Hydrology and Earth System Sciences (HESS) – Manuscript Revisions

Manuscript # hess-2015-374

Title: New interpretation of the role of water balance in an extended Budyko hypothesis in arid regions

Authors: C. Du, F. Sun, J. Yu, X. Liu, and Y. Chen

Dear Editor and Reviewers,

The authors thank the two reviewers for comments that we believe have improved the manuscript. According to the reviewers' comments, we have made revisions and corrections meet with the requirements of Journal Hydrology and Earth System Sciences. In this revised version, point by point responses to reviewers' comments are marked in colored text in the PDF documents of Hydrol. Earth Syst. Sci. Discuss 12, 11013-11052.

Response to Reviewer #1

1. "*Section 2.3: Equation (9) of the abcd model is a Budyko equation. Wang and Tang (2014) derived a one-parameter Budyko equation base on the generalized proportionality hypothesis originated from the SCS method. This Budyko equation has the same functional form of abcd model for monthly water balance. In abcd model, P+S0 is partitioned into E+S1 and Q. S0 is initial storage for a month; S1 is the ending storage of the month. E+S1 has an upper bound b; but Q has no upper bound. Equation (9) is for (E+S1)/(P+S0) versus b/(P+S0), i.e., a Budyko equation at the monthly scale.*
For mean annual water balance, P is partitioned into E and Q, where E has an upper bound of (E0) but Q has no upper bound. Budyko equation is for E/P versus Ep/P. When S0=0 and S1=0, b becomes Ep and Equation (9) becomes Budyko equation for mean annual water balance. Therefore, the original Budyko equation for mean annual water balance is a special case of Equation (9) for monthly water balance. The meaning of the parameter for mean annual water balance is explained in Wang and Tang (2014)."

Thank you.

We agree that Equation (9) is equivalent to Budyko type equation as well demonstrated by Wang and Tang (2014). We have modified the relevant part in the text (See Line 7, Page 11022).

The point about defining the equivalent precipitation as the sum of the precipitation and the initial soil water content ($P+S_0$) is interesting and was also used in the abcd model (Thomas 1981). Following up that suggestion and the tradition, we examine the relationship between $(ET+S_1)/(P+S_0)$ VS $b/(P+S_0)$, where b is the maximum of evaporation opportunity in Figure

A1. In Figure A1, we plot Region I (the upper sub-basin), Region III (the middle sub-basin), and Region V (also the middle sub-basin) and Region VI (the lower sub-basin). We found that those points for Region III, V and VI can exceed the water limit and the relationship of $(ET+S_1)/(P+S_0)$ VS $b/(P+S_0)$ do not follow the original Budyko hypothesis.

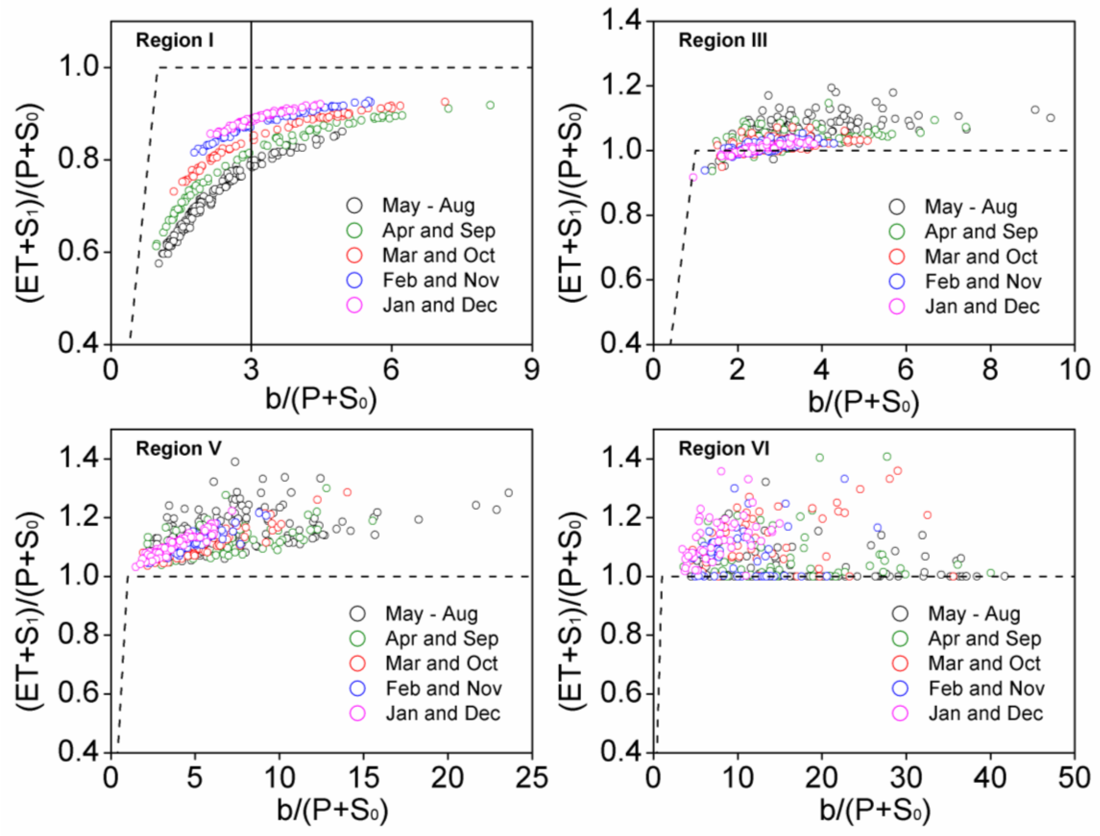


Figure A1. The relationship of $(ET+S_1)/(P+S_0)$ VS $b/(P+S_0)$.

We further examined the relationship of $ET/(P+S_0)$ VS $ET_0/(P+S_0)$ for the same four sub-basins in Figure A2. Compared with Figure 9 in the main text, the points of $ET/(P+S_0)$ VS $ET_0/(P+S_0)$ are scattered and do not fit the Budyko type equations.

Over last several years, theory and case studies about the Budyko hypothesis has well developed worldwide. Our study aims at extending the original Budyko framework to extremely arid conditions, on which discussion is rare. Therefore it becomes crucial to make the Budyko hypothesis applicable and suitable under that extreme condition.

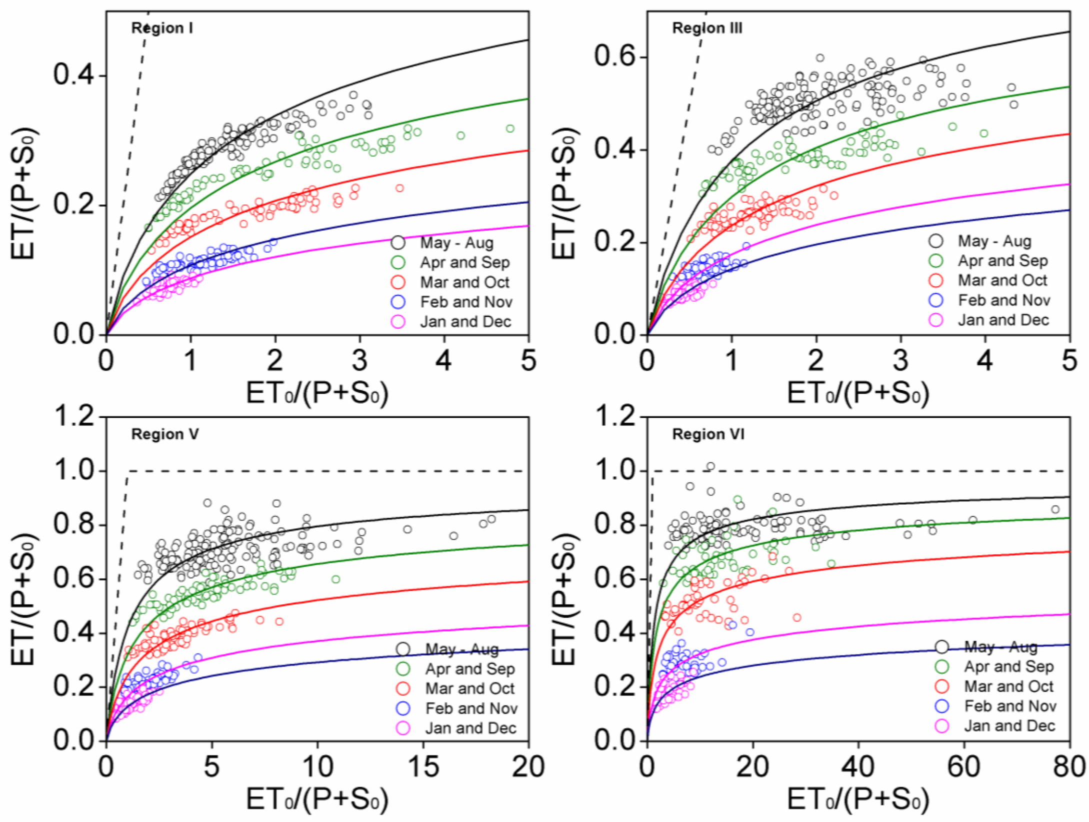


Figure A2. The relationship of $ET/(P+S_0)$ VS $ET_0/(P+S_0)$.

2. *Lines 18-19 on page 11015: "These equations with parameters" to "These equations with a single parameter"*

Done.

3. *Line 5 on page 11016: "expand" to "expanded"?*

Done.

4. *Line 8 on page 11016: "so large-scale human actives" to "large-scale human activities"*

Done.

5. *Line 23 on page 11016: "and found that"*

Done.

6. *Line 2 on page 11017: "grant" to "grand"?*

Done.

7. *Line 11 on page 11017: "flowing" to "flow"?*

Done.

8. *Line 14-22 on page 11017: The sentence is too long. It is can be break into short sentences.*

Thanks. We now break the sentence into three short sentences.

In this study, our aim is threefold. (1) We first test whether the BH is applicable to the unsteady state condition in extremely arid basins. (2) If not, we further improve the original BH by including observed water balance. (3) We finally extend the applicability of the BH in unclosed basins at annual or monthly time scales.

9. *Line 11 on page 11018: "interference" to "interferences"*

Done.

10. *Line 20 on page 11018: I am not convinced by the definition of available water inequation (2), i.e., delta_G is not included for the definition of Pe. See my comment on abcd model.*

We examined this point in Figure 11 to consider the effects of the ΔG in defining the available water. But it does not work well. We suspect that it is to do with that the ΔG does not contribute the evaporation.

11. *Equation (4) on page 11019: Remove "C" in the equation?*

Thanks. This comment is related to the Comment #12. The constant "C" was derived in the original derivation of Fu (1981) (equation (20)). Over long term mean annual time scale, when the boundary ($P\rightarrow0$, ET=0) applies, "C" become zero constant in the original equation and followers of Fu's equation.

However, for much finer timescale like yearly or monthly as investigated here, that boundary ($P\rightarrow0$, ET=0) is hard to meet in practice. Therefore in our study, we keep the constant "C" in Fu's equation to have one more freedom. To make a distinction from the widely used form of Fu's equation (without the "C"), here we call it a new Fu-type Budyko equation under unsteady state condition (with the C). As shown in the results we found the new form works better than the original Fu equation (Figure 9). We will test it in our future work over catchments worldwide.

For the physical meaning of the C and its dimensionless form lambda (λ), it is still an open question and requires further investigation. In our current preliminary understanding, over timescales like weekly, or monthly, we expect that it is more to do with rainfall intensity and hydraulic conductivity of soil.

We add a sentence in Line 4 Page 11021 that "We speculate that the λ may be related to rainfall intensity or hydraulic conductivity of soil".

12. *Line 16 on page 11019: "separating the local precipitation" to "mean annual ET"?*

Yes. "separating the local precipitation" has been replaced by "separating the ET from the local precipitation".

13. *It will be helpful to explain the meaning of lambda.*

See our response to Comment #11.

14. *Lines 17-18 on page 11021: Actually, the abcd model was original developed andapplied for monthly water balance instead of annual.*

After examining the literature, we agree. We have modified it in line 17 page 11021.

15. "*Thank authors for your responses. I have a quick comment for Figure A1. Since S0 and S1, and b are from abcd model, Figure A1 is plotting the equation (9) in the manuscript (W=P+S0; Y=ET+S1). For a given parameter value of a, equation (9) follows a Budyko curve. Authors may explain the reason that data points in Figure A1 do not following Budyko-type curve (i.e., equation (9)). E.g., Why is (ET+S1)/(P+S0)>1? (Since Q>=0, (ET+S1)/(P+S0)<=1).*

The reason for that $(ET+S_1)/(P+S_0)>1$ is that $(ET+S_1)/(P+S_0)$ in Figure A1 did not consider the inflow water from the upper sub-basin. The available water (W=P+S0) in abcd model is the sum of the local precipitation and soil moisture storage at the beginning of a period, that did not include the inflow water from the upper sub-basin. With the inflow water from the upper sub-basin considered in the available water, the $(ET+S_1)/(P+S_0)$ should be less than one. We plotted the relationships of $(ET+S_1)/(P+Q_{in}+S_0)$ VS $b/(P+Q_{in}+S_0)$ are plotted in Figure B1. That is in fact one of our contributions to applying Budyko-type curves in unclosed basins, like in this study area.

Hope our explanation above can make the necessary clarification.

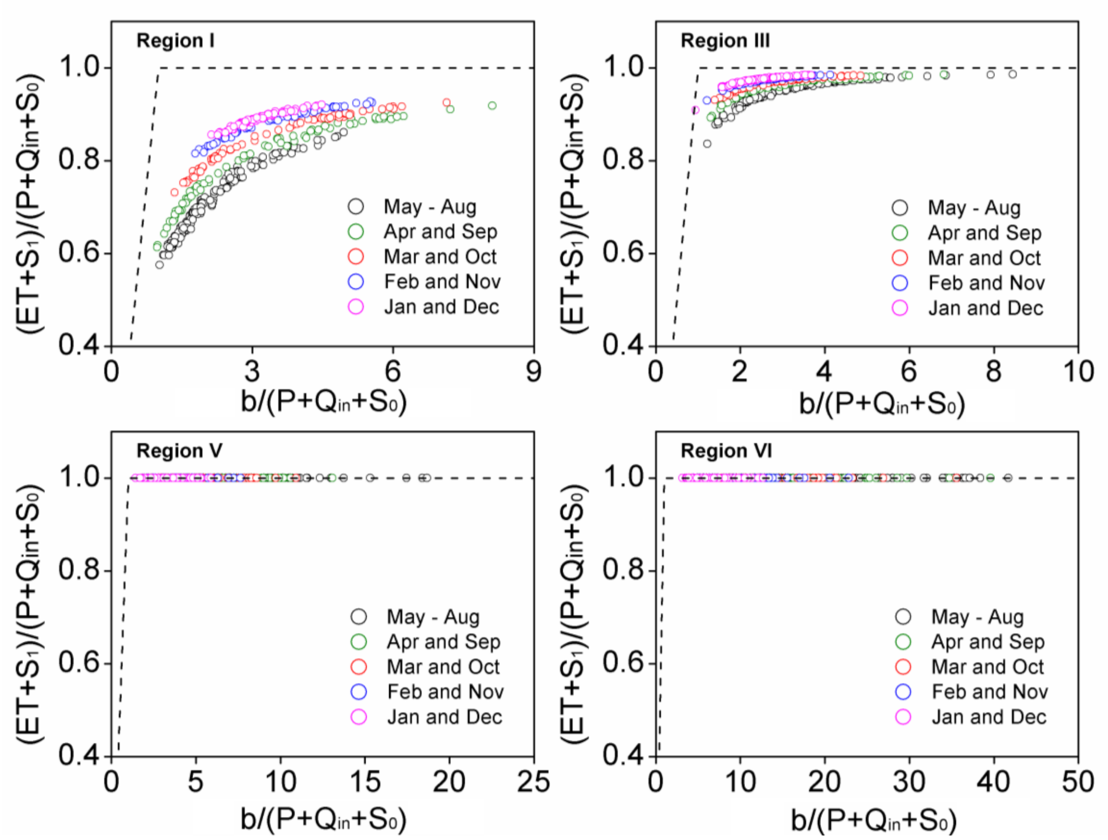


Figure B1. The relationship of $(ET+S_1)/(P+Q_{in}+S_0)$ VS $b/(P+ Q_{in}+S_0)$.

Reference:

Fu, B. P.: On the calculation of the evaporation from land surface, Scientia Atmospherica Sinica, 5, 23–31, 1981 (in Chinese).

Thomas, H. A.: Improved methods for national water assessment, water resources contract: WR15249270, Washington, D.C., Water Rsources Council, 59, 1981.

Wang, D., and Tang, Y.: A one-parameter Budyko model for water balance captures emergent behavior in darwinian hydrologic models, Geophys Res Lett, 41, 4569-4577, doi: 10.1002/2014gl060509, 2014.

Thank you a lot.

Response to Reviewer #2

1. *The authors extended the Budyko hypothesis (BH) to an arid basin, by taking inflow and storage change into account. Considering inflow from upper basins as part of the total water supply to evaporation in an unclosed basin is quite novel. The authors found that accounting for the inflow in BH is beneficial to improve BH performance both in annual and monthly time scale. This paper is well written, clearly structured. And it matches well with the scope of HESS, will be of interest for HESS readers. I recommend minor revision. But some parts need further clarification before it can be published.*

Thank you.

2. *Clarify how the authors calculate the Q_{in}. Accounting Q_{in} in BH is the most novel part of this paper, to my point of view. Maybe I missed it, but I did not get the clear definition of Q_{in} and how the authors do the calculation, especially for the basins in downstream. I guess the authors estimated Q_{in} by channel inflow and outflow, which implicitly takes all types of water consumption into account, such as irrigation, domestic and industrial water usage, but neglect the capillary rise from groundwater reservoir. I personally suggest the authors confirm this important term is well defined and clearly described.*

Yes, good idea. As guessed by the referee, we estimated $Q_{in.}$ by channel inflow that is from the upper basin and/or inter-basin water transfer. And in the estimate of the available water we neglected the capillary rise from groundwater reservoir.

To make a clarification, we rewrite the part in defining $Q_{in.}$ Page 11018, line 9, we replace "the transferring water with other basins" by "channel inflow that is from the upper basin and/or inter-basin water transfer".

3. *Page 11018 Line 9: The term deltaS is not well defined. It is defined as "soil water storage change" in the article. But what is the depth of the soil? Since soil depth in some places may be deeper than 50 meters, i.e. on Loess Plateau, which is obviously not what the authors intended to say. And I think the authors mean the top soil layers which can be used for ecosystem to absorb water by roots. I recommend using root zone storage (Gao et al., 2014) change as a clearer definition, which is the dynamic part in soil.*

Good idea, we agree and we specifically mean "root zone water storage". We have replaced "soil water" by "root zone water storage" in the relevant part of our manuscript.

4. *Page 11024 Line 13-14: ": : :The runoff data set includes monthly runoff at 4 stations located at the inflow or outflow of the six sub-basins." How 4 gauge stations can observe the inflow and outflow of six sub-basins? Please clarify.*

Sorry, we think it is to do with the way how we wrote the manuscript. Now we clarify them. For the two upper streams, Gauge #S1 controls Region I and Gauge #S2 controls Region II. For the two middle streams, Gauge #S1 and #S3 control Region III, and Gauge #S3 and Gauge #S4 control Region IV. For the two downstream Region V and Region VI without any runoff flowing out, Gauge #S2 and Gauge #S4 control their inflow respectively. (See Figure 3 in the main text). We clarify it in the main text (Line 14 Page 11024)

5. *Section 4.1. The role of abcd model in this study. To my understanding, it provides the time series of monthly "soil water storage change" and "monthly inflow" in downstream basins (due to the influence of hydraulic engineering), which are necessary for the new BH model. Am I right? Please do clarification in the revised manuscript. And how was the abcd model applied to calculate the water balance of Regions III and IV? The authors used evaporation observation to do calibration in Regions III and IV. But basin III and IV is the downstream of basin I and II. How does the abcd model handle the inflow from upstream? And how is the abcd model conducted in Region V? I did find it in Section 4.1.*

The outputs from the abcd model being used include soil water storage and actual evapotranspiration. Only over the two upper sub-basins (Region I and Region II), the streamflow is used for the calibration and validation purpose.

Over Regions III, IV, and also V being concerned by the referee, large areas of artificial oasis (cropland) is distributed and the streamflow was intensely disturbed by hydraulic engineering. Hence it becomes almost impossible to validate the abcd model by directly comparing the simulated and observed streamflow. Instead, we used the actual evapotranspiration by remote sensing to calibrate and validate the abcd model. For the new BH over Regions III, IV, and V, we use the observed Q_{in} from the upper sub-basin as the input. That is to be consistent with the remote sensing data, which are observed and hence human disturbed.

A key point of this paper is that inflows from upper sub-basins can be considered as a new and important source of available water for evapotranspiration.

To clarify it, we modify Lines 21 to 27 in page 11025.

6. *Page 11025 Line 21: ": : : completely controlled by hydrological stations: : :". Do you mean "hydraulic engineering"?*

We accept this comment in the manuscript.

Hydrol. Earth Syst. Sci. Discuss., 12, 11013–11052, 2015
www.hydrol-earth-syst-sci-discuss.net/12/11013/2015/
doi:10.5194/hessd-12-11013-2015

Hydrology and Earth System Sciences
Open Access
Discussions

New interpretation of the role of water balance in an extended Budyko hypothesis in arid regions

C. Du[1,2], F. Sun[1], J. Yu[1], X. Liu[1], and Y. Chen[3]

[1]Key Laboratory of Water Cycle and Related Land Surface Processes, Institute of Geographic Science and Natural Resources Research, Chinese Academy of Sciences, Beijing 100101, China
[2]University of Chinese Academy of Sciences, Beijing, 100049, China
[3]State Key Laboratory of Desert and Oasis Ecology, Xinjiang, Institute of Ecology and Geography, Chinese Academy of Sciences, Urumqi, 830011, China

Received: 31 August 2015 – Accepted: 11 September 2015 – Published: 27 October 2015

Correspondence to: F. Sun (sunfb@igsnrr.ac.cn)

Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

The Budyko hypothesis (BH) is an effective approach to investigating long-term water balance at large basin scale under steady state. The assumption of steady state prevents applications of the BH to basins, which is unclosed, or with significant variations in soil water storage, i.e., under unsteady state, such as in extremely arid regions. In this study, we choose the Heihe River Basin (HRB) in China, an extremely arid inland basin, as the study area. We firstly use a calibrated and then validated monthly water balance model, i.e., the *abcd* model to quantitatively determine annual and monthly variations of water balance for the sub-basins and the whole catchment of the HRB and find that the role of soil water storage change and that of inflow from upper sub-basins in monthly water balance are significant. With the recognition of the inflow water from other regions and the soil water storage change as additional possible water sources to evapotranspiration in unclosed basins, we further define the equivalent precipitation (P_e) to include local precipitation, inflow water and soil water storage change as the water supply in the Budyko framework. With the newly defined water supply, the Budyko curve can successfully describe the relationship between the evapotranspiration ratio and the aridity index at both annual and monthly timescales, whilst it fails when only the local precipitation being considered. Adding to that, we develop a new *Fu*-type Budyko equation with two non-dimensional parameters (ω and λ) based on the deviation of *Fu*'s equation. Over the annual time scale, the new *Fu*-type Budyko equation developed here has more or less identical performance to *Fu*'s original equation for the sub-basins and the whole catchment. However, over the monthly time scale, due to large seasonality of soil water storage and inflow, the new *Fu*-type Budyko equation generally performs better than *Fu*'s original equation. The new *Fu*-type Budyko equation (ω and λ) developed here enables one to apply the BH to interpret regional water balance over extremely dry environments under unsteady state (e.g., unclosed basins or sub-annual timescales).

HESSD
12, 11013–11052, 2015

1 Introduction

The Budyko Hypothesis (hereafter BH) was postulated by a Russian climatologist Mikhail Ivanovich Budyko to analyze regional differences in long-term annual water and energy balance (Budyko, 1948). The BH's mean annual water balance is described by only the evapotranspiration ratio and the climate aridity index. The BH becomes an effective approach to investigating the influence of climate change on mean annual runoff and evapotranspiration (Donohue et al., 2011; Xiong et al., 2014). There are various equations to describe the BH. Some empirical equations without parameters were proposed by Schreiber (1904), Ol'dekop (1911), Budyko (1948) and Pike (1964) (see Table 1). These equations explicitly include climate variations (radiation, precipitation, and evapotranspiration and air temperature) and do not deal with recently recognized important catchment properties, such as characteristics of groundwater system, vadoze zone properties, and vegetation. Hence, attempts have been made to introduce physical parameters in these empirical equations (Mezentsev, 1955; Fu, 1981; Milly, 1993; Zhang et al., 2001; Yang et al., 2007, 2008). These physical parameters are a collection of myriad catchment characteristics (topography, vegetation, soil, and groundwater etc.) and are therefore difficult to measure (Gerrits et al., 2009). These equations with parameters, however, provide the flexibility of using the BH over long-term time-scales.

The BH assumes steady state conditions. Firstly, the studied basin must be natural and closed, which means that the local precipitation is the only water source available to evapotranspiration. Recently, the BH has been widely used to investigate the inter-annual variability of precipitation partitioning (Gerrits et al., 2009), separation of runoff trends (Li et al., 2014; Xiong et al., 2015), evapotranspiration change (Savenije, 1997) and water storage change (Istanbulluoglu et al., 2012). These studies show that hydrological processes have been greatly affected by the climate change and intensive change of land cover owing to human activities. These human activities such as urbanization, withdrawing groundwater, hydraulic engineering, deforestation etc.

are significantly changing natural hydrological cycle and breaking the original water balance to form a new balance under the new hydroclimatic conditions. For example, the transferring water becomes the new water source of the basin to evapotranspiration due to the implemented inter-basin water transfer project (Bonacci and Andric, 2010). In dry regions, croplands expand with irrigation, which increased water availability for evapotranspiration (Gordon et al., 2005). Land use/cover changes have also caused the change of runoff (Li et al., 2014). Nowadays, most of the inhabited basins have been developed or disturbed by so large-scale human actives. Therefore, lots of basins were no longer closed or natural and the relationship between annual evapotranspiration ratio and potential evapotranspiration ratio hardly meet the first condition of the BH, which presents great challenge in applying the BH in unclosed basins.

Secondly, water storage change can be assumed to be negligible at the basin scale and at long-term time scale. However, over finer temporal scales, it becomes increasingly concerned of the importance of water storage in water balance in the Budyko framework. For example, Wang et al. (2009) found that the inter-annual water storage change should be considered due to the hysteresis response of the base flow to the inter-annual precipitation change in Nebraka Sand Hills. Zhang et al. (2008) considered the impacts of soil water and groundwater storage and developed a monthly water balance model based on the BH with application in 265 catchments in Australia. Yokoo et al. (2008) highlighted the importance of soil water storage change in determining both annual and seasonal water balances. Wang (2012) evaluated changes in inter-annual water storage at 12 watersheds in Illinois using the field observation of long-term groundwater and soil water found that the impact of inter-annual water storage changes on the water supply in the BH need to be considered. Chen et al. (2013) defined the difference between rainfall and storage change as effective precipitation to develop a seasonal model for construction long-term evapotranspiration. Therefore, water storage change should be taken into account as the important part of the steady state assumption of the BH (Zhang et al., 2008).

HESSD
12, 11013–11052, 2015

In summary, it has been more and more recognized that water systems are no longer natural to different extents (Sivapalan et al., 2011). Hence, it presents a grant challenge to apply the BH to unsteady state conditions (unclosed basins or intense water storage changes). The BH has been widely applied to mild arid basins with precipitation of 300–400 mm and aridity index of less than, for example, five, such as over northern China (Yang et al., 2007), the southwest regions of MOPEX catchments (Gentine et al., 2012; Carmona et al., 2014) and the west of Australia (Zhang et al., 2008). However, it is rare of applying the BH in extremely arid environments (say, the aridity index over five), where water systems are typically unclosed with intense human interference and irrigation. For example, rivers in the arid region of Northwestern China are typically from upper mountains with little human interference, and flowing through middle regions with intensive irrigation and human interferences and finally into extremely dry desert plains. To investigate it in more detail, we choose the Heihe River Basin (HRB), the second largest arid inland basin in northwestern China (mean annual aridity index = 10). Being an inland basins, the HRB consists of six sub-basins with different landscapes and climate conditions, where the upper mountainous basins are closed and natural with little human interference (long term mean annual water storage change approaches zero), the middle basins are arid and intensively irrigated plain with strong human interference (mean annual evapotranspiration is higher than the local precipitation), and the lower basin is extremely dry Gobi desert plain without any runoff flowing out (evapotranspiration is mainly the local precipitation, mean annual evapotranspiration approaches to mean annual precipitation). We aim at (1) testing whether the BH is applicable for the unsteady state condition in extremely arid basins, (2) improving the original BH by including observed water balance and (3) extending its applicability at unclosed basin scale and annual or monthly time scales.

HESSD
12, 11013–11052, 2015

2 Theory and method

2.1 Annual and monthly water balance analysis

In the original BH, the basin is a natural hydrologic unit, and the only possible water source to evapotranspiration is the local precipitation. Annual or monthly water balance equation can be written as.

$$P = \mathrm{ET} + Q_{\mathrm{out}} - Q_{\mathrm{in}} + \Delta S + \Delta G \quad (1)$$

where, P is the local annual or monthly precipitation (mm); ET is the sum of soil evaporation and vegetation transpiration (mm); Q_{out} is the outflow away from a basin (mm); Q_{in} is the transferring water with other basins (mm); ΔS is the soil water storage change (mm); ΔG is the groundwater storage change (mm).

Because of human interference (land cover change, dams, irrigation and other withdrawals) to the hydrologic system worldwide, the water supply to evapotranspiration in a basin has changed. Local groundwater and soil water and external water transfer also become new possible water sources. However, that new non-ignorable part of available water for evapotranspiration has yet been explicitly considered in the Budyko framework in an unclosed basin. More specifically, the inflow or/and inter-basin water transfer may affect the available water for evapotranspiration largely. By considering that, here we rearrange Eq. (1) as $P + Q_{\mathrm{in}} - \Delta S = \mathrm{ET} + Q_{\mathrm{out}} + \Delta G$ the available water for evapotranspiration in Eq. (1) as

$$P_{\mathrm{e}} = P + Q_{\mathrm{in}} - \Delta S \quad (2)$$

Where, the total water supply to evapotranspiration in an unclosed basin is denoted as P_{e} and for simplicity, P_{e} hereafter is defined as the equivalent precipitation of the BH at finer time scales. If ΔS is more than zero, it means the surplus water is stored in the vadose zone, which should be deducted from the water sources. If ΔS is less than zero, it means soil water contributes to the evapotranspiration consumption. Note

HESSD
12, 11013–11052, 2015

that the change of groundwater storage (ΔG) is the result of the exchange between groundwater and baseflow and is not directly interacted with evapotranspiration, so that ΔG is not included into the defined P_e in Eq. (2). It will be discussed in the results section.

2.2 Budyko hypothesis model at annual and monthly scale

As discussed above, in the original Budyko framework, the water supply to land evapotranspiration is mean annual precipitation, and the energy supply to land evapotranspiration is estimated by mean annual potential evapotranspiration. The general Budyko equation can be written as:

$$\frac{\mathrm{ET}}{P} = F\left(\frac{\mathrm{ET}_0}{P}\right) \tag{3}$$

where, $\frac{\mathrm{ET}}{P}$ is the evapotranspiration ratio; $\frac{\mathrm{ET}_0}{P}$ is the aridity index. $F()$ is the function to be determined. The general analytical solution to Eq. (3) over mean annual timescales is derived by Fu (1981) and is written as follows:

$$\mathrm{ET} = \mathrm{ET}_0 + P - \left(\mathrm{ET}_0^{\varpi} + P^{\varpi} + C\right)^{1/\varpi} \tag{4}$$

where, ϖ is the parameter, which reflects the integrated effects of soil, vegetation and topography on separating the local precipitation. If the local precipitation is zero, evapotranspiration approaches to zero due to no available water, C is zero constant. Note that another form of the BH is also given by Mezentsev (1955) (later, Choudhury, 1999 and Yang et al., 2008), which is, in fact, identical to *Fu*'s equation (Zhou et al., 2015) with the parameters linearly related ($R^2 = 0.9997$) (Sun, 2007).

Water balance analysis in Sect. 2.1 concludes that the water supply in the BH under the unsteady state condition is the equivalent precipitation instead of the local precipitation. So the annual (or monthly) evapotranspiration ratio is redefined as the ratio of annual (or monthly) evapotranspiration and equivalent precipitation,

HESSD
12, 11013–11052, 2015

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
◀| |▶
◀ ▶
Back | Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

and the annual (or monthly) aridity index is redefined as the ratio of annual (or monthly) potential evapotranspiration and equivalent precipitation. They are described as follows:

$$\frac{\mathrm{ET}}{P_\mathrm{e}} = \frac{\mathrm{ET}}{P + Q_\mathrm{in} - \Delta S} \tag{5}$$

$$\frac{\mathrm{ET}_0}{P_\mathrm{e}} = \frac{\mathrm{ET}_0}{P + Q_\mathrm{in} - \Delta S} \tag{6}$$

If the equivalent precipitation can be evaporated by enough available energy ($\mathrm{ET}_0/P_\mathrm{e} \to \infty$), then annual (or monthly) evapotranspiration may approach annual (or monthly) precipitation ($\mathrm{ET}/P_\mathrm{e} \to 1$). Such condition is moisture – constrained. While, if the available energy to evaporate the annual (or monthly) precipitation is limited ($\mathrm{ET}_0/P_\mathrm{e} \to 0$), the annual (or monthly) evapotranspiration may approach annual (or monthly) potential evapotranspiration ($\mathrm{ET}/\mathrm{ET}_0 \to 1$). Such condition is energy-constrained. Figure 1 describes partitioning of the equivalent precipitation into evapotranspiration and streamflow, which follows the BH. The Budyko equation under unsteady state assumption can be written as,

$$\frac{\mathrm{ET}}{P_\mathrm{e}} = F\left(\frac{\mathrm{ET}_0}{P_\mathrm{e}}\right) \tag{7}$$

Under the unsteady state conditions for a region, when the local precipitation in the origin *Fu*'s equation is zero, evapotranspiration may not be zero due to other water sources (e.g. inter-basin water transfer), so following the derivation of *Fu*, 1981. Equation (4) can be rewritten as,

$$\frac{\mathrm{ET}}{P_\mathrm{e}} = 1 + \frac{\mathrm{ET}_0}{P_\mathrm{e}} - \left[1 + \left(\frac{\mathrm{ET}_0}{P_\mathrm{e}}\right)^{\omega} + \lambda\right]^{1/\omega} \tag{8}$$

where, ω and λ are two fitting parameters and both non-dimensional. ω has been widely discussed and is greater than 1 (Fu, 1981; Yang et al., 2007). By meeting the

HESSD
12, 11013–11052, 2015

Title Page
Abstract Introduction
Conclusions References
Tables Figures
◀◀ ▶▶
◀ ▶
Back Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

constraints formed by the BH, we can derive that $\lambda \geq -1$ (see Appendix A). When $\lambda = 0$ (Fig. 2a), Eq. (8) is the same as the *Fu*'s Equation in its original form (Fu, 1981; Zhang et al., 2004; Yang et al., 2007). For λ becomes positive, e.g., 1, the lower end of the Budyko curve adjusts to the right (Fig. 2b, c). And $\lambda = -1$ ~~(Fig. 2c, d)~~ sets up the upper theoretical constraint of the Budyko curve (Fig. 2c, d).

2.3 A monthly water balance model: *abcd* model

Regional evapotranspiration and soil water cannot be measured directly and they are usually provided by ~~using~~ monthly water balance models. Monthly water balance models were first developed in the 1940s. From that, many models have been developed in hydrological studies, such as *T* model, *T α* model, *P* model, *abc* model and *abcd* model are often popular due to relatively simple structure and fewer parameters (Fernandez et al., 2000).

Among these monthly models, the *abcd* model was proposed by Thomas (1981) has been widely applied to assess regional water resources due to its explicit model structure and only four parameters, of which two parameters pertain to runoff characteristics and the other two relate to groundwater sound physical meanings. The model was originally applied at the annual time scale and later extended to the monthly time scale (Alley, 1984). Moreover, Savenije (1997) has verified that the *abcd* model to derive expressions for the evapotranspiration ratio has better agreement with observations than Budyko-type curves. Inputs to the *abcd* model are monthly precipitation and potential evapotranspiration. Outputs include monthly runoff (direct and indirect), soil water, groundwater storage and actual evapotranspiration. Therefore, this study employs the *abcd* model to provide monthly actual evapotranspiration and soil water storage.

The partitioning of monthly precipitation P_t in the model is as follows; runoff Q_t (direct and indirect), evapotranspiration ET_t, soil water storage S_t, and groundwater storage G_t. The partitioning is controlled by the magnitude of precipitation P_t, potential evapotranspiration ET_{0t}, and the initial storages in soil S_{t-1} and groundwater G_{t-1}. The

HESSD
12, 11013–11052, 2015
New interpretation of water balance in BH in arid regions
C. Du et al.

following equation controls the partitioning:

$$Y_t(W_t) = \frac{W_t + b}{2a} - \sqrt{\left(\frac{W_t + b}{2a}\right)^2 - \frac{W_t b}{a}} \tag{9}$$

where Y_t is the sum of monthly evapotranspiration and soil water storage at the end of the month, namely evapotranspiration opportunity. W_t is the sum of monthly precipitation and initial soil moisture, named as available water. The parameter a (0 ~ 1) means the propensity in a catchment for runoff to occur before the soil becomes saturated. The parameter b is the maximum value of Y_t. Available water partitioning between ET_t and S_t is controlled by the assumption that the loss rate of actual evaporation from soil water storage is proportional to the evapotranspiration capacity. So the soil water storage at the end of period t is written as:

$$S_t = Y_t \exp(-\text{ET}_{0t}/b) \tag{10}$$

The actual evapotranspiration at the period t is the difference between evapotranspiration opportunity and soil water storage ($Y_t - S_t$). The streamflow, including direct runoff and groundwater recharge, is determined by the difference between available water and evapotranspiration opportunity ($W_t - Y_t$). The parameter c separates the direct runoff $(1-c)(W_t - Y_t)$ and groundwater recharge $c(W_t - Y_t)$. Groundwater discharge dG_t as the base flow is determined by the parameter d and groundwater storage at the end of period t. The streamflow is sum of direct runoff and the base flow. For a given set of, b, c and d and initial soil water storage and groundwater storage, the allocation of monthly precipitation can be computed one by one.

HESSD
12, 11013–11052, 2015

3 Study area and data

3.1 Study area

The HRB, originating from Qilian Mountains, is the second largest inland river basin in the arid area of the northwestern China (Fig. 3). The drainage map and the basin border are extracted using a 90 m resolution digital elevation model (DEM) data from the Shuttle Radar Topography Mission (SRTM) website of the NASA (http://srtm.csi.cgiar.org/SELECTION/inputCoord.asp) (basin length: 820 km; total area: 143 044 km^2; elevation: 870–5545 m). The HRB is in the middle of Eurasia and away from oceans, characterized with dry and windy climate, and very limited precipitation (mean annual precipitation:126 mm yr^{-1}) but plentiful radiation (mean annual solar radiation: 1780 MJ m^{-2} yr^{-1}, $\sim$ 660 mm yr^{-1} in the unit of evaporation).

The HRB is divided into six sub-basins according to basin characteristics, distributing along eastern and western tributaries, shown in Fig. 3. Regions I and II are upper mountainous regions with the elevation of 3000–5500 m and belong to the cold and semiarid mountainous zone dominated by shrubs and trees with mean annual temperature of less than 2 °C and annual precipitation of 200 $\sim$ 400 mm. And these two sub-basins are the water source area to the middle and lower reaches and have little interference of human activities. Regions III and V with annual precipitation of 100–250 mm are the main irrigation zone and residential area with more than 90 % of total population of the HRB. The two sub-basins are the main water-consuming regions and largely disturbed by human activities. Regions IV and VI located at lower reaches are extremely arid and the mean annual precipitation is less than 100 mm.

3.2 Data

The required data for Eq. (8) and the *abcd* model include monthly precipitation, potential evapotranspiration and runoff from those sub-basins in the HRB.

HESSD
12, 11013–11052, 2015

The daily precipitation data of all stations during 1978–2012 are obtained from the year book hydrology of China including 28 rainfall stations and the China Administration of Meteorology including 19 meteorological stations (Fig. 3). The monthly precipitation of each station is calculated by summing daily precipitation. The gridded data set with 1 km resolution across the whole basin is obtained by interpolation of the site data. The monthly precipitation of the six sub-basins is obtained by the extraction from the monthly precipitation in the whole basin. Daily meteorological data of 19 stations during 1978 and 2012 are also available. Daily potential evapotranspiration is estimated in each station using the FAO Penman–Monteith equation recommended by Allen et al. (1998). The monthly ET_0 at each station is the sum of the daily ET_0 and then interpolated to the whole basin. Finally, annual runoff, precipitation and potential evapotranspiration are obtained by summing monthly data.

The runoff data set includes monthly runoff at 4 stations located at the inflow or outflow of the six sub-basins. The red points in Fig. 3 locate the hydrological stations. Monthly runoff data are obtained from the year book of hydrology of China and are intended for calibrate the *abcd* model. The annual runoff is obtained by summing monthly runoff. The data time series for Regions I and III are from 1978 to 2012. The same period is for Regions II and V but with the period of 1998–2006 missing. The length of data time series for Regions IV and VI is from 1988 to 2012.

The natural runoff in Regions III and IV were strongly disturbed by human activities and there is no runoff for the Regions V and VI and the whole basin. To validate the outputs of the *abcd* model for those regions, this study employs the evapotranspiration of remote sensing products from Heihe Plan Science Data Center (Wu et al., 2012) as a reference. The same data have been widely used as a reference for modeling evaluations and is supported by a State Key Research Program-Heihe Eco-hydrological Research Project of National Natural Science Foundation of China (Yan et al., 2014; Yao et al., 2014). The monthly evapotranspiration datasets (2000–2012) with 1 km spatial resolution over the HRB (http://westdc.westgis.ac.cn), are estimated by ETWatch model based on multi-source remote sensing data (Wu et al., 2012).

HESSD
12, 11013–11052, 2015

4 Results

4.1 Calibration of the *abcd* model

In extremely dry basins like the HRB, the lack of observed hydro-climatic data presents great challenge. A monthly water balance model becomes an effective tool to estimate actual evapotranspiration, change of soil water storage and change of groundwater storage. This study employs the *abcd* water balance model due to its simple and sound physical structure tested and recommended by Alley (1984) and Fernandez et al. (2000). We calibrate and validate the *abcd* model using monthly time series of precipitation, potential evapotranspiration and runoff at each of the seven regions (the six sub-basins and the whole basin) using the generalized pattern search optimization method. Nash–Sutcliffe efficiency (NSE) is used to assess the goodness of fit of the monthly water balance for the seven regions.

Figure 4 shows the results of the modeled streamflow at monthly time scale in Regions I and II. Regions I and II are the water source area of the whole basin with little interference of human activities and both keep relatively natural steady state. The NSE for the Regions I and II is for 0.92 and 0.83, respectively. The results illustrate that the simulated monthly streamflow agrees well with the observation and other modeled components can be reasonable estimates, for instance, monthly actual evapotranspiration, soil water storage change and groundwater storage change in the two sub-basins.

~~The streamflow in Regions III and IV was completely controlled by hydrological stations due to water resources allocation, so the observed monthly streamflow cannot be directly used to validate the simulations of the *abcd* model. To validate the results of actual evapotranspiration, we compare the simulated ET by the *abcd* model and remote sensing ET (Fig. 5) calculated by remote sensing data during 2000–2012. The NSE values for Regions III–VI and the whole basin are not less than 0.88, which illustrate that simulated ET have good agreement with ET by remote sensing.~~

HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

4.2 Annual and monthly water balance analysis

To test the "steady state" assumption of the Budyko framework, it is vital to examine whether changes in mean annual soil water storage in water balance approach to zero. By using the monthly runoff, evapotranspiration, soil water and groundwater storage change from the *abcd* model and the observed monthly precipitation, the mean annual water balance of all regions are summarized in the Table 2. Region I and Region II are located in mountainous area, where the mean soil water storage changes are almost zero with both 0.0 % of the corresponding precipitation. The mean annual soil water storage change in Regions III and IV are relatively significant. For Region V, Region VI and the whole basin without any outflow, the mean annual soil water storage and groundwater storage changes both approach zero. In conclusion, the mean annual soil water storage changes for all regions are very small and can be ignored in mean annual water balance ~~analysis~~. These sub-basins and the whole basin keep natural basin characteristics and meet the second assumption of the BH that mean annual soil water storage can be ignored. However, no inflow only exists in Regions I and II, which meets the first assumption of the BH that the local precipitation is the only potential water source to evapotranspiration. In other regions, water supply conditions have been changed by considerable inflow generally from upper sub-basins.

Because this study focuses on the application of the BH at the annual and monthly time scales, the annual and monthly water balance analysis is very critical to understanding the role of water storage and water source change in the BH. Figure 6 describes the variation of annual water balance for the six sub-basins and the whole basin. The most obvious in Fig. 6 is that the proportion of soil water storage change in annual water balance is small compared with the annual precipitation. So the impact of soil water storage change on annual water balance is ~~very~~ insignificant and can be also neglected. Moreover, annual evapotranspiration is higher than annual precipitation in the Regions III–VI and approaches to annual precipitation over the whole basin. For water-limited regions, when inflow from other regions is available, the

HESSD
12, 11013–11052, 2015

actual evapotranspiration increases with the increased water supply so that the actual evapotranspiration is more than the local precipitation. For the whole basin of the inland HRB, there is no water transferring with other basins, so the evapotranspiration almost approaches to the precipitation at the annual time scale due to little variations in the soil water storage changes. In conclusion, the facts that soil water storage change in all basins can be ignored in annual water balance meet the second assumption of the BH, and the results that the annual water balance in Regions III–VI and the whole basin have been disturbed do not meet the first assumption of the BH. Therefore, except for the Regions I and II, the original BH cannot be directly used for those sub-basins and the whole basin.

Different from the annual timescale, the impacts of monthly changes of soil water storage and groundwater storage behave differently (Fig. 7). The variations of monthly groundwater storage change for all regions are similar to those of runoff (Fig. 7). For those regions with no runoff (Regions V, VI and the whole basin), the modeled groundwater storage change is almost zero. This means that the groundwater storage can hardly contribute to the evapotranspiration while the variation of soil water storage is tightly coupled with the evapotranspiration (Fig. 7). For Regions I and II and during the winter season, the evapotranspiration is more than the precipitation; the extra water source required by the evapotranspiration is from soil water storage. After the summer season, the precipitation sharply decreases, but the evapotranspiration slowly decreases by consuming the soil water storage recharged during the summer season. For Regions III–VI, the water supply is more complicated by the interference of monthly inflow, and the monthly variations of soil water storage. As shown in Fig. 7, it can be concluded that both the soil water storage change and inflow have obvious effect on the monthly water balance, whilst the impact of monthly groundwater storage change is negligible.

In summary, due to the complications of the water transfer and soil water storage change, the two assumption conditions for applying the original BH are difficult to meet

HESSD
12, 11013–11052, 2015

for the sub-basins and the whole HRB on the monthly timescales, which in turn requires new treatments in the BH as further investigated in following sections.

4.3 The annual Budyko curve analysis

Figure 8 (left panel) plots the original Budyko curves for the six sub-basins and the whole basin. For Regions I and II, the points of annual evapotranspiration ratio and aridity index fall in the domain of water and energy limit boundary and they can be well fitted by *Fu*'s equation. The relationship between water and energy in Regions I and II can be described by the original BH as expected in the section above. However, the points of evapotranspiration ratio and aridity index for other regions exceed the water limit boundary. And the results show the relationship of water and energy in Regions III–VI and the whole basin is inconsistent with the original BH. After using the equivalent precipitation instead of the local precipitation, the new *Fu*-type Budyko curves (Eq. (8) with $\lambda = 0$) for all regions are shown in Fig. 8 (middle panel). Compared with the original Budyko curve, the new curves for Regions I and II did not behave differently, because the two basins are natural and closed. The obvious change between the improved and original Budyko curves are for the Regions III and IV. For the whole basin and Regions V and VI, the new curves fall on the upper limit of $\mathrm{ET}/P_\mathrm{e} = 1$ due to no runoff flowing out. These improved Budyko curves can be fitted using *Fu*'s equation and the parameters are listed in Table 3. Interestingly for the annual time scale, the fitted performances of *Fu*'s equation and Eq. (8) are almost identical. Therefore, the new *Fu*-type Budyko curves (Eq. 8) with fitted values of λ (right panel, Fig. 8) do not show much difference from those curves with λ set zero.

In summary, if a basin (sub-basin) is closed, the original BH can be applicable at the annual time scale. However under unsteady state, the new *Fu*-type BH, instead of the original BH is more applicable to describe the annual water balance.

HESSD
12, 11013–11052, 2015

4.4 The monthly Budyko curves analysis

Again as expected based on the monthly water balance analysis, the points of monthly evapotranspiration ratio and aridity index exceed the water limit boundary for all the basins (Fig. 9, left panel). The value of evapotranspiration ratio can be up to 40, which means that the local precipitation in original water balance is well below the actual water supply to the evapotranspiration. The new *Fu*-type Budyko curves at the monthly timescale are shown in Fig. 9 on the middle panel (Eq. 8 with setting $\lambda = 0$) and on the right panel (Eq. 8 with calibrated λ). It is remarkable that the points of monthly evapotranspiration ratio and aridity index distribute regularly in the Budyko framework (in Fig. 9, middle panel and right panel). The improved Budyko curves with calibrated λ perform better than *Fu*'s original equation (i.e., $\lambda = 0$) by 5–10 % in terms of NSF. The fitting parameter λ introduced in this study (Eq. 8) can add further improvement to the BH, in despite of obviously deserving further investigations.

The fitted values of the parameters in the Budyko curves for Regions I to VI are listed in the Table 4. These curves and the parameters have significantly seasonal characteristics. For example, the Budyko curves in Regions I and II can be divided to five groups (Fig. 9). The values of the integrated parameter ϖ in Eq. (8) gradually decrease from the summer months to winter months. The absolute values of parameters λ gradually increase, which illustrates that the points in summer months are more centralized than those in winter months. Moreover, in Regions V and VI and the whole basin, all the equivalent precipitation is consumed by evapotranspiration, and therefore the ratio of evapotranspiration to the equivalent precipitation is almost one.

4.5 Storage change and inflow water impact on the BH

In this study, we intended to extend the BH to the annual and sub-annual time scales by explicitly considering the soil water storage and new water source from other regions. To further investigate it, we choose Region I and Region III as typical cases in Fig. 10. In Region I, as there is no inflow into the region, we can separate the impact of soil

HESSD
12, 11013–11052, 2015

water storage and groundwater storage on the BH (Fig. 10a). With subtle difference, the impacts of changes in soil water storage and groundwater storage on water balance can be almost ignored at annual scales. Region III is another extreme case where only if the role of the inflow water being considered, the BH can perform well under unsteady state (Fig. 10b).

In Fig. 11, we further adopted the approach presented by Chen et al. (2013) to examine the impacts of soil water storage, groundwater storage and inflow water on monthly water balance. We test different combinations in monthly water balance in Region III, a midstream sub-basin of the HRB (Fig. 11a–c) and found that when the equivalent precipitation includes the soil water storage change the BH performs well at the monthly scale. However, the inclusion of the groundwater storage change into the equivalent precipitation does not improve as much (Fig. 11b, c). By examining the impact of monthly inflow water on the BH in Region III (Fig. 11d, f), we find that inflow water at the monthly scale has as much impact as that at annual scale. The results presented above highlight the fact that the water supply cannot be the local precipitation only, but should have included soil water storage change and inflow water.

5 Conclusions

The Budyko Hypothesis (BH) is a useful approach to depicting and understanding the long term mean water balance at large basin scale under steady state condition. However, river systems worldwide have in fact been disturbed by human to different extents. That is important for extremely arid environments (say, the aridity index over five) especially in China, where water systems are typically unclosed with intense human inference and irrigation. That presents grand challenge if one is applying the BH to those regions under unsteady state e.g., unclosed or significant variation in soil water storage, or those time scales finer than a year.

To investigate it, we choose an extremely arid inland basin, the Heihe River Basin in China as the study area, which is divided into six sub-basin based on catchment hydrologic characteristics. We first calibrate and validate a widely used monthly water balance model, i.e., the *abcd* model. For the two upper sub-basins, the simulated monthly water balance is compared against monthly streamflow from hydrological gauges, and for the other sub-basins and the whole catchment, the simulated evapotranspiration is compared with widely used remote sensing ET products in the HRB. The *abcd* model can successfully simulate the monthly water balance and capture the inter-annual variations (NSE over 0.85). Based on that, we found that the role of soil water storage change in monthly water balance is significant but almost negligible over timescales longer than a year. And the impact of inflow from upper sub-basins is also significant and does not rely on the timescale. We concluded that the upstream basin in the HRB are almost closed basins, which meet the two steady state conditions of the BH and other sub-basins become an unclosed basin due to impact of the inflow water and human interference.

With the recognition that the inflow water from other regions and the water storage change are both new possible water sources to evapotranspiration in unclosed basins, we define the equivalent precipitation (P_e) including the local precipitation, inflow water and water storage change as the water supply, instead of just the local precipitation, in the Budyko framework. (The evapotranspiration ratio and the aridity index are also redefined using the equivalent precipitation.) In addition to the new definition of the water supply, we develop a new *Fu*-type Budyko equation with two non-dimensional parameters (ω and λ) based on the deviation by *Professor Baopu Fu*, i.e., *Fu*'s equation to consider the effect of the change of soil water storage and the inflow water on the water and energy constraints. Over the annual time scale, the new *Fu*-type Budyko equation developed here has more or less identical performance to the *Fu*'s equation for the sub-basins and the whole catchment. However, for the monthly time scale, the new *Fu*-type Budyko equation performs better than *Fu*'s original equation when the ratio of evapotranspiration to equivalent precipitation less than one, and performs the same when the evapotranspiration ratio is very close to one. The new *Fu*-type Budyko equation (ω and λ) developed in this study enables one to apply the BH to interpret

HESSD

12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page
Abstract Introduction
Conclusions References
Tables Figures
◀◀ ▶▶
◀ ▶
Back Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

regional water balance over extremely dry environments under unsteady state (e.g., unclosed basins or sub-annual timescales).

Appendix A

For an unclosed basin or region, the water supply to evapotranspiration is defined as equivalent precipitation ($P_e = P + Q_{in} - \Delta S$). Evapotranspiration ratio: $\varepsilon = \mathrm{ET}/P_e$ and aridity index: $\phi = ET_0/P_e$. The Budyko equation is written the same as Eq. (8)

$$\varepsilon = 1 + \phi - (1 + \phi^{\omega} + \lambda)^{1/\omega} \quad \text{(A1)}$$

According to the constrained boundary of the BH, Eq. (1) evapotranspiration ratio is less than or equal to aridity index, namely $\varepsilon \leq \phi$, and Eq. (1) the evapotranspiration ratio is no more than 1, i.e., $\varepsilon \leq 1$,

With $\varepsilon \leq \phi$, we can have,

$$1 + \phi - (1 + \phi^{\omega} + \lambda)^{1/\omega} \leq \phi \quad \text{(A2)}$$

Therefore,

$$\phi^{\omega} + \lambda \geq 0 \quad \text{(A3)}$$

where $\phi \geq 0$ and $\omega > 1$.

For the other constraint, $\varepsilon \leq 1$ we can derive,

$$1 + \lambda \geq 0\,. \quad \text{(A4)}$$

Acknowledgements. This research was supported by the National Natural Science Foundation of China (41271049), Chinese Academy of Sciences (CAS) Pioneer Hundred Talents Program, and State Key Laboratory of Desert and Oasis Ecology, Xinjiang, Institute of Ecology and Geography, Chinese Academy of Sciences. The authors would like to appreciate Hubert H. G. Savenige, Wang Ping for his comments and support in this research.

References

Allen, R. G., Pereira, L. S., Raes, D., and Smith, M.: Crop evapotranspiration – Guidelines for computing crop water requirements – FAO Irrigation and drainage paper 56, Food and Agric. Organ., Rome, 24, 55–56, 1998.

Alley, W. M.: On the treatment of evapotranspiration, soil moisture accounting, and aquifer recharge in monthly water balance models, Water Resour. Res., 20, 1137–1149, doi:10.1029/WR020i008p01137, 1984.

Bonacci, O. and Andric, I.: Impact of an inter-basin water transfer and reservoir operation on a karst open streamflow hydrological regime: an example from the Dinaric karst (Croatia), Hydrol. Process., 24, 3852–3863, doi:10.1002/hyp.7817, 2010.

Budyko, M. I.: Evaporation Under Natural Conditions, Gedrometeoizdat, St. Petersburg, Russia, 29 pp., 1948 (English translation, Israel Program for Scientific Translations, Jerusalem, 1963).

Carmona, A. M., Sivapalan, M., Yaeger, M. A., and Poveda, G.: Regional patterns of interannual variability of catchment water balances across the continental US: a Budyko framework, Water Resour. Res., 50, 9177–9193, doi:10.1002/2014wr016013, 2014.

Chen, X., Alimohammadi, N., and Wang, D.: Modeling interannual variability of seasonal evaporation and storage change based on the extended Budyko framework, Water Resour. Res., 49, 6067–6078, doi:10.1002/wrcr.20493, 2013.

Choudhury, B. J.: Evaluation of an empirical equation for annual evaporation using field observations and results from a biophysical model, J. Hydrol., 216, 99–110, doi:10.1016/s0022-1694(98)00293-5, 1999.

Donohue, R. J., Roderick, M. L., and McVicar, T. R.: Assessing the differences in sensitivities of runoff to changes in climatic conditions across a large basin, J. Hydrol., 406, 234–244, doi:10.1016/j.jhydrol.2011.07.003, 2011.

Fernandez, W., Vogel, R. M., and Sankarasubramanian, A.: Regional calibration of a watershed model, Hydrol. Sci. J., 45, 689–707, doi:10.1080/02626660009492371, 2000.

Fu, B. P.: On the calculation of the evaporation from land surface, Scientia Atmospherica Sinica, 5, 23–31, 1981 (in Chinese) .

Gentine, P., D'Odorico, P., Lintner, B. R., Sivandran, G., and Salvucci, G.: Interdependence of climate, soil, and vegetation as constrained by the Budyko curve, Geophys. Res. Lett., 39, L19404, doi:10.1029/2012gl053492, 2012.

Gerrits, A. M. J., Savenije, H. H. G., Veling, E. J. M., and Pfister, L.: Analytical derivation of the Budyko curve based on rainfall characteristics and a simple evaporation model, Water Resour. Res., 45, W04403, doi:10.1029/2008wr007308, 2009.

Gordon, L. J., Steffen, W., Jonsson, B. F., Folke, C., Falkenmark, M., and Johannessen, A.: Human modification of global water vapor flows from the land surface, P. Natl. Acad. Sci. USA, 102, 7612–7617, doi:10.1073/pnas.0500208102, 2005.

Istanbulluoglu, E., Wang, T., Wright, O. M., and Lenters, J. D.: Interpretation of hydrologic trends from a water balance perspective: the role of groundwater storage in the Budyko hypothesis, Water Resour. Res., 48, W00H16, doi:10.1029/2010wr010100, 2012.

Li, H.-Y., Sivapalan, M., Tian, F., and Harman, C.: Functional approach to exploring climatic and landscape controls of runoff generation: 1. Behavioral constraints on runoff volume, Water Resour. Res., 50, 9300–9322, doi:10.1002/2014WR016307, 2014.

Li, J., Tan, S., Chen, F., and Feng, P.: Quantitatively analyze the impact of land use/land cover change on annual runoff decrease, Nat. Hazards, 74, 1191–1207, doi:10.1007/s11069-014-1237-x, 2014.

Mezentsev, V. S.: More on the calculation of average total evaporation, Meteorologia i Gidrologiya, 5, 24–26, 1955.

Milly, P. C. D.: An analytic solution of the stochatic storage problem applicable to soil water, Water Resour. Res., 29, 3755–3758, doi:10.1029/93wr01934, 1993.

Ol'dekop, E. M.: On evaporation from the surface of river basins, Transactions on Meteorological Observations, 4, 200 pp., 1911.

Pike, J. G.: The estimation of annual run-off from meteorological data in a traopical climate, J. Hydrol., 2, 116–123, 1964.

Porporato, A., Daly, E., and Rodriguez-Iturbe, I.: Soil water balance and ecosystem response to climate change, Am. Nat., 164, 625–632, doi:10.1086/424970, 2004.

Savenije, H. H. G.: Determination of evaporation from a catchment water balance at a monthly time scale, Hydrol. Earth Syst. Sci., 1, 93–100, doi:10.5194/hess-1-93-1997, 1997.

Schreiber, P.: Ueber die Beziehungen zwischen dem Niederschlag und der Wasserfvhrung der flvsse in Mitteleuropa, Z. Meteorol., 21, 442–452, 1904.

Sivapalan, M., Yaeger, M. A., Harman, C. J., Xu, X., and Troch, P. A.: Functional model of water balance variability at the catchment scale: 1. Evidence of hydrologic similarity and space–time symmetry, Water Resour. Res., 47, W02522, doi:10.1029/2010wr009568, 2011.

Sun, F.: Study on Watershed Evapotranspiraiton based on the Budyko Hypothesis, PhD thesis, Tsinghua University, Beijing, 147 pp., 2007 (in Chinese).
Thomas, H. A.: Improved methods for national water assessment, contract: WR15249270, Water Resources Council, Washington D.C., 59, 1981.
Wang, D.: Evaluating interannual water storage changes at watersheds in Illinois based on long-term soil moisture and groundwater level data, Water Resour. Res., 48, W03502, doi:10.1029/2011WR010759, 2012.
Wang, T., Istanbulluoglu, E., Lenters, J., and Scott, D.: On the role of groundwater and soil texture in the regional water balance: an investigation of the Nebraska Sand Hills, USA, Water Resour. Res., 45, W10413, doi:10.1029/2009wr007733, 2009.
Wu, B., Yan, N., Xiong, J., Bastiaanssen, W. G. M., Zhu, W., and Stein, A.: Validation of ETWatch using field measurements at diverse landscapes: a case study in Hai Basin of China, J. Hydrol., 436, 67–80, doi:10.1016/j.jhydrol.2012.02.043, 2012.
Xiong, L. H., Yu, K. X., and Gottschalk, L.: Estimation of the distribution of annual runoff from climatic variables using copulas, Water Resour. Res., 50, 7134–7152, doi:10.1002/2013wr015159, 2014.
Yan, H., Zhan, J., Liu, B., and Yuan, Y.: Model estimation of water use efficiency for soil conservation in the lower Heihe River Basin, northwest China during 2000–2008, Sustainability, 6, 6250–6266, doi:10.3390/su6096250, 2014.
Yang, D., Sun, F., Liu, Z., Cong, Z., Ni, G., and Lei, Z.: Analyzing spatial and temporal variability of annual water-energy balance in nonhumid regions of China using the Budyko hypothesis, Water Resour. Res., 43, W04426, doi:10.1029/2006wr005224, 2007.
Yang, H., Yang, D., Lei, Z., and Sun, F.: New analytical derivation of the mean annual water-energy balance equation, Water Resour. Res., 44, W03410, doi:10.1029/2007wr006135, 2008.
Yao, Y., Liang, S., Xie, X., Cheng, J., Jia, K., Li, Y., and Liu, R.: Estimation of the terrestrial water budget over northern China by merging multiple datasets, J. Hydrol., 519, 50–68, doi:10.1016/j.jhydrol.2014.06.046, 2014.
Yokoo, Y., Sivapalan, M., and Oki, T.: Investigating the roles of climate seasonality and landscape characteristics on mean annual and monthly water balances, J. Hydrol., 357, 255–269, doi:10.1016/j.jhydrol.2008.05.010, 2008.

HESSD
12, 11013–11052, 2015

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
◀| |▶
◀ ▶
Back | Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

Zhang, L., Dawes, W. R., and Walker, G. R.: Response of mean annual evapotranspiration to vegetation changes at catchment scale, Water Resour. Res., 37, 701–708, doi:10.1029/2000wr900325, 2001.

Zhang, L., Hickel, K., Dawes, W. R., Chiew, F. H. S., Western, A. W., and Briggs, P. R.: A rational function approach for estimating mean annual evapotranspiration, Water Resour. Res., 40, W02502, doi:10.1029/2003wr002710, 2004.

Zhang, L., Potter, N., Hickel, K., Zhang, Y., and Shao, Q.: Water balance modeling over variable time scales based on the Budyko framework – model development and testing, J. Hydrol., 360, 117–131, doi:10.1016/j.jhydrol.2008.07.021, 2008.

Zhou, S., Yu, B., Huang, Y., and Wang, G.: The complementary relationship and generation of the Budyko functions, Geophys. Res. Lett., 42, 1781–1790, doi:10.1002/2015gl063511, 2015.

Table 1. Different Budyko equations for mean annual water–energy balance.

Number	Equation	Parameter	Reference
1	$\varepsilon = 1 - \exp(-\varphi)$	none	Schreiber (1904)
2	$\varepsilon = \varphi \tanh(1/\varphi)$	none	Ol'dekop (1911)
3	$\varepsilon = \left\{\varphi[1 - \exp(-\varphi)]\tanh(1/\varphi)\right\}^{0.5}$	none	Budyko (1958, 1974)
4	$\varepsilon = (1 + \varphi^{-2})^{-0.5}$	none	Pike (1964)
5	$\varepsilon = (1 + \varphi^{-\alpha})^{-1/\alpha}$	α – calibration factor	Mezentsev (1955); Chouldhury (1999); Yang et al. (2008)
6	$\varepsilon = \frac{1+\varpi\varphi}{1+\varpi\varphi+\varphi^{-1}}$	ϖ – coefficient of vegetation and water supply	Zhang et al. (2001)
7	$\varepsilon = \frac{\exp[\gamma(1-1/\varphi)]-1}{\exp[\gamma(1-1/\varphi)]-\varphi^{-1}}$	γ – the ratio of the soil water storage capacity to precipitation	Milly (1993); Porporato et al. (2004)
8	$\varepsilon = 1 + \varphi - (1 + \varphi^{\varpi})^{1/\varpi}$	ϖ – a constant of integration	Fu (1981); Zhang et al. (2004); Yang et al. (2007)

Note: $\varepsilon = ET/P$ evapotranspiration ratio (the ratio of mean annual evapotranspiration to mean annual precipitation); $\varphi = ET_0/P$, aridity index (the ratio of mean annual potential evapotranspiration to mean annual precipitation).

HESSD
12, 11013–11052, 2015

Table 2. The mean annual water balance of all regions.

Region	*P* (mm)	Q_{in} (mm)	ET (mm)	Q_{out} (mm)	ΔS (mm)	PWS (%)
I	351.9	–	165.3	169.3	0.0	0.0
II	220.7	–	143.9	85.2	0.1	0.0
III	223.6	66.1	253.2	37.5	−2.1	−0.9
IV	73.5	74.0	103.4	47.5	1.0	1.3
V	117.3	39.6	156.7	–	0.2	0.1
VI	66.8	7.9	74.7	–	0.0	0.0
Whole basin	125.8	–	125.5	–	0.2	0.2

"–" means no runoff.
PWS represents the proportion of the soil water storage change in the total precipitation.

HESSD
12, 11013–11052, 2015

Table 3. The fitting parameters of *Fu*'s equation at annual scales.

Region	I	II	III	IV	V	VI	whole basin
ϖ [a]	1.34	1.45	2.05	1.42	20.28	13.05	17.60
ϖ [b]	1.45	1.69	2.34	1.44	1.07	10.8	1.09
λ [b]	0.25	0.67	0.62	0.08	−1.00	−1.00	−1.00

[a] means the calibrated values of ϖ in *Fu*'s equation; Eq. (8) when $\lambda = 0$.
[b] means the calibrated values of ϖ and λ in Eq. (8).

HESSD
12, 11013–11052, 2015

Table 4. The fitting parameters of the improved Budyko equation at the monthly scales.

Region	Parameter	May–Aug	Apr and Sep	Mar and Oct	Feb and Nov	Jan and Dec
I	ϖ [a]	1.40	1.39	1.35	1.28	1.22
	ϖ [b]	1.50	1.51	1.48	1.40	1.33
	λ [b]	0.16	0.24	0.31	0.32	0.28
II	ϖ [a]	1.54	1.53	1.47	1.37	1.29
	ϖ [b]	1.70	1.72	1.67	1.57	1.48
	λ [b]	0.34	0.54	0.63	0.66	0.60
Region	Parameter	April–Sep	Mar and Oct	Feb and Nov	Jan and Dec	
III	ϖ [a]	2.20	2.05	1.86	1.71	
	ϖ [b]	2.31	2.15	1.97	1.90	
	λ [b]	0.18	0.19	0.22	0.39	
IV	ϖ [a]	1.51	1.42	1.33	1.25	
	ϖ [b]	1.75	1.59	1.51	1.41	
	λ [b]	0.92	0.53	0.56	0.41	
Region	Parameter	May–Aug	Apr and Sep	Mar and Oct	Feb and Nov	Jan and Dec
V	ϖ [a]	35.5	29.8	28.0	22.5	23.7
	ϖ [b]	1.02	1.03	1.03	1.04	1.04
	λ [b]	−1	−1	−1	−1	−1
VI	ϖ [a]	17.3	18.9	15.5	12.7	13.1
	ϖ [b]	1.02	1.02	1.03	1.03	1.04
	λ [b]	−1	−1	−1	−1	−1
The whole basin	ϖ [a]	28.5	22.4	18.8	16.3	15.7
	ϖ [b]	1.02	1.02	1.04	1.04	1.03
	λ [b]	−1	−1	−1	−1	−1

[a] means the calibrated values of ϖ in *Fu*'s equation; Eq. (8) when $\lambda = 0$.
[b] means the calibrated values of ϖ and λ in Eq. (8).

HESSD
12, 11013–11052, 2015

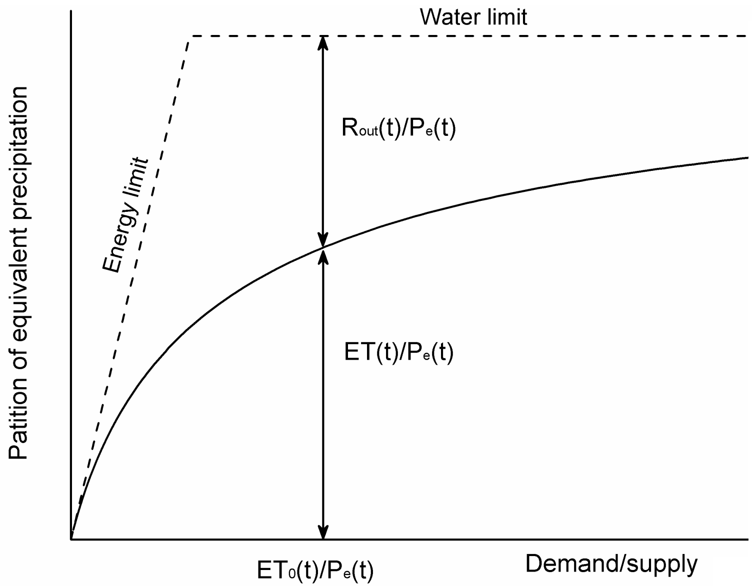


Figure 1. A schematic diagram of the BH under the unsteady state condition.

HESSD
12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

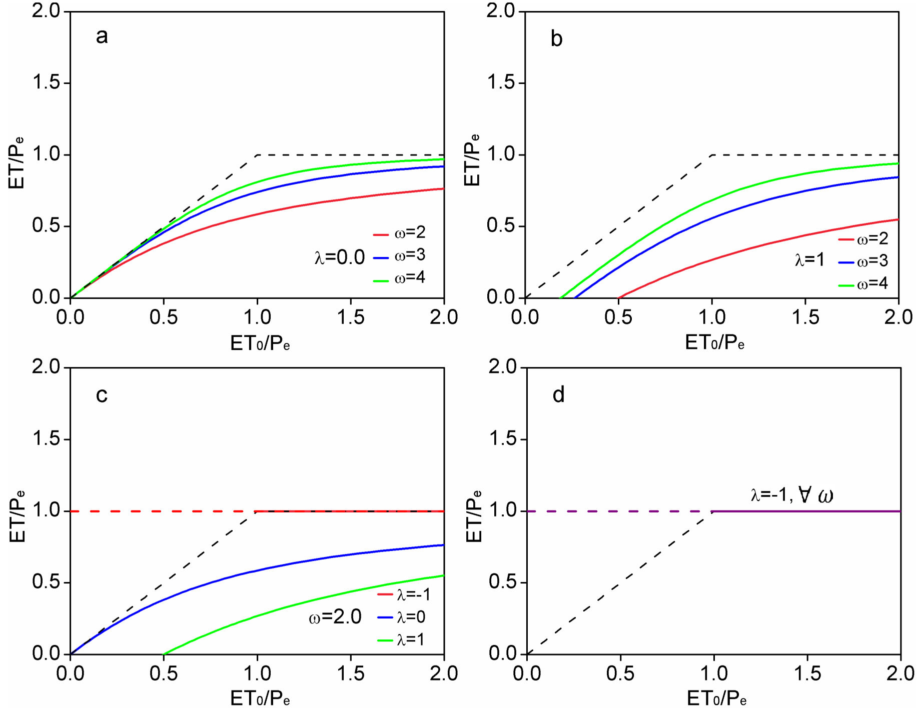


Figure 2. The Budyko curves in Eq. (8) with different combinations of parameters ϖ and λ.

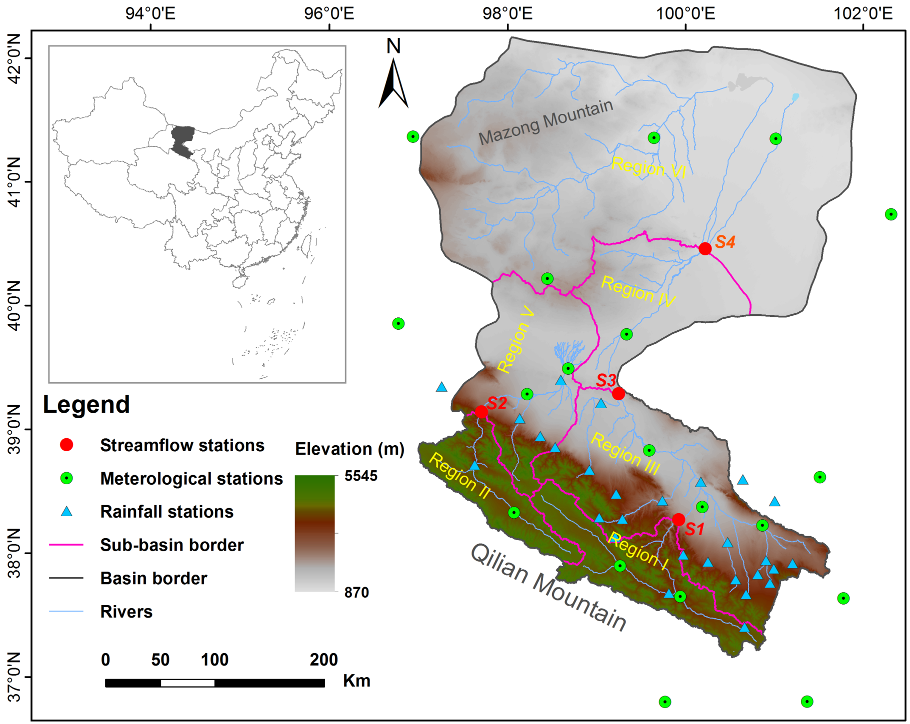


Figure 3. Location of study area and the distribution of hydrological stations and meteorological stations.

HESSD
12, 11013–11052, 2015

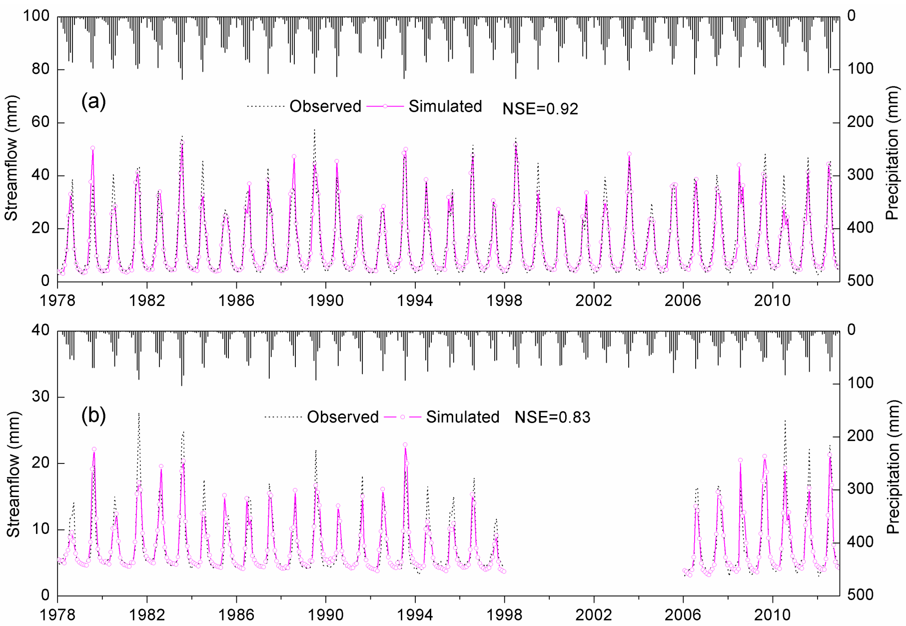


Figure 4. Time series of observed and simulated monthly streamflow using the *abcd* model in the Region I **(a)** and Region II **(b)** during 1978–2012.

HESSD
12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

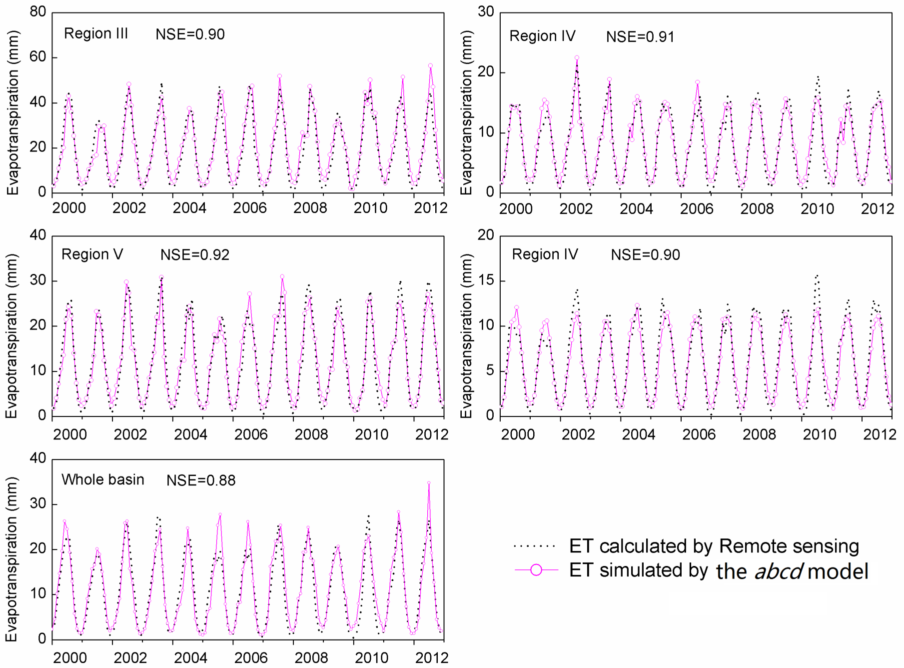


Figure 5. Comparison between ET simulated by the *abcd* model and ET calculated by remote sensing data for Regions III–VI and the whole basin during 2000–2012. "WBM" denotes the *abcd* water balance model.

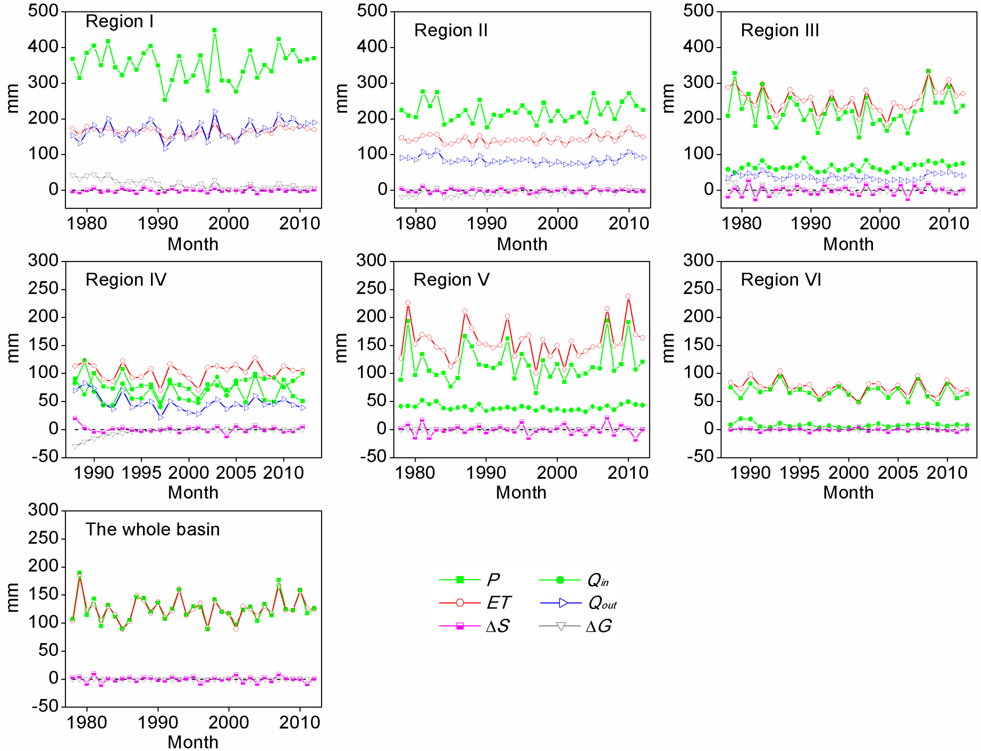


Figure 6. Variation of annual water balance for all the regions ~~simulated~~ using the *abcd* model.

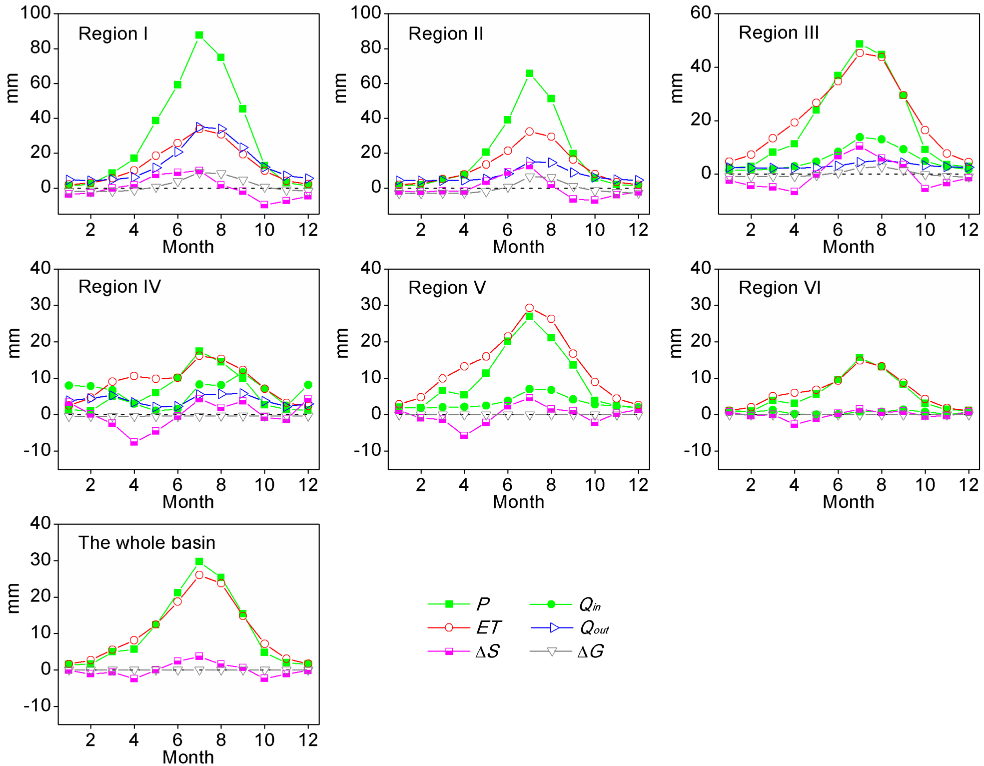


Figure 7. Variation of average monthly water balance for all regions using the *abcd* model.

HESSD
12, 11013–11052, 2015

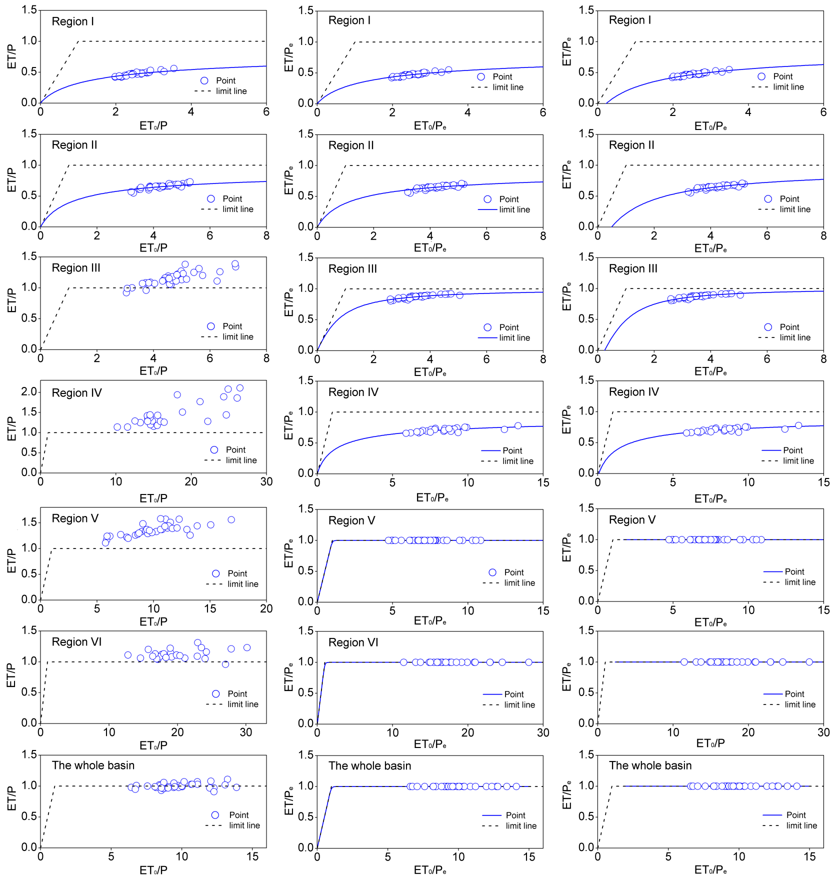


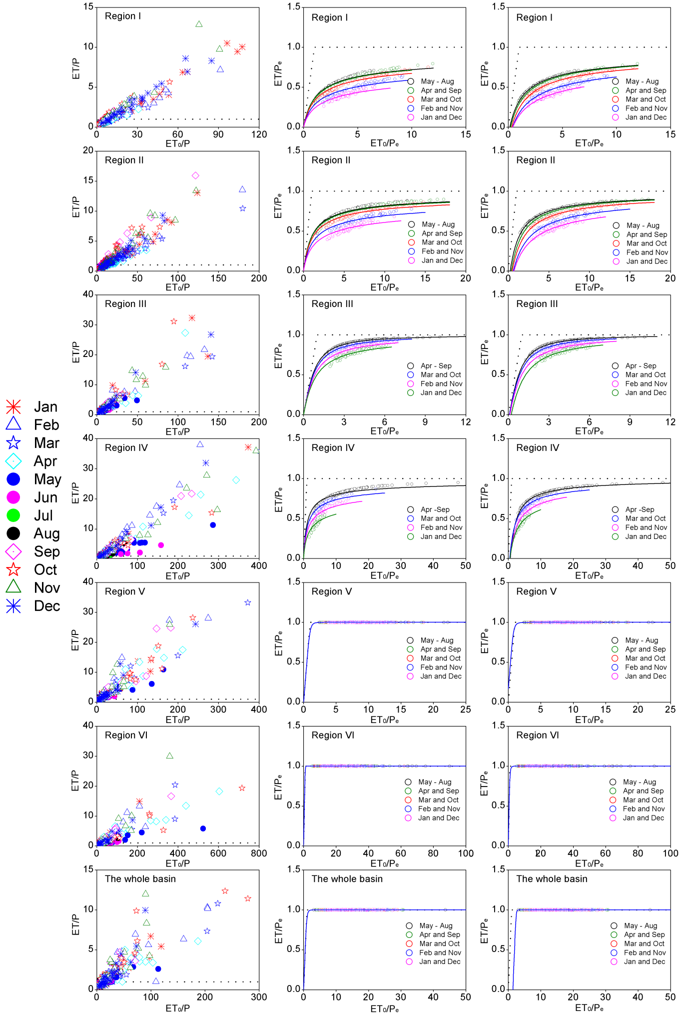
Figure 8. Comparison of the original Budyko curves (left panel) and the new *Fu*-type Budyko curves (middle panel, with $\lambda = 0$) and the new *Fu*-type Budyko curves (right panel, with $\lambda > 0$) for Regions I–VI and the whole basin at the annual time scale.

HESSD
12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
◀| |▶
◀ ▶
Back | Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion



HESSD
12, 11013–11052, 2015

New interpretation of water balance in BH in arid regions

C. Du et al.

Title Page
Abstract
Introduction
Conclusions
References
Tables
Figures
◀◀
▶▶
◀
▶
Back
Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

Figure 9. Comparison of the original Budyko curves (left panel) and the new *Fu*-type Budyko curves (middle panel, with $\lambda = 0$) and the new *Fu*-type Budyko curves (Right panel, with $\lambda > 0$) for Regions I–VI and the whole basin at monthly time scale.

HESSD
12, 11013–11052, 2015

C. Du et al.

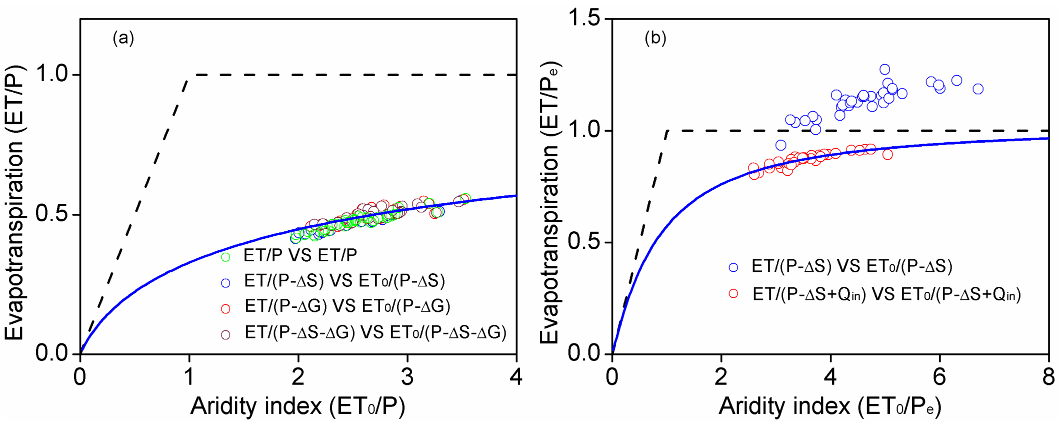


Figure 10. Different presentations of annual water balance for **(a)** Region I and **(b)** Region III.

HESSD
12, 11013–11052, 2015

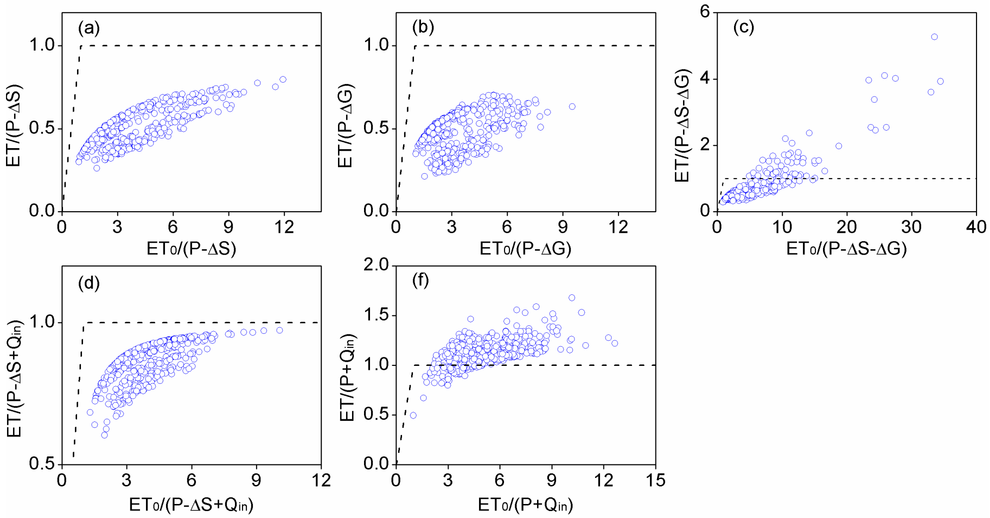


Figure 11. Five presentations of monthly water balance for Region III considering different combinations in the water supply to evapotranspiration.

HESSD
12, 11013–11052, 2015