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A continuous rainfall model based on vine copulas

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Abstract

Copulas have already proven their flexibility in rainfall modelling. Yet, their use is generally restricted to the description of bivariate dependence. Recently, vine copulas have been introduced, allowing multi-dimensional dependence structures to be described on the basis of a stage by stage mixing of two-dimensional copulas. This paper explores the use of such vine copulas in order to incorporate all relevant dependencies between the storm variables of interest. On the basis of such fitted vine copulas, an external storm structure is modeled. An internal storm structure is superimposed based on Huff curves, such that a continuous time series of rainfall is generated. The performance of the rainfall model is evaluated through a statistical comparison between an ensemble of synthetical rainfall series and the observed rainfall series and through the comparison of the annual maxima.

1 Introduction

Rainfall serves as an important base for many studies involving hydrological applications including flood risk estimation, the design of hydraulic structure and urban drainage systems or the evaluation of hydrological effects of climate change. Ideally, one should then have extensive observed rainfall time series at hand, both in time and space and at different time scales. Therefore, several rainfall modelling approaches have been proposed during the last decades (e.g. Kavvas and Delleur, 1981; Rodriguez-Iturbe et al., 1987a, b; Katz and Parlange, 1998; Menabde and Sivapalan, 2000; Willems, 2001; Evin and Favre, 2008; Gyasi-Agyei, 2011; Viglione et al., 2012), which can be subdivided in models that generate design storms and models that allow for the simulation of continuous time series at a point or spatially distributed. Design storms are generally developed for a given return period and storm duration. The corresponding rainfall volume, obtained from e.g. intensity-duration-frequency (IDF) curves is then assigned to the design storm according to a temporal rainfall pattern or inter-

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nal storm structure (Chow et al., 1988). Continuous rainfall models can be classified in four categories (Onof et al., 2000): (1) physically-based meteorological models, (2) stochastic multi-scale models that allow for modelling the spatial evolution of the rainfall process, (3) statistical models, preserving trends in precipitation and (4) stochastic process models that mimic the hierarchical structure of the rainfall process using a limited number of model parameters.

The variables that characterize a storm, i.e. the storm intensity, duration and volume, mostly exhibit some kind of mutual dependence: a long storm duration is more likely to be associated with a low storm intensity than with a high one. The marginal probability distribution functions of these variables are different and largely skewed (Vandenberghe et al., 2010b), i.e. there is a large deviation from the normal distribution. In frequency analysis studies, in the context of extremes, it is important to construct joint probability distribution functions in order to calculate joint probabilities, e.g. the probability of occurrence of a storm with a specific duration and intensity, which was facilitated by the introduction of copulas in hydrology.

Copulas are functions that couple the marginal distribution functions of the random variables into their joint distribution function and therefore describe the dependence structure between these random variables (Sklar, 1959). The great advantage of copulas is that the joint distribution function is built based on two independent tasks comprising the modelling of the dependence and the modelling of the marginal distribution functions. As this property allows for modelling a large variety of joint probability functions, copulas have been used within an increased number of publications in recent years. Pioneering work with respect to applying copulas in hydrology was performed by De Michele and Salvadori (2003), Salvadori and De Michele (2004), Favre et al. (2004) and De Michele et al. (2005). Concerning rainfall modelling, copulas offer a great flexibility in the modelling of high-dimensional dependence structures, however, determining parametric distributions for high-dimensional random vectors is complex (Aas and Berg, 2009). Copulas can, for instance, improve many rainfall models that mimic the external rainfall process, i.e. the process of storm arrival, duration and mean in-

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tensity at the coarse scale (Salvadori and De Michele, 2007). These models mostly consider rainfall as a sequence of rectangular pulses, having a certain duration and mean intensity, followed by a specific dry period.

This paper explores how a point-scale rainfall model can be constructed using multivariate copulas, in order to incorporate all relevant dependencies between the storm variables of interest. The application of multivariate copulas in hydrology is, in contrast to the application of bivariate copulas, a less explored domain. Some applications can be found in the modelling of trivariate rainfall (Zhang and Singh, 2007; Kao and Govindaraju, 2008; Salvadori and De Michele, 2006; Grimaldi and Serinaldi, 2006), trivariate floods (Serinaldi and Grimaldi, 2007; Genest et al., 2007; Ganguli and Reddy, 2013) and trivariate droughts (Song and Singh, 2010; Wong et al., 2010; Ma et al., 2013). Copula applications in hydrology that go beyond the trivariate case are still very scarce. One example is De Michele et al. (2007) who provide a study on constructing a copula for a four-dimensional sea storm phenomenon. The lack of successful applications of multivariate copulas in hydrology is, of course, influenced by the progress made in the theory of multivariate copulas. Due to the increase in dimensionality, the study of copulas becomes more complicated than in the bivariate case. Therefore, the question of how to construct a copula family that is sufficiently flexible to model the complete dependence structure is a very vivid one in theoretical research. Recently, a flexible construction method, based on mixing (conditional) bivariate copulas, has been introduced, which holds a large potential for many hydrological applications. In literature, this method is referred to as the vine-copula (or pair-copula) construction method (see e.g. Kurowicka and Cooke, 2007; Aas et al., 2009; Aas and Berg, 2009; Hobæk Haff et al., 2010). The underlying theory for this method is given by Bedford and Cooke (2001, 2002) and stems from Joe (1997), which also forms the basis for the method of “conditional mixtures”, as applied by De Michele et al. (2007).

The model that is developed in this paper consists of two submodels. In a first submodel, the vine-copulas-model, three- and four-dimensional vine copulas are used to describe the dependence between the storm duration, storm volume, the interstorm period following the storm and, in case a four-dimensional vine-copula is used, also the dry fraction within the storm. In a second submodel, the intrastorm-generating-model, the intrastorm variability is obtained based on Huff curves (Huff, 1967). Candela et al. (2014) also used Huff curves to generate an internal storm structure on the basis of external storm variables. Before introducing the model, Sect. 2 provides some background on the construction of vine copulas and the simulation using vine copulas. Section 3 briefly introduces the historical time series, while Sect. 4 describes the rainfall model. In Sect. 5 the model performance is assessed and a comparison with a state-of-the-art stochastic rainfall model is performed to further assess the performance of the newly introduced model.

2 Vine copulas

2.1 Construction

A vine copula mixes (conditional) bivariate copulas stage by stage in order to build a high-dimensional copula, i.e. the full density function is decomposed into a product of low-dimensional density functions. Consider the case of two random variables X and Y describing a phenomenon (e.g. storm duration and storm volume). Using their marginal distribution functions F_X and F_Y, the values of both random variables are transformed into values, respectively U and V, in the real unit interval $\mathbb{I} = [0,1]$:

$$\begin{cases} u = F_X(x) \\ v = F_Y(y) \end{cases} \Leftrightarrow \begin{cases} x = F_X^{-1}(u) \\ y = F_Y^{-1}(v) \end{cases}, \tag{1}$$

where x, y, u and v are the values of the corresponding variables X, Y, U and V. U and V are uniformly distributed on $\mathbb{I}$. F_X^{-1} and F_Y^{-1} are the (quasi-)inverse functions of the distribution functions F_X and F_Y (Nelsen, 2006).

A bivariate copula or a 2-copula is a function $C : \mathbb{I} \times \mathbb{I} \to \mathbb{I}$ that satisfies:

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1. for all $u, v \in \mathbb{I}$,

$$C(u,0) = 0 \quad \text{and} \quad C(0,v) = 0$$
$$C(u,1) = u \quad \text{and} \quad C(1,v) = v \tag{2}$$

2. for all $u_1, u_2, v_1, v_2 \in \mathbb{I}$ for which $u_1 \leq u_2$ and $v_1 \leq v_2$:

$$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \geq 0. \tag{3}$$

The extension of this definition to k dimensions results in a k-copula (see Nelsen, 2006, for the definition and a detailed explanation). The theorem of Sklar (1959) relates bivariate copulas and bivariate distribution functions and states that for any two continuous random variables X_1 and X_2, with continuous marginal cumulative distribution functions F_1 and F_2, a unique bivariate copula C_{12} exists such that

$$F_{12}(x_1,x_2) = C_{12}(F_1(x_1),F_2(x_2)) = C_{12}(u,v), \tag{4}$$

where F_{12} is the joint cumulative distribution function of X_1 and X_2. This theorem thus formulates that a copula couples the marginal cumulative distribution functions of two random variables into a joint cumulative distribution function F_{12}. This theorem can be extended to k dimensions and hence relates a k-dimensional cumulative distribution function $F_{12\ldots k}$ to k marginal distribution functions (Sklar, 1959): for k continuous random variables X_1, X_2, ..., X_k, with continuous marginal distributions functions F_1, F_2, ..., F_k, there exists a unique k-copula $C_{12\ldots k}$ such that:

$$F_{12\ldots k}(x_1,x_2,\ldots,x_k) = C_{12\ldots k}(F_1(x_1),F_2(x_2),\ldots,F_k(x_k)). \tag{5}$$

In order to explain the construction of vine copulas, the construction of a three-dimensional vine copula is first explained. The joint probability density function (PDF) f_{123} of a random vector (X_1,X_2,X_3) can be decomposed as follows:

$$f_{123}(x_1,x_2,x_3) = f_{13|2}(x_1,x_3|x_2) \cdot f_2(x_2), \tag{6}$$

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where $f_{13|2}$ is the joint PDF of X_1 and X_3, given $X_2 = x_2$ and f_2 is the marginal PDF of X_2. The joint cumulative distribution function (CDF) F_{123} is then obtained by integration, following the conditional mixtures approach (De Michele et al., 2007):

$$\begin{aligned} F_{123}(x_1,x_2,x_3) &= \int_{-\infty}^{x_1}\int_{-\infty}^{x_2}\int_{-\infty}^{x_3} f_{123}(r,s,t)\mathrm{d}r\mathrm{d}s\mathrm{d}t \\ &= \int_{-\infty}^{x_1}\int_{-\infty}^{x_2}\int_{-\infty}^{x_3} f_{13|2}(r,t|s)f_2(s)\mathrm{d}r\mathrm{d}s\mathrm{d}t \\ &= \int_{-\infty}^{x_2}\left[\int_{-\infty}^{x_1}\int_{-\infty}^{x_3} f_{13|2}(r,t|s)\mathrm{d}r\mathrm{d}t\right] f_2(s)\mathrm{d}s \\ &= \int_{-\infty}^{x_2} F_{13|2}(x_1,x_3|s)\mathrm{d}F_2(s) \\ &= \int_{-\infty}^{x_2} C_{13|2}(F_{1|2}(x_1|s),F_{3|2}(x_3|s))\mathrm{d}F_2(s). \end{aligned} \tag{7}$$

The conditional cumulative distribution functions $F_{1|2}(x_1|x_2)$ and $F_{3|2}(x_3|x_2)$ can also be expressed in terms of copulas:

$$\begin{aligned} F_{1|2}(x_1|x_2) &= \frac{\partial}{\partial u_2}C_{12}(u_1,u_2); \\ F_{3|2}(x_3|x_2) &= \frac{\partial}{\partial u_2}C_{23}(u_2,u_3), \end{aligned} \tag{8}$$

with $u_1 = F_1(x_1)$, $u_2 = F_2(x_2)$ and $u_3 = F_3(x_3)$. When instead of X_1, X_2 and X_3, their transformed uniform random variables on $\mathbb{I}$, U_1, U_2 and U_3, are considered, Eq. (7) can

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be expressed as follows:

$$C_{123} = \int_0^{u_2} C_{13|2}\left(\frac{\partial}{\partial s}C_{12}(u_1, s), \frac{\partial}{\partial s}C_{23}(s, u_3)\right) \mathrm{d}s. \tag{9}$$

In the theory of vine copulas, the same decomposition of the density function is performed, but instead of using cumulative probability functions, all equations are rather expressed in terms of density functions and the full density function c_{123} of the three-dimensional copula is then given by:

$$c_{123}(u_1, u_2, u_3) = c_{13|2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)) \cdot c_{12}(u_1, u_2) \cdot c_{23}(u_2, u_3). \tag{10}$$

Similarly, for a random vector (X_1, X_2, X_3, X_4), the joint probability density function f_{1234} can, for instance, be decomposed as follows:

$$f_{1234}(x_1, x_2, x_3, x_4) = f_{14|23}(x_1, x_4|x_2, x_3) \cdot f_{23}(x_2, x_3). \tag{11}$$

The joint cumulative distribution function F_{1234} is then obtained by integration, similarly as in Eq. (7):

$$F_{1234}(x_1, x_2, x_3, x_4) = \int_{-\infty}^{x_2}\int_{-\infty}^{x_3} C_{14|23}(F_{1|23}(x_1|s,t), F_{4|23}(x_4|s,t))\mathrm{d}F_{23}(s,t). \tag{12}$$

Herein, the derivative of the bivariate CDF $F_{23}(x_2, x_3)$ is expressed as $\mathrm{d}F_{23}(x_2, x_3) = f_{23}(x_2, x_3)\mathrm{d}x_2\mathrm{d}x_3$. The functions $F_{1|23}(x_1|x_2, x_3)$ and $F_{4|23}(x_4|x_2, x_3)$ are conditional cumulative distribution functions, and can also be expressed in terms of copulas:

$$\begin{aligned} F_{1|23}(x_1|x_2, x_3) &= \frac{\partial C_{12|3}(F_{1|3}(x_1|x_3), F_{2|3}(x_2|x_3))}{\partial F_{2|3}(x_2|x_3)}, \\ F_{4|23}(x_4|x_2, x_3) &= \frac{\partial C_{24|3}(F_{2|3}(x_2|x_3), F_{4|3}(x_4|x_3))}{\partial F_{2|3}(x_2|x_3)}, \end{aligned} \tag{13}$$

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where the conditional CDFs $F_{1|3}$, $F_{2|3}$ and $F_{4|3}$ are calculated as in Eq. (8).

2.2 Fitting a three- or four-dimensional vine copula

Figures 1 and 2 illustrate the principle of constructing a three- respectively four-dimensional vine copula. Consider tree 1 in Figs. 1 and 2 where three (respectively four) uniform (on $[0,1]$) random variables U_1, U_2 and U_3 (or U_1, U_2, U_3 and U_4) are given and their pairwise dependencies are described by the bivariate copulas C_{12} and C_{23} (respectively C_{12}, C_{23} and C_{34}). Given a specific value of the second variable, these bivariate copulas can be conditioned (cf. dashed arrows in Figs. 1 and 2) through partial differentiation (Aas et al., 2009), resulting in the conditional cumulative distribution functions $F_{1|2}$ and $F_{3|2}$ (respectively $F_{1|2}$, $F_{3|2}$, $F_{2|3}$ and $F_{4|3}$). The pairwise dependences between these conditional cumulative distribution functions are then captured by the bivariate copula $C_{13|2}$ (respectively the copulas $C_{13|2}$ and $C_{24|3}$). See tree 2 in Figs. 1 and 2. These latter copulas can then also be conditioned by partial differentiation to obtain $F_{3|12}$ (respectively $F_{3|12}$ and $F_{4|23}$). For the four-dimensional vine copula, another bivariate copula $C_{14|23}$ captures the pairwise dependence between these conditional cumulative distribution functions and can on its turn be partially differentiated to obtain $F_{4|123}$. See tree 3 in Fig. 2. The conditional cumulative distribution functions $F_{3|12}$ and $F_{4|123}$ (of the three- and four-dimensional vine, respectively) will be of use for simulation purposes (Aas et al., 2009). It should be noted that the hierarchical nesting of bivariate (conditional) copulas as presented here is just one of the possibilities and corresponds to what is called a D-vine (Aas et al., 2009).

In practice, the bivariate copulas in a higher tree of the vine (e.g. $C_{13|2}$) are fitted as follows. Consider a set of n data points, for all triplets $(u_{1,i}, u_{2,i}, u_{3,i})$ (or for all quadruplets $(u_{1,i}, u_{2,i}, u_{3,i}, u_{4,i})$), $i = 1, \ldots, n$, the conditional CDF values (e.g. the CDF values according to $F_{1|2}$ and $F_{3|2}$ in the case of $C_{13|2}$) are calculated. The bivariate copulas (e.g. $C_{13|2}$) are then fitted to these “conditioned observations”, which are again approximately uniformly distributed on $\mathbb{I}$.

2.3 Simulating samples out of the vine

A general simulation algorithm is presented next, borrowed from the theory on conditional mixtures. The literature on vine copulas reports very similar simulation algorithms (Aas and Berg, 2009; Aas et al., 2009). In order to simulate a random sample (u_1,u_2,u_3) (or (u_1,u_2,u_3,u_4)) out of the three- or four-dimensional conditional mixture copula, i.e. U_1, U_2, U_3 and U_4 are uniformly distributed on $\mathbb{I}$, the one-dimensional conditional cumulative distribution function (CCDF) is highly important and is defined as (De Michele et al., 2007):

$$\begin{aligned} G_{k|1\ldots k-1}(u_k|u_1,\ldots,u_{k-1}) &= \mathbb{P}(U_k \leq u_k \,|\, U_1 = u_1,\ldots,U_{k-1} = u_{k-1}) \\ &= \frac{\frac{\partial^{k-1}}{\partial u_1\ldots\partial u_{k-1}} C_{1\ldots k}(u_1,\ldots,u_k)}{\frac{\partial^{k-1}}{\partial u_1\ldots\partial u_{k-1}} C_{1\ldots k-1}(u_1,\ldots,u_{k-1})}. \end{aligned} \quad (14)$$

Herein, the numerator is the mixed partial derivative of the k-dimensional copula with respect to the conditioning variables. The denominator is the copula density of the $(k-1)$-dimensional copula of the conditioning variables. In order to simulate a random sample out of the three- (respectively four-) dimensional conditional mixture copula, a random sample (t_1,t_2,t_3) (or (t_1,t_2,t_3,t_4)) should be first generated from (T_1,T_2,T_3) (respectively (T_1,T_2,T_3,T_4)) which are uniformly distributed random variables on $\mathbb{I}$, and serve as random probability levels of the CCDFs in the simulation algorithm which is listed next (of course for generating a three-dimensional sample, step 4 should not be performed):

1. $u_1 = t_1$;
2. $u_2 = G^{-1}_{2|1}(t_2|u_1)$, where

$$G_{2|1}(u_2|u_1) = \frac{\partial}{\partial u_1} C_{12}(u_1,u_2); \quad (15)$$

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3. $u_3 = G^{-1}_{3|12}(t_3|u_1,u_2)$, where

$$G_{3|12}(u_3|u_1,u_2) = \frac{\frac{\partial^2}{\partial u_1 \partial u_2} C_{123}(u_1,u_2,u_3)}{\frac{\partial^2}{\partial u_1 \partial u_2} C_{12}(u_1,u_2)}; \tag{16}$$

4. $u_4 = G^{-1}_{4|123}(t_4|u_1,u_2,u_3)$, where

$$G_{4|123}(u_4|u_1,u_2,u_3) = \frac{\frac{\partial^3}{\partial u_1 \partial u_2 \partial u_3} C_{1234}(u_1,u_2,u_3,u_4)}{\frac{\partial^3}{\partial u_1 \partial u_2 \partial u_3} C_{123}(u_1,u_2,u_3)}. \tag{17}$$

The calculation of some partial derivatives, necessary for obtaining the CCDF $G_{4|123}$ is given below:

$$\frac{\partial^3}{\partial u_1 \partial u_2 \partial u_3} C_{1234}(u_1,u_2,u_3,u_4) = \frac{\partial}{\partial u_1} C_{14|23}(G_{1|23}(u_1|u_2,u_3), G_{4|23}(u_4|u_2,u_3)), \tag{18}$$

with

$$\begin{aligned} G_{1|23}(u_1|u_2,u_3) &= \frac{\frac{\partial^2}{\partial u_2 \partial u_3} C_{123}(u_1,u_2,u_3)}{\frac{\partial^2}{\partial u_2 \partial u_3} C_{23}(u_2,u_3)} \\ &= \frac{\frac{\partial}{\partial u_3} C_{13|2}\left(\frac{\partial}{\partial u_2} C_{12}(u_1,u_2), \frac{\partial}{\partial u_2} C_{23}(u_2,u_3)\right)}{\frac{\partial^2}{\partial u_2 \partial u_3} C_{23}(u_2,u_3)}, \end{aligned} \tag{19}$$

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and

$$G_{4|23}(u_4|u_2,u_3) = \frac{\frac{\partial^2}{\partial u_2 \partial u_3} C_{234}(u_2,u_3,u_4)}{\frac{\partial^2}{\partial u_2 \partial u_3} C_{23}(u_2,u_3)} = \frac{\frac{\partial}{\partial u_3} C_{24|3}\left(\frac{\partial}{\partial u_3} C_{23}(u_2,u_3), \frac{\partial}{\partial u_3} C_{34}(u_3,u_4)\right)}{\frac{\partial^2}{\partial u_2 \partial u_3} C_{23}(u_2,u_3)}. \tag{20}$$

Once u_1, u_2, u_3 and u_4 are simulated, the corresponding values of x_1, x_2, x_3 and x_4 can be calculated by means of the inverse marginal cumulative distribution functions F_1^{-1}, F_2^{-1}, F_3^{-1} and F_4^{-1}, respectively.

3 Historical time series characteristics

The time series used in this paper for fitting the model consists of a 105 year 10 min rainfall record of Uccle, Belgium. These data were obtained by a Hellmann–Fuess pluviograph, installed in and operated by the Royal Meteorological Institute at Uccle near Brussels, Belgium (Demarée, 2003). This exceptional time series has been used in several studies, albeit with varying lengths, concerning statistical analyses (Vaes et al., 2002; De Jongh et al., 2006; Ntegeka and Willems, 2008; Vandenberghe et al., 2010b) and stochastic rainfall modelling (Verhoest et al., 1997; Vandenberghe et al., 2011; Vanhaute et al., 2012; Evin and Favre, 2013; Pham et al., 2013). For the current study, storms were first selected and their characteristics of interest calculated. Storms were selected on the basis of a minimal dry duration that separates two storms. Any dry period shorter than this threshold is thus considered to be part of a storm (Bonta and Rao, 1988). Similar to Verhoest et al. (1997) and Vandenberghe et al. (2010a, b, 2011), a dry period of 24 h was chosen, as this period assures that the arrival times of independent storms are Poisson distributed Restrepo-Posada and Eagleson (1982). In this

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way, 8665 storms were selected and the following characteristics calculated: the onset of a storm (day, month, year), the storm volume V [mm], the storm duration W [h], the dry duration after the storm D [h], and the fraction dry within the storm p_d. It was observed that events with identical values of variables occur in the observed time series. As this is not desirable for a copula-based analysis, in which ranks are important, random noise, uniformly distributed between −0.1 and +0.1 mm was introduced to all strictly positive 10 min observations as described and motivated in Vandenberghe et al. (2010b). The choice of 0.1 mm was based on the pluviograph's resolution. We also refer to Vandenberghe et al. (2010b) for a profound analysis of the dependencies between the variables W, V, and D. Vandenberghe et al. (2010b) ascerted that the hypothesis of stationarity of storms on the Uccle time series is fulfilled through seasonally subdividing the storms allowing copulas to be fitted per season. Therefore, the storm characteristics of the 105 year 10 min rainfall time series are subdivided according to the season in which the storms occurred. To this end, similarly as in Vandenberghe et al. (2010b), winter is defined as the months December, January and February, spring as the months March, April and May, summer as the months June, July and August and autumn as the months September, October and November. It was furthermore noticed that some of the storms have no internal dry 10 min intervals, i.e. $p_d = 0$. The set of storms within each season is then further subdivided into a subset of storms for which $p_d = 0$ and a subset of storms for which $p_d \neq 0$. The observed probability of $p_d = 0$ for the different seasons is listed in Table 1.

A kernel-smoothed distribution function was fitted to the observed values of W, V, D and p_d as none of the commonly used probability distributions fitted well the data. As D has a theoretical minimum of 24 h, due to the selection criterion, the distribution was fit to $D - 24$ h values, and afterwards, 24 h were added.

As the storm characteristics V, W and D do not reveal any information on the internal storm structure, and p_d only gives partial information, Huff curves, as derived in Vandenberghe et al. (2010a) are employed to provide statistical information on the internal structure. Huff or mass curves present the normalized cumulative storm depth

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since the beginning of a storm w.r.t. the normalized time. Given the 105 year time series at hand, empirical Huff curves can be obtained by partitioning each storm in the time series in e.g. 20 identical time intervals at every 5 % of the total storm duration. Furthermore, storms were classified into seasons, and quartile groups according to the quartile of the storm that received the largest amount of rainfall. For each season and each quartile group the corresponding Huff curves were obtained by visualizing the 10 and 90 % percentiles of the distribution. In this way, 16 Huff curves were obtained (four quartile groups per season). As an example, Fig. 3 illustrates the 10 and 90 % percentile curves of the second quartile autumn storms. Vandenberghe et al. (2010a) showed that these curves are independent of the extremity of the storm.

4 Description of the rainfall model

4.1 The vine-copulas-submodel: construction and use of vine copulas in the generation of a time series

By examining the storm characteristics of the historical time series, it is observed that some storms have internal dry 10 min intervals while others have not. It was decided to fit, for each season, a four-dimensional vine copula to the values of W, V, D and the non-zero values of p_d. Furthermore, for each season, a three-dimensional vine copula was fitted to the values of W, V and D in case $p_d = 0$. In this way, dependencies between the variables for $p_d = 0$ and $p_d \neq 0$ are taken into account and four three-dimensional and four four-dimensional vine copulas are obtained. The three- (W, V, and D) and four-dimensional (p_d, W, V, D) vine copulas are fitted stage by stage, following the method explained in Sect. 2.2. We are aware that different copula families could be used to describe the dependencies between the different variables (cf. Vandenberghe et al., 2010b). Yet, in this study, we opted to restrict to the Frank copula family to describe the (conditional) bivariate dependencies within the vine copulas, because of its ability to represent positive or negative dependence. Furthermore, this fam-

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ily is frequently applied to describe bivariate hydrological phenomena (Pan et al., 2013). Alternative families could better fit the different dependencies within the vine copula, however, the search for the best fitting copula was out of the scope of the current study. It should be remarked that two different ways of parameterizing the Frank copula exist. In this paper, the one that has the dependence parameter range of $[-\infty, +\infty]$ is employed. The fitted Frank copula parameters are presented in Table 2. Then, two samples, with size 10 000, are simulated using the three- and four-dimensional vine copulas, respectively, based on the method described in Sect. 2.3. Table 3 shows the good correspondence between the observed and simulated pairwise dependencies. To be able to transform samples from the fitted copula to real samples, the (inverse) marginal cumulative distribution functions of p_d, W, V and D are employed.

The inverse CDFs are then used to transform simulated uniformly distributed values in $\mathbb{I}$ to values in $\mathbb{R}$. Once the simulated values of the quadruplet (p_d, W, V, D) are known, a time series of 105 years of rectangular rainfall pulses with duration W and height $I = V/W$, being separated by a dry duration D, is obtained. Of course, these rectangular pulses only correspond to the external storm structure. At this stage, only the timing of the beginning and the end of a storm is important. The pulse itself, characterized by W, V and p_d, needs to be further disaggregated into finer scale rainfall, which is elaborated upon in the next section.

4.2 The intrastorm-generating-submodel: disaggregation of rectangular pulses by means of Huff curves

In order to employ Huff curves in the disaggregation of the rectangular pulses, a random quartile group is first assigned based on the probabilities of occurrence of the four quartile groups, as indicated in Table 4. By assigning the first pulse to the beginning of the year, the season of each simulated pulse is easily derived. On the basis of the probability of $p_\mathrm{d} = 0$, the three- or four-dimensional vine copula for the corresponding season is employed to obtain simulated storm pulses. Next, as the season and the quartile group are known, a random internal storm structure can be assigned to each

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simulated storm pulse on the basis of the corresponding 10 and 90 % Huff curves. To this end, time steps corresponding to the end of each 10 min interval within the storm are selected. The 10 and 90 % curves are then interpolated such that the values of the normalized cumulative storm depth for each of these time steps (expressed as a percentage of the total storm duration) are obtained.

Time intervals having zero rainfall are randomly assigned within the storm such that the sampled fraction of dry periods p_d is respected. In this assignment, the first and last interval cannot be selected in order to preserve the duration W of the storm. When only one wet 10 min interval should be assigned, which could be caused by a value of p_d close to one, or due to the combination of a short duration and a high p_d, the rainfall depth is evenly divided among the first and last 10 min intervals. Furthermore, the total length of a dry spell within a storm is constrained to 23 h, i.e. one hour less than the selection criterion, in order to avoid that one storm would result in two different storms when the same storm selection criterion is applied on the simulated rainfall series. Furthermore, storms that have a duration smaller than 40 min and for which $p_d \neq 0$, are disregarded in the generation of the rainfall series, because of the inability to assure the generation of the imposed quartile storm.

The internal storm structure is generated by randomly selecting cumulative storm depths as follows. The procedure starts at the first time step within the pulse (i.e. the end of the first 10 min interval). During the generation of the internal storm structure, three cases may occur, requiring different sampling strategies. Time step b denotes in each of these cases the current time step in the storm at which a value of cumulative storm depth needs to be sampled. These cases are:

1. Time step b is situated in between two consecutive wet 10 min intervals. In this case, a cumulative storm depth is randomly selected between the 10 % and the 90 % percentile curves, ensuring that the cumulative storm depths do not decrease in time. Figure 4a illustrates this step, where time step a denotes the previous time step at which a value $V_{rc}(a)$ (i.e. the relative cumulative volume at time step a) was selected. The current time step b, at which a value $V_{rc}(b)$ is to be

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selected, is also indicated. $V_{rc}(b)$ is randomly selected between a minimal and maximal value, indicated in the figure.

2. Time step b corresponds to the end of a dry period. In this case, depicted in Fig. 4b, no sampling has to be performed (as the time interval between a and b should be dry), and thus the cumulative storm depth takes the value of the previous time step, i.e. $V_{rc}(b) = V_{rc}(a)$, where $V_{rc}(b)$ may be situated outside the percentile curves.

3. Time step b corresponds to the end of a wet period. In the third case, depicted in Fig. 4c and d, the dry period ends at time step c and two sampling strategies are possible. It is either allowed that a cumulative storm depth smaller or larger than the value of the corresponding 10, respectively 90 %, Huff curve is chosen. Both strategies are chosen with equal probability. The first strategy thus allows that a value lower than $H_{10}(c)$ is chosen. $V_{rc}(b)$ is then chosen in the range defined by the interval $[\max(V_{rc}(a), H_{10}(b)), H_{90}(b)]$ (cf. Fig. 4c). The second strategy allows that a value above the 90 % Huff curve is chosen. In this case, $V_{rc}(b)$ is drawn from the interval $[\max(V_{rc}(a), H_{10}(c)), H_{90}(c)]$ (cf. Fig. 4d). Such flexibility is required as the fraction of dry spells often does not allow to remain between the 10 and 90 % boundaries. However, this flexibility is not a major problem, as there are always 20 % of the historical relative cumulative storm depths outside these boundaries by definition.

Based on the historical time series, it was observed that the increment in cumulative storm depth between two subsequent time steps in a Huff curve is not uniformly distributed (this observation was neglected in Vandenberghe et al., 2010a). Smaller increments occur more often than large increments. This behaviour is simulated by first establishing a cumulative probability distribution of strictly positive increments on the basis of the 105 year time series. To this end, for all storms in a particular season and quartile group, the frequencies of normalized (strictly positive) increments of cumulative rainfall depths between two subsequent wet periods were recorded. This empirical

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cumulative distribution function for the respective season and quartile group is then used to randomly select a normalized cumulative storm depth for the subsequent wet time step. Figure 5 illustrates the use of the cumulative distribution in the sampling procedure. In this figure, the minimum and maximum bounds of the increments are first determined on the basis of the Huff curves, as explained above. These bounds are then transferred to the cumulative distribution of normalized increments between which a value is randomly selected by uniformly sampling within the sampling range and calculating the corresponding difference percentage of total storm depth.

5 Results

It is common to validate the performance of a model through comparing statistics of one modeled time series to those calculated on the observed time series. However, given that the model has a stochastic nature, the statistics of the simulated time series will show some variability. To account for these stochastic effects, the model described in the previous section is employed to generate an ensemble of 100 time series of 105 years of 10 min rainfall (i.e. similar to the length of the observed time series). In order to evaluate whether the model performs well in the reproduction of aggregated rainfall statistics, the 100 time series are furthermore regarded as equally probable realisations and the statistics are calculated on a yearly basis. The traditional first and second order statistical moments (i.e. mean and variance), autocorrelation (AC) at different time lags and the zero depth probability (ZDP) are calculated along with the third order central moment (skewness). These statistics are calculated on a yearly basis for each ensemble member at aggregation levels of 1/6, 1, 3, 6, 12 and 24 h. Thus, for an aggregation level, 100×105 values of each of these statistics are obtained, such that a bundle of 100 empirical cumulative distributions can be established, i.e. one distribution per ensemble member. The empirical cumulative distribution of the values of the statistics of the observed time series can then be compared with this bundle. If the empirical cumulative distribution function of the observed statistics is situated within the bundle of distributions obtained by the model, the model performes well.

Figure 6b visualises the bundles of the 100 empirical cumulative distributions of the yearly statistics of the time series generated by the copula-based model and the empirical cumulative distribution of the yearly statistics of the observed time series for a 10 min aggregation level. Figure 6b displays the comparison for a 1 h aggregation level. These figures show that the observed mean is well represented by the copula-based model. For a 10 min aggregation level, also the ZDP is fairly well represented. The variance and the third central moment are overestimated, whereas the lag-1 and lag-2 covariances are underestimated. For the 1 h aggregation level, the ZDP and the lag-1 autocovariance are underestimated, the other statistics are fairly well represented. For the other aggregation levels, only the mean is well represented. The other statistics are sometimes well represented, under- or overestimated. W.r.t. the ZDP-statistic, the fact that this statistic is well represented at a 10 min aggregation level, yet, underestimated at higher aggregation levels (except for a 24 h aggregation level), is probably due to the selection of the dry periods within the storm. For storms that have a duration of more than one hour, these zero intervals are probably not connected as no temporal correlation is taken into account during the selection of dry periods, such that less dry periods are obtained after the aggregation than what is observed in the Uccle time series. Future research will further elaborate on a better selection of dry periods within the storm.

As simulated time series are often used to simulate extreme discharges (Verhoest et al., 2010), the behaviour of the modelled extreme rainfall was also assessed. Figure 7 shows the annual maximum rainfall depths of the ensemble and of the observed rainfall series related to empirical return periods, considering six different aggregation levels. This figure shows that the extrema are well modelled albeit the model overestimates extremes at short aggregation levels and tends to underestimate extremes at larger aggregation levels. Notwithstanding the shortcomings highlighted, this novel modelling concept seems to perform well. It should furthermore be stressed that other

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stochastic models such as the state-of-the-art Bartlett–Lewis models also are not able to preserve all statistics.

6 Conclusions

This study is the first in its kind in which a continuous stochastic rainfall generator is developed that uses vine copulas to describe the storms and their arrival process. The internal storm structure is based on the concept of Huff curves, while the fraction of dry periods within the storm is determined by the copulas. The main advantage of this approach is that the model is completely data driven and does not need any calibration.

The model applies three- and four-dimensional vine copulas to describe the dependence between the different storm characteristics. The three-dimensional vine copulas are employed to describe the seasonal dependence between storm duration, storm volume, and the interstorm period for storms that have no dry fraction within the storm. The four-dimensional vine copulas are employed to describe the seasonal dependence between these storm characteristics and the dry fraction within the storm. These vine copulas were fitted to the observed storm characteristics of a 105 years time series of 10 min rainfall. Because of its frequent successful application in hydrological applications, the Frank copula family was chosen to be used within the vine copulas. On the basis of these vine copulas, values of these four storm characteristics were drawn, representing the external storm structure, ensuring a time series of 105 year of rectangular rainfall pulses. According to their seasonal probability of occurence, storms with zero dry fractions were sampled from the three-dimensional vine copulas. The internal storm structure of the rectangular pulses is superimposed based on Huff curves, which were identified on the basis of the observed time series, leading to the generation of continuous 10 min rainfall time series. In the generation of the internal storm structure, it is ensured that the fraction of dry periods within the storm as drawn from the vine copulas, is maintained. The internal storm structures are furthermore generated according to the probability of occurrence of the quartile storms in the observed

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time series, and the season in which they occur. In order to determine the difference of cumulative storm depths in the internal storm structure, the empirical cumulative probability distribution function of increments between two subsequent wet periods in the storm is employed. In this way it is guaranteed that smaller increments occur more often than larger increments, as was observed in the measured time series.

In order to evaluate the performance of the rainfall model, an ensemble of 100 time series of ca. 105 year 10 min rainfall was generated, such that stochastic effects were accounted for. The results show that the copula-based rainfall model represents the mean value of the time series well, whereas the other statistics are either represented (fairly) well, over- or underestimated, depending on the aggregation level. A second evaluation of the generated ensemble encompassed the calculation of the annual maximum series, for different aggregation levels. It was observed that the annual maxima simulated by means of the copula-based model were larger than the observed maxima for an aggregation level of 10 min, and the moderate return period of the 24 h aggregation level. For aggregation levels of 1–12 h and the smaller and larger return periods of an aggregation level of 24 h, a good correspondence between the simulated and observed extremes was observed. Future research will reveal whether the representation of the ZDP-statistic for larger aggregation levels by the copula-based model can be improved by better selecting the internal dry storm periods. The performance of the copula-based model will also be compared to state-of-the-art stochastic rainfall generators.

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Table 1. Observed probability of $p_d = 0$ for the storms in the different seasons.

	Probability of $p_d = 0$
Winter	8.75 %
Spring	11.21 %
Summer	13.13 %
Autumn	12.14 %

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Table 2. Parameters of the bivariate Frank copulas in the construction of the three- and four-dimensional vine copulas.

	parameter	(W,V,D)	(p_d,W,V,D)
season 1	θ_{12}	–	2.9632
	θ_{23}	4.3567	8.9789
	θ_{34}	−0.7276	−0.6651
	$\theta_{13\|2}$	–	−9.1797
	$\theta_{24\|3}$	−0.4077	0.4199
	$\theta_{14\|23}$	–	−0.4670
season 2	θ_{12}	–	3.4077
	θ_{23}	6.3457	7.6379
	θ_{34}	−0.6992	−0.7037
	$\theta_{13\|2}$	–	−7.6601
	$\theta_{24\|3}$	0.2150	0.2348
	$\theta_{14\|23}$	–	−0.2741
season 3	θ_{12}	–	4.2445
	θ_{23}	6.1971	6.8144
	θ_{34}	0.2221	−0.7737
	$\theta_{13\|2}$	–	−6.0083
	$\theta_{24\|3}$	−0.3429	0.0194
	$\theta_{14\|23}$	–	−0.2419
season 4	θ_{12}	–	3.1574
	θ_{23}	5.5352	8.0877
	θ_{34}	−0.9686	−0.7041
	$\theta_{13\|2}$	–	−8.1955
	$\theta_{24\|3}$	−1.1409	−0.0918
	$\theta_{14\|23}$	–	−0.0958

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Table 3. Correspondence between the observed and simulated pairwise dependencies among (p_d, W, V, D), expressed as Kendall's tau τ_K.

		observed 3-D	observed 4-D	simulated 3-D	simulated 4-D
season 1	τ_{12}	–	0.3040	–	0.3066
	τ_{23}	0.4140	0.6361	0.4118	0.6305
	τ_{34}	−0.0804	−0.0736	−0.0754	−0.0790
	τ_{13}	–	−0.0336	–	−0.0092
	τ_{24}	−0.0831	−0.0341	−0.0760	−0.0425
	τ_{14}	–	0.0259	–	0.0111
season 2	τ_{12}	–	0.3418	–	0.3423
	τ_{23}	0.5318	0.5888	0.5314	0.5866
	τ_{34}	−0.0773	−0.0778	−0.0811	−0.0649
	τ_{13}	–	−0.0183	–	−0.0018
	τ_{24}	−0.0397	−0.0410	−0.0421	−0.0416
	τ_{14}	–	0.0221	–	0.0026
season 3	τ_{12}	–	0.4060	–	0.4132
	τ_{23}	0.5243	0.5540	0.5248	0.5483
	τ_{34}	0.0247	−0.0855	0.0222	−0.0820
	τ_{13}	–	0.0411	–	0.0602
	τ_{24}	−0.0079	−0.0613	−0.0109	−0.0672
	τ_{14}	–	−0.0126	–	−0.0332
season 4	τ_{12}	–	0.3208	–	0.3276
	τ_{23}	0.4887	0.6058	0.4948	0.6016
	τ_{34}	−0.1066	−0.0779	−0.1084	−0.0816
	τ_{13}	–	−0.0287	–	−0.0080
	τ_{24}	−0.1377	−0.0636	−0.1680	−0.0737
	τ_{14}	–	−0.0086	–	−0.0099

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Table 4. Probability of a storm to belong to a certain quartile group.

quartile group	probability [–]
first	0.3930
second	0.2063
third	0.1826
fourth	0.2181

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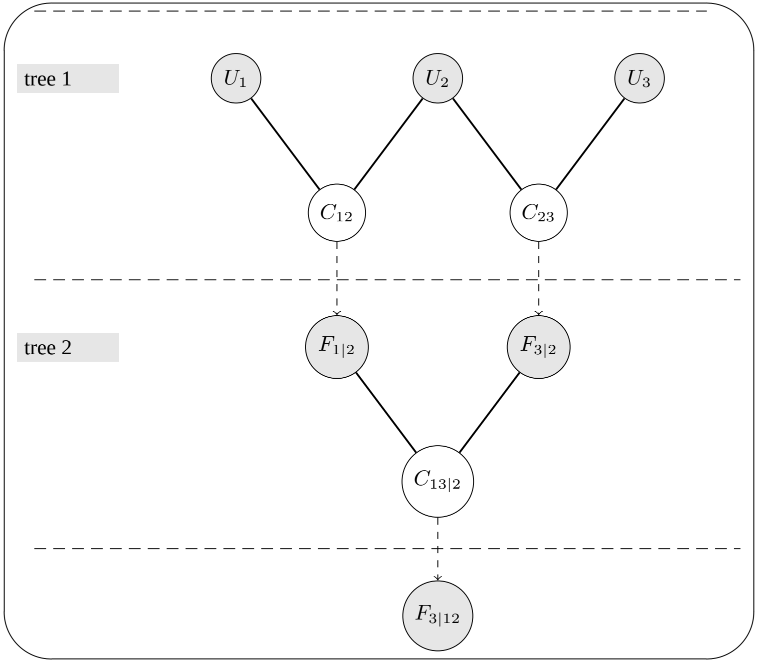

Figure 1. Hierarchical nesting of bivariate copulas in the construction of a three-dimensional vine copula through conditional mixtures.

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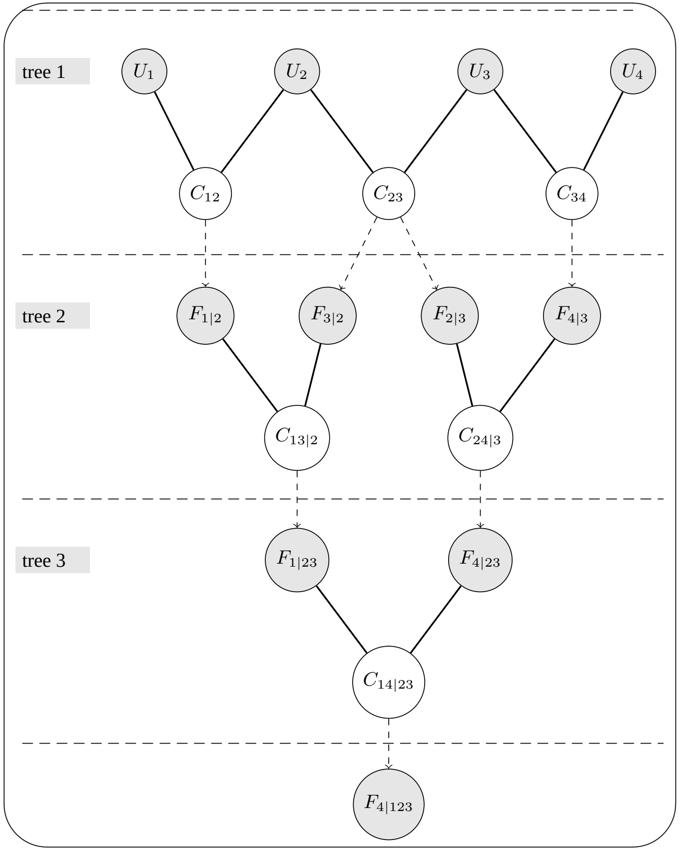

Figure 2. Hierarchical nesting of bivariate copulas in the construction of a four-dimensional vine copula through conditional mixtures.

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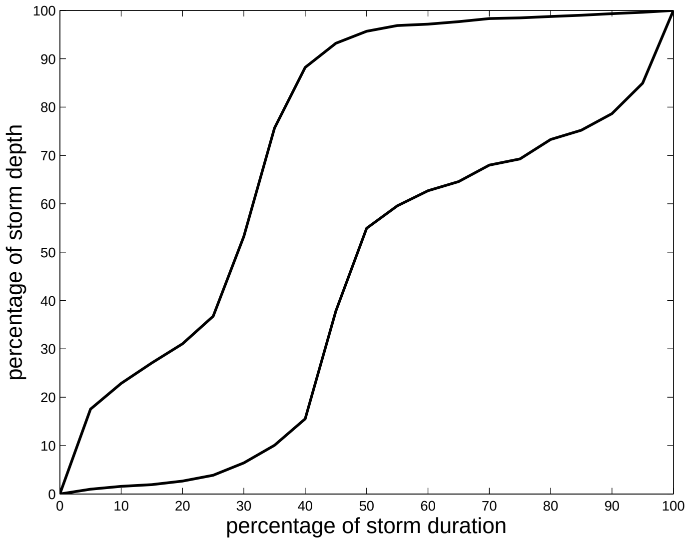

Figure 3. Huff curves for the second quartile autumn storms. The 10 and 90 % percentile curves are given.

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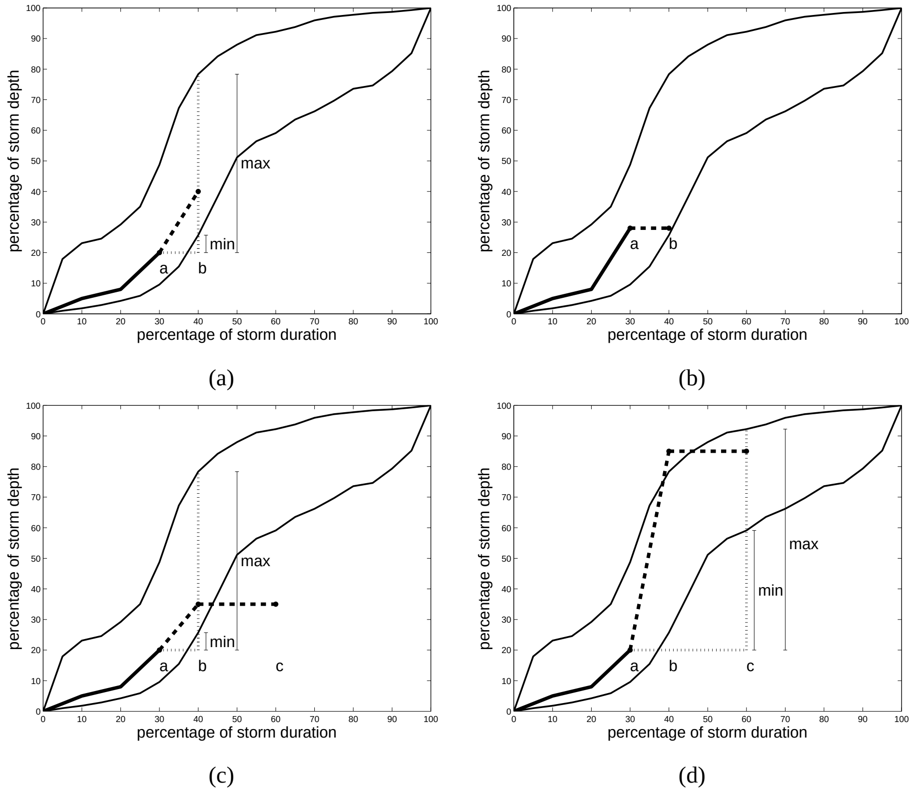

Figure 4. Illustration of the different possibilities in the generation of an internal storm structure. Sampling at the end of a wet period followed by a wet period **(a)**, sampling at the end of a dry period **(b)**, sampling at the end of a wet period followed by a dry period with a selection on the basis of the current time step **(c)** and with a selection on the basis of the last time step in the dry period **(d)**.

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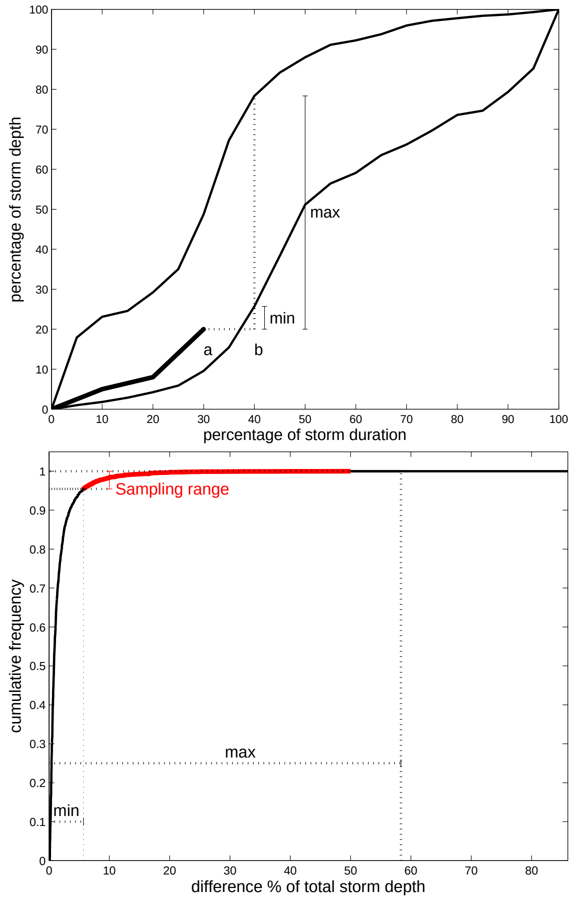

Figure 5. Illustration of the procedure to sample the storm depth at the next time step. Selection of the minimal and maximal difference percentage of storm depth between which the storm depth can be selected (top panel). Corresponding sampling range in the cumulative distribution function of normalized increments (bottom panel).

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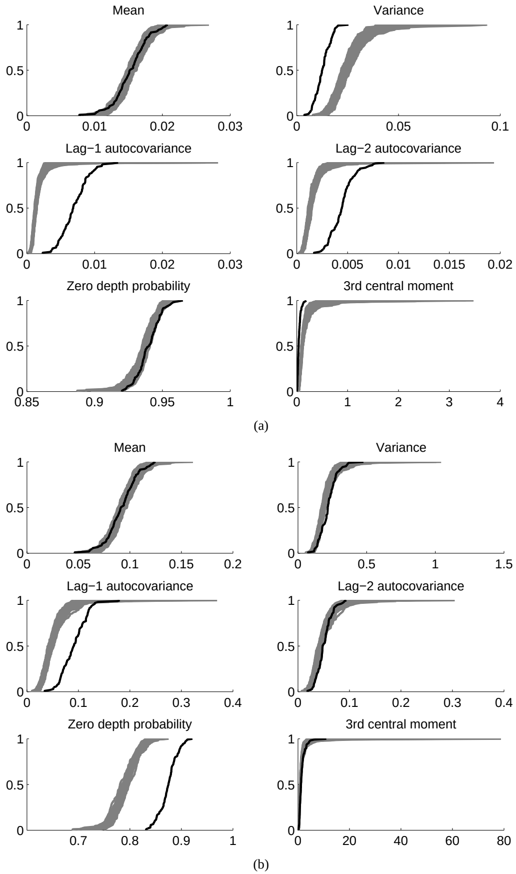

Figure 6. Comparison of the empirical cumulative distributions of the yearly statistics of the observed time series (black line) and the bundle of empirical cumulative distributions of synthetic time series generated by means of the copula-based model (grey) at a 10 min **(a)** and a 1 h aggregation level **(b)**.

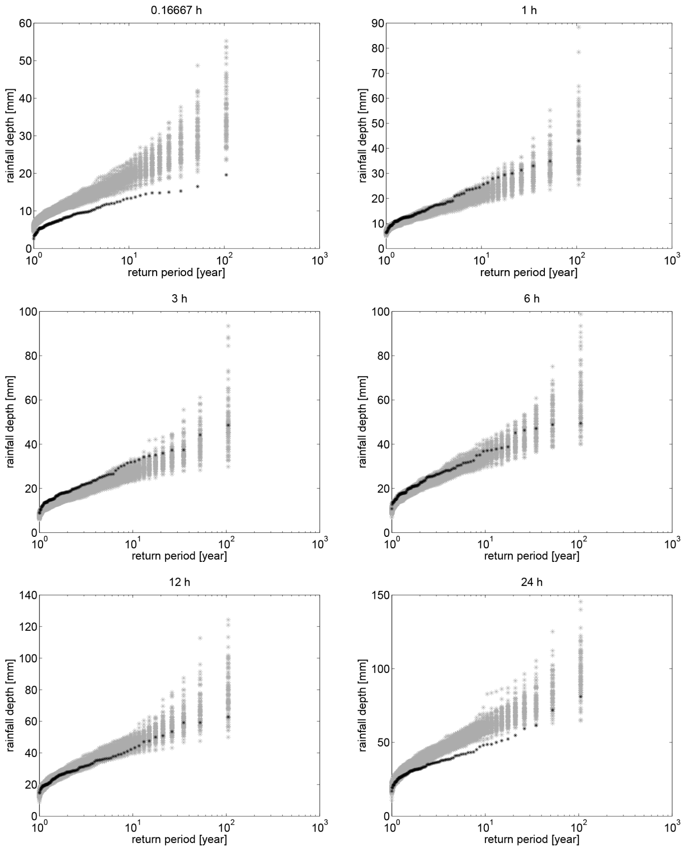

Figure 7. Comparison of empirically derived annual maxima related to the empirical return periods for different aggregation levels on the observed (black asterisks) and ensemble of synthetic time series generated by means of the copula-based rainfall model (grey asterisks).

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