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Interactive comment on "Estimation of heterogeneous aquifer parameters using centralized and decentralized fusion of hydraulic tomography data from multiple pumping tests" *by* A. H. Alzraiee et. al

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RESPONSE TO COMMENTS FROM REVIEWER # 2

Note: We use text with black color for reviewer comments (RC), blue color for author

comments (AC), and red color for modified text.

General comments from Authors

[RC#1] The main purpose of the work is to invert hydraulic tomography (duplicating interference pumping tests) by means of linear Kalman filtering. Though groundwater flow inversion did not widely rely on Kalman filtering yet, the major contribution, here, is in the comparison of how tomographic data can be handled. On the one hand, all interference drawdown curves as the responses to all pumping stresses can be gathered into a single set of data, the resulting flowing scenario being inverted in a single step. On the other hand, each pumping scenario can be inverted one at a time. The resulting sets of plausible solutions (ensemble Kalman filtering provides an ensemble of solutions, not a single one) are then fused."

[AC#1] [We would like to thank Reviewer #2 for his/her insightful comments on our paper. We addressed all of the comments and suggestions as described in this document as well as in responses to comments provided by two other reviewers.]

[RC#2] Because ensemble Kalman filtering has to duplicate the so-called states of the system including both the state variables (here, hydraulic interferences) and the parameters (hydraulic conductivity and specific storage capacity), the technique may result in quite intractable computations if interference testing is fully represented by hydraulic head variations in time at specified spatial locations. Interference information is therefore aggregated by calculating zeroth- and first-order temporal moments of the drawdown curves. These moments drastically diminish the volume of information needed to represent the hydrodynamic responses, at the cost however of a rough depiction of the system's transients.

[AC#2] [We added a new experiment as suggested by this reviewer to explore the effect of information loss brought about by the temporal moment formulation. Please refer to RC#4 and AC#4 for more details.]

[RC#3] “In general, the paper is well written and documented, even though one might find in some places too much (and useless) equations encapsulated in the lines of the main text and chopping the reading. With no doubt, the main topic addressed in the paper is appealing to the HESS readership, and methodologies and results are interesting. In my opinion, this paper could be accepted for publication in HESS after moderate revision.

[AC#3] [No reply is required].

[RC#4] My main general concerns about the content are twofold, though they do not jeopardize tentative publication” First, the system reduction when passing from a full representation of drawdown interferences to temporal moments is not discussed at all in terms of the incidence on the inverse sought solutions. As mentioned above, one can hear that Kalman filtering on drawdowns may become cumbersome, but temporal moments are also a severe reduction of the information on the system. It would have been interesting to know whether this reduction has some incidence on the inverse solutions (local parameter values, parameter variances and covariances, identification of the spatial structure (covariance) of parameters, etc.). If the problem is too much complicated when handling a complete tomographic exercise, perhaps one could limit the investigation to a single interference testing with one pumped well and drawdown responses at n observed wells. Incidentally, in view of the results provided by the authors, the identification of the specific storage capacity seems to be accurate. This parameter is reputed weakly sensitive (and difficult to identify) when dealing with classical

head transients. Is this result the consequence of using moments, the zeroth moment pre-evaluating the conductivity field, the second step of inversion relying on the first moments and re-handling the "pre-inversed" conductivity and the specific storage capacity? Stated differently, is the two-step inversion conducive to a better evaluation of the specific storage capacity because the second step of inversion emphasizes its incidence and just adds cosmetics to the conductivity field sought within the first step? If this assertion is right, it could be documented and diminishing the volume of information by using temporal moments would become a true benefit.”

[AC#4] [To address this comment, and as suggested by Reviewer #2 (this reviewer), we have conducted a new experiment (Experiment 3) and included its results in Section 4.3 of the revised manuscript. This experiment aims explicitly at comparing the performances of the EnKF algorithm when "raw" transient head data are assimilated and when zeroth and first moment data are assimilated. The results of these tests shown in a new Table 5 show that the use of the moment formulation produces a relatively significant loss of information, but the drastic reduction in CPU times achieved using this approach makes it, in our view, far more attractive. The new section reads as:

“ **Assimilating transient head data versus assimilating temporal moments:** While assimilating temporal moments instead of the transient data itself provides a significant saving in CPU time, it is important to verify to what extent this option affects the accuracy of the estimation. To do so, we conduce an experiment whose goal is to compare the performances of the EnKF when temporal moments are assimilated and when the "raw" transient hydraulic head data are assimilated. In this experiment, we use data from a single pumping test at well No. 1 in Figure 2. Using data from a single pumping test allows for reducing the scale of the data assimilation problem, thereby limiting the associated computational effort, without affecting the generality of conclusions drawn from the experiment.

Table 5 summarizes the performance statistics of the two approaches. One can observe that assimilating the transient data leads to better results compared with assimilating the temporal moments. This observation can be seen for the estimation of both Y and Z fields. This effect can be explained by information loss resulting from lumping transient head data into low order temporal moments. However, while assimilating transient head data provides a better characterization than using temporal moments, the associated computational cost is drastically higher. For example, in the case investigated here, the overall CPU time required by the transient data formulation is about 40 times larger than that required by the temporal moment formulation.

It is worth noting that the correlation coefficient r for estimation of Y resulting from a single pumping test ($r = 0.908$ in Table 5) is higher than that resulting from five pumping tests ($r = 0.825$ in Table 3), while the L_1 and L_2 statistics are better (lower) for multiple pumping tests. This due to the fact that the correlation coefficient r is invariant with respect to linear transformation of the two fields, and thus r provides a measure of similarity in the structure of spatial variability with no information about the Euclidean distance between the two fields, which is provided by L_2.

[RC#5] The second point is that the paper mainly discusses on THE optimal inverse solution as the ensemble mean of the equiprobable realizations manipulated by the Kalman filtering procedure. It is also discussed on the variance of parameters but the ensemble of solutions (updated parameter fields) is hardly exploited. For example, the ensemble covariance matrix of parameters is not compared, by any means, with the covariance prescribed in the synthetic problem serving as reference. However, if the ensemble of solutions is good, each solution should have a covariance of spherical type as the one prescribed in the reference problem and the ensemble covariance matrix should also be spherical. This feature is simply stemming from the fact that, in theory, the ensemble of solutions, even updated by the Kalman filter, is a set of equiprobable realizations of the same spatially distributed random function. Fortunately, one may conceive that some violations of the theory are acceptable. Up to which point? Another

example; the authors state that the parameters of the covariance of the log conductivity (Y) and log- specific storage capacity (Z) fields can be introduced in the matrix system updated by the Kalman filtering. What happens when these updated parameters yield a covariance model weakly compatible with the ensemble covariance calculated over the n updated realizations of Y and Z? It is interesting to handle inversion problems in a Bayesian framework provided one explores the benefits of getting multiple solutions instead of a single one. In my opinion, this exploration is not enough developed in the present version of the manuscript.

[AC#5][We understand from this comment that the reviewer is asking if augmenting the mean, standard deviation, and the correlation length would produce update geostatistical parameters that are compatible with parameters computed from the update ensemble of the fields. We agree with the reviewer that this point is very important and worth further investigation; however, and as reviewer #1 suggests, the estimation of geostatistical parameters needs more investigation and extensive literature review, which might make the reading of the paper more difficult and the main focus of the work might be lost. We agree with reviewer #1 that addressing the estimation of GSP in a separate paper would allow us enough space to explore different aspects of the problem. Accordingly, we have removed this section in the revised version of this paper.]

Editorial suggestions

[RC#6] 1. P. 4171. I do not understand how the moments quoted in Expressions (3) and (4) could correspond to the temporal moment definition in (2). Basically, the steady-state differential equations proposed in (6), (7), and (8) and obtained by Laplace transform or direct temporal integration of the groundwater flow equation would calculate temporal moments obeying to the definition in (2).

[AC#6] [Derivation of these equations were the contribution of Lie et al. (2005) as cited in the manuscript. To clarify the the reviewers question, we report the derivation only in this response letter and not in the manuscript.

In analogy to equation (1) in the paper, the steady state drawdown resulting from continuous extraction can be expressed as

$$s\left(x;t=\infty\right)=Q\int_0^{t=\infty}\theta_s(x;t)dt \tag{1}$$

where $\theta_s(x;t)$ is the response impulse function of the drawdown to unit extraction. The kth temporal moment of $\theta_s(x;t)$ is

$$m_k\left(x\right)=\int_0^{\infty}t^k\theta_s(x;t)dt \tag{2}$$

From equation [2], the zeroth temporal moments is

$$m_0\left(x\right)=\int_0^{\infty}\theta_s(x;t)dt \tag{3}$$

Substituting $\int_0^{t=\infty}\theta_s(x;t)dt$ obtained from equations [1] in [3], results in

$$m_0\left(x\right)=\frac{s\left(x;t=\infty\right)}{Q} \tag{4}$$

where the steady state drawdown is calculated as $s\left(x,t=\infty\right)=h\left(x;0\right)-h(x;\infty)$. Likewise we show that

$$\frac{m_1\left(x\right)}{m_0\left(x\right)}=\frac{m_0^{\Delta h}(x)}{s\left(x;t=\infty\right)} \tag{5}$$

Where $m_0^{\Delta h}(x)$ is the zeroth temporal moment of $\Delta h\left(x;t\right)$, which is equal to $m_0^{\Delta h}\left(x\right)=\int_0^{\infty}\Delta h\left(x;t\right)dt$. Substituting $m_0\left(x\right)$ from equation [4] in [5] results in

$$m_1\left(x\right)=\frac{\int_0^{\infty}\Delta h\left(x;t\right)dt}{Q} \tag{6}$$

[RC#7] P. 4173 Expression (9). If the notation $p(\phi,I)$ denotes for the authors the simple prior density function of ϕ (and not a joint density), then (9) is wrong and should be rewritten as $p\left(\phi\,|\,m,I\right)=\frac{p(m|\phi,I)p(\phi,I)}{p(m,I)}$. Notably, if I is, as stated by the authors, a "generic" information, it could be dropped in the notations, yielding the classical formulation $p\left(\phi\,|\,m\right)=\frac{p(m|\phi)p(\phi)}{p(m)}$.

[AC#7] [We agree with the review. Equation (9) has been modified accordingly.]

[RC#8] P.4182 line 10-12. When it is referred to the geometry of the aquifer, the parameters listed in the text and those concealed in Table 2 are not similar. We have a 10 m thick confined aquifer in the text and a 50 m thick in Table 2. In the same vein, Table 2 quotes that the cell size is 10 m along the vertical direction where it should be 50 for asingle-layer aquifer of 50 m thickness.

[AC#7][We agree with the reviewer, the dimensions were corrected. The thickness of the aquifer is 10m.]

[RC#9]4-P. 4183 line 22 and Fig. 2. I do not see in Fig (2) the 36 observation points mentioned in the text. Fig (2) shows 25 observation points and 4 pumping wells.

[AC#9][Figure 2 has been modified in response to this observation from the reviewer.]

[RC#10]5- P. 4183 line 2. I do not understand the sentence on how are calculated the mean and variance of errors on temporal moments.

[AC#10][We assume reviewer # 2 refers to Line 25 rather than Line 2. When measured data are heads (or drawdowns), measurement errors reflect our confidence in data. If observations are unbiased, we may assume errors to have a mean equal to zero and a standard deviation that reflects the accuracy of measuring tool. In this case, we have some sense of the error, say for example plus or minus 2 cm. In the case of temporal moments, selecting an error standard deviation is far less intuitive, therefore this is assigned equal to 1% of the square root of the prior ensemble variance of temporal moments, which is the diagonal coefficient of the prior covariance matrix.]

[RC#11] 6- Fig. (3) The map (d) should be labeled as being the "Estimated Z field"

[AC#11][We agree. Figure is corrected.]

[RC#12]7- P. 4184, Expression (25). I do not understand the definition given to the correlation coefficient between a true and an estimated field. To make it short and simple, let us take two variables x and y of zero mean. Defining the correlation coefficient as being $r = cov(xy)/(\sigma_x^2\sigma_y^2)^{0.5}$, yields $\sum\limits_i x_i y_i/(\sum_i x_i^2 \sum\limits_i y_i^2)^{0.5}$ I do not see why one observes double sums over indexes $i$and $j$in (25) .

[AC#12][We agree with the reviewer that Eq. (25) is not clear enough. This equation reports the regular Pearson's correlation coefficient, which slightly modified to calculate the correlation between two 2D images instead of two 1D vectors. If we reshape the 2D images into 1D vector we can use the equation suggested by the reviewer. However, the two equations are exactly the same. In the revised manuscript, we have rewritten

Equation (25) accordingly with the reviewer comment.]