

This discussion paper is/has been under review for the journal Hydrology and Earth System Sciences (HESS). Please refer to the corresponding final paper in HESS if available.

New interpretation of the role of water balance in an extended Budyko hypothesis in arid regions

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Received: 31 August 2015 – Accepted: 11 September 2015 – Published: 27 October 2015

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Published by Copernicus Publications on behalf of the European Geosciences Union.

HESSD

12, 11013–11052, 2015

New interpretation of
water balance in BH
in arid regions

C. Du et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Abstract

The Budyko hypothesis (BH) is an effective approach to investigating long-term water balance at large basin scale under steady state. The assumption of steady state prevents applications of the BH to basins, which is unclosed, or with significant variations in soil water storage, i.e., under unsteady state, such as in extremely arid regions. In this study, we choose the Heihe River Basin (HRB) in China, an extremely arid inland basin, as the study area. We firstly use a calibrated and then validated monthly water balance model, i.e., the *abcd* model to quantitatively determine annual and monthly variations of water balance for the sub-basins and the whole catchment of the HRB and find that the role of soil water storage change and that of inflow from upper sub-basins in monthly water balance are significant. With the recognition of the inflow water from other regions and the soil water storage change as additional possible water sources to evapotranspiration in unclosed basins, we further define the equivalent precipitation (P_e) to include local precipitation, inflow water and soil water storage change as the water supply in the Budyko framework. With the newly defined water supply, the Budyko curve can successfully describe the relationship between the evapotranspiration ratio and the aridity index at both annual and monthly timescales, whilst it fails when only the local precipitation being considered. Adding to that, we develop a new *Fu*-type Budyko equation with two non-dimensional parameters (ω and λ) based on the deviation of *Fu*'s equation. Over the annual time scale, the new *Fu*-type Budyko equation developed here has more or less identical performance to *Fu*'s original equation for the sub-basins and the whole catchment. However, over the monthly time scale, due to large seasonality of soil water storage and inflow, the new *Fu*-type Budyko equation generally performs better than *Fu*'s original equation. The new *Fu*-type Budyko equation (ω and λ) developed here enables one to apply the BH to interpret regional water balance over extremely dry environments under unsteady state (e.g., unclosed basins or sub-annual timescales).

New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



1 Introduction

The Budyko Hypothesis (hereafter BH) was postulated by a Russian climatologist Mikhail Ivanovich Budyko to analyze regional differences in long-term annual water and energy balance (Budyko, 1948). The BH's mean annual water balance is described by only the evapotranspiration ratio and the climate aridity index. The BH becomes an effective approach to investigating the influence of climate change on mean annual runoff and evapotranspiration (Donohue et al., 2011; Xiong et al., 2014). There are various equations to describe the BH. Some empirical equations without parameters were proposed by Schreiber (1904), Ol'dekop (1911), Budyko (1948) and Pike (1964) (see Table 1). These equations explicitly include climate variations (radiation, precipitation, and evapotranspiration and air temperature) and do not deal with recently recognized important catchment properties, such as characteristics of groundwater system, vadoze zone properties, and vegetation. Hence, attempts have been made to introduce physical parameters in these empirical equations (Mezentsev, 1955; Fu, 1981; Milly, 1993; Zhang et al., 2001; Yang et al., 2007, 2008). These physical parameters are a collection of myriad catchment characteristics (topography, vegetation, soil, and groundwater etc.) and are therefore difficult to measure (Gerrits et al., 2009). These equations with parameters, however, provide the flexibility of using the BH over long-term time-scales.

The BH assumes steady state conditions. Firstly, the studied basin must be natural and closed, which means that the local precipitation is the only water source available to evapotranspiration. Recently, the BH has been widely used to investigate the inter-annual variability of precipitation partitioning (Gerrits et al., 2009), separation of runoff trends (Li et al., 2014; Xiong et al., 2015), evapotranspiration change (Savenije, 1997) and water storage change (Istanbulluoglu et al., 2012). These studies show that hydrological processes have been greatly affected by the climate change and intensive change of land cover owing to human activities. These human activities such as urbanization, withdrawing groundwater, hydraulic engineering, deforestation etc.

HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



are significantly changing natural hydrological cycle and breaking the original water balance to form a new balance under the new hydroclimatic conditions. For example, the transferring water becomes the new water source of the basin to evapotranspiration due to the implemented inter-basin water transfer project (Bonacci and Andric, 2010).

In dry regions, croplands expand with irrigation, which increased water availability for evapotranspiration (Gordon et al., 2005). Land use/cover changes have also caused the change of runoff (Li et al., 2014). Nowadays, most of the inhabited basins have been developed or disturbed by so large-scale human activities. Therefore, lots of basins were no longer closed or natural and the relationship between annual evapotranspiration ratio and potential evapotranspiration ratio hardly meet the first condition of the BH, which presents great challenge in applying the BH in unclosed basins.

Secondly, water storage change can be assumed to be negligible at the basin scale and at long-term time scale. However, over finer temporal scales, it becomes increasingly concerned of the importance of water storage in water balance in the Budyko framework. For example, Wang et al. (2009) found that the inter-annual water storage change should be considered due to the hysteresis response of the base flow to the inter-annual precipitation change in Nebraska Sand Hills. Zhang et al. (2008) considered the impacts of soil water and groundwater storage and developed a monthly water balance model based on the BH with application in 265 catchments in Australia. Yokoo et al. (2008) highlighted the importance of soil water storage change in determining both annual and seasonal water balances. Wang (2012) evaluated changes in inter-annual water storage at 12 watersheds in Illinois using the field observation of long-term groundwater and soil water found that the impact of inter-annual water storage changes on the water supply in the BH need to be considered. Chen et al. (2013) defined the difference between rainfall and storage change as effective precipitation to develop a seasonal model for construction long-term evapotranspiration. Therefore, water storage change should be taken into account as the important part of the steady state assumption of the BH (Zhang et al., 2008).

HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



In summary, it has been more and more recognized that water systems are no longer natural to different extents (Sivapalan et al., 2011). Hence, it presents a grant challenge to apply the BH to unsteady state conditions (unclosed basins or intense water storage changes). The BH has been widely applied to mild arid basins with precipitation of 300–400 mm and aridity index of less than, for example, five, such as over northern China (Yang et al., 2007), the southwest regions of MOPEX catchments (Gentine et al., 2012; Carmona et al., 2014) and the west of Australia (Zhang et al., 2008). However, it is rare of applying the BH in extremely arid environments (say, the aridity index over five), where water systems are typically unclosed with intense human interference and irrigation. For example, rivers in the arid region of Northwestern China are typically from upper mountains with little human interference, and flowing through middle regions with intensive irrigation and human interferences and finally into extremely dry desert plains. To investigate it in more detail, we choose the Heihe River Basin (HRB), the second largest arid inland basin in northwestern China (mean annual aridity index = 10). Being an inland basins, the HRB consists of six sub-basins with different landscapes and climate conditions, where the upper mountainous basins are closed and natural with little human interference (long term mean annual water storage change approaches zero), the middle basins are arid and intensively irrigated plain with strong human interference (mean annual evapotranspiration is higher than the local precipitation), and the lower basin is extremely dry Gobi desert plain without any runoff flowing out (evapotranspiration is mainly the local precipitation, mean annual evapotranspiration approaches to mean annual precipitation). We aim at (1) testing whether the BH is applicable for the unsteady state condition in extremely arid basins, (2) improving the original BH by including observed water balance and (3) extending its applicability at unclosed basin scale and annual or monthly time scales.

that the change of groundwater storage (ΔG) is the result of the exchange between groundwater and baseflow and is not directly interacted with evapotranspiration, so that ΔG is not included into the defined P_e in Eq. (2). It will be discussed in the results section.

2.2 Budyko hypothesis model at annual and monthly scale

As discussed above, in the original Budyko framework, the water supply to land evapotranspiration is mean annual precipitation, and the energy supply to land evapotranspiration is estimated by mean annual potential evapotranspiration. The general Budyko equation can be written as:

$$\frac{ET}{P} = F\left(\frac{ET_0}{P}\right) \quad (3)$$

where, $\frac{ET}{P}$ is the evapotranspiration ratio; $\frac{ET_0}{P}$ is the aridity index. $F()$ is the function to be determined. The general analytical solution to Eq. (3) over mean annual timescales is derived by Fu (1981) and is written as follows:

$$ET = ET_0 + P - (ET_0^\omega + P^\omega + C)^{1/\omega} \quad (4)$$

where, ω is the parameter, which reflects the integrated effects of soil, vegetation and topography on separating the local precipitation. If the local precipitation is zero, evapotranspiration approaches to zero due to no available water, C is zero constant. Note that another form of the BH is also given by Mezentsev (1955) (later, Choudhury, 1999 and Yang et al., 2008), which is, in fact, identical to Fu's equation (Zhou et al., 2015) with the parameters linearly related ($R^2 = 0.9997$) (Sun, 2007).

Water balance analysis in Sect. 2.1 concludes that the water supply in the BH under the unsteady state condition is the equivalent precipitation instead of the local precipitation. So the annual (or monthly) evapotranspiration ratio is redefined as the ratio of annual (or monthly) evapotranspiration and equivalent precipitation,

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



constraints formed by the BH, we can derive that $\lambda \geq -1$ (see Appendix A). When $\lambda = 0$ (Fig. 2a), Eq. (8) is the same as the *Fu*'s Equation in its original form (Fu, 1981; Zhang et al., 2004; Yang et al., 2007). For λ becomes positive, e.g., 1, the lower end of the Budyko curve adjusts to the right (Fig. 2b, c). And $\lambda = -1$ (Fig. 2c, d) sets up the upper theoretical constraint of the Budyko curve (Fig. 2c, d).

2.3 A monthly water balance model: *abcd* model

Regional evapotranspiration and soil water cannot be measured directly and they are usually provided by using monthly water balance models. Monthly water balance models were first developed in the 1940s. From that, many models have been developed in hydrological studies, such as *T* model, *T α* model, *P* model, *abc* model and *abcd* model are often popular due to relatively simple structure and fewer parameters (Fernandez et al., 2000).

Among these monthly models, the *abcd* model was proposed by Thomas (1981) has been widely applied to assess regional water resources due to its explicit model structure and only four parameters, of which two parameters pertain to runoff characteristics and the other two relate to groundwater sound physical meanings. The model was originally applied at the annual time scale and later extended to the monthly time scale (Alley, 1984). Moreover, Savenije (1997) has verified that the *abcd* model to derive expressions for the evapotranspiration ratio has better agreement with observations than Budyko-type curves. Inputs to the *abcd* model are monthly precipitation and potential evapotranspiration. Outputs include monthly runoff (direct and indirect), soil water, groundwater storage and actual evapotranspiration. Therefore, this study employs the *abcd* model to provide monthly actual evapotranspiration and soil water storage.

The partitioning of monthly precipitation P_t in the model is as follows; runoff Q_t (direct and indirect), evapotranspiration ET_t , soil water storage S_t , and groundwater storage G_t . The partitioning is controlled by the magnitude of precipitation P_t , potential evapotranspiration ET_{0t} , and the initial storages in soil S_{t-1} and groundwater G_{t-1} . The

New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



following equation controls the partitioning:

$$Y_t(W_t) = \frac{W_t + b}{2a} - \sqrt{\left(\frac{W_t + b}{2a}\right)^2 - \frac{W_t b}{a}} \quad (9)$$

where Y_t is the sum of monthly evapotranspiration and soil water storage at the end of the month, namely evapotranspiration opportunity. W_t is the sum of monthly precipitation and initial soil moisture, named as available water. The parameter a ($0 \sim 1$) means the propensity in a catchment for runoff to occur before the soil becomes saturated. The parameter b is the maximum value of Y_t . Available water partitioning between ET_t and S_t is controlled by the assumption that the loss rate of actual evaporation from soil water storage is proportional to the evapotranspiration capacity. So the soil water storage at the end of period t is written as:

$$S_t = Y_t \exp(-ET_{0t}/b) \quad (10)$$

The actual evapotranspiration at the period t is the difference between evapotranspiration opportunity and soil water storage ($Y_t - S_t$). The streamflow, including direct runoff and groundwater recharge, is determined by the difference between available water and evapotranspiration opportunity ($W_t - Y_t$). The parameter c separates the direct runoff $(1 - c)(W_t - Y_t)$ and groundwater recharge $c(W_t - Y_t)$. Groundwater discharge dG_t as the base flow is determined by the parameter d and groundwater storage at the end of period t . The streamflow is sum of direct runoff and the base flow. For a given set of, b , c and d and initial soil water storage and groundwater storage, the allocation of monthly precipitation can be computed one by one.

New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



3 Study area and data

3.1 Study area

The HRB, originating from Qilian Mountains, is the second largest inland river basin in the arid area of the northwestern China (Fig. 3). The drainage map and the basin border are extracted using a 90 m resolution digital elevation model (DEM) data from the Shuttle Radar Topography Mission (SRTM) website of the NASA (<http://srtm.csi.cgiar.org/SELECTION/inputCoord.asp>) (basin length: 820 km; total area: 143 044 km²; elevation: 870–5545 m). The HRB is in the middle of Eurasia and away from oceans, characterized with dry and windy climate, and very limited precipitation (mean annual precipitation: 126 mm yr⁻¹) but plentiful radiation (mean annual solar radiation: 1780 MJ m⁻² yr⁻¹, ~ 660 mm yr⁻¹ in the unit of evaporation).

The HRB is divided into six sub-basins according to basin characteristics, distributing along eastern and western tributaries, shown in Fig. 3. Regions I and II are upper mountainous regions with the elevation of 3000–5500 m and belong to the cold and semiarid mountainous zone dominated by shrubs and trees with mean annual temperature of less than 2 °C and annual precipitation of 200 ~ 400 mm. And these two sub-basins are the water source area to the middle and lower reaches and have little interference of human activities. Regions III and V with annual precipitation of 100–250 mm are the main irrigation zone and residential area with more than 90 % of total population of the HRB. The two sub-basins are the main water-consuming regions and largely disturbed by human activities. Regions IV and VI located at lower reaches are extremely arid and the mean annual precipitation is less than 100 mm.

3.2 Data

The required data for Eq. (8) and the *abcd* model include monthly precipitation, potential evapotranspiration and runoff from those sub-basins in the HRB.

HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

⏴

⏵

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



**New interpretation of
water balance in BH
in arid regions**C. Du et al.

[Title Page](#)[Abstract](#)[Introduction](#)[Conclusions](#)[References](#)[Tables](#)[Figures](#)[⏪](#)[⏩](#)[◀](#)[▶](#)[Back](#)[Close](#)[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)

The daily precipitation data of all stations during 1978–2012 are obtained from the year book hydrology of China including 28 rainfall stations and the China Administration of Meteorology including 19 meteorological stations (Fig. 3). The monthly precipitation of each station is calculated by summing daily precipitation. The gridded data set with 1 km resolution across the whole basin is obtained by interpolation of the site data. The monthly precipitation of the six sub-basins is obtained by the extraction from the monthly precipitation in the whole basin. Daily meteorological data of 19 stations during 1978 and 2012 are also available. Daily potential evapotranspiration is estimated in each station using the FAO Penman–Monteith equation recommended by Allen et al. (1998). The monthly ET_0 at each station is the sum of the daily ET_0 and then interpolated to the whole basin. Finally, annual runoff, precipitation and potential evapotranspiration are obtained by summing monthly data.

The runoff data set includes monthly runoff at 4 stations located at the inflow or outflow of the six sub-basins. The red points in Fig. 3 locate the hydrological stations. Monthly runoff data are obtained from the year book of hydrology of China and are intended for calibrate the *abcd* model. The annual runoff is obtained by summing monthly runoff. The data time series for Regions I and III are from 1978 to 2012. The same period is for Regions II and V but with the period of 1998–2006 missing. The length of data time series for Regions IV and VI is from 1988 to 2012.

The natural runoff in Regions III and IV were strongly disturbed by human activities and there is no runoff for the Regions V and VI and the whole basin. To validate the outputs of the *abcd* model for those regions, this study employs the evapotranspiration of remote sensing products from Heihe Plan Science Data Center (Wu et al., 2012) as a reference. The same data have been widely used as a reference for modeling evaluations and is supported by a State Key Research Program-Heihe Eco-hydrological Research Project of National Natural Science Foundation of China (Yan et al., 2014; Yao et al., 2014). The monthly evapotranspiration datasets (2000–2012) with 1 km spatial resolution over the HRB (<http://westdc.westgis.ac.cn>), are estimated by ETWatch model based on multi-source remote sensing data (Wu et al., 2012).

4 Results

4.1 Calibration of the *abcd* model

In extremely dry basins like the HRB, the lack of observed hydro-climatic data presents great challenge. A monthly water balance model becomes an effective tool to estimate actual evapotranspiration, change of soil water storage and change of groundwater storage. This study employs the *abcd* water balance model due to its simple and sound physical structure tested and recommended by Alley (1984) and Fernandez et al. (2000). We calibrate and validate the *abcd* model using monthly time series of precipitation, potential evapotranspiration and runoff at each of the seven regions (the six sub-basins and the whole basin) using the generalized pattern search optimization method. Nash–Sutcliffe efficiency (NSE) is used to assess the goodness of fit of the monthly water balance for the seven regions.

Figure 4 shows the results of the modeled streamflow at monthly time scale in Regions I and II. Regions I and II are the water source area of the whole basin with little interference of human activities and both keep relatively natural steady state. The NSE for the Regions I and II is for 0.92 and 0.83, respectively. The results illustrate that the simulated monthly streamflow agrees well with the observation and other modeled components can be reasonable estimates, for instance, monthly actual evapotranspiration, soil water storage change and groundwater storage change in the two sub-basins.

The streamflow in Regions III and IV was completely controlled by hydrological stations due to water resources allocation, so the observed monthly streamflow cannot be directly used to validate the simulations of the *abcd* model. To validate the results of actual evapotranspiration, we compare the simulated ET by the *abcd* model and remote sensing ET (Fig. 5) calculated by remote sensing data during 2000–2012. The NSE values for Regions III–VI and the whole basin are not less than 0.88, which illustrate that simulated ET have good agreement with ET by remote sensing.

HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[⏴](#)

[⏵](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



for the sub-basins and the whole HRB on the monthly timescales, which in turn requires new treatments in the BH as further investigated in following sections.

4.3 The annual Budyko curve analysis

Figure 8 (left panel) plots the original Budyko curves for the six sub-basins and the whole basin. For Regions I and II, the points of annual evapotranspiration ratio and aridity index fall in the domain of water and energy limit boundary and they can be well fitted by Fu 's equation. The relationship between water and energy in Regions I and II can be described by the original BH as expected in the section above. However, the points of evapotranspiration ratio and aridity index for other regions exceed the water limit boundary. And the results show the relationship of water and energy in Regions III–VI and the whole basin is inconsistent with the original BH. After using the equivalent precipitation instead of the local precipitation, the new Fu -type Budyko curves (Eq. (8) with $\lambda = 0$) for all regions are shown in Fig. 8 (middle panel). Compared with the original Budyko curve, the new curves for Regions I and II did not behave differently, because the two basins are natural and closed. The obvious change between the improved and original Budyko curves are for the Regions III and IV. For the whole basin and Regions V and VI, the new curves fall on the upper limit of $ET/P_e = 1$ due to no runoff flowing out. These improved Budyko curves can be fitted using Fu 's equation and the parameters are listed in Table 3. Interestingly for the annual time scale, the fitted performances of Fu 's equation and Eq. (8) are almost identical. Therefore, the new Fu -type Budyko curves (Eq. 8) with fitted values of λ (right panel, Fig. 8) do not show much difference from those curves with λ set zero.

In summary, if a basin (sub-basin) is closed, the original BH can be applicable at the annual time scale. However under unsteady state, the new Fu -type BH, instead of the original BH is more applicable to describe the annual water balance.

HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



hydrologic characteristics. We first calibrate and validate a widely used monthly water balance model, i.e., the *abcd* model. For the two upper sub-basins, the simulated monthly water balance is compared against monthly streamflow from hydrological gauges, and for the other sub-basins and the whole catchment, the simulated evapotranspiration is compared with widely used remote sensing ET products in the HRB. The *abcd* model can successfully simulate the monthly water balance and capture the inter-annual variations (NSE over 0.85). Based on that, we found that the role of soil water storage change in monthly water balance is significant but almost negligible over timescales longer than a year. And the impact of inflow from upper sub-basins is also significant and does not rely on the timescale. We concluded that the upstream basin in the HRB are almost closed basins, which meet the two steady state conditions of the BH and other sub-basins become an unclosed basin due to impact of the inflow water and human interference.

With the recognition that the inflow water from other regions and the water storage change are both new possible water sources to evapotranspiration in unclosed basins, we define the equivalent precipitation (P_e) including the local precipitation, inflow water and water storage change as the water supply, instead of just the local precipitation, in the Budyko framework. (The evapotranspiration ratio and the aridity index are also redefined using the equivalent precipitation.) In addition to the new definition of the water supply, we develop a new *Fu*-type Budyko equation with two non-dimensional parameters (ω and λ) based on the deviation by Professor Baopu Fu, i.e., *Fu*'s equation to consider the effect of the change of soil water storage and the inflow water on the water and energy constraints. Over the annual time scale, the new *Fu*-type Budyko equation developed here has more or less identical performance to the *Fu*'s equation for the sub-basins and the whole catchment. However, for the monthly time scale, the new *Fu*-type Budyko equation performs better than *Fu*'s original equation when the ratio of evapotranspiration to equivalent precipitation less than one, and performs the same when the evapotranspiration ratio is very close to one. The new *Fu*-type Budyko equation (ω and λ) developed in this study enables one to apply the BH to interpret

HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



regional water balance over extremely dry environments under unsteady state (e.g., unclosed basins or sub-annual timescales).

Appendix A

For an unclosed basin or region, the water supply to evapotranspiration is defined as equivalent precipitation ($P_e = P + Q_{in} - \Delta S$). Evapotranspiration ratio: $\varepsilon = ET/P_e$ and aridity index: $\phi = ET_0/P_e$. The Budyko equation is written the same as Eq. (8)

$$\varepsilon = 1 + \phi - (1 + \phi^\omega + \lambda)^{1/\omega} \quad (A1)$$

According to the constrained boundary of the BH, Eq. (1) evapotranspiration ratio is less than or equal to aridity index, namely $\varepsilon \leq \phi$, and Eq. (1) the evapotranspiration ratio is no more than 1, i.e., $\varepsilon \leq 1$,

With $\varepsilon \leq \phi$, we can have,

$$1 + \phi - (1 + \phi^\omega + \lambda)^{1/\omega} \leq \phi \quad (A2)$$

Therefore,

$$\phi^\omega + \lambda \geq 0 \quad (A3)$$

where $\phi \geq 0$ and $\omega > 1$.

For the other constraint, $\varepsilon \leq 1$ we can derive,

$$1 + \lambda \geq 0. \quad (A4)$$

Acknowledgements. This research was supported by the National Natural Science Foundation of China (41271049), Chinese Academy of Sciences (CAS) Pioneer Hundred Talents Program, and State Key Laboratory of Desert and Oasis Ecology, Xinjiang, Institute of Ecology and Geography, Chinese Academy of Sciences. The authors would like to appreciate Hubert H. G. Savenige, Wang Ping for his comments and support in this research.

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New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



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New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



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**New interpretation of
water balance in BH
in arid regions**

C. Du et al.

[Title Page](#)[Abstract](#)[Introduction](#)[Conclusions](#)[References](#)[Tables](#)[Figures](#)[|◀](#)[▶|](#)[◀](#)[▶](#)[Back](#)[Close](#)[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)

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C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



Table 1. Different Budyko equations for mean annual water–energy balance.

Number	Equation	Parameter	Reference
1	$\varepsilon = 1 - \exp(-\varphi)$	none	Schreiber (1904)
2	$\varepsilon = \varphi \tanh(1/\varphi)$	none	Ol'dekop (1911)
3	$\varepsilon = \{\varphi[1 - \exp(-\varphi)] \tanh(1/\varphi)\}^{0.5}$	none	Budyko (1958, 1974)
4	$\varepsilon = (1 + \varphi^{-2})^{-0.5}$	none	Pike (1964)
5	$\varepsilon = (1 + \varphi^{-\alpha})^{-1/\alpha}$	α – calibration factor	Mezentsev (1955); Chouldhury (1999); Yang et al. (2008)
6	$\varepsilon = \frac{1 + \omega\varphi}{1 + \omega\varphi + \varphi^{-1}}$	ω – coefficient of vegetation and water supply	Zhang et al. (2001)
7	$\varepsilon = \frac{\exp[\gamma(1-1/\varphi)] - 1}{\exp[\gamma(1-1/\varphi)] - \varphi^{-1}}$	γ – the ratio of the soil water storage capacity to precipitation	Milly (1993); Porporato et al. (2004)
8	$\varepsilon = 1 + \varphi - (1 + \varphi^\omega)^{1/\omega}$	ω – a constant of integration	Fu (1981); Zhang et al. (2004); Yang et al. (2007)

Note: $\varepsilon = ET/P$ evapotranspiration ratio (the ratio of mean annual evapotranspiration to mean annual precipitation); $\varphi = ET_0/P$, aridity index (the ratio of mean annual potential evapotranspiration to mean annual precipitation).

New interpretation of water balance in BH in arid regions

C. Du et al.

Table 2. The mean annual water balance of all regions.

Region	P (mm)	Q_{in} (mm)	ET (mm)	Q_{out} (mm)	ΔS (mm)	PWS (%)
I	351.9	–	165.3	169.3	0.0	0.0
II	220.7	–	143.9	85.2	0.1	0.0
III	223.6	66.1	253.2	37.5	–2.1	–0.9
IV	73.5	74.0	103.4	47.5	1.0	1.3
V	117.3	39.6	156.7	–	0.2	0.1
VI	66.8	7.9	74.7	–	0.0	0.0
Whole basin	125.8	–	125.5	–	0.2	0.2

“–” means no runoff.

PWS represents the proportion of the soil water storage change in the total precipitation.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Table 4. The fitting parameters of the improved Budyko equation at the monthly scales.

Region	Parameter	May–Aug	Apr and Sep	Mar and Oct	Feb and Nov	Jan and Dec
I	ω^a	1.40	1.39	1.35	1.28	1.22
	ω^b	1.50	1.51	1.48	1.40	1.33
	λ^b	0.16	0.24	0.31	0.32	0.28
II	ω^a	1.54	1.53	1.47	1.37	1.29
	ω^b	1.70	1.72	1.67	1.57	1.48
	λ^b	0.34	0.54	0.63	0.66	0.60
Region	Parameter	April–Sep	Mar and Oct	Feb and Nov	Jan and Dec	
III	ω^a	2.20	2.05	1.86	1.71	
	ω^b	2.31	2.15	1.97	1.90	
	λ^b	0.18	0.19	0.22	0.39	
IV	ω^a	1.51	1.42	1.33	1.25	
	ω^b	1.75	1.59	1.51	1.41	
	λ^b	0.92	0.53	0.56	0.41	
Region	Parameter	May–Aug	Apr and Sep	Mar and Oct	Feb and Nov	Jan and Dec
V	ω^a	35.5	29.8	28.0	22.5	23.7
	ω^b	1.02	1.03	1.03	1.04	1.04
	λ^b	–1	–1	–1	–1	–1
VI	ω^a	17.3	18.9	15.5	12.7	13.1
	ω^b	1.02	1.02	1.03	1.03	1.04
	λ^b	–1	–1	–1	–1	–1
The whole basin	ω^a	28.5	22.4	18.8	16.3	15.7
	ω^b	1.02	1.02	1.04	1.04	1.03
	λ^b	–1	–1	–1	–1	–1

^a means the calibrated values of ω in Fu's equation; Eq. (8) when $\lambda = 0$.

^b means the calibrated values of ω and λ in Eq. (8).

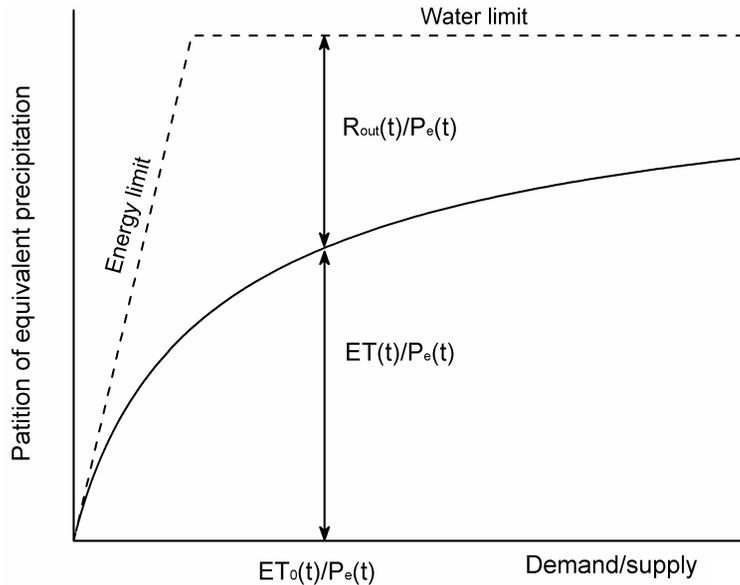


Figure 1. A schematic diagram of the BH under the unsteady state condition.

New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#) [Introduction](#)

[Conclusions](#) [References](#)

[Tables](#) [Figures](#)

[◀](#) [▶](#)

[◀](#) [▶](#)

[Back](#) [Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



New interpretation of water balance in BH in arid regions

C. Du et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)

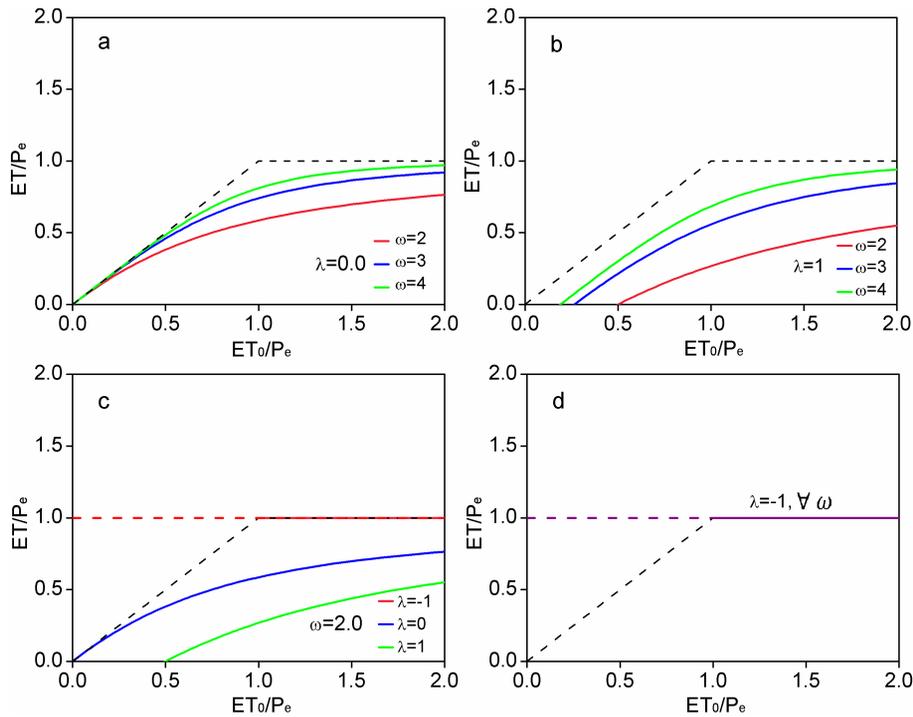


Figure 2. The Budyko curves in Eq. (8) with different combinations of parameters ω and λ .

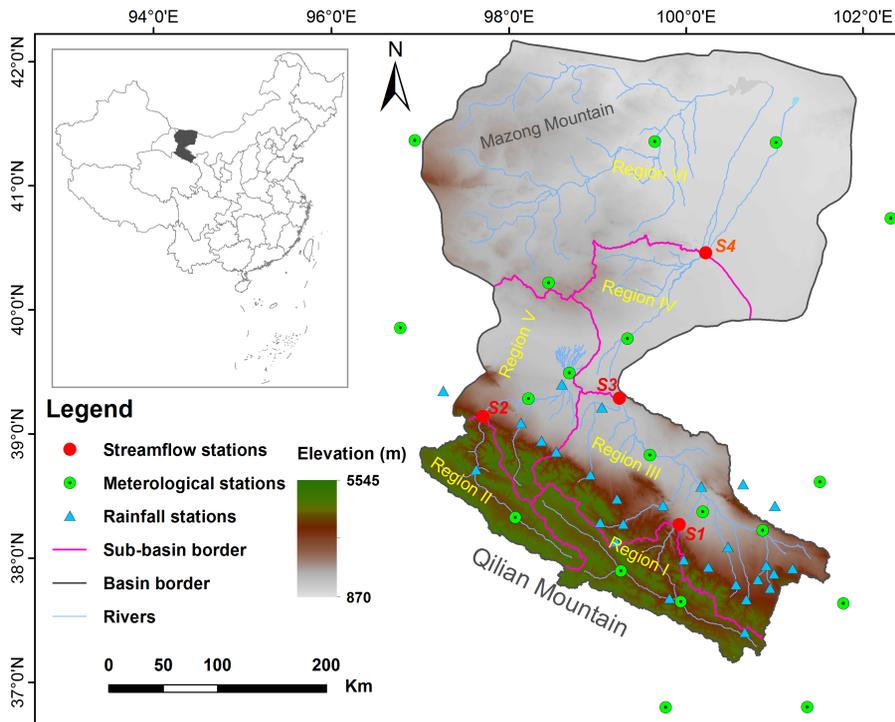


Figure 3. Location of study area and the distribution of hydrological stations and meteorological stations.

New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



New interpretation of water balance in BH in arid regions

C. Du et al.

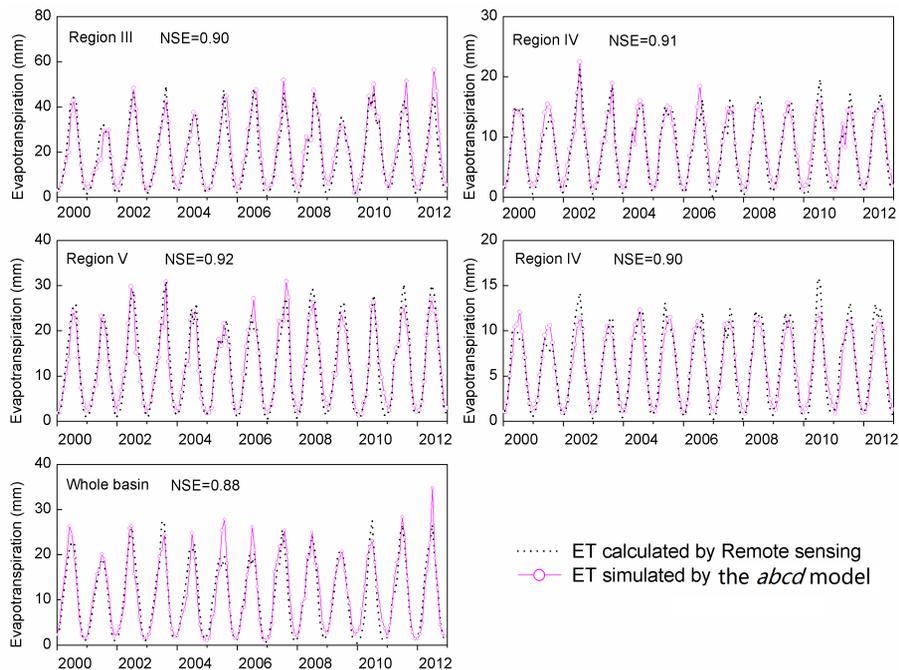


Figure 5. Comparison between ET simulated by the *abcd* model and ET calculated by remote sensing data for Regions III–VI and the whole basin during 2000–2012. “WBM” denotes the *abcd* water balance model.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



New interpretation of water balance in BH in arid regions

C. Du et al.

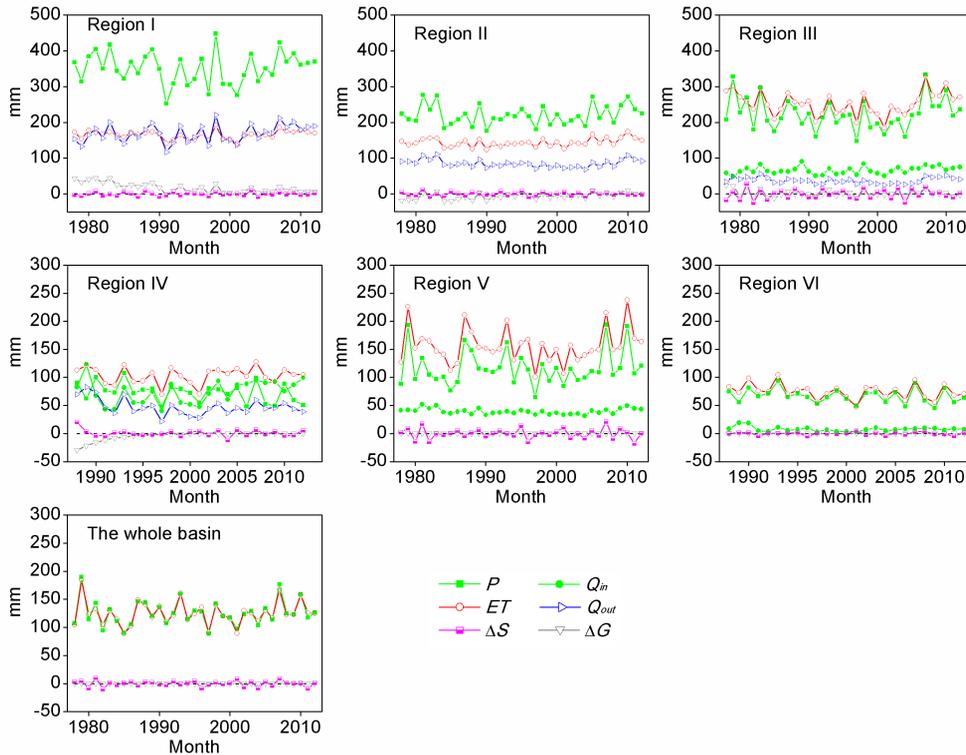


Figure 6. Variation of annual water balance for all the regions simulated using the *abcd* model.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



New interpretation of water balance in BH in arid regions

C. Du et al.

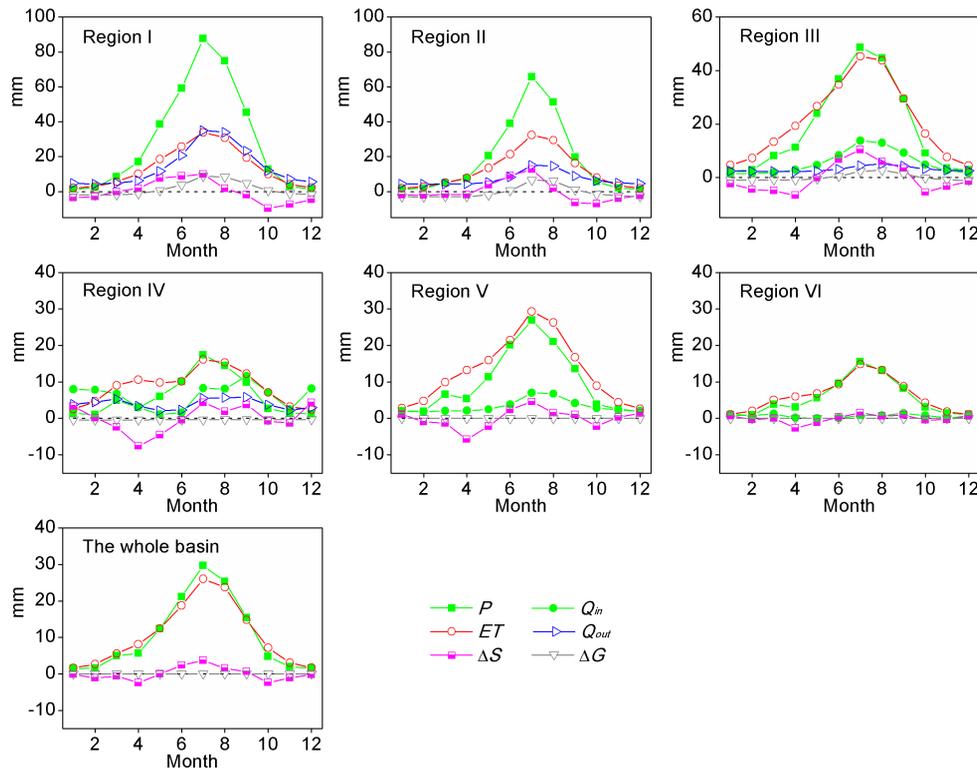


Figure 7. Variation of average monthly water balance for all regions using the *abcd* model.

[Title Page](#)

[Abstract](#) [Introduction](#)

[Conclusions](#) [References](#)

[Tables](#) [Figures](#)

[◀](#) [▶](#)

[◀](#) [▶](#)

[Back](#) [Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



New interpretation of water balance in BH in arid regions

C. Du et al.

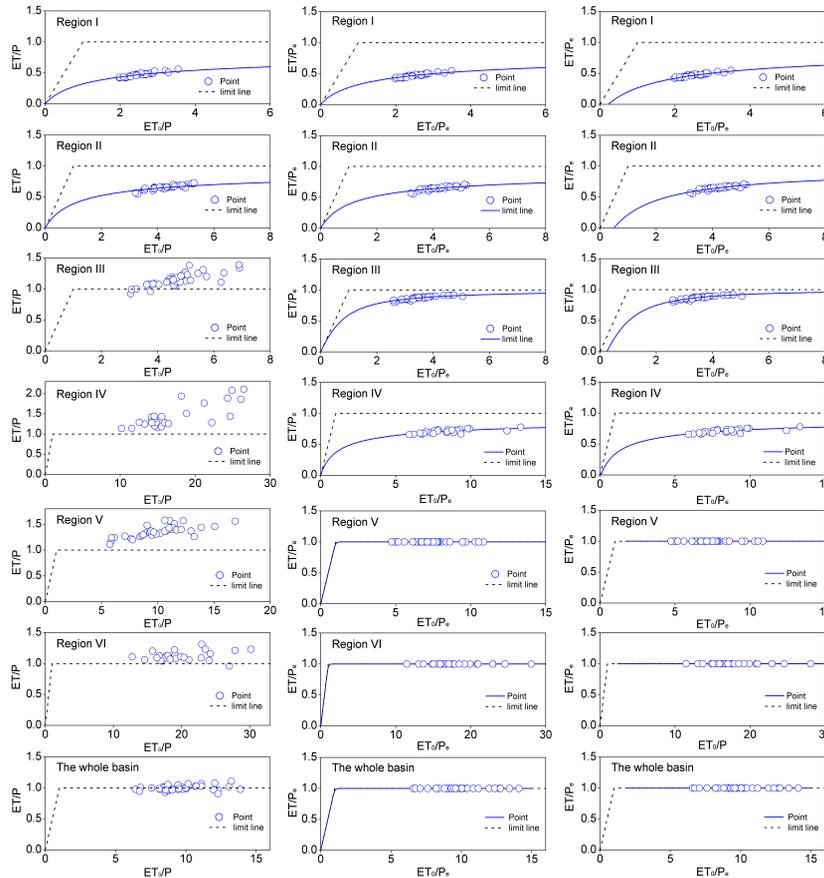


Figure 8. Comparison of the original Budyko curves (left panel) and the new F_u -type Budyko curves (middle panel, with $\lambda = 0$) and the new F_u -type Budyko curves (right panel, with $\lambda > 0$) for Regions I–VI and the whole basin at the annual time scale.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

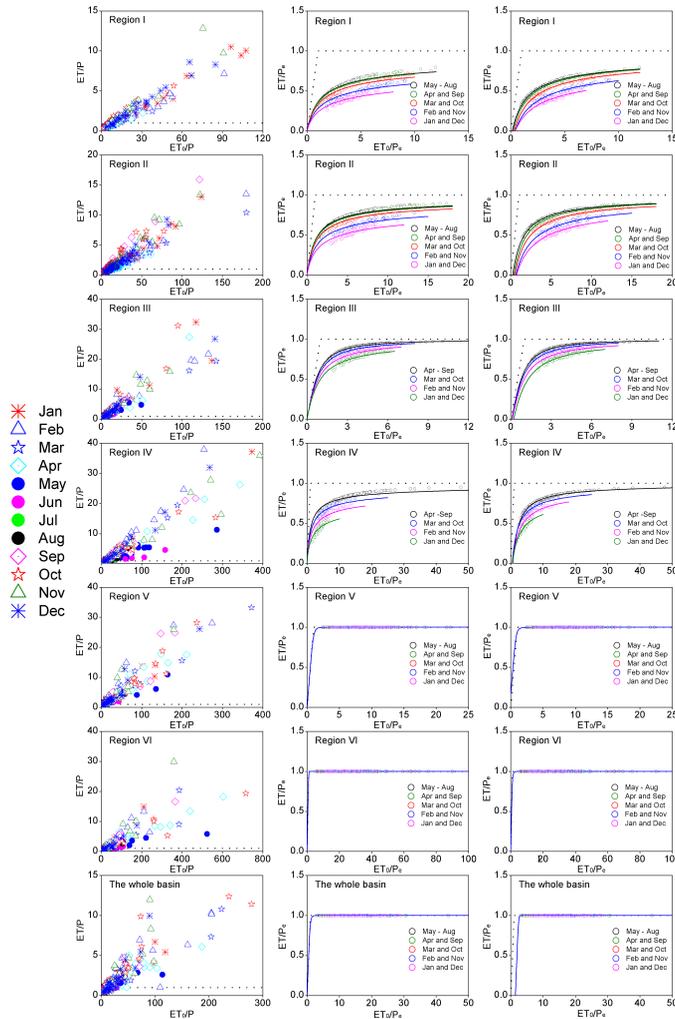
Printer-friendly Version

Interactive Discussion



New interpretation of water balance in BH in arid regions

C. Du et al.



Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



New interpretation of water balance in BH in arid regions

C. Du et al.

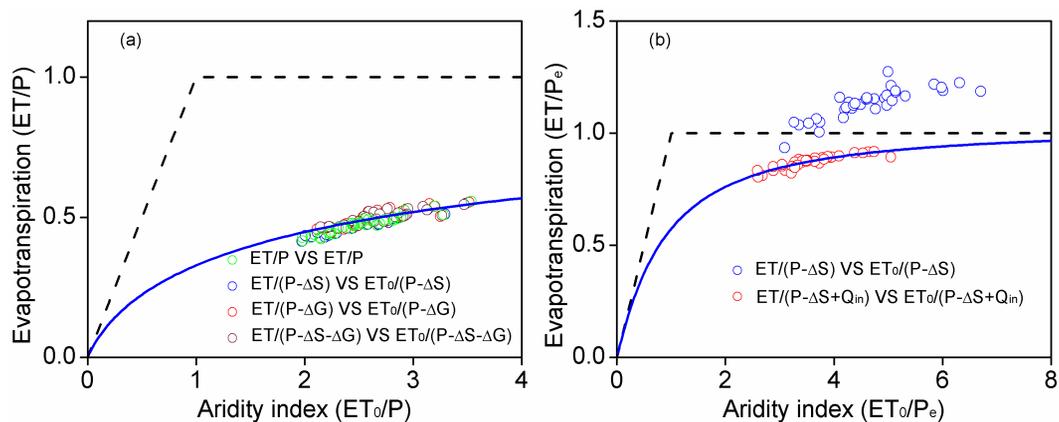


Figure 10. Different presentations of annual water balance for **(a)** Region I and **(b)** Region III.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



New interpretation of water balance in BH in arid regions

C. Du et al.

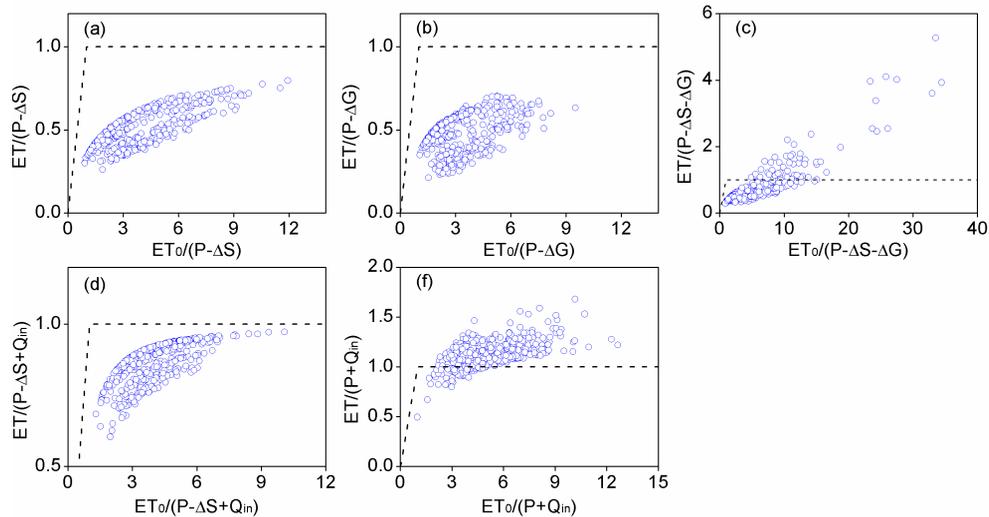


Figure 11. Five presentations of monthly water balance for Region III considering different combinations in the water supply to evapotranspiration.

[Title Page](#)
[Abstract](#)
[Introduction](#)
[Conclusions](#)
[References](#)
[Tables](#)
[Figures](#)
[◀](#)
[▶](#)
[◀](#)
[▶](#)
[Back](#)
[Close](#)
[Full Screen / Esc](#)
[Printer-friendly Version](#)
[Interactive Discussion](#)
