the following we provide the complete solution in order to obtain equation 12.

The value of m is the slope of the linear function $m\Phi + 1$ that forms the asymptote to $F(\Phi, \kappa, y_0)$ given by equation 9. Hence,

$$335 \quad \lim_{\Phi \rightarrow \infty} [F(\Phi, \kappa, y_0) - (m\Phi + 1)] = 0. \quad (\text{B1})$$

Using equation 9 and dividing by Φ yields

$$\lim_{\Phi \rightarrow \infty} \left[\frac{(1 + (1 - y_0)^{\kappa-1} (\Phi)^\kappa)^{\frac{1}{\kappa}}}{\Phi} + 1 - m \right] = 0. \quad (\text{B2})$$

By raising the term in brackets to the power of κ one obtains

$$\lim_{\Phi \rightarrow \infty} [(1 - m)^\kappa - \Phi^{-\kappa} (1 + \Phi^\kappa (1 - y_0)^{\kappa-1})] = 0, \quad (\text{B3})$$

340 and it follows

$$\lim_{\Phi \rightarrow \infty} [(1 - m)^\kappa - (1 - y_0)^{\kappa-1} - \Phi^{-\kappa}] = 0. \quad (\text{B4})$$

Since $\Phi^{-\kappa} \rightarrow 0$ for $\Phi \rightarrow \infty$ we obtain

$$(1 - m)^\kappa = (1 - y_0)^{\kappa-1}. \quad (\text{B5})$$

Solving for m yields

$$345 \quad m = (1 - y_0)^{1 - \frac{1}{\kappa}}. \quad (\text{B6})$$

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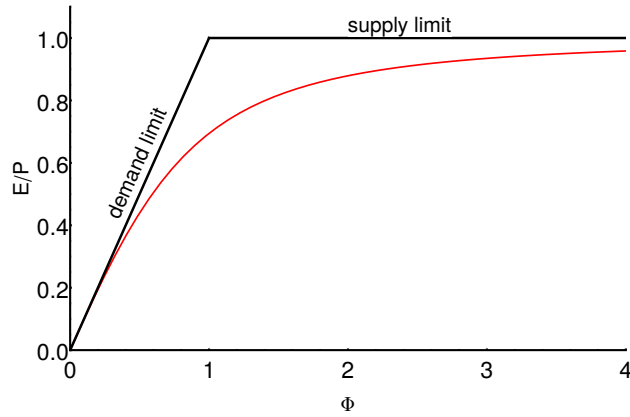


Figure 1. The original Budyko (1956) curve (red), limited by both the demand limit ($E = E_p$) and the supply limit ($E = P$).

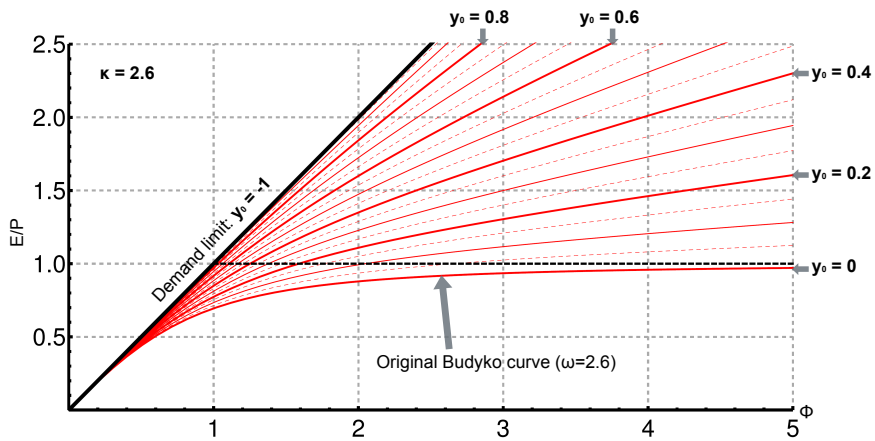


Figure 2. Set of curves of the new framework for $\kappa = 2.6$ and different y_0 . Note that the obtained curve for the parameter set $(\kappa, y_0) = (2.6, 0)$ corresponds to the original Budyko curve ($\omega = 2.6$). The supply limit (dashed black line) is systematically exceeded if $y_0 > 0$ and the demand limit (solid black line) is reached if $y_0 = 1$.

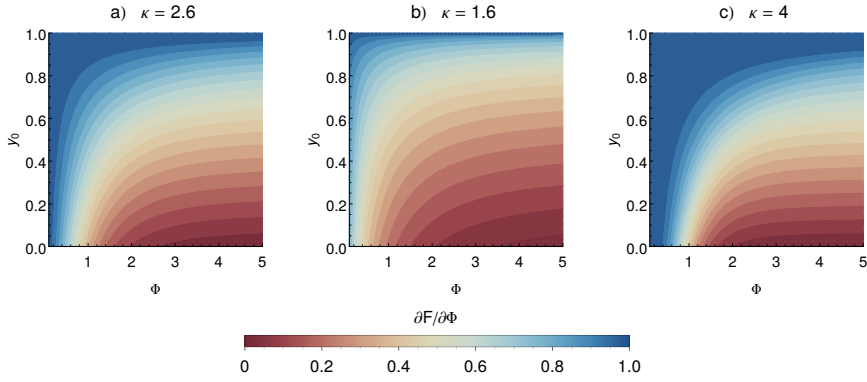


Figure 3. The sensitivity $\partial F/\partial\Phi$ under varying y_0 , for $\kappa = 2.6$ (left, similar to the original Budyko framework if $y_0 = 0$), $\kappa = 1.6$ (center) and $\kappa = 4$ (right). Blueish colors denote high, reddish colors low sensitivity.

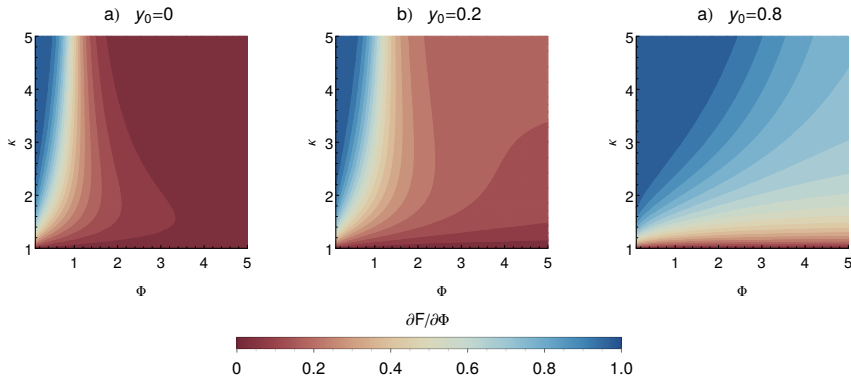


Figure 4. The sensitivity $\partial F/\partial\Phi$ under varying κ , for $y_0 = 0$ (left), $y_0 = 0.2$ (center) and $y_0 = 0.8$ (right). Blueish colors denote high, reddish colors low sensitivity.

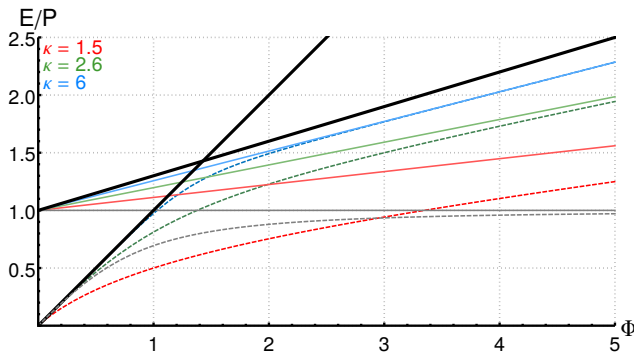


Figure 5. Difference between the actual (solid colored lines) and maximum slope (solid black line) of the supply limit for different values of κ (red: $\kappa = 1.5$, green: $\kappa = 2.6$ and blue: $\kappa = 6$) and $y_0 = 0.3$. The maximum slope ($m = y_0 = 0.3$) is reached if $\kappa \rightarrow \infty$.

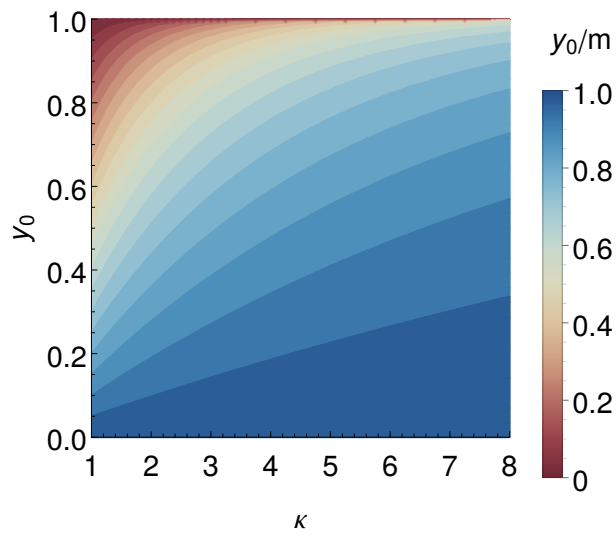


Figure 6. The ratio y_0/m as a function of both y_0 and κ estimated from equation 13.

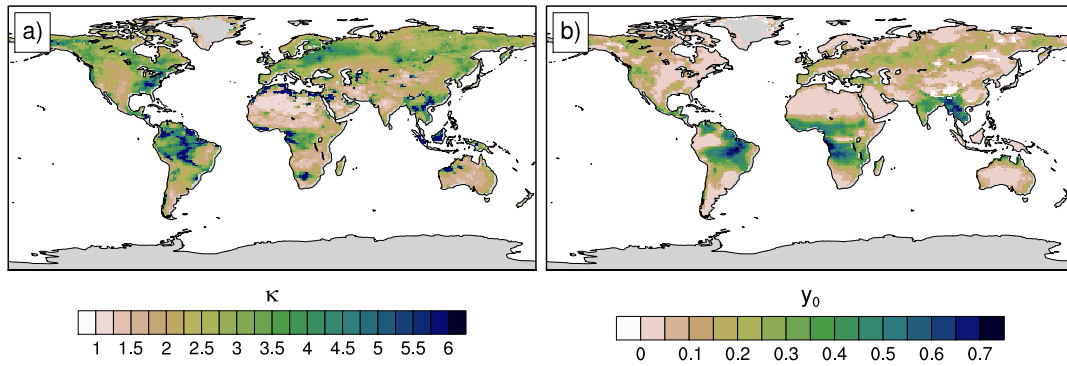
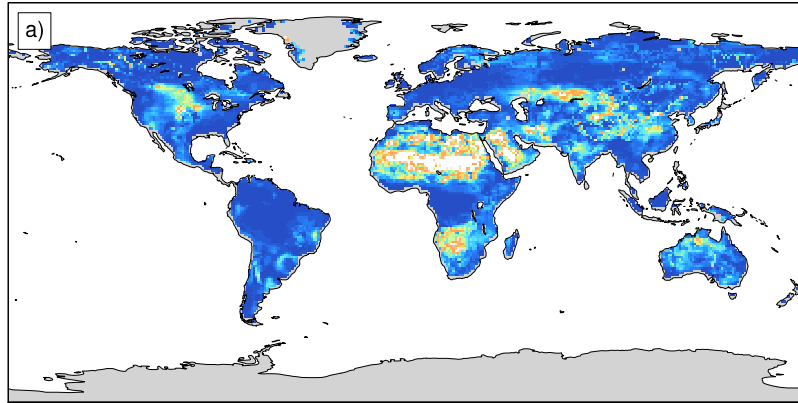


Figure 7. Estimated values of κ (subfigure a) and y_0 (subfigure b) estimated in a least squares fitting using standard monthly datasets of P , E and E_p within the 1990-2000 period.

calibrated y_0



$y_0 = 0$ (Fu, 1981)

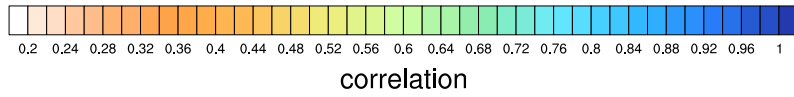
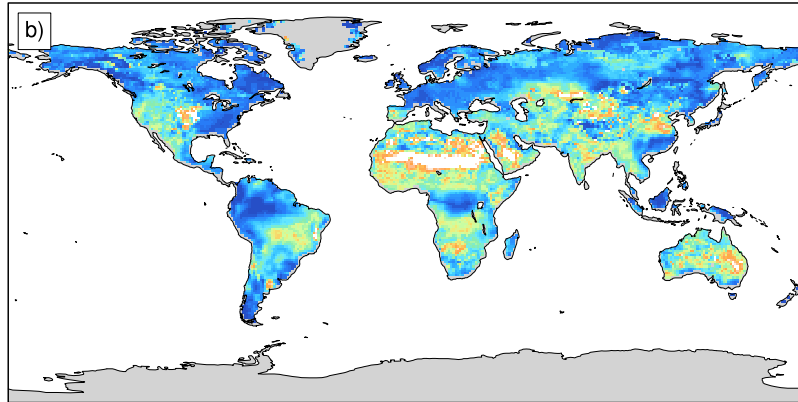


Figure 8. Correlation between the mean seasonal cycle of E/P computed from equation 9 and observed E/P for a) a grid-point specific parameter set (κ, y_0) and b) $(\kappa, 0)$ (Fu's equation).

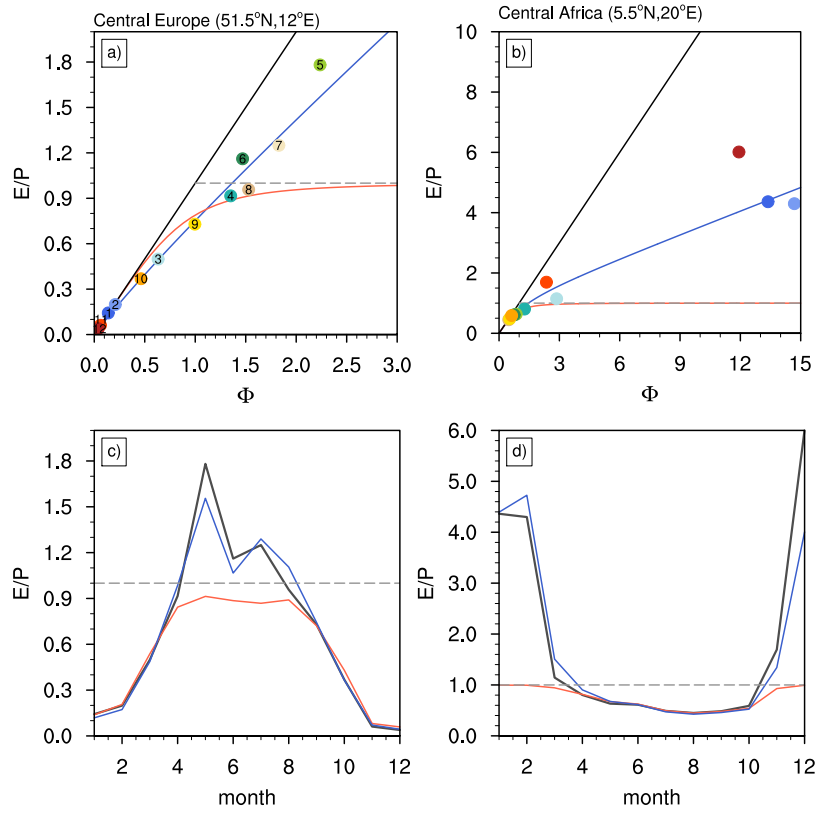


Figure 9. Data cloud of monthly climatologies within the Budyko space for a gridpoints in a) central Europe ($51.5^{\circ}N$, $12^{\circ}E$) and b) central Africa ($5.5^{\circ}N$, 20°). The black solid line denotes the demand limit, the dashed line denotes the original supply limit. The blue line depicts the obtained curve using the modified formulation of Fu's equation, whereas the red line shows the original Fu curve. Numbers within the dots denote the particular month of the year. c), d) Observed (grey) and computed mean seasonal cycles at both gridpoints. The blue line depicts the obtained seasonal cycle using the modified formulation of Fu's equation, whereas the red line shows the seasonal cycle obtained using Fu's equation. Please note that axes are different in each plot.