Technical Note for Hillslope Tiling Implementation in LM3

Zack Subin
GFDL

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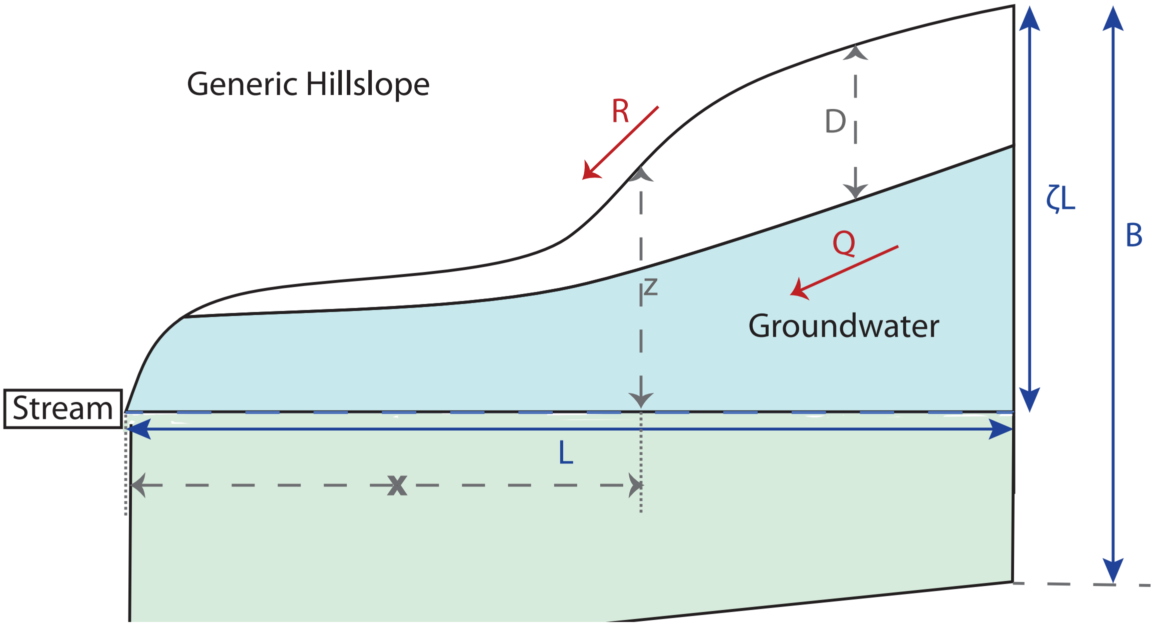

Figure 1: Representative conceptual hillslope unit, profile view. x and z are the horizontal and vertical distance from the stream; L and ζL are the maximum length and height of the hillslope, respectively; D is the water table depth; and R is the surface runoff. The groundwater flux Q is a vector. B is the resolved depth to impermeable bedrock, constant along the hillslope.

1 Overview

We extend the LM3 subgrid tiling of vegetated areas to encompass additional dimensions of variability between units of the landscape, beyond the current implementation that considers only anthropogenic land use history and disturbance. Each tile represents a characteristic ecosystem or geomorphological unit with a length scale of 0.1-10 km that is associated with a distinct 1-dimensional (1D) vegetation state, soil state, and surface fluxes. We primarily focus on topography as a predictor of tile variability: this includes both the overall geometry of the landscape terrain (e.g., mountainous, glacial, or coastal) and the position of a particular tile within this geometry (i.e. upland, valley, or lowland). The topography can be divided into a macroscale component comparable in size to or much larger than an individual tile and a microscale component much smaller than an individual tile. Other dimensions of tile variation include substrate characteristics and disturbance history. Substrate characteristics, such as bedrock and mineral soil permeability, may influence wetland dynamics, although global data is presently limited. In addition to anthropogenic disturbance history, natural disturbance, such as fire, and intrinsic dynamics causing evolution into disparate ecosystem types in similar locations (i.e., bogs and fens) can be represented as a dimension of time variability.

1.1 Hillslope Unit

The macroscale topography determines the topology of relationships between tiles by defining the directions of groundwater and surface water flow. Tiles exchange water and associated advection of heat, with later development to include advection of tracers such as dissolved organic carbon. To conceptualize this topology of water flow, we consider a 2D (1 horizontal and 1 vertical dimension) hillslope element (Figure 1). The streambed is at $x = 0$, and the hillslope extends horizontally to $x = L$. The hillslope profile is assumed to be monotonically increasing towards $x = L$, with maximum elevation ζL. A point on the hillslope surface (i.e., as a function of x) at elevation above the stream z is associated with a water table depth D, defined by the lowermost point where the water pressure equals the atmospheric pressure, or the matric potential $\psi = 0$.

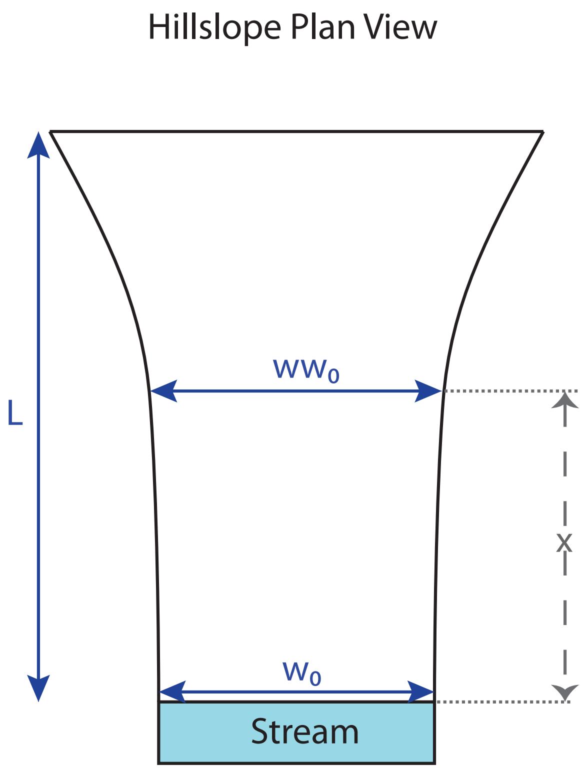

Figure 2: Representative conceptual hillslope unit, plan view. x is the horizontal distance from the origin along the length of the hillslope; w is the normalized width of the hillslope at a given x, assumed to have constant elevation above the stream z.

The depth to impermeable bedrock B is a constant throughout the hillslope. As the hydraulic conductivity will decay to a small value K_b below the soil surface in poorly drained areas, the dynamics are relatively insensitive to the choice of B. The groundwater flow Q (including unsaturated water flux above the water table) is defined by Darcy's Law and forms a 2D flow-field. In practice, we will decompose the flow into a vertical component that will be solved on a short timestep to allow rapid fluctuations around hydrostatic equilibrium, and a slower horizontal component along the hillslope between land model tiles. (The slope of the land-surface is not considered elsewhere in the land model, including in the surface flux solution, so we will consider these components to be orthogonal, although the slope is included as an adjustment to the horizontal length between tiles in the groundwater solution (Eq. 10).)

We allow the horizontal width of the hillslope to vary (Figure 2). Defining w_0 as the width at $x = 0$, w is the normalized width at distance x, with $w > 0$. Only the normalized w will affect the dynamics, and we assume that z is constant for a given w for simplicity.

2 Tiling Implementation

Three "dimensions" of vegetated tiles are allowed in each gridcell. One dimension allows for variation in hillslope geometry, or in other properties varying among hillslopes such as subsurface properties or large-scale disturbance history. Tiles in each of these hillslopes are associated with a hillslope-index k, and interactions between tiles in different hillslopes interact only via the river network.

Within each hillslope indexed by k, tiles are grouped in the hillslope by location along the dimension x, associated with an index $1 \leq j \leq N_{vc}$ following a gradient from lowland to upland (Figure 3). Finally, we allow multiple tiles to exist at each level j, differing according to their disturbance history or natural

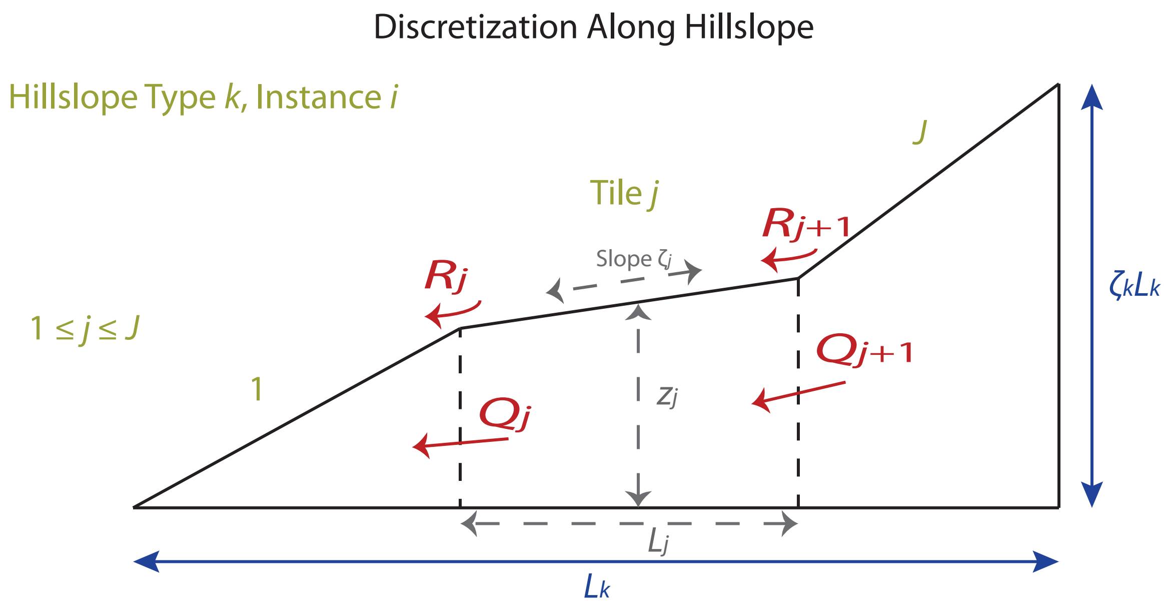

Figure 3: Discretization by vertical elevation along hillslope type k. Each vertical tile in cluster j is associated with a length L_j, an elevation above the stream z_j, and a slope ζ_j. Groundwater flow Q flows from tiles in j to tiles in $j-1$. Surface runoff R is routed directly to the stream.

ecosystem successional characteristics. These tiles will be defined by an area fraction within the cluster of tiles indexed by j and may differ in soil water state. They do not have explicit topological relationships within the tile cluster, but they are defined by a length-scale of disturbance y that defines how strongly they interact with each other.

Each tile interacts hydrologically with other tiles in the same j and in neighboring j. The tiles at the top of the hillslope have a zero-flux boundary condition upslope. At the bottom of the hillslope, the lowland tiles interact with a stream with $\psi = 0$ at the surface, and a hydrostatic pressure head beneath. The stream is assumed to have a limited penetration depth z_{str}, associated with a decline in hydraulic conductivity $\kappa_s(d)$ [dimensionless] of

$$\kappa_s(d) = \exp\left\{-\frac{d}{z_{str}}\right\}, \tag{1}$$

where d is the depth below the local surface.

Each tile n in the gridcell is associated with an area fraction A_n, and according to its location j along hillslope k, it has a length L_j, a slope ζ_j, an elevation Z_j, a normalized width w_j, and other hillslope-specific characteristics (i.e., groundwater permeability). A_j is the sum of the areas of tiles in hillslope cluster j, and within the gridcell, $A_j \propto w_j$ and $A_j \propto L_j$.

2.1 Geometry

Hillslopes vary independently in their profile (Figure 5) and plan (Figure 6) geometries. In each gridcell, we allow up to N_{topo} different geometries. We assume that the profile of hillslope-type k is given by

$$z(x) = L_k \zeta_k x^{\beta_k}, \beta_k > 0. \tag{2}$$

where L_k, ζ_k, and β_k are defined separately for each hillslope in the gridcell according to surface data. We assume that the plan of hillslope-type k is given by

$$w(x) = 1 + \alpha_k \frac{x}{L_k}, \alpha_k > -1. \tag{3}$$

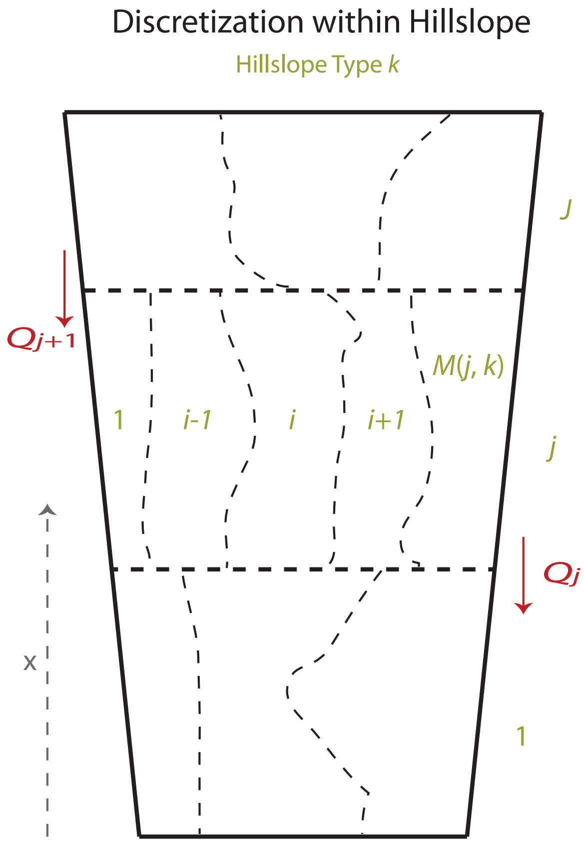

Figure 4: Each cluster of tiles j in hillslope type k can be subdivided into tiles that share macrotopographic characteristics but differ in disturbance or successional history. The curved dotted lines between the tiles within each j merely suggest subdivided spatial area but do not represent a physically meaningful interface or ordering among them; their interaction is characterized by a length-scale between tile centers y.

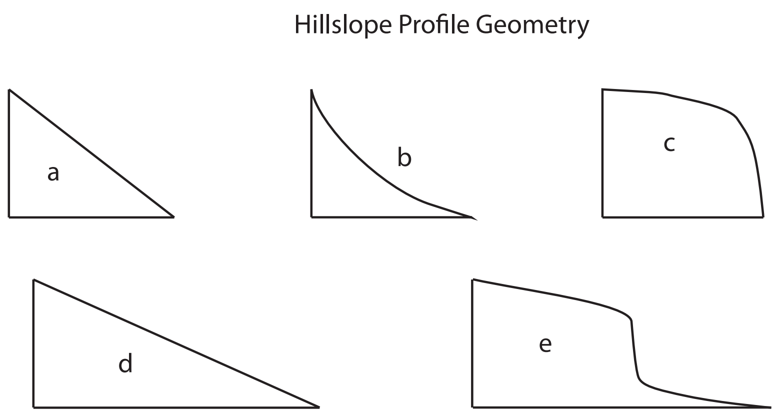

Figure 5: Representative Hillslope Profile Geometries: (a) neutral; (b) concave; (c) convex; (d) long [short not shown]; and (e) sigmoidal.

Hillslope Plan Geometry

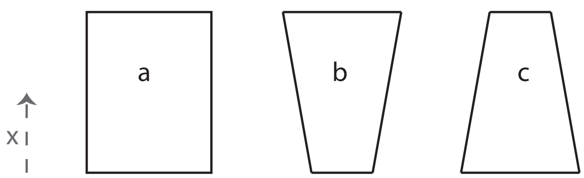

Figure 6: Representative Hillslope Plan Geometries: (a) neutral; (b) converging; and (c) diverging.

2.2 Initialization

At the start of the model run, tiles are created for each hillslope type k in the gridcell provided by the surface data, up to the maximum N_{topo} allowed for the run. The areas of the hillslopes are normalized so that $\sum_{k=1}^{N_{topo}} A_k$ is equal to the total soil area for that gridcell. These areas are defined, as elsewhere in the land model, with respect to a horizontal projection of the earth's surface. Within each hillslope, N_{vc} vertical tiles are created, with areas given by $A_j = A_k \frac{L_j}{L_k} f_c$; $L_j = \frac{L_k}{N_{vc}}$ by default but may be prescribed otherwise at runtime; where f_c is a geometric factor associated with the trapezoidal shape of the tiles due to the convergence or divergence of the hillslope. If tile lengths are equal, then f_c is given by

$$f_c = 2\frac{\frac{2N_{vc}-1-2(j-1)}{2N_{vc}} + w_L \frac{1+2(j-1)}{2N_{vc}}}{N_{vc}(1+w_L)}, \tag{4}$$

where w_L is the width at the top of the hillslope of type k (at $x = L_k$), normalized by the width at the stream ($x = 0$).

Hillslope-dependent properties, such as substrate characteristics, are initialized accordingly from surface data. Properties that are a function of position within the hillslope are initialized using the hillslope geometry given by Eq. 2, and by linearizing the hillslope profile into a series of line segments whose endpoints match the curve given by Eq. 2, with tile properties being defined at the center, based on the average of the values at the endpoints. Thus, the hillslope horizontal position x_j of tile j is given by $\sum_{i=1}^{j-1} L_i + \frac{L_j}{2}$, where L_j is the length of tile j. The vertical elevation Z_j above the stream is the mean of the elevations at the tile endpoints (dropping subscripts k):

$$Z_j = \zeta L \frac{(\frac{x_j - \frac{L_j}{2}}{L})^\beta + (\frac{x_j + \frac{L_j}{2}}{L})^\beta}{2}, \tag{5}$$

where ζ is the slope of the hillslope. The width w_j is set similarly, using Eq. 3.

3 Hydrology

The mass fluxes of water (and the fluxes of energy and other associated tracers) between tiles within each hillslope are determined according to Darcy's Law:

$$\vec{q}(x,z) = -\vec{k}(x,z)\frac{\partial h}{\partial \vec{r}}, \tag{6}$$

where $\vec{q}(x,z)$ [kg/(m^2 s)] is the water flux per unit area at point $\vec{r}$, and h is the total pressure head $h = \psi + z$, where ψ is the soil matric potential and z is the local elevation. Point $\vec{r}$ may be above the water table ($\psi < 0$) or below the water table ($\psi > 0$). We assume that the vertical hydraulic dynamics within each tile, occurring over a scale of meters and parallel to the effects of gravity, can be solved separately from the horizontal exchanges of water between tiles, which occur on a scale of hundreds of meters and will generally involve more slight head gradients arising from topography or the resulting differences between tile surface fluxes. Consequently, the solution procedure is as follows: at each timestep, the explicit flux tendencies are evaluated between each pair of neighboring tiles. These fluxes are then imposed as a boundary condition (a source or sink term) in the vertical Richards equation implicit solution.

We allow fluxes between tiles with the same j or between tiles with j differing by 1, and between the tile at $j = 1$ and the stream. These fluxes are discretized vertically by calculating separate fluxes between each enumerated pair l of vertical soil layers of the tiles.

3.1 Fluxes Between Tiles of Different j

We first consider the simplest case, where two tiles each are alone in their hillslope clusters j_a and j_b (and $|j_a - j_b| = 1$). For convenience, we will identify the tiles as a and b (Figure 7). Tile a has length L_a, width w_a, elevation z_a, and at each l, hydraulic conductivity $k_{a,l}$ and hydraulic head $h_{a,l}$; tile b has likewise. We define flows between the midpoints (along the hillslope) of the tiles. For each l, we define the following interface properties: a harmonic mean conductivity, a mean length, and an interface width.

$$\hat{k} = \frac{k_a k_b (L_a + L_b)}{k_b L_a + k_a L_b} \tag{7}$$

$$\hat{L} = \frac{L_a + L_b}{2} \tag{8}$$

$$\hat{w} = \frac{L_a w_b + L_b w_a}{L_a + L_b}. \tag{9}$$

There is an option to consider the vertical slope in the calculation of $\hat{L}$. (Otherwise we are assuming perpendicular coordinates with intertile flows being horizontal and intratile flows being vertical. This is consistent with the surface flux calculation being based on the horizonal projection of the land area only.) In this case,

$$\hat{L} = \sqrt{\left(\frac{L_a + L_b}{2}\right)^2 + (z_b - z_a)^2} \tag{10}$$

Finally, we define the head gradient

$$\Delta h = \Delta\psi + \Delta z = \psi_{b,l} - \psi_{a,l} + z_b - z_a. \tag{11}$$

The flow per unit length Q [kg/(m s)] at the interface from tile b to tile a, for soil layer l, is given by:

$$Q = \frac{\hat{k}\Delta h}{\hat{L}} \Delta z_l, \tag{12}$$

where Δz_l is the vertical thickness of soil layer l.

To convert Q into the water flux $q_{a,b}$ into a from b, per unit area of a, we need to multiply Q by the ratio of the interface length to the area of a: $\frac{\hat{w}}{L_a w_a}$. Finally, if tile b is not the only tile in hillslope cluster j, then we consider the contributing area of b compared to the entire area of A_{j_b}, making no assumptions about the relative spatial locations of tile b with respect to a. We need not consider the area of tile a here, as we are calculating the flux into tile a per unit area of a; the total flow into the cluster j_a will sum over the other tiles, if present, proportional to their respective areas. This yields the following:

$$q_{a,b} = \frac{\hat{k}\Delta h}{\hat{L}} \Delta z_l \frac{A_b}{A_{j_b}} \frac{\hat{w}}{L_a w_a}. \tag{13}$$

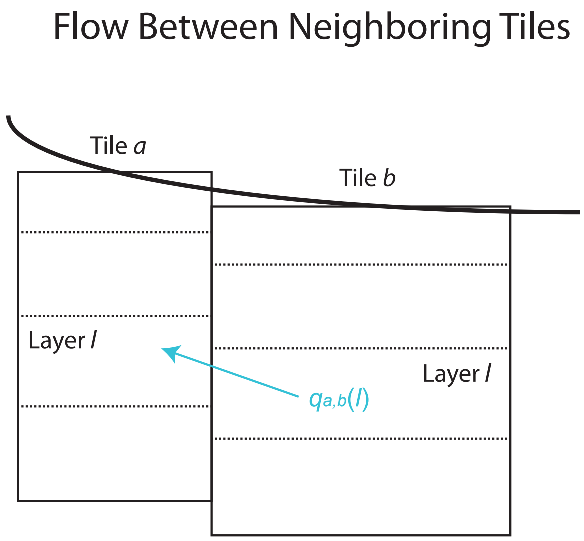

Figure 7: Flow Between Tile a and Tile b Immediately Downslope. Each tile has an elevation defined as the mean of the elevation at its endpoints. Flow $q_{a,b}(l)$ [kg/m^2/s], into tile a per unit area a is discretized by soil layer and defined between each pair of soil layers in tiles a and b of the same level. Each of these pairs has the same elevation difference but may have a different total hydraulic head difference due to variations in local matric potential.

3.2 Fluxes Between Tiles of Same j

Multiple tiles, with the same length and width, may occupy hillslope cluster j, due to disturbance or successional history. These tiles generally will have the same elevation above the stream (although, for instance, thermokarst or peat processes may cause slight differences in tile surface height), but they may have distinct hydrologic states because they have different vegetation and surface fluxes. Consider two tiles, labeled a and b, with areas A_a and A_b that are subsets of the total cluster area A_j. In order to define the fluxes between tiles, we are agnostic about the spatial arrangement of tiles within the cluster except for defining an interaction or disturbance lengthscale y on the order of the distance between centers of the elements of the landscape represented by the different tiles. So, for instance, if harvesting patches were of size 100 m, then $y \sim$ 100m. Whether patches are circular, rectangular, etc., will affect the fluxes with a factor of order unity. For simplicity, we define y to represent the case where the width of the hillslope is subdivided into rectangular strips of two alternating tiles, where the distance between the centers of the strips is y (Figure 8). The number of repetitions of strips of each tile is equal to $\frac{W_j}{2y}$, where W_j [m] is the non-normalized width of the hillslope. In this case, in the limit $y << W_j$, we can show that fluxes are independent of L_j and W_j and depend only on the lengthscale y and the area fractions. This yields the following modification of Eq. 13 for the flux from b to a for soil layer l, per unit area a:

$$q_{a,b} = \frac{\hat{k}\Delta h}{2y^2}\Delta z_l \frac{A_b}{A_j}. \tag{14}$$

This equation can be evaluated even if tiles a and b are the same, yielding zero flux. Note that the larger the fraction of area of tile a within cluster j, the less total flux will be exchanged with other tiles per unit area a.

3.3 Fluxes to Stream

For tiles a in hillslope cluster $j = 1$, flows to the stream occur, based on the stream height h_s, the conductivities $k_{a,l}$, and the stream conductivities $\kappa_{s,l}$ [dimensionless] defined by the depth of soil layer l and Eq.

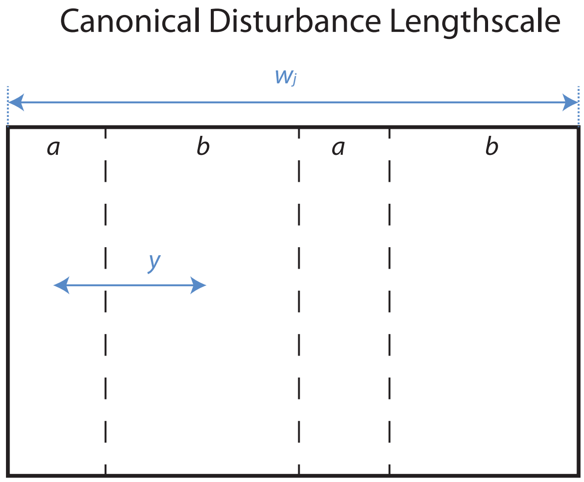

Figure 8: The hillslope cluster j is defined by a disturbance lengthscale y and a physical width W_j. In the limit of $y << W_j$, the total interface length between tiles is proportional to $\frac{W_j}{2y}$. As the length between tile centers is also proportional to y, the strength of fluxes between tiles a and b is proportional to $\frac{1}{y^2}$.

1. The interaction length is assumed to be $\frac{L_a}{2}$. This yields the following equation for the flux out of a from layer l to the stream, per unit area of a:

$$q_{a,s} = \frac{2k_{a,l}\kappa_{s,l}\Delta h \Delta z_l}{L_a^2}. \tag{15}$$

Here, $\Delta h = \psi_{a,l} + z_a - \psi_s - z_l$, where z_l is the depth of soil layer l. In the static stream configuration, the stream state is prescribed to be $\psi_s = 0$, and flows are only allowed if $\Delta h > 0$.

3.4 Advection of Heat

The advection of enthalpy between tiles due to intertile water fluxes is tracked, consistent with the definition of enthalpy used elsewhere in the land model:

$$H = (W_l c_l + W_i c_i)(T - T_f), \tag{16}$$

where H [J/m^2] is the enthalpy per unit area; W_l and W_i [kg/m^2] are the liquid water and ice contents per unit area, respectively; T [K] is the local temperature; c_l and c_i [J/(kg K)] are the specific heat per unit mass of liquid water and ice, respectively; and T_f [K] is the freezing temperature of fresh water. For Eq. 16 to conserve energy in the land model, all phase change must occur at the same temperature. The enthalpy advected during water flow is associated with the enthalpy content of the water moving from the high-head to low-head tile. Only liquid water is allowed to flow between tiles. Consequently, the enthalpy flux $E_{a,b}$ [J/(m^2 s)] into tile a from tile b, per unit area a, for soil layer l, is given by:

$$E_{a,b} = \begin{Bmatrix} (T_{b,l} - T_f)c_l q_{a,b}, & q_{a,b} \geq 0 \\ (T_{a,l} - T_f)c_l q_{a,b}, & q_{a,b} < 0 \end{Bmatrix}, \tag{17}$$

where $q_{a,b}$ is the flow into tile a from tile b for soil layer l, and $T_{a,l}$ and $T_{b,l}$ are the temperatures of soil layer l in tiles a and b, respectively.

3.5 Solution and Energy Conservation

For each tile, the fluxes out of the tile for each other interacting tile are summed to yield a total flux per unit tile area. Then, water and energy conservation are checked according to:

$$\sum_a \sum_l \{A_a [q_a(l) - q_{a,s}(l)]\} = 0 \tag{18}$$

$$\sum_a \sum_l \{A_a [E_a(l) - E_{a,s}(l)]\} = 0, \tag{19}$$

where $q_a(l)$ and $E_a(l)$ are the total water and energy fluxes out of tile a for soil layer l.

3.6 Integration into the Vertical Soil Physics Within Each Tile

The water fluxes $q_a(l)$ are prescribed as explicit divergence source or sink terms in the vertical Richards Eq. solution for each tile. This is identical to the solution evaluated in LM3.1 (Milly et al., 2014) except that this divergence may now be positive or negative. The prescribed enthalpy fluxes require modification of the LM3.1 solution, whose formulation required that flows only be out of tiles to the stream.

The governing equation for the 2D advection of heat by soil water flow in x and z is:

$$\frac{\partial}{\partial t}\left[(c_s + c_{l,v}\theta_l)T\right] = -\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial x}\right)(c_{l,v} q_v T), \tag{20}$$

where c_s is the volumetric heat capacity of the solid (mineral and ice) soil [J/(m^3 K)]; $c_{l,v}$ is the volumetric heat capacity of water [J/(m^3 K)]; θ_l is the volumetric fraction of soil water content [-]; T is the soil temperature [K]; and q_v is the volumetric flow of water [m/s]. In LM3.1, the right-most term $-\frac{\partial}{\partial x}(c_{l,v} q_v T)$ is identified as a divergence term associated with groundwater runoff. Here, we replace this term with $\frac{\partial E}{\partial z} - \frac{\partial}{\partial x}(c_{l,v} q_v T_f)$, where E is the prescribed divergence of enthalpy from the explicit horizontal flow calculation Eq. 17.

Following the solution in LM3.1, we discretize using implict timestepping, divide both sides by c_l, switch from q_v to q [kg/m^2], and identify: $w_0 = \frac{c_s}{c_l}\Delta z$, and $w_i = \frac{\theta_l c_{l,v}}{c_l}\Delta z = \rho \Delta z$, where w_0 is related to the constant heat content, and w_i is the time-varying water content [kg/m^2]. This yields:

$$(w_0 + w_i^{k+1})T^{k+1} - (w_0 + w_i^k)T^k = \Delta t(q_{i-\frac{1}{2}} T_{i-\frac{1}{2}}^{k+1} - q_{i+\frac{1}{2}} T_{i+\frac{1}{2}}^{k+1}) - \frac{\Delta t}{c_l} E_i - \Delta t D_i T_f, \tag{21}$$

where k and $k+1$ index the current and subsequent timesteps; $i - \frac{1}{2}$ and $i + \frac{1}{2}$ represent the flow at the interface above and below the soil layer; and D_i and E_i represent the prescribed horizontal water [kg/(m^2 s)] and energy flux [W/m^2] into the soil layer, respectively. We decompose the interface temperatures as $T_{i-\frac{1}{2}} = u_i^- T_{i-1} + (1 - u_i^-)T_i$, and $T_{i+\frac{1}{2}} = u_i^+ T_i + (1 - u_i^+)T_{i+1}$, where $u_i^+ = u_{i+1}^-$ and is equal to 1 if flow is downward at the interface and zero otherwise. Next, we identify $w_i^{k+1} = w_i^k + \delta w_i$ and $T_i^{k+1} = T_i^k + \delta T_i$. Expanding, simplifying, and regrouping, this yields the following tridiagonal set of equations for the δT_i:

$$a_i \delta T_{i-1} + b_i \delta T_i + c_i \delta T_{i+1} = d_i \tag{22}$$

$$a_i = -\Delta t q_{i-\frac{1}{2}} u_i^- \tag{23}$$

$$b_i = w_0 + w_i^k + \delta w_i - \Delta t q_{i-\frac{1}{2}}(1 - u_i^-) + \Delta t q_{i+\frac{1}{2}}(1 - u_i^+) \tag{24}$$

$$c_i = \Delta t q_{i+\frac{1}{2}} u_i^+ \tag{25}$$

$$\begin{aligned} d_i =& \Delta t q_{i-\frac{1}{2}} u_i^- T_{i-1} + \\ & \left[\Delta t q_{i-\frac{1}{2}}(1 - u_i^+) - \Delta t q_{i+\frac{1}{2}}(1 - u_i^+) - \delta w_i\right] T_i \\ & - \Delta t q_{i+\frac{1}{2}} u_i^+ T_{i+1} - \frac{\Delta t}{c_l} E_i - \Delta t D_i T_f. \end{aligned} \tag{26}$$

For boundary conditions, $\delta T_0 = 0$ and $q_{N+\frac{1}{2}} = 0$, where N is the number of soil layers. However, the temperature of any inflowing water at the soil surface is prescribed for the solution as T_0 in d_1.

Energy is conserved as:

$$\Delta \sum_{i=1}^{N} \left[c_{s,i}\Delta z_i + c_f w_{s,i} + c_l w_i\right](T_i - T_f) = H_s - H_r, \tag{27}$$

where $c_{s,i}$ is the volumetric heat capacity [J/(m^3 K)] of the mineral soil in layer i; c_f is the heat capacity of frozen water [J/(kg K)]; $w_{s,i}$ is the frozen water content of soil layer i; and H_s and H_r are the energy fluxes in at the surface and lost to runoff, respectively, defined by

$$H_s = c_l \Delta t q_{\frac{1}{2}} \left[u_1^- T_0 + (1 - u_1^-)T_1 - T_f\right] \tag{28}$$

$$H_r = c_l \Delta t \sum_{i=1}^{N} D_i (T_i - T_f), \tag{29}$$

where T_i is evaluated at time $k+1$.