

# The Budyko framework beyond stationarity

by P. Greve, L. Gudmundsson, B. Orlowsky, and S. I. Seneviratne

Hydrol. Earth Syst. Sci. Discuss., 12, 6799-6830, 2015

Referee comment

## Referee #1:

We sincerely thank Fernando Jaramillo for his positive, extensive and helpful review. We overall agree with his general comments and revised the text accordingly. In combination with his comprehensive list of specific comments, the manuscript significantly benefited from his review. We will first address the general comments of Fernando Jaramillo, followed by a response to the main specific comments that are not related to the general comments and are not too technical or related to typos and grammatical issues. **Please note: due to a late comment by another reviewer the manuscript was changed substantially and the following response partly differs from that provided beforehand.**

## General comments

*1) It needs to state from the beginning that it only deals with temporal climatic intra-annual non-stationarity, not with landscape non-stationarity due to water use, land use and land cover change. The authors imply in the beginning that the term “stationarity” only relates to the climate component, but this is not true (See Milly et al., 2008, Science). Landscape non-stationarity is a fact and has been shown to have major implications for E/P at basin scales, I mention some references. I know the authors know this, but they should mention from the beginning what type of non-stationarity they are dealing with. They should mention from the beginning that their advance does not deal with the understanding of landscape non-stationarity.*

This is an important point which led to some confusion and we thank the reviewer for bringing this up. We clarify in the revised version of the manuscript that we are only considering conditions of non-negligible storage change. The additional parameter  $y_0$  is simply representing the amount of additional water (besides P) that is available to E (see Eq. 4b). In our understanding this includes all processes (such as e.g. intra-annual and inter-annual storage changes, but also landscape changes e.g. from irrigation, etc.) constituting storage changes. We substantially changed the wording and the title of the manuscript..

*2) The methodology is also rather cryptic and confusing. I think they are missing a complete methods section where they explain how they estimate their  $y_0$  and K parameters at the global scale, boot strapping, resampling, calibration, validation, etc. If they want other scientists to use their model or “framework”, they should clearly state what is the procedure to derive  $y_0$  and K. Importantly, they also need to be more precise in introduction and conclusion on what ways this study differentiates from the former works (e.g., empirical Milly, 1993, Potter and Zhang, 2007, Zhang, 2008; and stochastic Zanardo et al, WWR, 2012) that also tackle climatic intra-annual non-stationarity. In other words why is their work “the robust, theoretical incorporation into the Budyko framework*

*that is missing". Right now, it is not very clear.*

**Thanks! We substantially revised the last part of the manuscript which now features a simple approach (least squares fitting) to determine the parameters. We also see the need of putting our work in context to previous assessments on similar topics and thus enhanced the respective part in the introduction and revise the conclusions.**

*3) Their exploration of their model (sensitivity analysis) is very complete and well developed, and the model appears to be robust based on the high correlations at the global scale and for the period 1990-2000. But in order to know really the advance that their models implies for the Budyko framework, it is necessary to show in what way it predicts E and E/P better than Fu 1981 or Zhang 2001,2004 and even better than Budyko's (1956, 1974). They indeed show good correlations between predicted and observed E at the global scale, but how much better than the previous models? For instance, Zhang's model 2004 was able to explain 89% of the variance, how much better is this one then? They should repeat their global analysis with the previous models they mention and compare. I would redo similar Figures 8 and 9 but with the former empirical models and compare.*

**The last part of the manuscript was substantially revised. We now validate the capability of the new formulation to represent additional water available to E besides P. We now directly compare the extended framework to the original Budyko framework.**

### **Specific comments**

*Suggestion for the title: The Budyko framework beyond climatic stationarity*

**Thanks for the nice suggestion! However, we now avoid the word stationarity and changed the title accordingly.**

*-Line 12- I don't think this study is a new framework but rather a good improvement or advance to Budyko's framework, or can this work compare to Budyko's framework to be also called a framework? But of course, this is the authors' choice. I think we scientists are now drowning in so many frameworks...*

This is a good point and also refers to some comments of the other reviewer. We changed the wording in the revised manuscript and introduced the new formulation rather as a modification of Fu's equation than as a new framework.

*-Page 6801 line 9 Water use should be included in the list of factors affecting the scatter in the Budyko space. Position in Budyko space is a cause, but also a consequence, of movement in Budyko Space. Movement in Budyko and hence non-stationarity is also attributed to human changes in landscape conditions by land use and water use or by changes in water phase (landscape changes; Jaramillo and Destouni, GRL, 2014 ), and this should be clearly stated here in this paragraph. Land use change is mentioned by the authors by including Donohue 2007, Zhang 2001, Li et al. 2013, however water use and most water phase changes are neglected. Hydropower and irrigation CAN affect, and rather substantially, the position of a basin in Budyko space, see as example Destouni et al. Nature Climate Change, 2013).*

Thanks! We are well aware of the related and interesting work of the reviewer and changed the text accordingly to mention issues of land-scape nonstationarity. See also our response to the first comment.

*-Zanardo et al., WRR, 2012 deals closely with what the authors deal here, but from a stochastic point of view and should be included in the introduction, I think.*

We added a sentence on the findings of Zanardo et al. (2012) to the revised version of the manuscript.

*-Page 6801. Line 23. Since the definition of “steady conditions” or “stationarity” is an important part of this study, the terms should be defined appropriately in the beginning. What do you mean by these terms? I assume the authors relate stationarity to steady state conditions. “Stationarity” is mentioned in the title of the manuscript but nowhere else in the text. Since steady-conditions instead are mentioned in several parts of the manuscript, I assume they mean stationarity as “steady conditions”, i.e. no change in the storage term of the water budget. The authors relate “stationarity” to that dealt in the manuscript, i.e., that of the intra-annual climatic conditions that may change water storage at the annual scale. However, again, the stationarity assumption is affected also “by water infrastructure, channel modifications, drainage works, and land-cover and land-use change” (Milly, 2008, Science) and Review Fernando Jaramillo changes in water phase (Jaramillo and Destouni, GRL, 2014). This last work shows that changes in the landscape were responsible for non-stationarity in up to 74% of the basins of a global study once intraannual climatic non-stationarity was coarsely ruled out. Since this is not explored in their manuscript, I would appreciate if the authors could be more specific and mention the type of stationarity that their framework is dealing with, i.e. changes in water storage due to intra-annual changes in climatic conditions ( $E_p$  and  $P$ ) as they mention in the first two lines of the Conclusions.*

**Thanks! As already mentioned in response to previous comments, we revised the text to make clear that our main intention is to explicitly represent the amount of water that is available to E besides P.**

*Page 6802, line 15 – Isn't the relationship found by the authors Eq. 9 also empirical? Please specify the difference between “empirical” and “analytical”, since this is a main justification of this work.*

An empirical relationship (or evidence) is usually derived directly from data or observations. Here we use very simple phenomenological assumptions, from which a mathematical relationship is derived analytically. We clarified this in the revised manuscript.

*Page 6804, line 2 – Why not  $\leq -1$ ? Please specify for the reader.*

This is based on the assumption that  $E \leq E_p$  and hence the minimum value of  $(P-E)/E_p$  is  $-1$  (if  $P=0$  and  $E=E_p$ ).

*Line 11 – Again, in relation to my recent question, it should state if additions of water due to changes in the landscape conditions of water phase (melting glaciers, thawing permafrost, closing stomata by rising CO<sub>2</sub> concentrations or systematic anthropogenic changes linked to water use, are accounted in this boundary condition  $y_0$ . Or if these additions/subtractions of*

*water are rather represented by changes in the mathematical constant  $k$ , following Zhang's  $w$ . Or if they are not accounted for at all.*

This is again related to the response to the first general comment. The parameter  $y_0$  is a measure of additional water that is besides  $P$  available to  $E$ . This does in our understanding not exclude other storage components. However, investigating controls and drivers of the two parameters is a complex task and clearly beyond the scope of this study.

*Figure 2 and 4 and text. There is something strange with the sign of  $y_0$  along the manuscript! In the*

We apologize for this unfortunate mistake! The parameter  $y_0$  is defined between 0 and 1. The figure captions are wrong and corrected accordingly.

*Line 7 to line 13 – Let's say I want to replicate the results. This explanation for the derivation of  $y_0$  and  $k$  for the global grid requires more wording because as it is now is rather cryptic. Forgive me if I understood incorrectly but since you use several combinations of  $P$ ,  $E$  and  $E_p$  for each grid cell to minimize and thus estimate  $y_0$  (Fig. 8 a and c), why do you then need the dataset values of  $P$ ,  $E$  and  $E_p$  at all? Also, please explain in more detail the resampling, bootstrapping and least-square fit. Maybe a flow diagram of the procedure would be helpful. Also the difference between panel a and c or between b and d in Fig. 8 should be better explained.*

Please see our comment to the second general comment.

*Line 23 and 25- This procedure also requires more information, it is difficult to understand what was done here, it is cryptic: "anomaly correlations between "detrended" time series with removed annual cycles???? Explain please.*

**We removed this part from the manuscript. Please see our comment to the second general comment.**

*Line 28, 29. I do not know how to see that "...the annual cycle is well represented by the model" by looking at the four panels.*

**We now explicitly compute the mean annual cycle. Please see our comment to the second general comment.**

**We further sincerely thank the reviewer for all comments related to typos, technical and grammatical issues. We changed the text accordingly.**

## The Budyko framework beyond stationarity

by P. Greve, L. Gudmundsson, B. Orlowsky, and S. I. Seneviratne

Hydrol. Earth Syst. Sci. Discuss., 12, 6799-6830, 2015

Referee comment

### Referee #2:

*The manuscript by Greve et al. extends the Budyko framework to include the effect of non steadiness and especially seasonality in water storage. Even though I think it is a good idea I have some major concerns regarding the derivation and some of the assumptions used. Also the time scale of analysis should be made very clear (this is similar to a comment posted by another reviewer). Also it should be made very clear that a major disadvantage is to add many parameters (many  $y_0$ 's for each month) to get the correct ET.*

### Major issues

*Assumption equation 7 seems incorrect. If  $E \leq E_p$  then  $-E = -E_p$  and  $(P-E)/E_p \geq -1$ . (It would be nice to add this directly in the text) but the second assumption of positive definiteness is not correct or at least it very much depends on the time scale you are considering. There should be much more substantial discussion on this. Also the assumption  $dE/dE_p = 0$  at  $-y_0$  is not valid (see several work on the Bouchet Morton relationship, e.g. by Brutsaert). - Equation 10 is not correct or at least again the assumptions behind it should be discussed. Indeed the min function is not linear. -  $y_0$  should be time varying and it is indeed what you present in Fig 9. There should be a clear discussion on this and the fact that to really represent the seasonal cycle we would need at least 12 values (in fact more because interannual variability would change  $y_0$ ). - Throughout the manuscript some physical interpretation is missing and some steps of the derivation could be added (see specific comments below). At this point the manuscript appears more as a manipulation of Fu's equation with little link to intrannual storage. - The manuscript is really ostly on seasonal storage effect so you should make this very clear upfront*

We thank the reviewer for her/his comments. It is our assessment that the criticism can be pinned down to three major issues. These will be addressed at first, followed by a point-to-point response to the specific comments. Unfortunately, it was sometimes hard to fully grasp the reviewers intent, due to incomplete sentences, typos and inconsistent punctuation. However, we attempt to respond to the concerns raised by the reviewer in the best possible way. **Please note: due to a late comment by another reviewer the manuscript was changed substantially and the following response partly differs from that provided beforehand.**

1. The reviewer argues that the modified boundary condition provided in equation 7 is not correct. However, a key assumption underlying the original Budyko framework is stationarity, i.e. water storage does not change over time. This assumption is further also expressed in the boundary conditions underlying Fu's equation. The aim of our study is to relax the stationarity assumption and to explicitly account for storage changes. This could be e.g. the case due to seasonal changes in water storage components such as soil moisture or groundwater, e.g. due to carry-over effects from wet to dry seasons. Hence, even under conditions of low water supply through P, high rates of E can be sustained and consequently  $E \geq P$ . This is also evident in Figure 10a, where springtime values of  $E/P$  usually exceed the steady-state supply limit. This results in the modified (and criticized) boundary condition. Since  $E \geq P$  is possible,  $y = (P-E)/E_p$  is not necessarily positive, but necessarily larger than -1 (since we assume that  $E \leq E_p$ ). At  $y_0$ , E is

limited by  $P$  and water storage changes and a further increase in  $E_p$  have no effect on  $E$ . This (and only this) is expressed in equation 7, after redefining the sign convention of the minimum value  $y_{\min}$  ( $y_0 = -y_{\min}$ ) for convenience. We regret that we did not manage to communicate this clearly in the submitted manuscript and we revised this accordingly. It is further not evident to us, why the Bouchet-Morton (or complementary) relationship does invalidate our very basic assumption  $dE/dE_p = 0$  at  $-y_0$  if we assume  $E \leq E_p$ . This would also imply that the well-established and widely-used assumptions of Fu (1981) and Zhang et al. (2004) are invalid. A more thorough explanation on the strong statement of the reviewer that our assumption is invalid would have been beneficial.

2. The reviewer criticized that many different  $y_0$ -parameters (for each individual month) are needed to compute  $E$ . However, this is only the case in monsoon regions with complex seasonal hydroclimatology. Figure 9a shows that for the vast majority of land regions a single  $y_0$ -value is sufficient to obtain high correlations between the modeled and observed time series. Also figure 10 shows that for a region in central Europe the model performs equally well with a single  $y_0$ -value compared to multiple  $y_0$ -values for each month. We revised the text to really make clear that there is in fact no need for monthly values of  $y_0$  in most land regions and a single value is sufficient. However, we also make clear that a single  $y_0$ -value does not permit an adequate assessment of the more complex seasonal pattern in monsoon climates and hence, more elaborate parameter sets are needed.

**However, due to criticism from other reviewers we now completely revised the last part and the mentioned climatology of  $y_0$ -parameters is not featured anymore.**

3. We further would like to stress that the manuscript is a manipulation of (or a development based on) Fu's equation, which is now also clearly communicated in the revised version.

### Specific points

The specific points displayed below do represent our attempt to reformat the formless structure of the text provided by the reviewer.

*Comment: p6800 lines 8-9: the water balance is always closed what you meant I think is negligible storage*

Response: Thanks! We revised the text accordingly.

*Comment: p6801 before equation 1 define the time scale of applicability and the fact that Budyko only applies on large-scale to remove groundwater contribution improve the clarity of paragraph*

Response: Thanks! However, we explicitly state that the framework is defined at mean-annual catchment scales. This mainly ensures the assumption of negligible water storage change (of which groundwater is just one potential component).

*Comment: after equation 1: why would it be only a problem with supply it is steady for both*

Response: We revised the text accordingly.

*Comment: line 4: You should cite Zhou et al. 2015 GRL, which is close to your derivation Change "complex hybrid of various"...*

Response: We cited Zhou et al. (2015) already in line 1, but we now cite it here as well. Zhou et al. (2015) is now cited several times throughout the manuscript. We also changed the text accordingly.

*Comment: Equation 2 is missing groundwater and you should state that it is valid at the watershed level otherwise you need to include lateral flow*

Response: Groundwater is included in  $S$ , which integrates all relevant terrestrial water storage components such as soil moisture, snow, water stored in vegetation, groundwater, etc. For the

purpose of our study it is not necessary to distinguish these (although it might be important for detailed investigations). We now also clarify that we consider catchment scales.

*Comment: p6802: in fact multi-year variability has to be avoided for the budyko framework to be valid, this should be discussed clearly line 6: even on interannual time scales Budyko is not valid (see carry over effect of water storage so that the budyko curve can be higher than 1 for instance)*

Response: We added this information to the revised text. It is further important to note that even though the physical processes leading to  $E \geq P$  are different at monthly, seasonal and multi-annual time scales, the response itself is similar (i.e. the exceedance of the supply limit). Hence, our modification of Fu's equation is in fact capable to also represent carry-over effects on multi-annual time scales.

*Comment: Equations 3 and 4: have a look at Zhou et al. 2015 GRL*

Response: As mentioned above we are aware of Zhou et al. (2015), which was also cited in the submitted version of the manuscript. However, we modify here the derivation of Fu's equation. Equation 3,4,5 and 6 do exactly represent the set of differential equations and boundary conditions as introduced by Fu (1981) and Zhang et al. (2004).

*Comment: p6805:  $y_0$  is really a function of time, you should comment on this*

Response: Not necessarily. In fact,  $y_0$  is clearly defined as the minimum value of equation 4a under given conditions. It thus represents the maximum amount of additional water besides  $P$  that is available to  $E$ . However,  $y_0$  is (similar to  $k$ ) also a function of several other (yet unknown) variables and quantities See also general comment 2.

*Comment: line 7:  $y_0=1$  you could mention that this is when  $E=E_{max}=E_p$ . Also you should clearly define  $E_p$  because  $E$  can be larger than  $E_p$  in many cases (because of roughness or large LAI)*

Response: We added this information to the text. We further refer to the very general, standard definition of  $E_p$  that ideally accounts for all potential controls (including roughness and LAI). We do not refer to any specific estimate of  $E_p$  (as e.g. Penman, Thornthwaite, etc.). Hence,  $E_p$  is always larger or equal to  $E$ .

*Comment: p6806 can you give more physical explanation after equation 11*

Response: We added an additional sentence to the revised manuscript, clarifying that  $y_0$  represents an estimate of the maximum amount of water that originates from storage changes and contributes to  $E$ . See also our response to a previous comment.

*Comment: section 5: please carefully discuss the time scale effect Also Penman Monteith should be Penman for potential evaporation. What is done for the reference roughness? What do you consider as a reference, this is important.*

Response: We use here a monthly, global dataset which can be downloaded here:

<http://hydrology.princeton.edu/data.pdsi.php> and which is based on a Penman-Monteith  $E_p$  algorithm. Details are provided in the Methods section of Sheffield et al., 2012. The Penman-Monteith equation is originally defined to compute actual  $E$ . However, if the stomatal resistance is set to zero, the Penman-Monteith equation provides an estimate of  $E_p$ . The aerodynamic resistance is further defined after: Shuttleworth, W. J. in *Handbook of Hydrology* (ed. Maidment, D. R.) 4.1–4.53 (see also Sheffield et al., 2012)

*Comment: p6809: define your frequency of  $y_0$  computation end of section 5: you should have a discussion on the fact that you have added many parameters to really retrieve  $E$  (see Fig 9) so it is not really a predictive model but more a model for physical interpretation. Also the main trouble is that we do not have access to  $E$  at the watershed scale so how can this be used on water shed data where  $P$  and  $Q$  only are measured?*

Response: Please see general comment 2. The new formulation is further not intended to solely work

for watershed data, but also for other spatial units (e.g. gridboxes). **However, please note that the last part of the manuscript was changed substantially.**

*Comment: p6811 line21: physically well defined: in fact it is more mathematically defined some physical explanation would be nice.*

Response: Thanks. We changed the wording accordingly.

*Comment: Equation A1: can you give further steps for this equation? Derivation A2 to A8 is very elegant.*

Response: Thanks! Equation A1 simply represents a necessary condition that needs to be fulfilled to solve the set of differential equations presented in the main text. This approach is, however, similar to the derivation presented in Fu (1981) and Zhang et al. (2004).

*Comment: Figure 9: we really need at least monthly estimates of  $y_0$  to get ET right so why not run a simple water balance model?*

Response: See general comment 2 and **please note that the last part of the manuscript was changed substantially.**

# The Budyko framework beyond stationarity

by P. Greve, L. Gudmundsson, B. Orlowsky, and S. I. Seneviratne

Hydrol. Earth Syst. Sci. Discuss., 12, 6799-6830, 2015

Referee comment

## Referee #3:

*This paper deals with the  $F_u$  water-balance formula. Basically, the work reported here aims at introducing in the formula a second parameter which accounts for catchment storage capacity. The authors do so by going back to the initial differential equations and reintegrate them by using a new boundary condition.*

*I was very interested by this work, and I honestly believe that there is a lot of potential. The reason why I am quite negative about it, is that I think that there is a fundamental confusion (in the way it is presented to the reader, and also in the solution proposed). Here is what I consider to be the major flaw: the  $F_u$  water balance equation is not a dynamic equation, and its partial derivatives do not refer to time. This equation explicitly deals with long-term fluxes. The very use of the word “stationarity” shows that the authors have made a confusion.*

We thank the reviewer for her/his critical, but very helpful comment. There was a clear misconception regarding the communication of our results. Most of the confusion was in our assessment due to the sloppy use of the word “stationarity” in the reviewed version of the manuscript.

We now have substantially revised the manuscript including the title. We avoid the use of the word “stationarity” and now clearly define that we are only considering conditions where evapotranspiration exceeds precipitation. We completely removed the part where we attempted to dynamically model ET. We now include a much shorter and easily-accessible example of how our modified formulation of  $F_u$ 's equation is capable to represent additional water sources besides P that are available to E.

*I also have doubts about a few choices made by the authors (such as  $P_{min}$  in Eq. 11, what if  $P_{min}$  is equal to zero?).*

Thanks! We now clarify in the revised version of the manuscript that equation 11 is not valid in case  $P_{min} = 0$ .

*Last, I believe that section 5 lacks enough information to explain how parameters  $k$  and  $y_0$  have been calibrated. I initially thought that the authors had used catchment (measured) data, I then understood that they have used simulations given by another model. There, I have a second major problem: what do you prove here by calibrating a model with another model? Isn't it quite a circular approach?*

We completely revised section 5, which is now much shorter and uses standard methods. We now only feature an example application to illustrate the capability of the presented, modified Budyko framework to represent water that is additionally available to E at seasonal time scales. We still use the same data to compute monthly climatologies. This data is, however,

based on models, which are driven and constraint by observations. Nonetheless, the used datasets are widely-used and well-established to provide reliable estimates of P, E, and Ep for hydroclimatological assessments.

#### Minor Remarks

*. P6599-L11: I understand what you mean by “Budyko curve” but I think that this concept is not properly defined.*

Thanks! We changed the wording accordingly

*. P6800-L25 : your historical introduction is wrong. Please read Schreiber’s paper, it has nothing to do with the ‘Budyko’ framework, Schreiber does not introduce the aridity index, he does not even use the notion of potential evaporation.*

We are well aware of Schreiber's paper and all authors of the manuscript have actually read it. All authors are also aware of Budyko (1974), where Budyko referred to both Schreiber (1904) and Oldekop (1911) in Chapter VI, part 2 (page 323), despite Schreiber himself was not aware of the concept of potential evaporation (not even of the concept of evaporation). However, Oldekop was and accordingly modified Schreiber's equation. Due to this we cautiously wrote that the formulation derived by Budyko is only “based on the findings” of (and not directly derived from) Schreiber and Oldekop.. Therefore we do not see the necessity to change the wording.

*. P6801-L20: again “the original Budyko curve” requires some explanation.*

Thanks! We changed the wording throughout the manuscript.

*. I do not understand equation A4*

Equation A4 is a simple expansion of P/Ep:

$$P/Ep = 1 * P/Ep = (Ep + P - E) / (Ep + P - E) * P/Ep = (Ep + P - E) / Ep / (Ep + P - E) / P$$

In a next step (P-E)/Ep was substituted by y and (Ep-E)/P was substituted by x.

# A two-parameter Budyko function to represent conditions under which evapotranspiration exceeds precipitation

Peter Greve<sup>1,2</sup>, Lukas Gudmundsson<sup>1</sup>, Boris Orlowsky<sup>1</sup>, and Sonia I. Seneviratne<sup>1</sup>

<sup>1</sup>Institute for Atmospheric and Climate Science, ETH Zurich, Zurich, Switzerland

<sup>2</sup>Center for Climate Systems Modeling (C2SM), ETH Zurich, Zurich, Switzerland

Correspondence to: Peter Greve (peter.greve@env.ethz.ch)

## Abstract.

~~Water availability is of major importance for a wide range of socio-economic sectors. Over land,~~  
A comprehensive assessment of the partitioning of precipitation ( $P$ ) into evapotranspiration ( $E$ )  
and runoff ( $Q$ ) is ~~the key process to assess hydrological conditions~~of major importance for a wide  
5 range of socio-economic sectors. For climatological averages, the Budyko framework provides a  
simple first order relationship to estimate ~~the evaporative index  $E/P$~~ water availability represented  
by the ratio  $E/P$  as a function of the aridity index ( $E_p/P$ , with  $E_p$  denoting potential evaporation).  
However, a major downside of the Budyko framework is its limitation to steady state conditions,  
being a result of ~~the assumption of a closed~~assuming negligible storage change in the land water  
10 ~~balance. Nonstationary processes coming into play at other than mean annual catchment scales are~~  
~~thus~~Processes leading to changes in the terrestrial water storage at any spatial and/or temporal scale  
are hence not represented. Here we propose an analytically derived ~~new formulation~~modification  
of the Budyko ~~curve including an additional parameter being implicitly related to the nonlinear~~  
~~storage term of the land water balance. The new framework including a new parameter explicitly~~  
15 representing additional water available to evapotranspiration besides precipitation. The modified  
framework is comprehensively ~~analysed~~analyzed, showing that the additional parameter leads to  
an upward rotation of the original ~~supply limit and therefore implicitly represents the amount of~~  
~~additional water available for evaporation. The obtained model is further validated using standard~~  
~~datasets of  $P$ ,  $E$  and  $E_p$ . It is shown that the model is capable to represent first-order seasonal~~  
20 ~~dynamics within the hydroclimatological system~~water supply limit. We further evaluate the new  
formulation in an example application at mean seasonal time scales, showing that the extended  
framework is able to represent conditions in which evapotranspiration exceeds precipitation.

## 1 Introduction

The Budyko framework serves as a tool to predict mean annual water availability as a function of aridity. It is widely-used and well-established within the hydrological community, both due to its simplicity and long history, combining experience from over a century of hydrological research. ~~Since Before and after~~ Budyko (1956, 1974) derived a formulation of the ~~curve-function~~ based on findings of Schreiber (1904) and Ol'Dekop (1911), several other formulations have been postulated, which however are numerically surprisingly similar (Schreiber, 1904; Ol'Dekop, 1911; Turc, 1955; Mezentsev, 1955; Pike, 1964; Fu, 1981; Choudhury, 1999; Zhang et al., 2001, 2004; Porporato et al., 2004; Yang et al., 2008; Donohue et al., 2012; Wang and Tang, 2014; Zhou et al., 2015b). Many of these formulations are empirically derived and only few are analytically determined from simple phenomenological assumptions (~~Fu, 1981; Milly, 1994; Porporato et al., 2004; Zhang et al., 2004; Yang et al., 2007~~). ~~Nonetheless, derived functional forms in all formulations are deterministic and assessments on~~ (Fu, 1981; Milly, 1994; Porporato et al., 2004; Zhang et al., 2004; Yang et al., 2007; Zhou et al., 2015b). ~~Numerous studies further assess~~ controls determining the observed systematic scatter ~~around the mean Budyko curve have been subject to numerous studies~~ within the Budyko space. A variety of catchment and climate characteristics, such as e.g. vegetation (Zhang et al., 2001; Donohue et al., 2007; Williams et al., 2012; Li et al., 2013; Zhou et al., 2015a), seasonality characteristics (Milly, 1994; Potter et al., 2005; Gentine et al., 2012; Chen et al., 2013; Berghuijs et al., 2014), soil properties (Porporato et al., 2004; Shao et al., 2012; Donohue et al., 2012), and topographic controls (Shao et al., 2012; Xu et al., 2013) have been proposed to exert a certain influence on the scatter within the Budyko space. Also ~~complex hybrids of more complex approaches to combine~~ various controls (Milly, 1994; Gentine et al., 2012; Donohue et al., 2012; Xu et al., 2013) have been considered, ~~but until present~~. ~~Nonetheless, until present~~ no conclusive statement on controls ~~were made~~. ~~determining the scatter within the Budyko space could be made. In a recent assessment, Greve et al. (2015) suggested a probabilistic Budyko framework by assuming that the combined influence of all possible controls follows a probability distribution.~~

In this study we make use of the formulation ~~derived~~ introduced by Fu (1981) and Zhang et al. (2004). They ~~derive the following~~ derived a functional form between  $E/P$  and  $\Phi = E_p/P$  ~~at mean annual catchment scales~~ analytically from simple physical assumptions:-

$$\frac{E}{P} = 1 + \Phi - (1 + (\Phi)^\omega)^{\frac{1}{\omega}}, \quad (1)$$

where  $\omega$  is a free model parameter ( ~~$\omega = 2.6$  results in the original Budyko curve~~). ~~The original formulation introduced by Budyko (1956, 1974) is numerically reproduced by setting  $\omega = 2.6$  (Zhang et al., 2004).~~ The obtained ~~curve-function~~ is subject to two physical constraints constituting both the water demand and supply ~~limit~~limits. The water demand limit represents  $E$  being limited by  $E_p$ , whereas the water supply limit determines  $E$  to be limited by  $P$  (see Fig. 1). ~~Hence, Regarding~~ the supply limit ~~requires~~

steady-state conditions. The storage term ( $dS/dt$ ) in the land water balance equation at catchment scales

$$60 \quad \frac{dS}{dt} = P - E - Q \quad (2)$$

is consequently neglected, a generally valid approach are required and the storage term ( $dS/dt$ ) is consequently assumed to be zero, which is generally a valid assumption at mean annual catchment scales. Although we note, that year-to-year changes in soil moisture may happen, e.g. under transient climate change (Wang, 2005; Orlowsky and Seneviratne, 2013) scales. However, the assumption of steady-state conditions does not permit the usage of the Budyko framework at monthly to seasonal time scales and negligible storage changes constitutes a major limitation of the framework. Only few assessments addressed this limitation. Potter and Zhang (2007) derived a formulation based on previous work by Milly (1993) in order to model interstorm to the original Budyko framework. As a consequence, the framework is not valid under conditions in which additional water (besides  $P$ ) is available to  $E$ . In a comprehensive top-down approach, Zhang et al. (2008) developed a water balance model for subannual to mean annual time scale. They suggested that model complexity has to increase at intrannual time scales to account for soil moisture dynamics, and they extended the Budyko model accordingly by introducing four additional parameters. Chen et al. (2013) extended the Budyko model to seasonal time scales by introducing a seasonal aridity index that accounts for storage changes. Although these approaches provide interesting insights on and  $E > P$ . Such conditions can occur e.g. at sub-annual or inter-annual time scales due to changes in terrestrial water storage terms such as soil moisture, groundwater or snow storage. Additional water might be also introduced by landscape changes (Jaramillo and Destouni, 2014), human interventions (Milly et al., 2008) or phase changes of water within the system or supplied through precipitation (Jaramillo and Destouni, 2014; Berghuijs et al., 2014). Also long-term changes in soil moisture may happen, e.g. under transient climate change (Wang, 2005; Orlowsky and Seneviratne, 2013). Only few assessments addressed this limitation and provided further insights on how the Budyko hypothesis at subannual time scales, they are still derived empirically. Nevertheless, all approaches agree on the necessity to include storage changes, but could be extended to conditions under which  $E$  exceeds  $P$  (Milly, 1993; Potter and Zhang, 2007; Zhang et al., 2008; Zarnado et al., 2012; Chen et al., 2013). Nonetheless, so far a robust, theoretical incorporation theoretical, rigorous incorporation of conditions in which  $E > P$  into the Budyko framework is missing.

In this work, Here we aim to analytically derive a new Budyko formulation for dynamic conditions at e.g. subannual time scales. Our approach is based on simple phenomenological assumptions in which the storage term is implicitly considered. This is achieved by reformulating the set of differential equations given in address this issue by analytically deriving a new, modified Budyko formulation from basic phenomenological assumptions by using the approach of Fu (1981) and Zhang et al. (2004) such that the water supply limit is no rigid physical constraint.

## 2 Deriving a new modified formulation

### 2.1 Preliminary Assumptions

95 ~~On the basis of~~ Fu (1981) and Zhang et al. (2004), ~~we postulate~~ suggested that for a given potential evaporation, the rate of change in evapotranspiration as a function of the rate of change in precipitation ( $\partial E/\partial P$ ) increases with residual potential evaporation ( $E_p - E$ ) and decreases with precipitation. Similar assumptions ~~are~~ were made regarding the rate of change in evapotranspiration as a function of the rate of change in potential evaporation ( $\partial E/\partial E_p$ ) by considering residual  
100 precipitation ( $P - E$ ). Hence, both ratios can be written as

$$\frac{\partial E}{\partial P} = f(x) \quad (3a)$$

$$\frac{\partial E}{\partial E_p} = g(y) \quad (3b)$$

with

$$x = \frac{E_p - E}{P} \quad (4a)$$

$$105 \quad y = \frac{P - E}{E_p} \quad (4b)$$

Considering  $E_p$  being a natural constraint of  $E$ , it follows

$$\left. \frac{\partial E}{\partial P} \right|_{x=0} = 0. \quad (5)$$

The original approach of Fu (1981) further assumes that  $P$  is a natural constraint of  $E$ , giving constituting the following boundary condition

$$110 \quad \left. \frac{\partial E}{\partial E_p} \right|_{y=0} = 0. \quad (6)$$

This assumption requires steady-state conditions and is consequently considered to be valid at mean annual catchment scales (such that  $P - E \geq 0$ ) only. However, ~~due to storage changes, on shorter time scales and smaller spatial scales~~  $E \geq P$  (respectively,  $y \leq 0$ ) is possible. ~~In this case as mentioned in the introduction, a wealth of possible mechanisms and processes can induce conditions~~  
115 in which  $E$  exceeds  $P$ . In such cases,  $E_p$  remains the only constraint of  $E$ . Consequently, since we

explicitly aim to account for conditions of  $E > P$ , the value  $y = (P - E)/E_p$  (see equation 4) is not necessarily positive (but larger than -1 since we assume that  $E < E_p$ ). The minimum value  $y_{min}$  of  $y$ , denoted as  $y_{min}$  (see equation 4), thus lies within the interval between -1 and 0 and depends on the additional amount of water being available for evaporation (and thus implicitly refers to the storage term in equation 2)  $E$  besides water supplied by  $P$ . For convenience we define  $y_0 = -y_{min}$  (and thus  $y_0 \in [0, 1]$ ). The As a consequence the boundary condition 6 is then redefined as

$$\left. \frac{\partial E}{\partial E_p} \right|_{-y_0} = 0. \tag{7}$$

## 2.2 Solution

Solving the system of the differential equations 3a,b using boundary condition 5 and the new condition 7 yields the following solution (details are provided in Appendix A):

$$E = E_p + P - ((1 - y_0)^{\kappa-1} E_p^\kappa + P^\kappa)^{\frac{1}{\kappa}} \tag{8}$$

with  $\kappa$  being a free model parametersparameter. It follows

$$\frac{E}{P} = F(\Phi, \kappa, y_0) = 1 + \Phi - (1 + (1 - y_0)^{\kappa-1} (\Phi)^\kappa)^{\frac{1}{\kappa}}. \tag{9}$$

Similar to the traditional Budyko approach a free model parameter (named  $\kappa$  to avoid confusion with the traditional  $\omega$ ) is obtained. The second parameter  $y_0$ , as introduced in the previous section, is directly related to the new boundary condition. Hence, in contrast to  $\kappa$ , which is a mathematical constant,  $y_0$  has an actual physical interpretation a physical interpretation as it accounts for additional water. However, similarly similar to the  $\omega$  parameter-parameter in Fu's equation,  $\kappa$  is potentially could be interpreted as an integrator of all other catchment properties the variety of catchment properties other than the aridity index.

## 3 Characteristics of the new modified framework

The obtained new formulation given in equation 9 newly derived formulation given (equation 9) is similar to equation 4 the classical solution (equation 1), but includes  $y_0$  as a new parameter (Assuming . Assuming e.g.  $\kappa = 2.6$  ; (corresponding to the original Budyko curve function with  $\omega = 2.6$  and an example set in Fu's equation) and example values of  $y_0$ -values, Fig. 2 shows a set of curves providing insights on the basic characteristics of the new equation) modified equation.

First, if In case  $y_0 = 0$  (being the original boundary condition), the obtained curve corresponds to the steady-state framework of Fu (1981) and Zhang et al. (2004), which is also evident from equation 9 and. This shows that both model formulations are consistently transferable. If  $y_0 > 0$ , the supply

145 limit is systematically exceeded. The exceedance of the supply limit increases with increasing  $y_0$ .  
 If further  $y_0 = 1$ , the ~~demand limit is reached~~ curve follows the demand limit. All curves are further  
 continuous and strictly increasing.

Taking a closer look at the underlying boundary conditions and definitions (see section 2.1) reveals  
 that  $y_0$  ~~implicitly~~ explicitly accounts for the amount of additional water (besides water supplied  
 150 through  $P$ ) available for  $E$ . Since  $y_{min}$  is ~~explicitly~~ defined to be the minimum of  $y = (P - E)/E_p$ ,  
 the quantity  $y_0 = -y_{min}$  physically represents the maximum fraction of  $E$  relative to  $E_p$ , which  
 is not originating from  $P$ . A larger fraction consequently results in higher  $y_0$ -values and thus in a  
 stronger exceedance of the original supply limit. Further details on  $y_0$  is provided in section 4.

The sensitivity  $\partial F(\Phi, \kappa, y_0)/\partial \Phi$  under varying  $\kappa$  and for three preselected values of  $y_0$  is illus-  
 155 trated in Fig. 3. The sensitivity  $\partial F(\Phi, \kappa, y_0)/\partial \Phi$  for different values of  $y_0$  and  $\kappa$  shows the effect of  
 the parameter choice on changes in  $E/P$  relative to changes in  $\Phi$ . In general, the sensitivity is largest  
 for small  $\Phi$  (humid conditions), due to the fact that changes in  $E/P$  basically follow the demand  
 limit (resulting in a sensitivity close to 1) regardless of parameter set  $(\kappa, y_0)$ . For different parameter  
 settings, the sensitivity generally decreases with increasing  $\Phi$ . For small values of  $y_0$  (close to zero),  
 160 sensitivity becomes smallest with increasing  $\Phi$ , since small values of  $y_0$  indicates ~~indicates~~ indicate  
 conditions similar to ~~steady-state conditions being constraint by the (horizontal and thus implying zero~~  
~~sensitivity)original supply limit~~ the classical solution (equation 1). Further, the smallest sensitivity is  
 reached for large values of  $\kappa$ . Large values of  $y_0$  (close to 1) indicate conditions mainly constrained  
 by the demand limit, thus implying a sensitivity close to 1.

165 A similar analysis is performed for varying values of  $\kappa$  under three preselected levels of  $y_0$   
 (see Fig. 4). For  $y_0 = 0$  (steady-state conditions), the sensitivity  $\partial F/\partial \Phi$  is under humid conditions  
 ( $\Phi < 1$ ) rather large, since changes in  $E/P$  are mainly constrained by demand limit. This espe-  
 cially applies for large values of  $\kappa$ . Under more arid conditions ( $\Phi > 1$ ), the Budyko curve slowly  
 converges towards the (horizontal) supply limit, resulting in a near-zero sensitivity. For  $y_0 = 0.2$ ,  
 170 denoting conditions relatively similar to steady-state conditions, the decrease in sensitivity with in-  
 creasing  $\Phi$  is weaker, whereas for  $y_0 = 0.8$ , denoting conditions where  $E$  is mainly constraint by the  
 demand limit, sensitivity is large for large  $\kappa$ -values and decreases rather slowly with increasing  $\Phi$ .

#### 4 Interpreting the new parameter $y_0$

The new parameter  $y_0$  is, in contrast to  $\kappa$ , physically well defined. The combination of equation 4b  
 175 and 7 shows that  $y_0$  is ~~implicitly~~ explicitly related to the amount of additional water (besides water  
 supplied through  $P$ ), which is available ~~for to~~ to  $E$ . If we rewrite equation 4b with respect to  $y_0$

$$y_0 = -y_{min} = -\left(\frac{P - E}{E_p}\right)_{min} = -\frac{P_{min} - E_{max}}{E_p}, \quad \text{if } P_{min} - E_{max} < 0, \quad (10)$$

where  $P_{min}$  and  $E_{max}$  are chosen in order to minimize  $y_{min}$  for a given  $E_p$ , we obtain a linear equation in terms of aridity index

$$180 \quad \left(\frac{E}{P}\right)_{max} = y_0 \left(\frac{E_p}{P_{min}}\right) + 1, \quad (11)$$

which constitutes the mathematical ~~and physical meaning interpretation~~ of  $y_0$  within the ~~new modified~~ framework. That is, that  $y_0$  determines the maximum slope of the upper limit, against which the obtained curve from equation 9 asymptotically converges to if  $\kappa \rightarrow \infty$  (see Fig. 5). Physically,  $y_0$  determines the maximum  $E/P$  that is reached in relation to  $\Phi$  within a certain time period and spatial domain. It thus represents an estimate of the maximum amount of additional water that contributes to  $E$  and originates from other sources than  $P$ . Technically speaking,  $y_0$  determines the slope of the upper limit such that all possible pairs  $(\Phi, E/P)$  are just below the line  $y_0\Phi + 1$ . It is further important to note that for mean annual conditions ( $P - E \geq 0$ ),  $y_0 = 0$  is considered, which results in a zero slope and thus determines the original supply limit of 1. Please also note, that this approach is not valid if  $P_{min} = 0$ .

However, the actual slope  $m$  of the upper limit is smaller than  $y_0$ , but directly related to both  $y_0$  and  $\kappa$  as follows (see Appendix B for more information)

$$m = 1 - (1 - y_0)^{1 - \frac{1}{\kappa}}. \quad (12)$$

The relative difference between the maximum slope  $y_0$  and the actual slope  $m$  of the upper limit (being the ratio of  $y_0/m$ ) is thus determined following the relationship

$$\frac{y_0}{m} = (1 - y_0)^{1/\kappa}. \quad (13)$$

The ratio  $y_0/m$  as a function of both  $y_0$  and  $\kappa$  is illustrated in Fig. 6. For small  $\kappa$  and large  $y_0$  (close to 1), the difference between the actual slope  $m$  and the maximum slope  $y_0$  is large, whereas for large  $\kappa$  the actual slope  $m$  converges towards  $y_0$ . However, in any case,  $y_0$  determines the maximum overshoot allowed with respect to the original supply limit at  $y_0 = 0$ .

~~Taking into account that  $y_0$  is well-defined by equation 10, the parameter is in the following estimated from data. In the following, we use standard datasets-~~

## 5 Example application: Seasonal carryover effects in terrestrial water storage

At monthly time scales, changes in terrestrial water storage (due to changes in water storage components such as soil moisture, snow or groundwater) potentially play an important role and are by no means negligible. Such changes can provide a significant source of additional water that is (besides  $P$ )

available to  $E$ . Here we analyse the multi-year mean seasonal cycle of  $E/P$  by using gridded, monthly data estimates of  $P$ ,  $E$  and  $E_p$ . This allows us to evaluate the performance capability of the obtained model described by equation 8. framework (given by equation 9) to represent additional water sources at such time scales.

## 6 Assessing the framework with observations

The new framework allows to compute  $E$  as a function of both  $P$  and  $E_p$ . Here we use We employ the following, well-established estimates of all three variables: gridded data products: (i) the Global Precipitation Climatology Project (GPCP) precipitation estimates,  $P$  dataset (Adler et al., 2003), (ii) an  $E_p$  dataset estimate (Sheffield et al., 2006, 2012) based on the Penman-Monteith method (Monteith, 1965), and  $E_p$  algorithm (Monteith, 1965; Sheffield et al., 2012), and (iii) the LandFlux-Eval  $E$  estimates (Mueller et al., 2013), for the 1990-2000 time period and dataset (Mueller et al., 2013). All data is bilinearly interpolated to a unified  $1^\circ$ -grid and the mean seasonal cycle for the 1990-2000 period is calculated at gridpoint-scale. Please note that the combination of datasets used here is arbitrary and only used to illustrate the capability of the newly developed framework to represent the multi-year mean annual cycle of  $E/P$ .

We estimate the parameters  $\kappa$  and  $y_0$  at gridpoint scale by determining  $y_0$  from data (using equation 10) in order to obtain a fixed parameter for the whole time period. After  $y_0$  is estimated,  $\kappa$  is estimated using a least squares fitting approach. However, estimating  $y_0$  from data requires to find the set of  $(P, E, E_p)$  that minimizes equation 10 and results in the maximum slope of the adjusted supply limit. In order to account for the underlying data uncertainty and potential outliers, bootstrapping is used. The data cloud of a particular gridpoint is resampled 1000 times and for each sample the set of  $(P, E, E_p)$  that maximizes  $y_0$  is selected. The median of all acquired  $y_0$ -values is further used to estimate  $\kappa$  in a least squares fit.

The estimates of We estimate the parameter set  $(\kappa, y_0)$  provide a fixed set of parameters that represents the whole time period and are illustrated in Fig. ???. The  $\kappa$ -parameter is rather small in most subtropical desert regions and somewhat larger in tropical regions. Relatively large values of  $\kappa$  are further found in mid to high latitude regions. For  $y_0$ , lowest values are found in tropical and midlatitude regions, whereas subtropical and also subpolar areas show somewhat higher values. In summary, dry regions tend to show values of  $y_0$  close to zero, denoting conditions similar to the original framework. It is further important to note that  $\kappa$  and  $y_0$  are spatially not correlated. from equation 9 by minimizing the residual sum of squares. This means that at every gridpoint 12 monthly climatologies of  $E/P$  (representing the mean seasonal cycle of  $E/P$ ) are used to determine a specific parameter set.

To validate the performance of the model given by equation 10 evaluate the modified framework, the derived set of parameters parameter sets at each gridpoint is used to model  $E$  within the calibration

period (1990-2000). Correlations between the modeled time series derived by using the parameter set are used in equation 9 to compute mean seasonal cycles of  $E/P$ . The correlation between the computed and the observed time series and anomaly correlations between 'detrended' time series with removed annual cycles are seasonal cycle is shown in Fig. 7.

Generally, a The correlations are relatively large in many regions, whereas anomaly correlations are smaller most regions. Largest correlations ( $>0.80$ ) are found in all most mid to high latitude regions. However, and tropical areas, clearly showing the capability of the most important feature regarding the time series of  $E$  in these regions is the annual cycle, which is well represented by the model. Hence, the first-order control on  $E$  regarding seasonal variations is robustly represented by variations in water supply  $P$  and demand  $E_p$ . Further, correlations are, despite being still positive and relatively large (around 0.5), smaller in the inner tropics (central Amazonia and Congo Basin). This is most probably due modified formulation to represent the seasonal cycle in  $E/P$ . Correlations are generally somewhat lower in drier regions, especially in parts of Africa and Central Asia, probably occurring due to more complex seasonal patterns in  $E/P$ . Using instead Fu's original equation (or setting  $y_0 = 0$ ) to weak seasonal variations of  $E$  and hence an increased importance of second-order controls on month-to-month changes in  $E$ .

Similar to the original Budyko framework, this is however not true for deviations from the mean, which are potentially subject to various second-order controls, as suggested by very small anomaly correlations in most regions. However, in some subtropical areas, anomaly correlations are reasonably large (up to 0.5).

An interesting feature is found regarding many monsoon regions (India, Southeast Asia, Northeast Brazil and the Sahel). The distinct difference between wet and dry seasons seems to prohibit the use of a fixed parameter set. The derived parameter set instead represents wet season characteristics as  $y_0$  and consequently overestimates dry season  $E$ . These issues could be circumvented by calibrating separate parameter sets for either each month of the year, or dry and wet seasons in particular. Using estimates of  $y_0$  derived from monthly climatologies and corresponding  $\kappa$ -values represents seasonal variations in the parameters themselves. By doing so, resulting correlations in monsoon regions are similar to those in mid and high latitude regions (see estimate the mean seasonal cycle of  $E/P$  shows overall lower correlations, especially in semi-arid regions (Fig. 7e). Interestingly, using the individual parameter sets derived from monthly climatologies instead of using a fixed parameter set for the whole time period, does not significantly increase the performance of the model in mid to high latitude areas. It does further not significantly increase the capability of the model to predict anomalies (comparing Fig. 7band db).

To further highlight the differences between midlatitude and monsoon regions, the model performance is analysed in more detail for two regions: Taking a closer look at the mean seasonal cycle for example gridpoints in (i) central Europe Central Europe (humid climate) and (ii) central Sahel (see Fig. 8, regions are highlighted in Fig. 7). The upper two plots of Africa (semi-arid climate) clearly

shows the improvement gained through the use of the modified formulation (Fig. 8) illustrate the  
280 ~~respective data cloud of monthly values for both regions within the Budyko space. To note first, it~~  
~~is evident, that the original supply limit does not hold at monthly time scales as it is systematically~~  
~~overshot. The data cloud for central Europe shows an almost linear increase of  $E/P$  with increasing  $\Phi$ .~~ In Central Europe,  
~~additional water is available in the early summer months due to e.g. depletion of soil moisture or~~  
~~snow melt, resulting in values of  $E/P$  with increasing  $\Phi$ , that is just slightly upset from the demand~~  
285 ~~limit (thus implying a rather large  $y_0$ ). For the central Sahel region, two regimes are noticeable. The~~  
~~first (during the winter months) being relatively similar to those of central Europe, with increasing~~  
 ~~$E/P$  close to the demand limit (large  $y_0$ ) and therefore depicting wet season conditions. The second~~  
~~regime (during spring and summer months) remains within the bounds of the original Budyko~~  
~~framework, hence depicting conditions of no additional water other than  $P$  available for  $E$  (therefore~~  
290 ~~implying  $y_0$  being close to zero).~~

The comparison between modeled and observed  $E$  reveals a rather good performance of the model  
for Central Europe ( $R^2 = 0.87$ , RMSE = 0.51). In the Sahel region, the fixed parameter set (see  
Fig. 8d) best represents the wet regime (as it determines the maximum slope), resulting in the model  
to overestimate dry season  $E$ . However, the model performs significantly better in the Sahel region if  
295 one explicitly accounts for seasonal variations in the parameter set (see Fig. 8f). For central Europe,  
however, it is evident that a monthly Climatology of parameter sets does not significantly improve  
the model performance.

It is further important to note, that in some instances also the demand limit is exceeded, occurring  
most probably due to data uncertainties regarding the  $E$  estimates and the  $E_p$  parametrization exceeding  
300 the original supply limit. The modified formulation has the ability to represent this exceedance,  
whereas the original formulation is naturally bounded to 1. This is even more evident for the example  
grid point in Africa, showing a large overshoot of the original supply limit under dry season conditions.

## 6 Conclusions

305 Our study introduces a new, two-parameter Budyko-like model, which is capable to represent non-stationary  
~~characteristics of  $E/P$  and~~ In conclusion we present an extension to the Budyko framework that  
~~explicitly accounts for conditions under which  $E$  is also driven by other water sources than  $P$ .~~ The  
original Budyko framework is ~~constrained limited~~ to mean annual catchment scales, ~~in order to~~  
~~ensure a steady-state water balance that constitute  $P$  and  $E_p$  to be natural constraints of  $E$ .~~ Here  
310 we assume, ~~that on most other spatio-temporal scales, that~~ the boundary condition constituted  
by ~~the atmospheric water demand remains  $E_p$  remains overall valid,~~ whereas the boundary con-  
dition constituted by ~~water supply is, besides  $P$ , is~~ also subject to ~~water added (or withdrawn) via~~  
~~storage changes. To account for this assumption, the additional water stemming from other sources.~~

Such additional water could e.g. originate from changes in the terrestrial water storage, landscape changes and human interventions.

315

In order to account for such additional water, we modified the set of equations underlying the derivation of Fu's equation (Fu, 1981; Zhang et al., 2004) ~~was modified accordingly and and obtained~~ a similar formulation including an additional parameter ~~is obtained. Although the parameter in the original and the first parameter of our formulation are purely mathematical, the~~. The additional parameter is physically well defined. ~~Technically, the parameter and technically~~ rotates the original supply limit upwards. ~~The framework was validated by using global, monthly, gridded standard estimates of  $P$ ,  $E$  and  $E_p$ . The prediction of  $E$  using the model did represent seasonal dynamics for many parts of the world well by using a fixed parameter set over the whole time period. However, in several monsoon regions, the distinct difference between wet and dry seasons required enhanced parameter sets to represent the particular hydrological conditions of each month/season.~~

320

325

Like the ~~Similar to the~~ original Budyko framework, the derived two-parameter Budyko model represents the influence of first-order controls (namely  $P$  and  $E_p$ , ~~or in combination aridity index~~) on water availability). ~~Also, the combined~~. The integrated influence of second-order controls (like e.g. vegetation, topography, etc.) ~~are~~is, comparable to Fu's equation, ~~integrated into represented by~~ the first parameter ~~of the framework ( $\kappa$  in the new framework,  $\omega$  in Fu's equation, respectively).~~ ~~Studying these~~. Analyzing such controls in Fu's formula was subject to numerous studies, but no conclusive assessment was conducted until present. Assessing the combined influence of climatic and catchment controls is hence clearly beyond the scope of this study. However, the additional second parameter of the ~~new-modified~~ formulation  $y_0$  ~~is physically well defined does have a clear~~ physical interpretation as it represents a measure of additional water being (besides  $P$ ) available ~~for~~ to  $E$ . ~~But the availability of additional water is itself subject to numerous controls and if no data is available, a direct estimation of the parameter is initially not possible. Assessing these controls is, however, subject to future research.~~

330

335

Finally we note that the available water that can compensate for lack ~~The framework was validated for the special case of average seasonal changes in water storage by using monthly climatologies of global, gridded standard estimates of  $P$ , i.e. soil moisture, ground water and other surface water sources can be more accurately assessed on a month-to-month basis when using a water balance model. The purpose of  $E$  and  $E_p$ . The computed gridpoint-specific seasonal cycle of  $E/P$  using the modified framework did adequately represent mean seasonal storage changes for many parts of the world. However,~~ the present formulation is not to replace such modeling approaches but to promote a general framework accounting for non-stationary conditions within the Budyko relationship.

340

345

Further, Greve et al. (2015) recently suggested a probabilistic Budyko framework by assuming that the free parameter in Fu's equation is distributed. Similar assumptions could be applied to the two-parameter Budyko curve in future assessments, to allow for a better statistical representation of the scatter around the obtained curve ~~application of the modified framework is by no means limited~~

350

to this case and could be extended to a variety of climatic conditions under which additional water besides  $P$  is available to  $E$ .

## Appendix A: Complete Solution

Equations 3, 5 and 7 form a system of differential equations. A necessary condition to solve this  
355 system is

$$\frac{\partial f(x)}{\partial E_p} + \frac{\partial f(x)}{\partial E} g(y) = \frac{\partial g(y)}{\partial P} + \frac{\partial g(y)}{\partial E} f(x) \quad (\text{A1})$$

Combining equation A1 with equation 4 yields

$$\frac{\partial f(x)}{\partial E_p} = \frac{\partial f(x)}{\partial E_p} \frac{\partial x}{\partial x} = \frac{1}{P} \left(1 - \frac{\partial E}{\partial E_p}\right) \frac{\partial f(x)}{\partial x} = \frac{1}{P} (1 - g(y)) \frac{\partial f(x)}{\partial x} \quad (\text{A2a})$$

$$\frac{\partial f(x)}{\partial E} = \frac{\partial f(x)}{\partial E} \frac{\partial x}{\partial x} = \frac{1}{P} \left(\frac{\partial E_p}{\partial E} - 1\right) \frac{\partial f(x)}{\partial x} = \frac{1}{P} \left(\frac{1}{g(y)} - 1\right) \frac{\partial f(x)}{\partial x} \quad (\text{A2b})$$

$$360 \quad \frac{\partial g(y)}{\partial P} = \frac{\partial g(y)}{\partial P} \frac{\partial y}{\partial y} = \frac{1}{E_p} \left(1 - \frac{\partial E}{\partial P}\right) \frac{\partial g(y)}{\partial y} = \frac{1}{E_p} (1 - f(x)) \frac{\partial g(y)}{\partial y} \quad (\text{A2c})$$

$$\frac{\partial g(y)}{\partial E} = \frac{\partial g(y)}{\partial E} \frac{\partial y}{\partial y} = \frac{1}{E_p} \left(\frac{\partial P}{\partial E} - 1\right) \frac{\partial g(y)}{\partial y} = \frac{1}{E_p} \left(\frac{1}{f(x)} - 1\right) \frac{\partial g(y)}{\partial y} \quad (\text{A2d})$$

Substituting the factors in equation A1 with those given in equations A2 gives:

$$365 \quad \begin{aligned} \frac{\partial f(x)}{\partial x} \left( (1 - g(y)) + \left(\frac{1}{g(y)} - 1\right) g(y) \right) &= \frac{P}{E_p} \frac{\partial g(y)}{\partial y} \left( (1 - f(x)) + \left(\frac{1}{f(x)} - 1\right) f(x) \right) \\ (1 - g(y)) \frac{\partial f(x)}{\partial x} &= \frac{P}{E_p} (1 - f(x)) \frac{\partial g(y)}{\partial y} \end{aligned} \quad (\text{A3})$$

Expanding  $P/E_p$  yields under consideration of equations 4

$$\frac{P}{E_p} = \frac{\frac{E_p + P - E}{E_p}}{\frac{E_p + P - E}{P}} = \frac{1 + \frac{P - E}{E_p}}{1 + \frac{E_p - E}{P}} = \frac{1 + y}{1 + x} \quad (\text{A4})$$

From equation A3 and equation A4 follows

$$370 \quad \begin{aligned} (1 - g(y)) \frac{\partial f(x)}{\partial x} &= \frac{1 + y}{1 + x} (1 - f(x)) \frac{\partial g(y)}{\partial y} \\ \frac{1 + x}{1 - f(x)} \frac{\partial f(x)}{\partial x} &= \frac{1 + y}{1 - g(y)} \frac{\partial g(y)}{\partial y} \end{aligned} \quad (\text{A5})$$

where each side is a function of  $x$  or  $y$  only. Assuming the result of each side is  $\alpha$  it follows

$$\frac{1+x}{1-f(x)} \frac{\partial f(x)}{\partial x} = \alpha \quad (\text{A6a})$$

$$\frac{1+y}{1-g(y)} \frac{\partial g(y)}{\partial y} = \alpha \quad (\text{A6b})$$

375 Integrating equation A6a under consideration of the boundary condition given by equation 5 leads to the following expression for  $f(x)$

$$\begin{aligned} \int_0^x \frac{1}{1-f(t)} \frac{\partial f(t)}{\partial t} dt &= \alpha \int_0^x \frac{1}{1-t} dt \\ [-\ln(1-f(t))]_0^x &= \alpha [\ln(1+t)]_0^x \\ \ln(1-f(x)) &= -\alpha \ln(1+x) \\ 380 \quad 1-f(x) &= (1+x)^{-\alpha} \\ f(x) &= 1 - (1+x)^{-\alpha} \end{aligned} \quad (\text{A7})$$

Integrating equation A6b is different from the traditional solution given in Zhang et al. (2004), as we are using the new boundary condition given by equation 7

$$\begin{aligned} 385 \quad \int_{-y_0}^y \frac{1}{1-g(t)} \frac{\partial g(t)}{\partial t} dt &= \alpha \int_{-y_0}^y \frac{1}{1-t} dt \\ [-\ln(1-g(t))]_{-y_0}^y &= \alpha [\ln(1+t)]_{-y_0}^y \\ \ln(1-g(y)) - \ln(1-g(-y_0)) &= \alpha (\ln(1-y_0) - \ln(1+y)) \\ \ln(1-g(y)) &= \alpha \ln \left( \frac{1-y_0}{1+y} \right) \\ 1-g(y) &= \left( \frac{1-y_0}{1+y} \right)^\alpha \\ 390 \quad g(y) &= 1 - \left( \frac{1-y_0}{1+y} \right)^\alpha \end{aligned} \quad (\text{A8})$$

Considering the expansion from equation A4 finally gives

$$\partial E / \partial P = 1 - (1+x)^{-\alpha} = 1 - \left( \frac{P}{E_p + P - E} \right)^\alpha \quad (\text{A9})$$

$$\partial E / \partial E_0 = 1 - (1-y_0)^\alpha (1+y)^{-\alpha} = 1 - (1-y_0)^\alpha \left( \frac{E_0}{E_0 + P - E} \right)^\alpha \quad (\text{A10})$$

395 In the next step, equation A9 is integrated over  $P$ . As equation A9 is identical to those in Zhang et al. (2004), we follow their substitution approach. It follows

$$E = E_0 + P - (k + P^{\alpha+1})^{\frac{1}{\alpha+1}} \quad (\text{A11})$$

where  $k$  is a function of  $E_0$  only. Differentiate equation A11 with respect to  $E_0$  gives an estimate of  $\partial E/\partial E_0$ , which used with equation A10 determines  $k$

$$400 \quad \frac{\partial E}{\partial E_0} = 1 - \frac{1}{\alpha+1} (k + P^{\alpha+1})^{-\frac{\alpha}{\alpha+1}} \frac{\partial k}{\partial E_0} = 1 - (1-y_0)^\alpha \left( \frac{E_0}{E_0 + P - E} \right)^\alpha \quad (\text{A12})$$

This leads under consideration of equation A11 to the following expression

$$\begin{aligned} \frac{\partial k}{\partial E_0} &= (\alpha+1)(1-y_0)^\alpha \left( \frac{E_0}{E_0 + P - E} \right)^\alpha (k + P^{\alpha+1})^{\frac{\alpha}{\alpha+1}} \\ &= (\alpha+1)(1-y_0)^\alpha \left( \frac{E_0}{E_0 + P - (E_0 + P - (k + P^{\alpha+1})^{\frac{1}{\alpha+1}})} \right)^\alpha (k + P^{\alpha+1})^{\frac{\alpha}{\alpha+1}} \\ &= (\alpha+1)(1-y_0)^\alpha E_0^\alpha \\ 405 \quad k &= (\alpha+1)(1-y_0)^\alpha \int E_0^\alpha dE_0 \\ k &= (1-y_0)^\alpha E_0^{\alpha+1} + C \end{aligned} \quad (\text{A13})$$

with  $C$  being an integration constant. Substituting equation A13 back into equation A11, one obtains the following expression

$$410 \quad E = E_0 + P - ((1-y_0)^\alpha E_0^{\alpha+1} + C + P^{\alpha+1})^{\frac{1}{\alpha+1}} \quad (\text{A14})$$

and as  $\lim_{P \rightarrow 0} E = 0$  follows  $C = 0$ . Substituting  $\kappa = \alpha + 1$  finally gives

$$E = E_p + P - ((1-y_0)^{\kappa-1} E_p^\kappa + P^\kappa)^{\frac{1}{\kappa}} \quad (\text{A15})$$

and further provides by writing  $\Phi = E_p/P$

$$\frac{E}{P} = 1 + \Phi - (1 + (1-y_0)^{\kappa-1} (\Phi)^\kappa)^{\frac{1}{\kappa}} \quad (\text{A16})$$

$$415 \quad F\left(\frac{E}{E_p}, \kappa, y_0\right) = \frac{E}{E_p} = 1 + \frac{P}{E_p} - \left( (1-y_0)^{\kappa-1} + \left(\frac{P}{E_p}\right)^\kappa \right)^{\frac{1}{\kappa}} \quad (\text{A17})$$

## Appendix B: Solution of the actual slope

The actual slope  $m$  of the upper limit against which the obtained Budyko curve is converging to is smaller than  $y_0$ . We introduced equation 12 to calculate  $m$  and in the following we provide the complete solution in order to obtain equation 12.

420 The value of  $m$  is the slope of the linear function  $m\Phi + 1$  that forms the asymptote to  $F(\Phi, \kappa, y_0)$  given by equation 9. Hence,

$$\lim_{\Phi \rightarrow \infty} [F(\Phi, \kappa, y_0) - (m\Phi + 1)] = 0. \quad (\text{B1})$$

Using equation 9 and dividing by  $\Phi$  yields

$$\lim_{\Phi \rightarrow \infty} \left[ \frac{(1 + (1 - y_0)^{\kappa-1} (\Phi)^\kappa)^{\frac{1}{\kappa}}}{\Phi} + 1 - m \right] = 0. \quad (\text{B2})$$

425 By raising the term in brackets to the power of  $\kappa$  one obtains

$$\lim_{\Phi \rightarrow \infty} [(1 - m)^\kappa - \Phi^{-\kappa} (1 + \Phi^\kappa (1 - y_0)^{\kappa-1})] = 0, \quad (\text{B3})$$

and it follows

$$\lim_{\Phi \rightarrow \infty} [(1 - m)^\kappa - (1 - y_0)^{\kappa-1} - \Phi^{-\kappa}] = 0. \quad (\text{B4})$$

Since  $\Phi^{-\kappa} \rightarrow 0$  for  $\Phi \rightarrow \infty$  we obtain

$$430 \quad (1 - m)^\kappa = (1 - y_0)^{\kappa-1}. \quad (\text{B5})$$

Solving for  $m$  yields

$$m = (1 - y_0)^{1 - \frac{1}{\kappa}}. \quad (\text{B6})$$

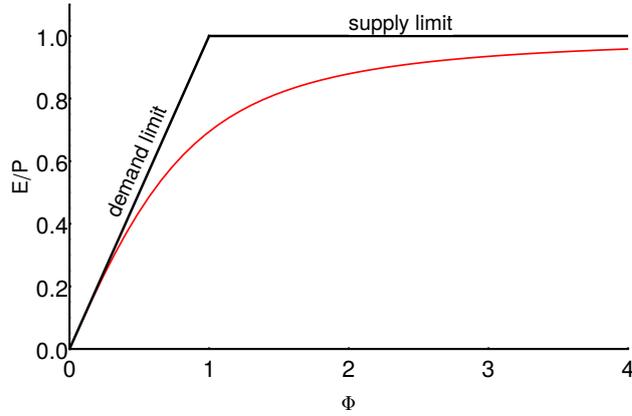
*Acknowledgements.* The Center for Climate Systems Modeling (C2SM) at ETH Zurich is acknowledged for providing technical and scientific support. We acknowledge partial support from the ETH Research Grant CH2-435 01 11-1, EU-FP7 EMBRACE and the ERC DROUGHT-HEAT Project.

## References

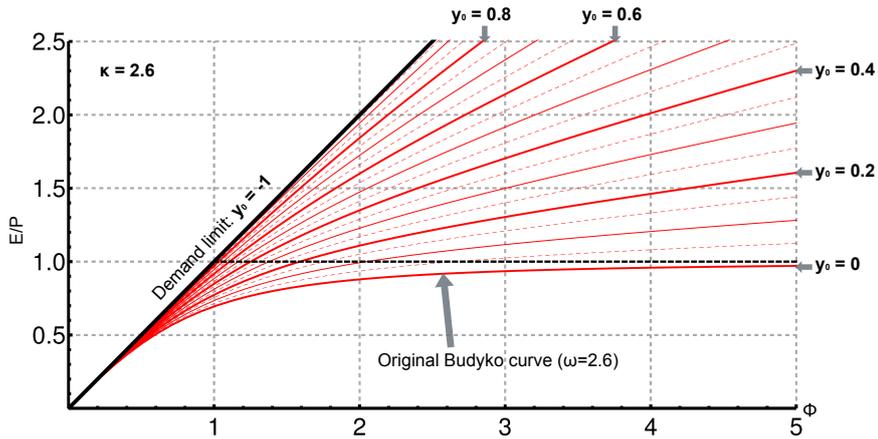
- Adler, R. F., Huffman, G. J., Chang, A., Ferraro, R., Xie, P., Janowiak, J., Rudolf, B., Schneider, U., Curtis, S., Bolvin, D., Gruber, A., Susskind, J., Arkin, P., and Nelkin, E. (2003). The version-2 global precipitation climatology project (GPCP) monthly precipitation analysis (1979-present). *J. Hydrometeor.*, 4:1147–1167.
- 440 Berghuijs, W. R., Woods, R. A., and Hrachowitz, M. (2014). A precipitation shift from snow towards rain leads to a decrease in streamflow. *Nature Climate Change*, 4(7):583–586.
- Budyko, M. (1956). The Heat Balance of the Earth's Surface. (in Russian).
- Budyko, M. (1974). Climate and life. *Academic Press*, (New York, USA).
- Chen, X., Alimohammadi, N., and Wang, D. (2013). Modeling interannual variability of seasonal evaporation and storage change based on the extended Budyko framework. *Water Resources Research*, 49(9):6067–6078.
- 445 Choudhury, B. (1999). Evaluation of an empirical equation for annual evaporation using field observations and results from a biophysical model. *Journal of Hydrology*, 216(1–2):99–110.
- Donohue, R. J., Roderick, M. L., and McVicar, T. R. (2007). On the importance of including vegetation dynamics in Budyko's hydrological model. *Hydrol. Earth Syst. Sci.*, 11(2):983–995.
- 450 Donohue, R. J., Roderick, M. L., and McVicar, T. R. (2012). Roots, storms and soil pores: Incorporating key ecohydrological processes into Budyko's hydrological model. *Journal of Hydrology*, 436–437:35–50.
- Fu, B. (1981). On the calculation of the evaporation from land surface (in Chinese)., *Sci. Atmos. Sin.*, 1(5):23–31.
- Gentine, P., D'Odorico, P., Lintner, B. R., Sivandran, G., and Salvucci, G. (2012). Interdependence of climate, soil, and vegetation as constrained by the Budyko curve. *Geophysical Research Letters*, 39(19)
- 455 Greve, P., Gudmundsson, L., Orlowsky, B., and Seneviratne, S. I. (2015). Introducing a probabilistic Budyko framework. *Geophysical Research Letters*, 42(7):2015GL063449.
- Jaramillo, F. and Destouni, G. (2014). Developing water change spectra and distinguishing change drivers worldwide. *Geophysical Research Letters*, page 2014GL061848.
- 460 Li, D., Pan, M., Cong, Z., Zhang, L., and Wood, E. (2013). Vegetation control on water and energy balance within the Budyko framework. *Water Resources Research*, 49(2):969–976.
- Mezentsev, V. (1955). More on the computation of total evaporation (Yechio raz o rastchetie srednevo sumarnovo ispareniiia). *Meteorog. i Gridrolog.*, 5:24–26.
- Milly, P. C. D. (1993). An analytic solution of the stochastic storage problem applicable to soil water. *Water Resources Research*, 29(11):3755–3758.
- 465 Milly, P. C. D. (1994). Climate, soil water storage, and the average annual water balance. *Water Resources Research*, 30(7):2143–2156.
- Milly, P. C. D., Betancourt, J., Falkenmark, M., Hirsch, R. M., Kundzewicz, Z. W., Lettenmaier, D. P., and Stouffer, R. J. (2008). Stationarity Is Dead: Whither Water Management? *Science*, 319(5863):573–574.
- 470 Monteith, J. (1965). Evaporation and environment. In *Symp. Soc. Exp. Biol*, volume 19, page 4.
- Mueller, B., Hirschi, M., Jimenez, C., Ciais, P., Dirmeyer, P. A., Dolman, A. J., Fisher, J. B., Jung, M., Ludwig, F., Maignan, F., Miralles, D. G., McCabe, M. F., Reichstein, M., Sheffield, J., Wang, K., Wood, E. F., Zhang, Y., and Seneviratne, S. I. (2013). Benchmark products for land evapotranspiration: LandFlux-EVAL multi-data set synthesis. *Hydrol. Earth Syst. Sci.*, 17(10):3707–3720.
- 475 Ol'Dekop, E. M. (1911). On Evaporation From the Surface of River Basins. *Univ. of Tartu, Tartu, Estonia*.

- Orlowsky, B. and Seneviratne, S. I. (2013). Elusive drought: uncertainty in observed trends and short- and long-term CMIP5 projections. *Hydrol. Earth Syst. Sci.*, 17(5):1765–1781.
- Pike, J. G. (1964). The estimation of annual run-off from meteorological data in a tropical climate. *Journal of Hydrology*, 2(2):116–123.
- 480 Porporato, A., Daly, E., and Rodriguez-Iturbe, I. (2004). Soil Water Balance and Ecosystem Response to Climate Change. *The American Naturalist*, 164(5):625–632.
- Potter, N. J. and Zhang, L. (2007). Water balance variability at the interstorm timescale. *Water Resources Research*, 43(5):W05405.
- Potter, N. J., Zhang, L., Milly, P. C. D., McMahon, T. A., and Jakeman, A. J. (2005). Effects of rainfall  
485 seasonality and soil moisture capacity on mean annual water balance for Australian catchments. *Water Resources Research*, 41(6):W06007.
- Schreiber, P. (1904). Ueber die Beziehungen zwischen dem Niederschlag und der Wasserfuehrung der Fluesse in Mitteleuropa. *Z. Meteorol.*, 21:441–452.
- Shao, Q., Traylen, A., and Zhang, L. (2012). Nonparametric method for estimating the effects of climatic and  
490 catchment characteristics on mean annual evapotranspiration. *Water Resources Research*, 48(3):W03517.
- Sheffield, J., Goteti, G., and Wood, E. F. (2006). Development of a 50-Year High-Resolution Global Dataset of Meteorological Forcings for Land Surface Modeling. *Journal of Climate*, 19(13):3088–3111.
- Sheffield, J., Wood, E. F., and Roderick, M. L. (2012). Little change in global drought over the past 60 years. *Nature*, 491(7424):435–438.
- 495 Turc, L.-H. (1955). *Le Bilan d'eau des sols: relations entre les précipitations, l'évaporation et l'écoulement...* Institut national de la recherche agronomique, Paris.
- Wang, D. and Tang, Y. (2014). A One-Parameter Budyko Model for Water Balance Captures Emergent Behavior in Darwinian Hydrologic Models. *Geophysical Research Letters*, page 2014GL060509.
- Wang, G. (2005). Agricultural drought in a future climate: results from 15 global climate models participating  
500 in the IPCC 4th assessment. *Climate Dynamics*, 25(7-8):739–753.
- Williams, C. A., Reichstein, M., Buchmann, N., Baldocchi, D., Beer, C., Schwalm, C., Wohlfahrt, G., Hasler, N., Bernhofer, C., Foken, T., Papale, D., Schymanski, S., and Schaefer, K. (2012). Climate and vegetation controls on the surface water balance: Synthesis of evapotranspiration measured across a global network of flux towers. *Water Resources Research*, 48(6)
- 505 Xu, X., Liu, W., Scanlon, B. R., Zhang, L., and Pan, M. (2013). Local and global factors controlling water-energy balances within the Budyko framework. *Geophysical Research Letters*, 40(23):6123–6129.
- Yang, D., Sun, F., Liu, Z., Cong, Z., Ni, G., and Lei, Z. (2007). Analyzing spatial and temporal variability of annual water-energy balance in nonhumid regions of China using the Budyko hypothesis. *Water Resources Research*, 43(4):W04426.
- 510 Yang, H., Yang, D., Lei, Z., and Sun, F. (2008). New analytical derivation of the mean annual water-energy balance equation. *Water Resources Research*, 44(3)
- Zanardo, s., Harman, C. J., Troch, P. A., Rao, P. S. C., and Sivapalan, M. (2012). Intra-annual rainfall variability control on interannual variability of catchment water balance: A stochastic analysis. *Water Resources Research*, 48(6):W00J16.

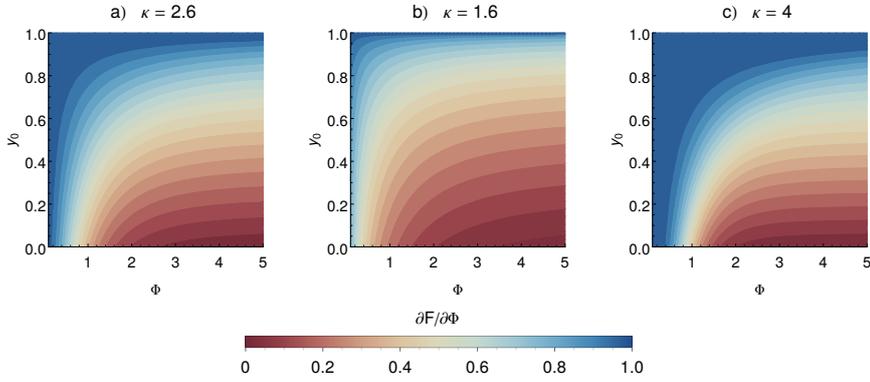
- 515 Zhang, L., Dawes, W. R., and Walker, G. R. (2001). Response of mean annual evapotranspiration to vegetation changes at catchment scale. *Water Resources Research*, 37(3):701–708.
- Zhang, L., Hickel, K., Dawes, W. R., Chiew, F. H. S., Western, A. W., and Briggs, P. R. (2004). A rational function approach for estimating mean annual evapotranspiration. *Water Resources Research*, 40(2)
- 520 Zhang, L., Potter, N., Hickel, K., Zhang, Y., and Shao, Q. (2008). Water balance modeling over variable time scales based on the Budyko framework – Model development and testing. *Journal of Hydrology*, 360(1–4):117–131.
- Zhou, G., Wei, X., Chen, X., Zhou, P., Liu, X., Xiao, Y., Sun, G., Scott, D. F., Zhou, S., Han, L., and Su, Y. (2015a). Global pattern for the effect of climate and land cover on water yield. *Nat Commun*, 6.
- 525 Zhou, S., Yu, B., Huang, Y., and Wang, G. (2015b). The complementary relationship and generation of the Budyko functions. *Geophysical Research Letters*, page 2015GL063511.



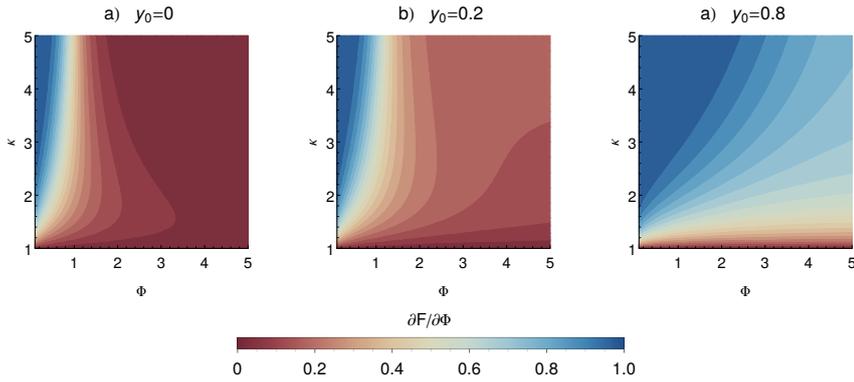
**Figure 1.** The original Budyko (1956) curve (red), limited by both the demand limit ( $E = E_p$ ) and the supply limit ( $E = P$ ).



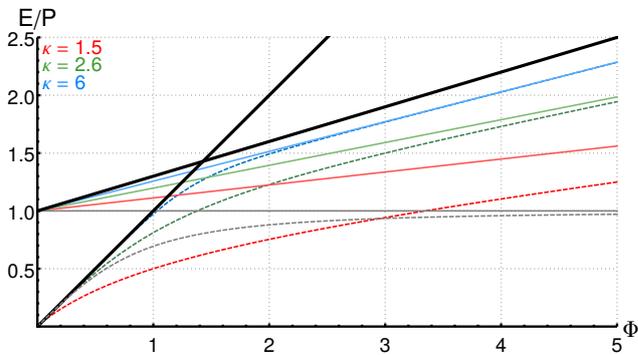
**Figure 2.** Set of curves of the new framework for  $\kappa = 2.6$  and different  $y_0$ . Note that the obtained curve for the parameter set  $(\kappa, y_0) = (2.6, 0)$  corresponds to the original Budyko curve ( $\omega = 2.6$ ). The supply limit (dashed black line) is systematically exceeded if  $y_0 > 0$  and the demand limit (solid black line) is reached if  $y_0 = 1$ .



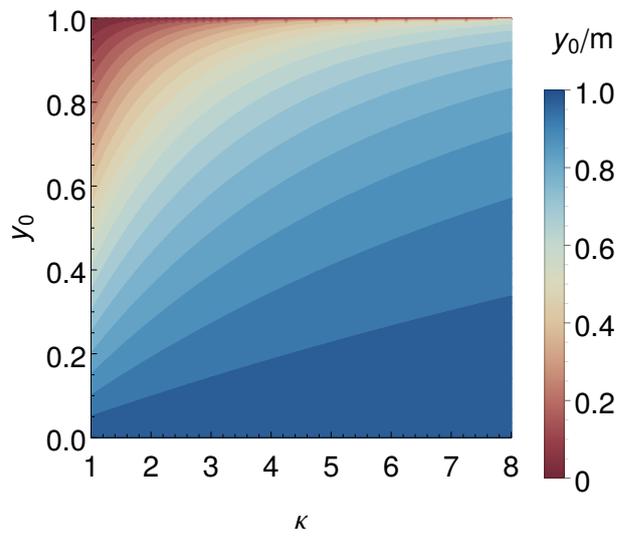
**Figure 3.** The sensitivity  $\partial F/\partial\Phi$  under varying  $y_0$ , for  $\kappa = 2.6$  (left, similar to the original Budyko framework if  $y_0 = 0$ ),  $\kappa = 1.6$  (center) and  $\kappa = 4$  (right). Blueish colors denote high, reddish colors low sensitivity.



**Figure 4.** The sensitivity  $\partial F/\partial\Phi$  under varying  $\kappa$ , for  $y_0 = 0$  (left),  $y_0 = 0.2$  (center) and  $y_0 = 0.8$  (right). Blueish colors denote high, reddish colors low sensitivity.

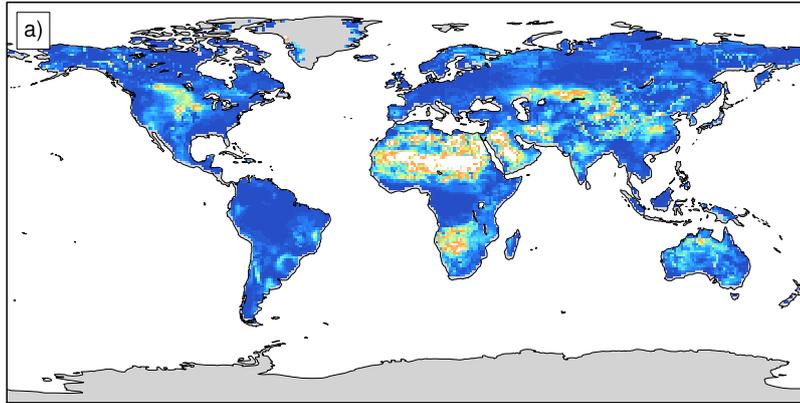


**Figure 5.** Difference between the actual (solid colored lines) and maximum slope (solid black line) of the supply limit for different values of  $\kappa$  (red:  $\kappa = 1.5$ , green:  $\kappa = 2.6$  and blue:  $\kappa = 6$ ) and  $y_0 = 0.3$ . The maximum slope ( $m = y_0 = 0.3$ ) is reached if  $\kappa \rightarrow \infty$ .

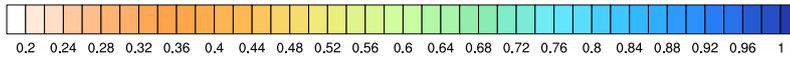
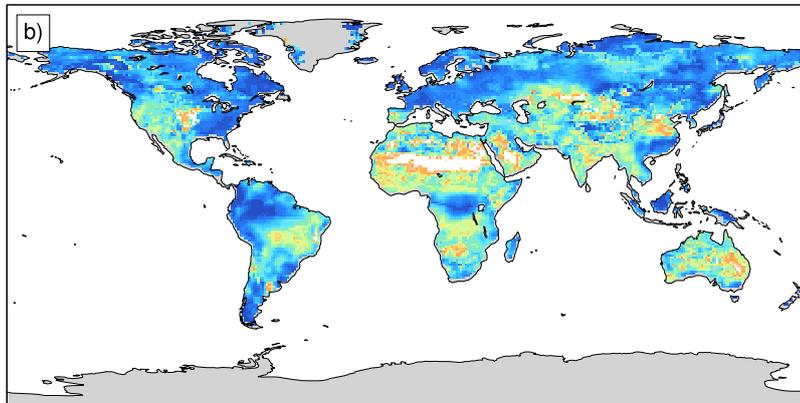


**Figure 6.** The ratio  $y_0/m$  as a function of both  $y_0$  and  $\kappa$  estimated from equation 13.

calibrated  $y_0$

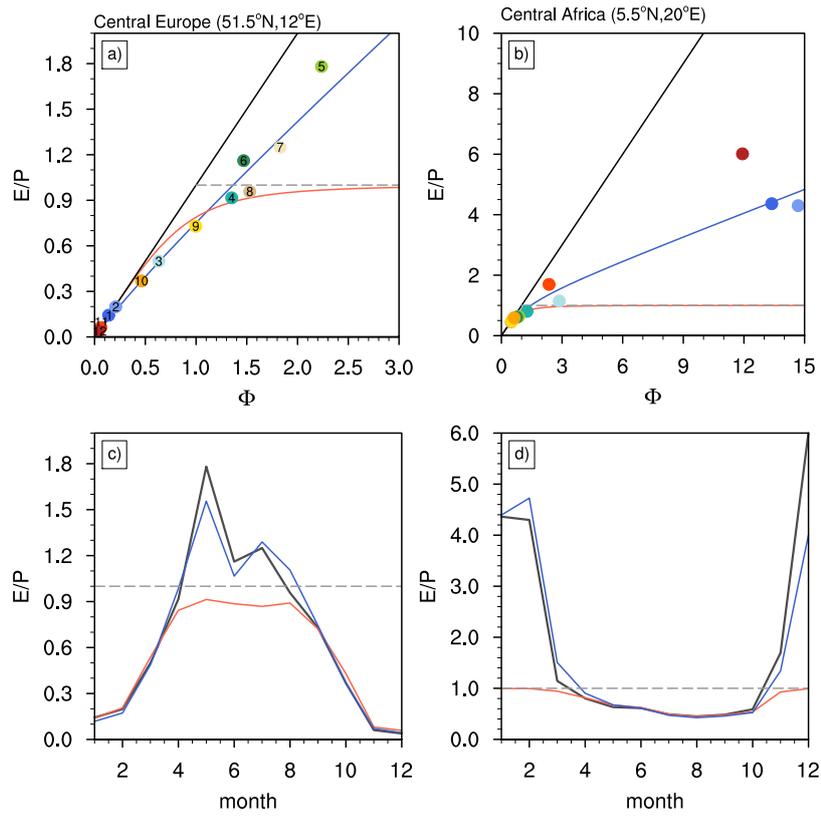


$y_0 = 0$  (original Budyko)



correlation

**Figure 7.** Correlation between the mean seasonal cycle of  $E/P$  computed from equation 9 and observed  $E/P$  for a) a grid-point specific parameter set  $(\kappa, y_0)$  and b)  $(\kappa, 0)$  (Fu's equation).



**Figure 8.** Data cloud of monthly climatologies within the Budyko space for a gridpoints in a) central Europe ( $51.5^{\circ}N$ ,  $12^{\circ}E$ ) and b) central Africa ( $5.5^{\circ}N$ ,  $20^{\circ}$ ). The black solid line denotes the demand limit, the dashed line denotes the original supply limit. The blue line depicts the obtained curve using the modified formulation of Fu's equation, whereas the red line shows the original Fu curve. Numbers within the dots denote the particular month of the year. c), d) Observed (grey) and computed mean seasonal cycles at both gridpoints. The blue line depicts the obtained seasonal cycle using the modified formulation of Fu's equation, whereas the red line shows the seasonal cycle obtained using Fu's equation. Please note that axes are different in each plot.