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A two-parameter Budyko function to represent conditions under which evapotranspiration exceeds precipitation

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Abstract. A comprehensive assessment of the partitioning of precipitation (P) into evapotranspiration (E) and runoff (Q) is of major importance for a wide range of socio-economic sectors. For climatological averages, the Budyko framework provides a simple first order relationship to estimate water availability represented by the ratio E/P as a function of the aridity index (E_p/P, with E_p denoting potential evaporation). However, a major downside of the Budyko framework is its limitation to steady state conditions, being a result of assuming negligible storage change in the land water balance. Processes leading to changes in the terrestrial water storage at any spatial and/or temporal scale are hence not represented. Here we propose an analytically derived modification of the Budyko framework including a new parameter explicitly representing additional water available to evapotranspiration besides precipitation. The modified framework is comprehensively analyzed, showing that the additional parameter leads to an upward rotation of the original water supply limit. We further evaluate the new formulation in an example application at mean seasonal time scales, showing that the extended framework is able to represent conditions in which evapotranspiration exceeds precipitation.

1 Introduction

The Budyko framework serves as a tool to predict mean annual water availability as a function of aridity. It is widely-used and well-established within the hydrological community, both due to its simplicity and long history, combining experience from over a century of hydrological research. Before and after Budyko (1956, 1974) derived a formulation of the function based on findings of Schreiber (1904) and Ol'Dekop (1911), several other formulations have been postulated, which however are numerically surprisingly similar (Schreiber, 1904; Ol'Dekop, 1911; Turc, 1955; Mezentsev, 1955; Pike, 1964; Fu, 1981; Choudhury, 1999; Zhang et al., 2001, 2004; Porporato et al., 2004; Yang et al., 2008; Donohue et al., 2012; Wang and Tang, 2014; Zhou et al., 2015b). Many of these formulations are empirically derived and only few are analytically determined from simple phenomenological assumptions (Fu, 1981; Milly, 1994; Porporato et al., 2004; Zhang et al., 2004; Yang et al., 2007; Zhou et al., 2015b). Numerous studies further assess controls determining the observed systematic scatter within the Budyko space. A variety of catchment and climate characteristics, such as e.g. vegetation (Zhang et al., 2001; Donohue et al., 2007; Williams et al., 2012; Li et al., 2013; Zhou et al., 2015a), seasonality characteristics (Milly, 1994; Potter et al., 2005; Gentine et al., 2012; Chen et al., 2013; Berghuijs et al., 2014), soil properties (Porporato et al., 2004; Shao et al., 2012; Donohue et al., 2012), and topographic controls (Shao et al., 2012; Xu et al., 2013) have been proposed to exert a certain influence on the scatter within the Budyko space. Also more complex approaches to combine various controls (Milly, 1994; Gentine et al., 2012; Donohue et al., 2012; Xu et al., 2013) have been considered. Nonetheless, until present no conclusive statement on controls determining the scatter within the Budyko space could be made. In a recent assessment, Greve et al. (2015) suggested a probabilistic Budyko framework by assuming that the combined influence of all possible controls follows a probability distribution.

In this study we make use of the formulation introduced by Fu (1981) and Zhang et al. (2004). They derived a functional form between E/P and $\Phi = E_p/P$ at mean annual catchment scales analytically from simple physical assumptions,

$$\frac{E}{P} = 1 + \Phi - \left(1 + (\Phi)^{\omega}\right)^{\frac{1}{\omega}}, \tag{1}$$

where ω is a free model parameter. The original formulation introduced by Budyko (1956, 1974) is numerically reproduced by setting $\omega = 2.6$ (Zhang et al., 2004). The obtained function is subject to two physical constraints constituting both the water demand and supply limits. The water demand limit represents E being limited by E_p, whereas the water supply limit determines E to be limited by P (see Fig. 1). Regarding the supply limit, steady-state conditions in the land water balance at catchment scales

$$\frac{dS}{dt} = P - E - Q \tag{2}$$

are required and the storage term (dS/dt) is consequently assumed to be zero, which is generally a valid assumption at mean annual scales. However, the assumption of negligible storage changes constitutes a major limitation to the original Budyko framework. As a consequence, the framework is not valid under conditions in which additional water (besides P) is available to E and $E > P$. Such conditions can occur e.g. at sub-annual or inter-annual time scales due to changes in terrestrial water storage terms such as soil moisture, groundwater or snow storage. Additional water might be also introduced by landscape changes (Jaramillo and Destouni, 2014), human interventions (Milly et al., 2008) or phase changes of water within the system or supplied through precipitation (Jaramillo and Destouni, 2014; Berghuijs et al., 2014). Also long-term changes in soil moisture may happen, e.g. under transient climate change (Wang, 2005; Orlowsky and Seneviratne, 2013). Only few assessments addressed this limitation and provided further insights on how the Budyko hypothesis could be extended to conditions under which E exceeds P (Milly, 1993; Potter and Zhang, 2007; Zhang et al., 2008; Zarnado et al., 2012; Chen et al., 2013). Nonetheless, so far a theoretical, rigorous incorporation of conditions in which $E > P$ into the Budyko framework is missing. Here we aim to address this issue by analytically deriving a new, modified Budyko formulation from basic phenomenological assumptions by using the approach of Fu (1981) and Zhang et al. (2004).

2 Deriving a modified formulation

2.1 Preliminary Assumptions

Fu (1981) and Zhang et al. (2004) suggested that for a given potential evaporation, the rate of change in evapotranspiration as a function of the rate of change in precipitation ($\partial E/\partial P$) increases with residual potential evaporation ($E_p - E$) and decreases with precipitation. Similar assumptions were made regarding the rate of change in evapotranspiration as a function of the rate of change in potential evaporation ($\partial E/\partial E_p$) by considering residual precipitation ($P - E$). Hence, both ratios can be written as

$$\frac{\partial E}{\partial P} = f(x) \tag{3a}$$

$$\frac{\partial E}{\partial E_p} = g(y) \tag{3b}$$

with

$$x = \frac{E_p - E}{P} \tag{4a}$$

$$y = \frac{P - E}{E_p} \tag{4b}$$

Considering E_p being a natural constraint of E, it follows

$$\left.\frac{\partial E}{\partial P}\right|_{x=0} = 0. \tag{5}$$

The original approach of Fu (1981) further assumes that P is a natural constraint of E, constituting the following boundary condition

$$\left.\frac{\partial E}{\partial E_p}\right|_{y=0} = 0. \tag{6}$$

This assumption requires steady-state conditions and is consequently considered to be valid at mean annual catchment scales (such that $P - E \geq 0$) only. However, as mentioned in the introduction, a wealth of possible mechanisms and processes can induce conditions in which E exceeds P. In such cases, E_p remains the only constraint of E. Consequently, since we explicitly aim to account for conditions of $E \geq P$, the value $y = (P - E)/E_p$ (see equation 4) is not necessarily positive (but larger than -1 since we assume that $E \leq E_p$ The minimum value of y, denoted as y_{min} (see equation 4), thus lies within the interval between -1 and 0 and depends on the additional amount of water being available for E besides water supplied by P. For convenience we define $y_0 = -y_{min}$ (and thus $y_0 \in [0, 1]$). As a consequence the boundary condition 6 is then redefined as

$$\left.\frac{\partial E}{\partial E_p}\right|_{-y_0} = 0. \tag{7}$$

2.2 Solution

Solving the system of the differential equations 3a,b using boundary condition 5 and the new condition 7 yields the following solution (details are provided in Appendix A):

$$E = E_p + P - ((1 - y_0)^{\kappa - 1} E_p^{\kappa} + P^{\kappa})^{\frac{1}{\kappa}} \tag{8}$$

with κ being a free model parameter. It follows

$$\frac{E}{P} = F(\Phi, \kappa, y_0) = 1 + \Phi - \left(1 + (1 - y_0)^{\kappa - 1} (\Phi)^{\kappa}\right)^{\frac{1}{\kappa}}. \tag{9}$$

Similar to the traditional Budyko approach a free model parameter (named κ to avoid confusion with the traditional ω) is obtained. The second parameter y_0, as introduced in the previous section, is directly related to the new boundary condition. Hence, in contrast to κ, which is a mathematical constant, y_0 has a physical interpretation as it accounts for additional water. However, similar to the ω-parameter in Fu's equation, κ could be interpreted as an integrator of the variety of catchment properties other than the aridity index.

3 Characteristics of the modified framework

The newly derived formulation given (equation 9) is similar to the classical solution (equation 1), but includes y_0 as a new parameter. Assuming e.g. $\kappa = 2.6$ (corresponding to the original Budyko function with $\omega = 2.6$ in Fu's equation) and example values of y_0-values, Fig. 2 shows a set of curves providing insights on the basic characteristics of the modified equation.

In case $y_0 = 0$ (being the original boundary condition), the obtained curve corresponds to the steady-state framework of Fu (1981) and Zhang et al. (2004). This shows that both model formulations are consistently transferable. If $y_0 > 0$, the supply limit is systematically exceeded. The exceedance of the supply limit increases with increasing y_0. If $y_0 = 1$, the curve follows the demand limit. All curves are further continuous and strictly increasing.

Taking a closer look at the underlying boundary conditions and definitions (see section 2.1) reveals that y_0 explicitly accounts for the amount of additional water (besides water supplied through P) available for E. Since y_{min} is defined to be the minimum of $y = (P - E)/E_p$, the quantity $y_0 = -y_{min}$ physically represents the maximum fraction of E relative to E_p, which is not originating from P. A larger fraction consequently results in higher y_0-values and thus in a stronger exceedance of the original supply limit. Further details on y_0 is provided in section 4.

The sensitivity $\partial F(\Phi, \kappa, y_0)/\partial\Phi$ under varying κ and for three preselected values of y_0 is illustrated in Fig. 3. The sensitivity $\partial F(\Phi, \kappa, y_0)/\partial\Phi$ for different values of y_0 and κ shows the effect of the parameter choice on changes in E/P relative to changes in Φ. In general, the sensitivity is largest for small Φ (humid conditions), due to the fact that changes in E/P basically follow the demand limit (resulting in a sensitivity close to 1) regardless of parameter set (κ, y_0). For different parameter settings, the sensitivity generally decreases with increasing Φ. For small values of y_0 (close to zero), sensitivity becomes smallest with increasing Φ, since small values of y_0 indicate conditions similar to the classical solution (equation 1). Further, the smallest sensitivity is reached for large values of κ. Large values of y_0 (close to 1) indicate conditions mainly constrained by the demand limit, thus implying a sensitivity close to 1.

A similar analysis is performed for varying values of κ under three preselected levels of y_0 (see Fig. 4). For $y_0 = 0$ (steady-state conditions), the sensitivity $\partial F/\partial\Phi$ is under humid conditions ($\Phi < 1$) rather large, since changes in E/P are mainly constrained by demand limit. This especially applies for large values of κ. Under more arid conditions ($\Phi > 1$), the Budyko curve slowly converges towards the (horizontal) supply limit, resulting in a near-zero sensitivity. For $y_0 = 0.2$, denoting conditions relatively similar to steady-state conditions, the decrease in sensitivity with increasing Φ is weaker, whereas for $y_0 = 0.8$, denoting conditions where E is mainly constraint by the demand limit, sensitivity is large for large κ-values and decreases rather slowly with increasing Φ.

4 Interpreting the new parameter y_0

The new parameter y_0 is, in contrast to κ, physically well defined. The combination of equation 4b and 7 shows that y_0 is explicitly related to the amount of additional water (besides water supplied through P), which is available to E. If we rewrite equation 4b with respect to y_0

$$y_0 = -y_{min} = -\left(\frac{P-E}{E_p}\right)_{min} = -\frac{P_{min}-E_{max}}{E_p}, \quad \text{if } P_{min}-E_{max}<0, \tag{10}$$

where P_{min} and E_{max} are chosen in order to minimize y_{min} for a given E_p, we obtain a linear equation in terms of aridity index

$$\left(\frac{E}{P}\right)_{max} = y_0\left(\frac{E_p}{P_{min}}\right)+1, \tag{11}$$

which constitutes the mathematical interpretation of y_0 within the modified framework. That is, that y_0 determines the maximum slope of the upper limit, against which the obtained curve from equation 9 asymptotically converges to if $\kappa \to \infty$ (see Fig. 5). Physically, y_0 determines the maximum E/P that is reached in relation to Φ within a certain time period and spatial domain. It thus represents an estimate of the maximum amount of additional water that contributes to E and originates from other sources than P. Technically speaking, y_0 determines the slope of the upper limit such that all possible pairs (Φ, E/P) are just below the line $y_0\Phi+1$. It is further important to note that for mean annual conditions ($P-E \geq 0$), $y_0 = 0$ is considered, which results in a zero slope and thus determines the original supply limit of 1. Please also note, that this approach is not valid if $P_{min} = 0$.

However, the actual slope m of the upper limit is smaller than y_0, but directly related to both y_0 and κ as follows (see Appendix B for more information)

$$m = 1-(1-y_0)^{1-\frac{1}{\kappa}}. \tag{12}$$

The relative difference between the maximum slope y_0 and the actual slope m of the upper limit (being the ratio of y_0/m) is thus determined following the relationship

$$\frac{y_0}{m} = (1-y_0)^{1/k}. \tag{13}$$

The ratio y_0/m as a function of both y_0 and κ is illustrated in Fig. 6. For small κ and large y_0 (close to 1), the difference between the actual slope m and the maximum slope y_0 is large, whereas for large κ the actual slope m converges towards y_0. However, in any case, y_0 determines the maximum overshoot allowed with respect to the original supply limit at $y_0 = 0$.

5 Example application: Seasonal carryover effects in terrestrial water storage

At monthly time scales, changes in terrestrial water storage (due to changes in water storage components such as soil moisture, snow or groundwater) potentially play an important role and are by no means negligible. Such changes can provide a significant source of additional water that is (besides P) available to E. Here we analyse the multi-year mean seasonal cycle of E/P by using gridded, monthly data estimates of P, E and E_p. This allows us to evaluate the capability of the obtained framework (given by equation 9) to represent additional water sources at such time scales.

We employ the following, well-established, gridded data products: (i) the Global Precipitation Climatology Project (GPCP) P dataset (Adler et al., 2003), (ii) an E_p estimate (Sheffield et al., 2006, 2012) based on the Penman-Monteith E_p algorithm (Monteith, 1965; Sheffield et al., 2012), and (iii) the LandFlux-Eval E dataset (Mueller et al., 2013). All data is bilinearly interpolated to a unified $1°$-grid and the mean seasonal cycle for the 1990-2000 period is calculated at gridpoint-scale. Please note that the combination of datasets used here is arbitrary and only used to illustrate the capability of the newly developed framework to represent the multi-year mean annual cycle of E/P.

We estimate the parameter set (κ, y_0) from equation 9 by minimizing the residual sum of squares. This means that at every gridpoint 12 monthly climatologies of E/P (representing the mean seasonal cycle of E/P) are used to determine a specific parameter set.

To evaluate the modified framework, the derived parameter sets at each gridpoint are used in equation 9 to compute mean seasonal cycles of E/P. The correlation between the computed and the observed seasonal cycle is shown in Fig. 7a. The correlations are relatively large in most regions. Largest correlations (>0.9) are found in most mid to high latitude and tropical areas, clearly showing the capability of the modified formulation to represent the seasonal cycle in E/P. Correlations are generally somewhat lower in drier regions, especially in parts of Africa and Central Asia, probably occurring due to more complex seasonal patterns in E/P. Using instead Fu's original equation (or setting $y_0 = 0$) to estimate the mean seasonal cycle of E/P shows overall lower correlations, especially in semi-arid regions (Fig. 7b).

Taking a closer look at the mean seasonal cycle for example gridpoints in (i) Central Europe (humid climate) and (ii) Africa (semi-arid climate) clearly shows the improvement gained through the use of the modified formulation (Fig. 8). In Central Europe, additional water is available in the early summer months due to e.g. depletion of soil moisture or snow melt, resulting in values of E/P exceeding the original supply limit. The modified formulation has the ability to represent this exceedance, whereas the original formulation is naturally bounded to 1. This is even more evident for the example grid point in Africa, showing a large overshoot of the original supply limit under dry season conditions.

6 Conclusions

In conclusion we present an extension to the Budyko framework that explicitly accounts for conditions under which E is also driven by other water sources than P. The original Budyko framework is limited to mean annual catchment scales that constitute P and E_p to be natural constraints of E. Here we assume that the boundary condition constituted by E_p remains overall valid, whereas the boundary condition constituted by P is also subject to additional water stemming from other sources. Such additional water could e.g. originate from changes in the terrestrial water storage, landscape changes and human interventions.

In order to account for such additional water, we modified the set of equations underlying the derivation of Fu's equation (Fu, 1981; Zhang et al., 2004) and obtained a similar formulation including an additional parameter. The additional parameter is physically well defined and technically rotates the original supply limit upwards. Similar to the original Budyko framework, the derived two-parameter Budyko model represents the influence of first-order controls (namely P and E_p) on water availability. The integrated influence of second-order controls (like e.g. vegetation, topography, etc.) is, comparable to Fu's equation, represented by the first parameter. Analyzing such controls in Fu's formula was subject to numerous studies, but no conclusive assessment was conducted until present. Assessing the combined influence of climatic and catchment controls is hence clearly beyond the scope of this study. However, the additional second parameter of the modified formulation y_0 does have a clear physical interpretation as it represents a measure of additional water being (besides P) available to E.

The framework was validated for the special case of average seasonal changes in water storage by using monthly climatologies of global, gridded standard estimates of P, E and E_p. The computed gridpoint-specific seasonal cycle of E/P using the modified framework did adequately represent mean seasonal storage changes for many parts of the world. However, the application of the modified framework is by no means limited to this case and could be extended to a variety of climatic conditions under which additional water besides P is available to E.

Appendix A: Complete Solution

Equations 3, 5 and 7 form a system of differential equations. A necessary condition to solve this system is

$$\frac{\partial f(x)}{\partial E_p} + \frac{\partial f(x)}{\partial E} g(y) = \frac{\partial g(y)}{\partial P} + \frac{\partial g(y)}{\partial E} f(x) \tag{A1}$$

Combining equation A1 with equation 4 yields

$$\frac{\partial f(x)}{\partial E_p} = \frac{\partial f(x)}{\partial E_p} \frac{\partial x}{\partial x} = \frac{1}{P}\left(1 - \frac{\partial E}{\partial E_p}\right) \frac{\partial f(x)}{\partial x} = \frac{1}{P}\left(1 - g(y)\right) \frac{\partial f(x)}{\partial x} \tag{A2a}$$

$$\frac{\partial f(x)}{\partial E} = \frac{\partial f(x)}{\partial E}\frac{\partial x}{\partial x} = \frac{1}{P}\left(\frac{\partial E_p}{\partial E} - 1\right)\frac{\partial f(x)}{\partial x} = \frac{1}{P}\left(\frac{1}{g(y)} - 1\right)\frac{\partial f(x)}{\partial x} \tag{A2b}$$

$$\frac{\partial g(y)}{\partial P} = \frac{\partial g(y)}{\partial P}\frac{\partial y}{\partial y} = \frac{1}{E_p}\left(1 - \frac{\partial E}{\partial P}\right)\frac{\partial g(y)}{\partial y} = \frac{1}{E_p}\left(1 - f(x)\right)\frac{\partial g(y)}{\partial y} \tag{A2c}$$

$$\frac{\partial g(y)}{\partial E} = \frac{\partial g(y)}{\partial E}\frac{\partial y}{\partial y} = \frac{1}{E_p}\left(\frac{\partial P}{\partial E} - 1\right)\frac{\partial g(y)}{\partial y} = \frac{1}{E_p}\left(\frac{1}{f(x)} - 1\right)\frac{\partial g(y)}{\partial y} \tag{A2d}$$

Substituting the factors in equation A1 with those given in equations A2 gives:

$$\begin{aligned} \frac{\partial f(x)}{\partial x}\left((1-g(y)) + \left(\frac{1}{g(y)} - 1\right)g(y)\right) &= \frac{P}{E_p}\frac{\partial g(y)}{\partial y}\left((1-f(x)) + \left(\frac{1}{f(x)} - 1\right)f(x)\right) \\ (1-g(y))\frac{\partial f(x)}{\partial x} &= \frac{P}{E_p}(1-f(x))\frac{\partial g(y)}{\partial y} \end{aligned} \tag{A3}$$

Expanding P/E_p yields under consideration of equations 4

$$\frac{P}{E_p} = \frac{\frac{E_p+P-E}{E_p}}{\frac{E_p+P-E}{P}} = \frac{1+\frac{P-E}{E_p}}{1+\frac{E_p-E}{P}} = \frac{1+y}{1+x} \tag{A4}$$

From equation A3 and equation A4 follows

$$\begin{aligned} (1-g(y))\frac{\partial f(x)}{\partial x} &= \frac{1+y}{1+x}(1-f(x))\frac{\partial g(y)}{\partial y} \\ \frac{1+x}{1-f(x)}\frac{\partial f(x)}{\partial x} &= \frac{1+y}{1-g(y)}\frac{\partial g(y)}{\partial y} \end{aligned} \tag{A5}$$

where each side is a function of x or y only. Assuming the result of each side is α it follows

$$\frac{1+x}{1-f(x)}\frac{\partial f(x)}{\partial x} = \alpha \tag{A6a}$$

$$\frac{1+y}{1-g(y)}\frac{\partial g(y)}{\partial y} = \alpha \tag{A6b}$$

Integrating equation A6a under consideration of the boundary condition given by equation 5 leads to the following expression for $f(x)$

$$\int_0^x \frac{1}{1-f(t)} \frac{\partial f(t)}{\partial t} dt = \alpha \int_0^x \frac{1}{1-t} dt$$
$$[-\ln(1-f(t))]_0^x = \alpha\,[\ln(1+t)]_0^x$$
$$\ln(1-f(x)) = -\alpha \ln(1+x)$$
$$1-f(x) = (1+x)^{-\alpha}$$
$$f(x) = 1-(1+x)^{-\alpha} \tag{A7}$$

Integrating equation A6b is different from the traditional solution given in Zhang et al. (2004), as we are using the new boundary condition given by equation 7

$$\int_{-y_0}^{y} \frac{1}{1-g(t)} \frac{\partial g(t)}{\partial t} dt = \alpha \int_{-y_0}^{y} \frac{1}{1-t} dt$$
$$[-\ln(1-g(t))]_{-y_0}^{y} = \alpha\,[\ln(1+t)]_{-y_0}^{y}$$
$$\ln(1-g(y)) - \ln(1-g(-y_0)) = \alpha\,(\ln(1-y_0) - \ln(1+y))$$
$$\ln(1-g(y)) = \alpha \ln\left(\frac{1-y_0}{1+y}\right)$$
$$1-g(y) = \left(\frac{1-y_0}{1+y}\right)^{\alpha}$$
$$g(y) = 1-\left(\frac{1-y_0}{1+y}\right)^{\alpha} \tag{A8}$$

Considering the expansion from equation A4 finally gives

$$\partial E/\partial P = 1-(1+x)^{-\alpha} = 1-\left(\frac{P}{E_p+P-E}\right)^{\alpha} \tag{A9}$$

$$\partial E/\partial E_0 = 1-(1-y_0)^{\alpha}(1+y)^{-\alpha} = 1-(1-y_0)^{\alpha}\left(\frac{E_0}{E_0+P-E}\right)^{\alpha} \tag{A10}$$

In the next step, equation A9 is integrated over P. As equation A9 is identical to those in Zhang et al. (2004), we follow their substitution approach. It follows

$$E = E_0 + P - (k + P^{\alpha+1})^{\frac{1}{\alpha+1}} \tag{A11}$$

where k is a function of E_0 only. Differentiate equation A11 with respect to E_0 gives an estimate of $\partial E/\partial E_0$, which used with equation A10 determines k

$$\frac{\partial E}{\partial E_0} = 1 - \frac{1}{\alpha+1}(k+P^{\alpha+1})^{-\frac{\alpha}{\alpha+1}} \frac{\partial k}{\partial E_0} = 1-(1-y_0)^{\alpha}\left(\frac{E_0}{E_0+P-E}\right)^{\alpha} \tag{A12}$$

This leads under consideration of equation A11 to the following expression

$$
\begin{aligned}
\frac{\partial k}{\partial E_0} &= (\alpha+1)(1-y_0)^{\alpha}\left(\frac{E_0}{E_0+P-E}\right)^{\alpha}(k+P^{\alpha+1})^{\frac{\alpha}{\alpha+1}} \\
&= (\alpha+1)(1-y_0)^{\alpha}\left(\frac{E_0}{E_0+P-(E_0+P-(k+P^{\alpha+1})^{\frac{1}{\alpha+1}})}\right)^{\alpha}(k+P^{\alpha+1})^{\frac{\alpha}{\alpha+1}} \\
&= (\alpha+1)(1-y_0)^{\alpha}E_0^{\alpha} \\
k &= (\alpha+1)(1-y_0)^{\alpha}\int E_0^{\alpha}\,dE_0 \\
k &= (1-y_0)^{\alpha}E_0^{\alpha+1}+C
\end{aligned}
\tag{A13}
$$

with C being an integration constant. Substituting equation A13 back into equation A11, one obtains the following expression

$$E = E_0 + P - ((1-y_0)^{\alpha}E_0^{\alpha+1} + C + P^{\alpha+1})^{\frac{1}{\alpha+1}} \tag{A14}$$

and as $\lim\limits_{P\to 0} E = 0$ follows $C = 0$. Substituting $\kappa = \alpha+1$ finally gives

$$E = E_p + P - ((1-y_0)^{\kappa-1}E_p^{\kappa} + P^{\kappa})^{\frac{1}{\kappa}} \tag{A15}$$

and further provides by writing $\Phi = E_p/P$

$$\frac{E}{P} = 1 + \Phi - \left(1 + (1-y_0)^{\kappa-1}(\Phi)^{\kappa}\right)^{\frac{1}{\kappa}} \tag{A16}$$

$$F\left(\frac{E}{E_p},\kappa,y_0\right) = \frac{E}{E_p} = 1 + \frac{P}{E_p} - \left((1-y_0)^{\kappa-1} + \left(\frac{P}{E_p}\right)^{\kappa}\right)^{\frac{1}{\kappa}} \tag{A17}$$

Appendix B: Solution of the actual slope

The actual slope m of the upper limit against which the obtained Budyko curve is converging to is smaller than y_0. We introduced equation 12 to calculate m and in the following we provide the complete solution in order to obtain equation 12.

The value of m is the slope of the linear function $m\Phi+1$ that forms the asymptote to $F(\Phi,\kappa,y_0)$ given by equation 9. Hence,

$$\lim_{\Phi\to\infty}\left[F(\Phi,\kappa,y_0) - (m\Phi+1)\right] = 0. \tag{B1}$$

Using equation 9 and dividing by Φ yields

$$\lim_{\Phi\to\infty}\left[\frac{\left(1+(1-y_0)^{\kappa-1}(\Phi)^{\kappa}\right)^{\frac{1}{\kappa}}}{\Phi}+1-m\right]=0. \tag{B2}$$

By raising the term in brackets to the power of κ one obtains

$$\lim_{\Phi\to\infty}\left[(1-m)^{\kappa}-\Phi^{-\kappa}(1+\Phi^{\kappa}(1-y_0)^{\kappa-1})\right]=0, \tag{B3}$$

and it follows

$$\lim_{\Phi\to\infty}\left[(1-m)^{\kappa}-(1-y_0)^{\kappa-1}-\Phi^{-\kappa}\right]=0. \tag{B4}$$

Since $\Phi^{-\kappa}\to 0$ for $\Phi\to\infty$ we obtain

$$(1-m)^{\kappa}=(1-y_0)^{\kappa-1}. \tag{B5}$$

Solving for m yields

$$m=(1-y_0)^{1-\frac{1}{\kappa}}. \tag{B6}$$

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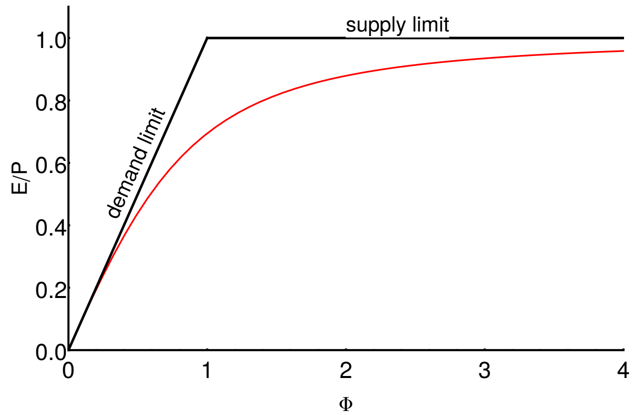

Figure 1. The original Budyko (1956) curve (red), limited by both the demand limit ($E = E_p$) and the supply limit ($E = P$).

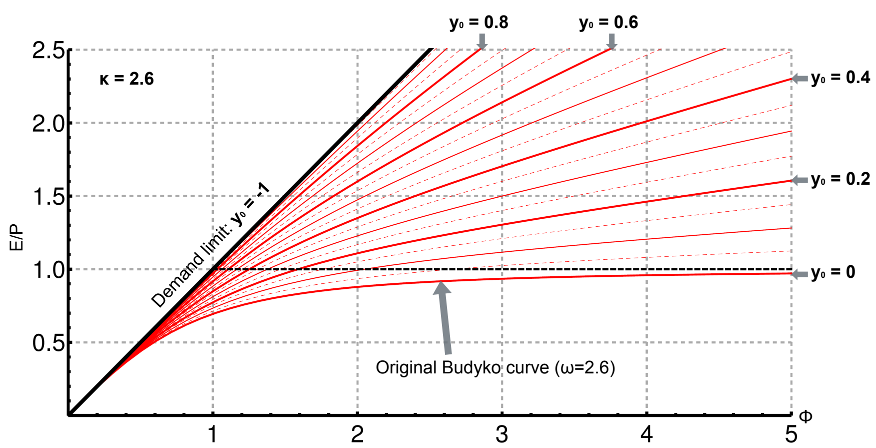

Figure 2. Set of curves of the new framework for $\kappa = 2.6$ and different y_0. Note that the obtained curve for the parameter set $(\kappa, y_0) = (2.6, 0)$ corresponds to the original Budyko curve ($\omega = 2.6$). The supply limit (dashed black line) is systematically exceeded if $y_0 > 0$ and the demand limit (solid black line) is reached if $y_0 = 1$.

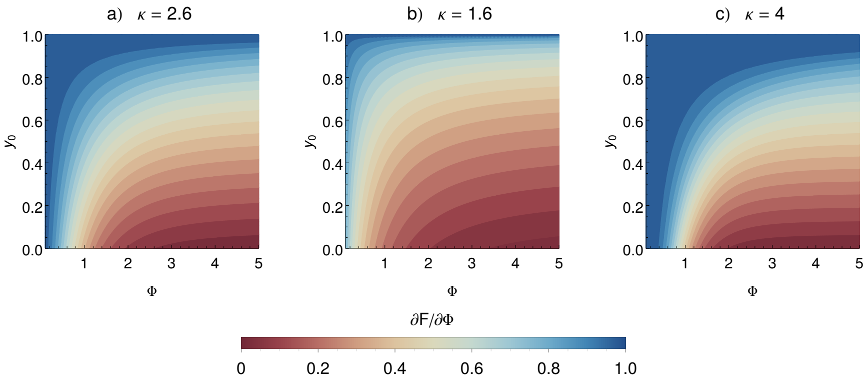

Figure 3. The sensitivity $\partial F/\partial\Phi$ under varying y_0, for $\kappa = 2.6$ (left, similar to the original Budyko framework if $y_0 = 0$), $\kappa = 1.6$ (center) and $\kappa = 4$ (right). Blueish colors denote high, reddish colors low sensitivity.

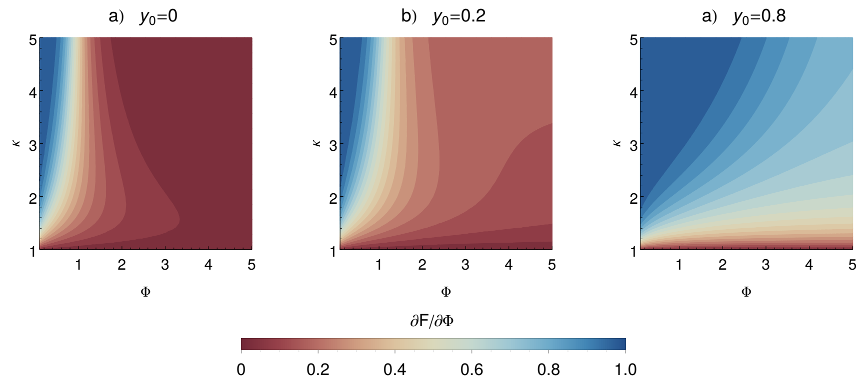

Figure 4. The sensitivity $\partial F/\partial\Phi$ under varying κ, for $y_0 = 0$ (left), $y_0 = 0.2$ (center) and $y_0 = 0.8$ (right). Blueish colors denote high, reddish colors low sensitivity.

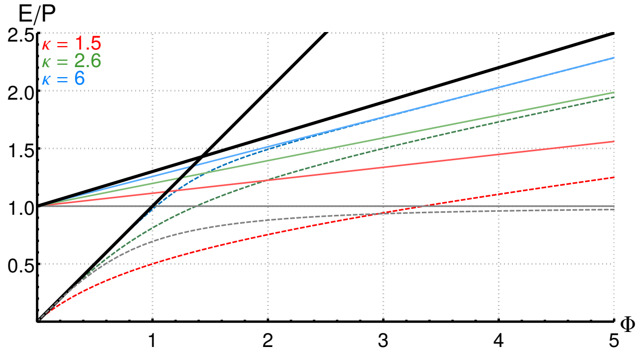

Figure 5. Difference between the actual (solid colored lines) and maximum slope (solid black line) of the supply limit for different values of κ (red: $\kappa = 1.5$, green: $\kappa = 2.6$ and blue: $\kappa = 6$) and $y_0 = 0.3$. The maximum slope ($m = y_0 = 0.3$) is reached if $\kappa \to \infty$.

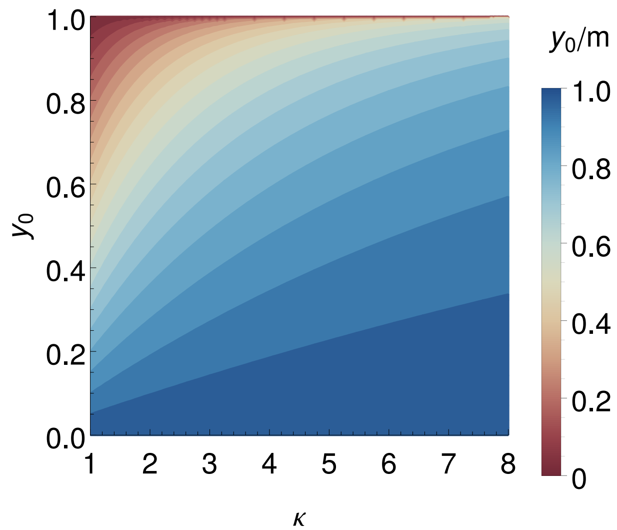

Figure 6. The ratio y_0/m as a function of both y_0 and κ estimated from equation 13.

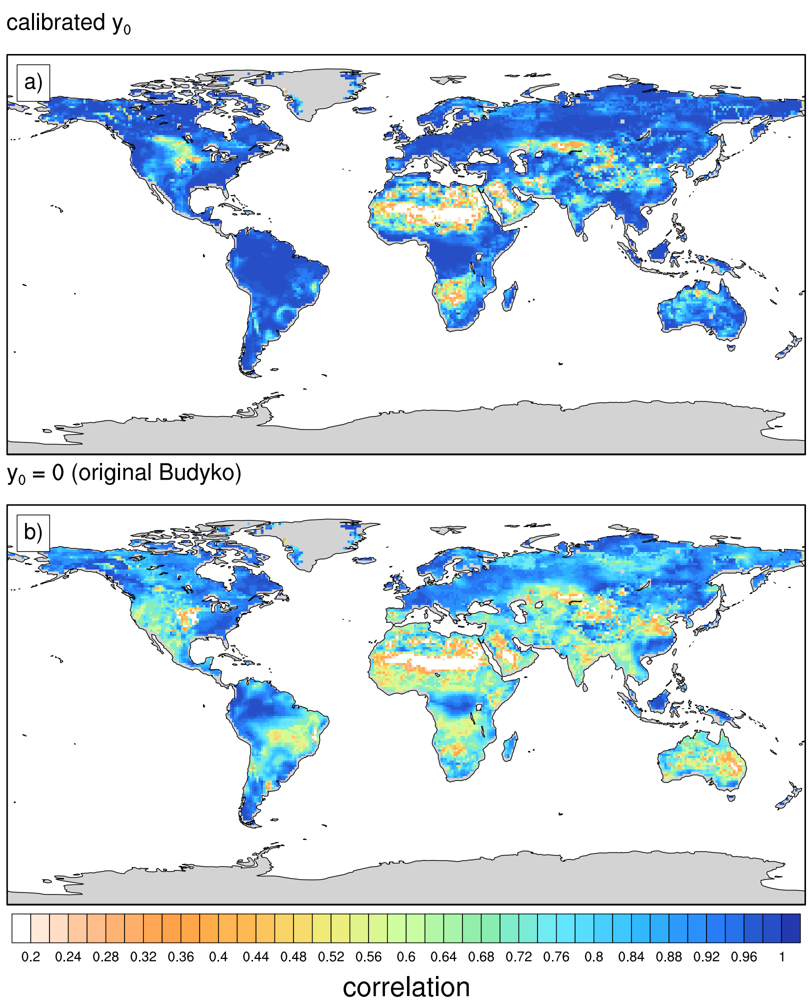

Figure 7. Correlation between the mean seasonal cycle of E/P computed from equation 9 and observed E/P for a) a grid-point specific parameter set (κ, y_0) and b) $(\kappa, 0)$ (Fu's equation).

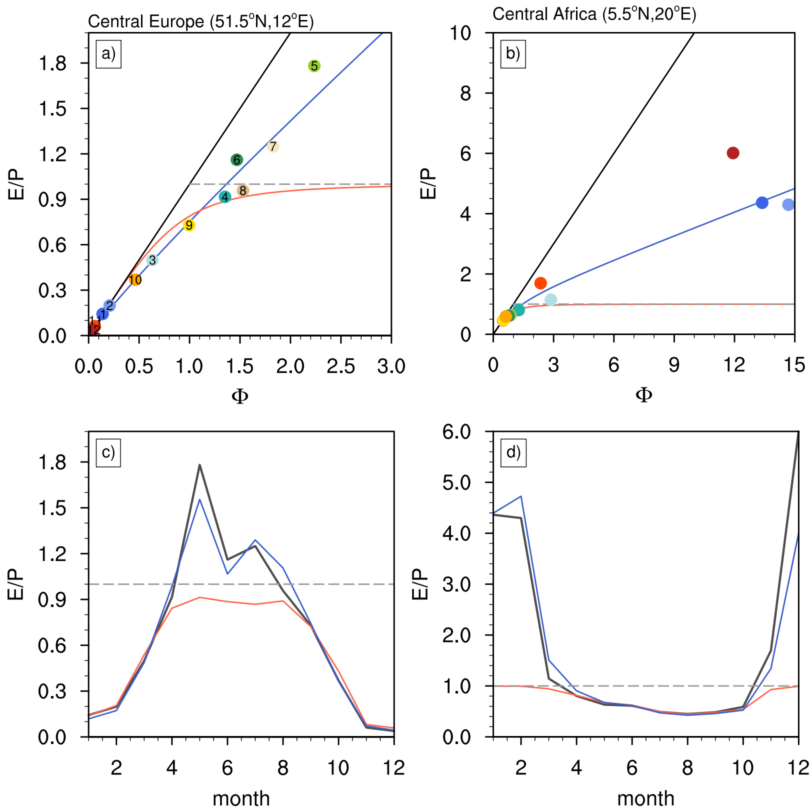

Figure 8. Data cloud of monthly climatologies within the Budyko space for a gridpoints in a) central Europe $(51.5^\circ N, 12^\circ E)$ and b) central Africa $(5.5^\circ N, 20^\circ)$. The black solid line denotes the demand limit, the dashed line denotes the original supply limit. The blue line depicts the obtained curve using the modified formulation of Fu's equation, whereas the red line shows the original Fu curve. Numbers within the dots denote the particular month of the year. c), d) Observed (grey) and computed mean seasonal cycles at both gridponts. The blue line depicts the obtained seasonal cycle using the modified formulation of Fu's equation, whereas the red line shows the seasonal cycle obtained using Fu's equation. Please note that axes are different in each plot.