Dear Prof. Günther Blöschl,

Thank you very much for your positive comments and appreciation of our paper. We are very grateful for the useful comments and recommendations from both reviewers on this paper. They have provided many constructive comments to improve the quality of our manuscript. Below are our point-by-point responses to each of the comments.

1st Reviewer's Comments

General comments:
The idea of estimating the freshwater flow through an estuarine cross-section using tidal theory and tidal analysis is not new [cf. Jay and Kulkulka, 2003], but it has only recently been presented with a careful verification and analysis of uncertainty [Moftakhari et al., 2013]. The latter authors also introduced the term "tidal discharge estimation" or TDE. As with any innovation, multiple approaches are useful, so the present contribution is a welcome addition to the field of applied tidal dynamics.

Our reply: We agree that the topic of predicting fresh water discharge through observed tidal water levels has been explored by many researchers (e.g., Jay and Kulkulka, 2003; Jay et al., 2006; Moftakhari et al., 2013). The aim of the present contribution is to propose an analytical relationship that can be used to predict fresh water discharge based on observed tidal water levels along the estuary axis. Such a relationship is derived based on the envelope theory developed by Savenije (2005, 2012) and Cai et al. (2014). It can be regarded as a modified Manning equation that includes the effects of residual water level slope (i.e., $\mathrm{d}\bar{h}/\mathrm{d}x$, where $\bar{h}$ is the tidally averaged depth) and tidal damping (i.e., $\mathrm{d}\eta/\mathrm{d}x$, where η is the tidal amplitude). A detailed derivation is provided as an appendix in the revised manuscript (See Appendix A below). Unlike the previous studies that make use of statistical and harmonic analyses relating the tidal water levels to the fresh water discharge, the proposed analytical relationship is a closed form equation, which can be easily implemented for given observed tidal water levels. But as the reviewer indicates, the methods are complementary and help to approach the issue from different angles.

Specific comments:
a: History of TDE
The history of the idea of using tidal theory and the fluvial modification of tidal properties to estimate river discharge needs some explanation, which is not provided here. This is most simply explained using the nomenclature "forward model" (determining tidal properties from river flow) and "inverse model"(determining river flow from tidal properties). Conceptually, the key idea is that river tides are very nonstationary, and that this non-stationarity, while complicating the prediction of tides, has many dynamical uses [Jay and Kulkulka, 2003], of which TDE is only one. There is an extensive literature on river tides dating to at least WWII, and I will not attempt to review it here. Jay and Flinchem [1997] added continuous wavelet transform (CWT) methods to the tidal analyst's tool kit and provided a simple forward model that related the tidal admittance (the complex ratio of tidal

amplitude and phase at any point in the river to the tidal amplitude and phase at the ocean entrance) to river flow. Kukulka and Jay [2003a,b] provided a better forward model. Jay and Kukulka [2003] then used an inverse model to hindcast river flow for the December 1964 Columbia River, USA flood. Because this flood resulted primarily from tributary inflow below the most seaward river gauge, our estimate of its flow history is the only instrumental "measurement" available, though the usual flow routing approaches have also been used. We also verified that the method worked in the Fraser River, British Columbia, Canada, though this work has not been published. Jay et al. [2006] then provided a hindcast of the history of inflow to San Francisco Bay, using the long (1858 to date) San Francisco tidal record. This is a useful step, because the inflow to San Francisco Bay through its complex delta was not gauged by the US Geological Survey until 1930. This 2006 AGU presentation also provided the first instrumental estimate of the magnitude of the great flood of January 1862, the largest in the last two centuries in San Francisco Bay. The inverse models used in these two studies added an innovation, in that they were based on a single tide gauge. When only one gauge is available, then the admittance is formed in one of two ways: (a) if the variations of a major constituent like M_2 are used, then an admittance is formed using the astronomical tidal potential; or (b) if an overtide like M_4 is used, then the ratio M_4/M_2^2 is employed as an ersatz admittance. This complex admittance can be separated into an amplitude ratio and phase difference. Tidal theory suggest that the M_4/M_2^4 ratio should be useful for low flows, while the M_2 admittance is best for high flows. Practice confirms this, at least for the Columbia River and San Francisco Bay. To minimize the impact of time errors inherent in historical tidal records we have used amplitude ratios, though Kulkulka and Jay [2003a] verified that the phase difference could also be represented by a forward model. More recently, Moftakhari et al. [2013] returned to the San Francisco Bay case to provide a revised estimate, a formal error analysis, and a discussion of long-term hydrologic change in the system. Also, if CWT methods are used to provide an estimate with a time-scale of a few days, the ratios actually involve the D_2 and D_4 tidal species, not the M_2 and M_4 constituents. If the M_2 and M_4 constituents are resolved via a properly windowed monthly harmonic analysis (as in Moftakhari et al. [2013]), then the time scale of flow estimates is ~18 days.

Our reply: Indeed, we shall provide a more detailed description of the history of "tidal discharge estimation" (TDE) in the new version of the manuscript. In particular, we have added a paragraph in the introduction to illustrate the history of TDE.

The new paragraph is as follows:
It is noted that several forward models (determing tidal properties from fresh water discharge) have been presented to investigate the interaction between fresh water discharge and tide in estuaries (e.g., Dronkers, 1964; Leblond , 1978; Godin, 1985, 1999; Jay, 1991, 2001; Jay and Flinchem, 1997; Kukulka and Jay, 2003a, 2003b; Horrevoets et al., 2004; Buschman et al., 2009; Cai et al., 2012b, 2014). Based on the tidal theory developed by Jay (1991, 2001), Jay and Flinchem (1997) and Kukulka and Jay (2003a, 2003b), Jay and Kukulka (2003) used an inverse model (determining fresh water discharge from tidal properties) to hindcast river flows for a very high-flow year (1948)

and for a low-flow year (1992) in Columbia River. The model was further successfully applied to estimate the history of inflow to San Francisco Bay using the available tidal records (Jay et al., 2006). Recently, Moftakhari et al. (2013), building on the earlier work by Jay and Kukulka (2003), revisted the method of predicting fresh water discharge by including a quantification of uncertainties. However, such an approach is based on statistical and harmonic analyses without using an analytical relationship between the fresh water discharge and other controlling parameters (such as water level and tidal damping). In this paper, we aim to establish an analytical equation relating tidal wave propagation to the fresh water discharge from upstream. Besides the general interest of establishing an analytical relation between wave cererity, phase lag, velocity amplitude, tidal damping, residual slope and river discharge, this relationship can be of practical use to estimate, in an inverse way, river discharge on the basis of observed tidal water levels along the estuary axis. Of course our method also has its disadvantages. It requires an exponential shape (as is the case in alluvial estuaries), it requires that the M_2 is dominant over other tidal constituents, and there should be a measurable influence of the river discharge (river discharge and tidal discharge being within the same order of magnitude). It should also be realised that in convergent estuaries of infinite length there is no reflected wave (see also Jay, 1991), but that it is essentially a single wave moving in upstream direction with a phase shift that depends on convergence and damping (according to the phase lag equation in Table 2).

b. Theoretical foundation

The theoretical foundation of the TDE is also not explained here. It uses the tidal propagation theory for convergent channels of Jay et al. [1991]. The key assumptions are that: (a) there is no reflected wave; (b) the wave is critically convergent so that the real and imaginary part of the complex wave number are equal (i.e., the scale length for damping is the same as the inverse wave number); (c) the tidal velocity amplitude and river flow velocity are of the same order; and (d) the channel geometry does not change drastically with river flow. In practice, the last two assumptions are the most restrictive, though both can be stretched. With these assumptions it is simple to express the tidal admittance in terms of the wave number, which can then be represented using the Dronkers [1964] cubic Tschebychev polynomial. The latter allows the admittance to be expressed in terms of the river flow and tidal amplitude at the ocean entrance. The tidal range terms recognizes that the relationship between river flow and damping of the tides varies over the neap-spring cycle. This is important for hindcasting flows on the scale of days, but not for hindcasts based on windowed monthly harmonic analyses. The relationship between admittance, river and tidal range is nonlinear and cannot be exactly inverted, but approximate inversion is simple, especially when windowed monthly harmonic analyses -- this scale of time averaging allows the tidal range term to be dropped. In practice, the coefficients in the equation for TDE are fit by regression using a calibration data set. On the whole, the analysis is just as rigorous as that proposed here. In both cases, the nonlinear bedstress term is approximated, and one or more constants must be determined from data.

Our reply: We agree that the theoretical foundation of the proposed approach should be explained in more detail. We use the envelope theory developed by Savenije (2005, 2012) for tidal wave propagation. The analytical model is further expanded by Cai et al. (2014) to account for the influence of river discharge. The basic assumptions made in the analytical model are that: (a) the longitudinal cross-sectional area can be described by an exponential function; (b) there is no reflected wave; (c) the ratio of tidal amplitude to depth ratio is less than unity. For predicting fresh water discharge, it is also required that the river discharge is at least in the same order of magnitude as the maximum tidal flow. In fact, we can see from the derivation below that the proposed analytical relationship relating the tidal wave propagation to the fresh water discharge is **a modified Manning equation** that accounts for the effects of residual water level slope and tidal damping (see detailed derivation in Appendix A below). We have clarified these theoretical foundation in the new version of the manuscript (see paragraph 3 in the introduction part).

c. Practical application

The approach presented here finds a closed form equation, which is an advantage for application. On the other hand, it is not obvious that the envelope tidal theory used here would work in tidal rivers where mixed tides are prominent. All three Eastern Pacific systems (the Fraser and Columbia Rivers and San Francisco Bay) we have examined have mixed tides. it is also unclear whether the present method could be used to estimate river flow variations on a scale of days, as is possible through use of the TDE method with CWT determination of tidal properties. In conclusion, any new methodology benefits from diverse approaches, and this is a useful contribution.

Our reply: It should be noted that the tidal theory used in this paper only focuses on a single dominated constituent (e.g., M_2). It is not applicable to tidal rivers where mixed tides are prominent. We have explicitly mentioned this limitation in the text (see paragraph 3 in the introduction part). On the other hand, the proposed analytical approach can be used to predict daily fresh water discharge for given observed tidal damping and residual water level on a scale of day. We are planning to collect more detailed tidal records and fresh water discharge (daily scale) in order to test the performance of the proposed method.

2nd Reviewer's Comments

General comments:

1. The objective is to predict river discharge from observed tidal water levels. The method of the authors is limited to upstream sections where river discharge is dominated over tidal discharge. The example of Datong represents a station at 600 km from the mouth where the tidal range is 0.1-0.2 m (Fig. 5). The measured values may be easily disturbed by ship motions and other variations. The authors should explain why this topic is so important. River discharges are very well known from upstream data. Discharge-stage relationships based on data are available for most rivers. These are easy to use. The method of the authors is fairly complicated and it

will be difficult to determine for which river section it will be sufficiently accurate. Figure 11 shows that the model is not so accurate at low river discharges. The authors should comment on these outlyers.

Our reply: We appreciate the comments given by the reviewer. It is true that the proposed approach is only applicable to upstream sections where river discharge is dominated over tidal discharge. For the Yangtze estuary, the model is applicable to the river section upstream from around 350 km in the dry season and upstream from 150 km in the flood season. This limitation is due to the fact that the fresh water discharge is usually small compared to the amplitude of the tidal discharge in the seaward sections of an estuary, where the cross-sectional area is generally orders of magnitude larger than the cross-section of the river. Thus the influence of river discharge on tidal dynamics in these downstream parts is usually negligible, which suggests that there is no significant correlation between observed tidal water levels and fresh water discharge. We note that this is also the case for other methods to predict fresh water discharge in estuaries, such as by Jay and Kukulka (2003) and Moftakhari et al. (2013).

We agree that the relatively small values of the tidal range used in the analytical model could affect the performance of the proposed method. To reduce the statistical uncertainties, we used the monthly averaged tidal range in Maanshan (x=430 km) and Wuhu (x=482 km) stations and estimated the fresh water discharge at the location in between (i.e., x=456 km).

In the new version of the manuscript, we have added a paragraph in the introduction to clarify the importance of our work:
"Due to the general dominance of tidal flows in the tidal region of an estuary, it is often difficult to determine the magnitude of the fresh water discharge accurately. Thus, discharge gauging stations are usually situated at locations outside the tidal region, even though there may be additional tributaries or drainage areas within the tidal region. Knowing the fresh water discharge within the tidal region, however, may be important for water resource assessment or flood hazard prevention (e.g., Madsen and Skltner, 2005; Erdal and Karakurt, 2013; Liu et al., 2014), or for the analyses of sediment supply (e.g., Syvitski et al., 2003; Prandle, 2004; Wang et al., 2008), or for irrigation or estimating the effect of water withdrawals on salt intrusion (e.g., MacCready, 2007; Gong and Shen, 2011; Zhang et al., 2012), and for assessing the impacts of future climate change (e.g., Kukulka and Jay, 2003a, 2003b; Moftakhari et al., 2013). Although it is possible to estimate river flow by upscaling the gauged part of a catchment, such an estimate may be inaccurate, especially in poorly gauged catchments or in high-precipitation coastal areas (Jay and Kukulka, 2003)".

Meanwhile, we have provided more explanations of the proposed analytical approach. In fact, we can see that the introduced damping equation (i.e., Eq. T4 in Table 2) is **a modified 'Discharge-stage relationship'** that accounts for the effects of residual water level slope (i.e., $\mathrm{d}\bar{h}/\mathrm{d}x$, where $\bar{h}$ is the tidally averaged depth) and tidal damping (i.e., $\mathrm{d}\eta/\mathrm{d}x$, where η is the tidal amplitude), while the resulted predictive Eq. (25) is **a modified Manning equation** that is applicable to estuaries. A detailed derivation can be found in Appendix A below. It is worth mentioning that such a modified Manning's equation does provide more insights into our understanding of the interaction between fresh water discharge and tide in estuaries.

In Figure 11 of the previous manuscript, the deviation from observations is mainly due to the statistical uncertainties in estimating tidal damping $\delta = \frac{1}{\eta}\frac{\mathrm{d}\eta}{\mathrm{d}x}\frac{c_0}{\omega}$, which is rather sensitive to changes in observed tidal amplitudes. In the revised paper, we propose to use a moving average filter to reduce the statistical uncertainties in the observed tidal damping δ (see Figure R1a below). It can be seen from Figure R1b that the correspondence with observations is significantly improved by using a moving average of 5 months.

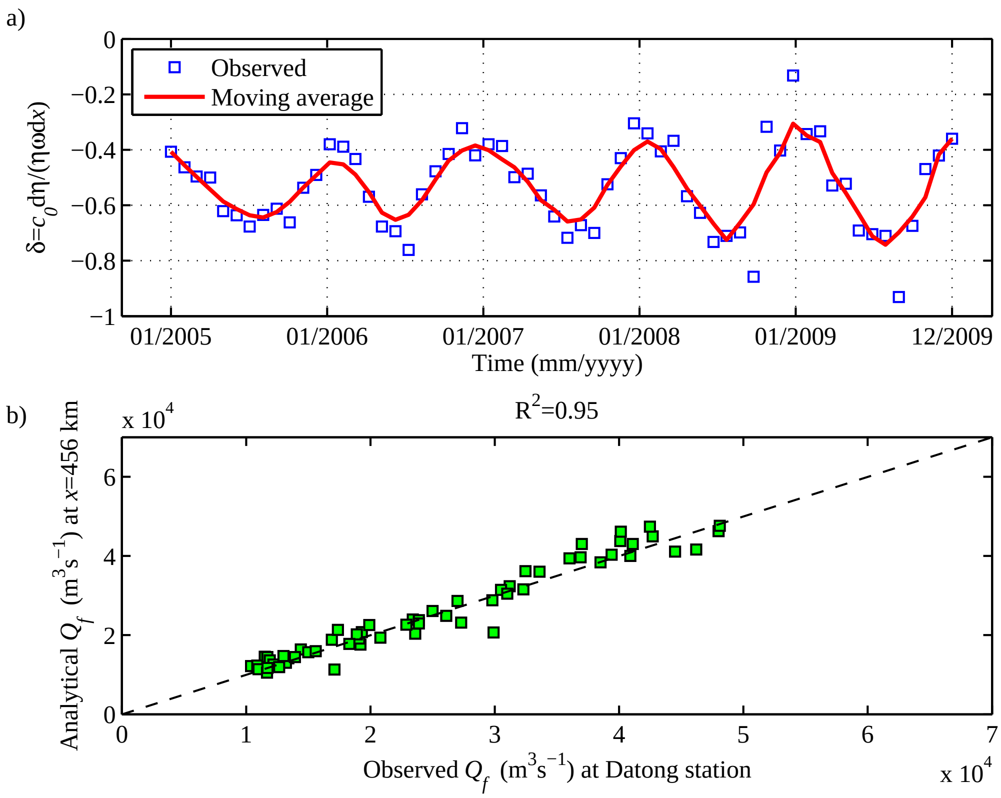

Figure R1. (a) Comparison between observed tidal damping δ and its corresponding moving average value with a window of 5 months; (b) Comparison between analytically predicted fresh water discharge and observations (at Datong tidal station) in the Yangtze estuary in different months of 2005–2009. R^2 is the coefficient of determination.

We also note that the cross-sectional area convergence a is no longer a constant at the studied position (x=456 km) due to the significant variation of the residual water level slope $\mathrm{d}\overline{h}/\mathrm{d}x$, which is implicitly included in the parameter of the parameter of a since $\frac{1}{a} = \frac{1}{b} + \frac{1}{d} = -\frac{1}{\overline{B}}\frac{\mathrm{d}\overline{B}}{\mathrm{d}x} - \frac{1}{\overline{h}}\frac{\mathrm{d}\overline{h}}{\mathrm{d}x}$. The seasonal variation of the a is given in Figure R2, where we see a larger value of a during wet season while a smaller value during dry season.

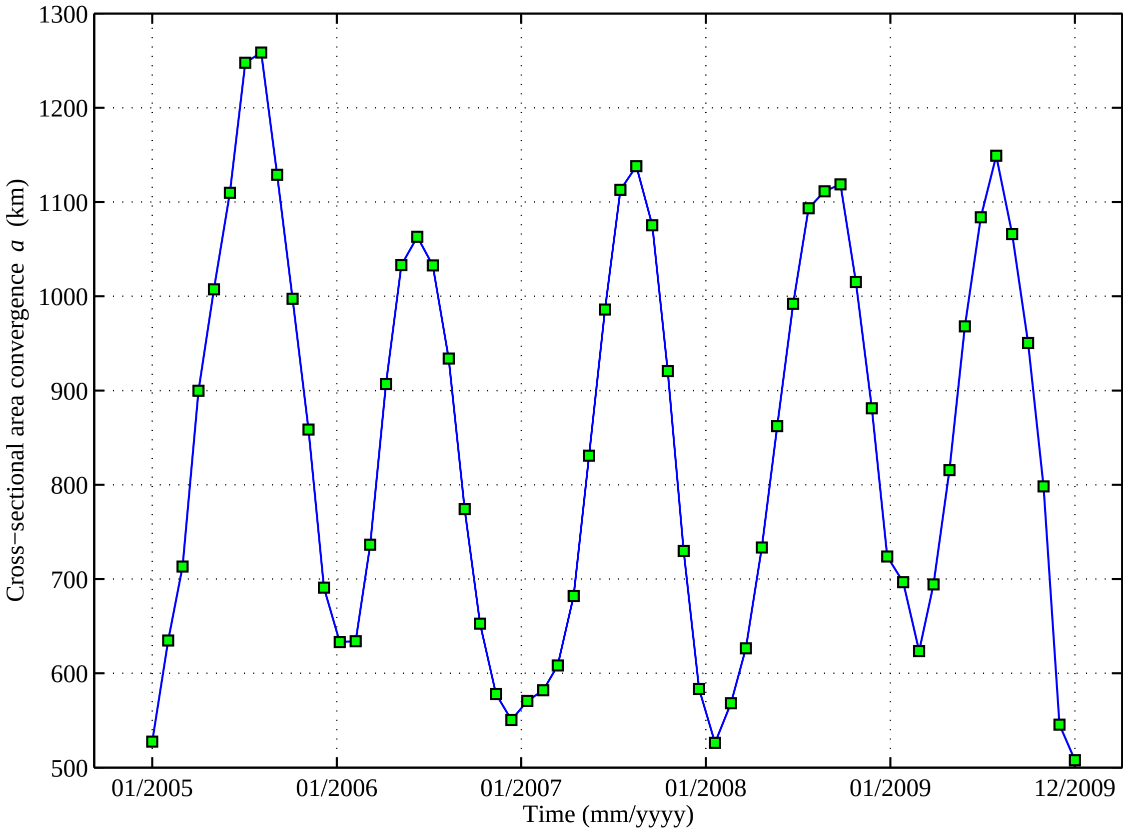

Figure R2. Seasonal variation of the cross-sectional area convergence *a* due to the changes in residual water level slope $\mathrm{d}\bar{h}/\mathrm{d}x$ at x=456 km in the Yangtze estuary.

2. The model equations can only be understood by a few specialists but not by a common reader. It is to the editor to decide whether the paper is intended for the audience of HESS. It is suggested to transfer all equations to an appendix. The text and figures should be given in physical descriptions and explanations. The model should be made available as e.g. a spreadsheet or otherwise (freeware) so that an interested reader can use and check the model. If the authors are unable to do so the I would advise to reject the paper (however to be decided by the editor).

Our reply: We apologize for the confusion of the many equations. It indeed takes time and effort to understand the whole story. Generally, this study is a subsequent contribution which builds on the previous work published in HESS entitled as: "Linking the river to the estuary: influence of river discharge on tidal damping" (Cai et al., 2014). Readers can obtain more details about the analytical model by reading this publication. We agree that the model should be made available for readers so that they could attempt to use the model. Two examples of Matlab scripts have been provided in the new version of the manuscript, including both the forward model (determing tidal properties from fresh water discharge) and the inverse model (determining fresh water discharge from tidal properties).

3. The authors should compare their model results to one-dimensional numerical

model results to show that their model is sufficiently accurate. 1D numerical models are widely available and easy to operate. A simple estuary can be modeled in a few days with such a model. The authors should made clear what are the advantages of their model compared to a 1D numerical model.

Our reply: We very much appreciate this comment, which was also raised in our previous publication in HESS, i.e., Cai et al., 2014. For detailed comparison between analytical results and 1D numerical model, readers can refer to Cai et al. (2014). Generally, the most important advantage of analytical tools is that they can offer a more efficient way of assessing the impact of future changes (e.g., fresh water withdrawal). Moreover, they provide direct insights into cause-effect relations, which generally are non-linear.

Specific comments:

1. Page 7064, line 6: please indicate what the phase lag is for a progressive wave. Do the authors refer to a frictionless progressive wave in a prismatic channel? The authors should further clarify whether the wave from their model is really progressive or not. In other words: is there only one wave travelling upstream or is there a second wave propagating in downstream direction due to continuous reflection by the convergence of the estuary. A discussion on this aspect would be very helpful in understanding tidal propagation in converging estuaries.

Our reply: Thank you very much for your suggestions. We have added a new paragraph to discuss the tidal character in convergent estuaries:
It is worth examining the tidal wave propagation in convergent estuaries with significant river discharge. We focus on analytical solutions for infinite length estuaries (long coastal plain estuaries), where there is no reflected wave (see also Jay, 1991). In this case, the value of the phase lag ε is always between 0 and $\pi/2$ (i.e., mixed wave, see Savenije, 2005, 2012). If $\varepsilon=\pi/2$, the tidal wave is a progressive wave, which corresponds to a frictionless wave in a prismatic channel. If $\varepsilon=0$, the tidal wave is an ``apparently standing” wave (the wave is not formally a standing wave generated by the superimposition of incident and reflected waves; rather it is an incident wave that mimics a standing wave with a phase difference of 90° between water level and velocity and a wave celerity tending to infinity).

2. Page 7064, line 16: It is no clear why the influence of river discharge is that of increasing friction (by comparing Eq. (19) with Eq. (14). This could be shown in more detail in the appendix.
Our reply: We shall clarify the difference between Eq. (19) and Eq. (14) by introducing an artificial friction number as $\chi_r=\kappa\chi$, where κ is a correction coefficient of the friction term due to river discharge. In particular, the following derivation will be included as an appendix in the revised paper.

In case of negligible river discharge, the damping equation is given by (see Cai et al., 2012a):

$$\delta=\frac{\mu^2}{1+\mu^2}\left[\gamma-\mu\lambda\chi\left(\frac{2}{3}\mu\lambda+\frac{8}{9\pi}\right)\right]. \quad (0.1)$$

To illustrate the influence of river discharge on the friction term, we introduce an artificial friction number χ_r due to river discharge. When accounting for the effect of river discharge, the damping Eq. (0.1) is modified as (see Cai et al., 2014):

$$\delta = \frac{\mu^2}{\beta\mu^2+1}\left[\gamma\theta - \mu\lambda\chi\left(\frac{2}{3}\mu\lambda+\frac{8}{9\pi}\right)\frac{\frac{2}{3}\mu\lambda\kappa_1+\frac{8}{9\pi}\kappa_2}{\frac{2}{3}\mu\lambda+\frac{8}{9\pi}}\right] = \frac{\mu^2}{\beta\mu^2+1}\left[\gamma\theta-\mu\lambda\left(\frac{2}{3}\mu\lambda+\frac{8}{9\pi}\right)\chi_r\right] \tag{0.2}$$

where β and θ are defined in Eq. (5) and the coefficients κ_1 and κ_2 are given by

$$\kappa_1 = \begin{cases} 1+\frac{8}{3}\zeta\frac{\varphi}{\mu\lambda}+\left(\frac{\varphi}{\mu\lambda}\right)^2 & \text{for } \varphi<\mu\lambda \\ \frac{4}{3}\zeta+2\frac{\varphi}{\mu\lambda}+\frac{4}{3}\zeta\left(\frac{\varphi}{\mu\lambda}\right)^2 & \text{for } \varphi\geq\mu\lambda \end{cases} \tag{0.3}$$

$$\kappa_2 = \frac{3\pi}{16}L_1 - \frac{\pi}{8}\frac{L_0\zeta}{\mu\lambda} \tag{0.4}$$

As can be seen from Eqs. (0.1) and (0.2), the influence of fresh water discharge is basically that of increasing friction by a factor which is a function of φ. Expressing the artificial friction number as $\chi_r=\kappa\chi$ provides an estimation of the correction of the friction term

$$\kappa = \frac{\chi_r}{\chi} = \frac{\frac{2}{3}\mu\lambda\kappa_1+\frac{8}{9\pi}\kappa_2}{\frac{2}{3}\mu\lambda+\frac{8}{9\pi}} \tag{0.5}$$

which is needed to compensate for the lack of considering fresh water discharge. It should be noted that both β and θ are equal to unity if $\varphi=0$. For $\varphi>0$, the correction factors θ and β have values smaller than unity, but are close to unity as long as $\zeta<<1$. Thus the influence of river discharge introduced by these parameters are less prominent compared with that of the friction term.

3. Page 7067, line 25: During calibration of the model the river discharge should be known. This is in contradiction to the conclusion that river discharges could be deduced from tidal water level observations only.

Our reply: Actually, there are two methods to determine the parameters r_S and K. If there are some measurements of fresh water discharge, then the parameters r_S and K can be determined by calibrating the analytical model (i.e., Eq. (25)) against observations. Otherwise, these two parameters can be obtained by calibrating the analytical model for tidal wave propagation without considering the effect of river discharge (e.g., Cai et al., 2012) against the observed tidal amplitude in the seaward part of the estuary, where the influence of river discharge on tidal damping is negligible. With these two calibrated parameters, the analytical model can be used to hindcast fresh water discharge based on the tidal water level observations. We have added a new paragraph to clarify this point in the revised paper.

4. Page 7068, Eq. (25): From Eq. (21) it follows that α_1 is always negative for relatively small values of ζ. If α_2 >0 (which is not trivial) then the solution given by

Eq. (25) is indeed positive (thus assuming ζ«1). Can the authors proof that the 2nd (positive) root never results in a real solution for?

Our reply: Indeed, α_1 is always negative (indicating the denominator of Eq. (25) is always negative). It should be noted that the critical value for ζ (tidal amplitude to depth ratio) is 0.75 due to the Taylor approximation of the exponent of the hydraulic radius in the friction term (see Eq. (4)). In fact, we can see from Eq. (22) that α_2 is also always negative since all the parameters are positive except δ for given $\zeta<0.75$. Consequently, $-\alpha_2+\sqrt{\alpha_2^2-4\alpha_1\alpha_3}$ is always positive. Thus the only positive solution can only be given by Eq. (25) with numerator of $-\alpha_2-\sqrt{\alpha_2^2-4\alpha_1\alpha_3}$. We have explicitly mentioned this in the revised paper.

Text comments:

We agree with the suggested corrections, which have been made in the revised paper. We thank the reviewer for the detailed reading.

Figure 5: how can the water depth decrease in upstream direction if there is a net river discharge? Table 3 suggests that a constant water depth for the 2 sections is being used (10.4 and 9.2 m). Or can the model handle a non-zero bed slope? Some explanation on this is required in the text.

Our reply: Thank you for pointing out this point. In Figure 5 of the previous manuscript, it presents the averaged water depth rather than the averaged water level. It can be seen from the Figure R3 below that the averaged water level would not decrease in upstream direction. In the new manuscript we used Figure R3 to avoid misunderstanding.

The averaged depths presented in Table 3 are only used to show the characterized depths over the corresponding reach. In the revised paper, we have clarified that the model uses a variable depth in order to account for along-channel variations of the estuarine sections.

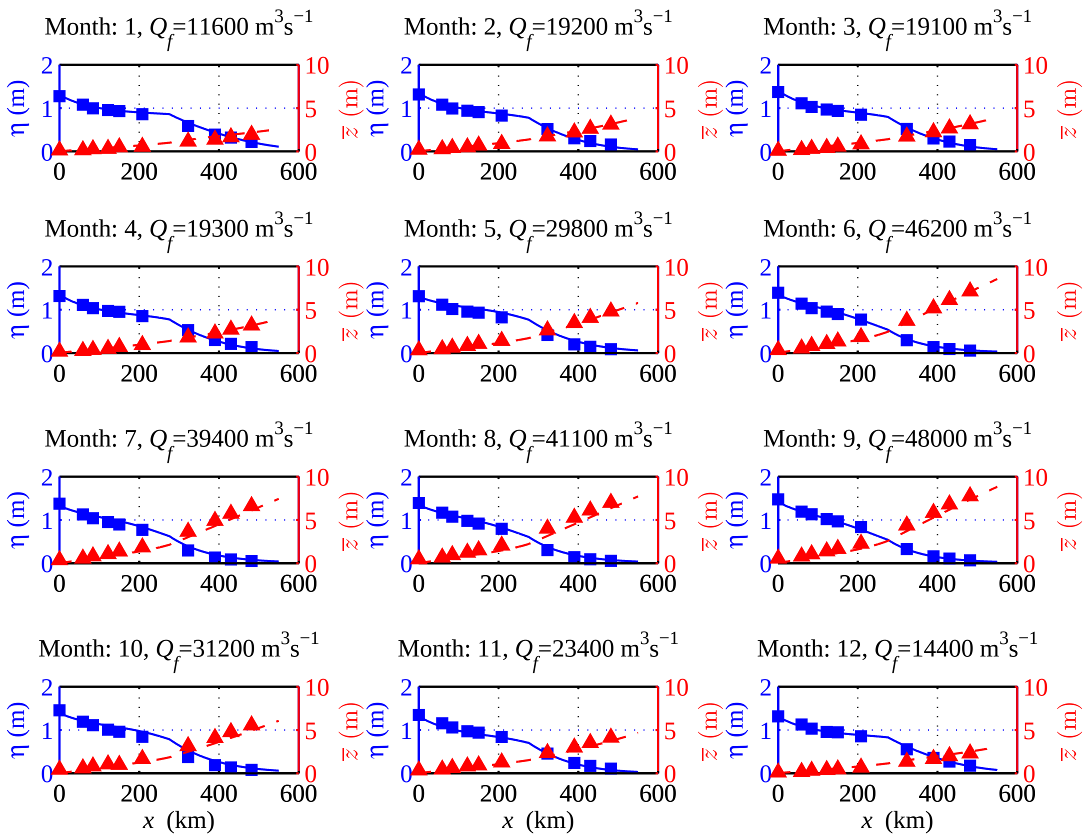

Figure R3. Comparison between analytically computed monthly-averaged values (left-hand vertical scale: tidal amplitude; right-hand vertical scale: residual water level) and observations in the Yangtze estuary in 2005.

We hope that these answers are satisfactory, and the revised manuscript will be acceptable for publication in *Hydrology and Earth System Sciences*. Thank you very much for your kind consideration.

With best regards,
Huayang Cai
Huub Savenije
Chenjuan Jiang

Appendix A: Revisiting the Manning equation

The momentum equation when written in a Lagrangean reference frame reads (Savenije, 2005, 2012):

$$\frac{\mathrm{d}V}{\mathrm{d}t}+g\frac{\partial h}{\partial x}+g\frac{\partial z_b}{\partial x}+g\frac{h}{2\rho}\frac{\partial \rho}{\partial x}+gn^2\frac{V\,|V\,|}{R^{4/3}}=0 \qquad \text{(R1)}$$

where h is the water depth, z_b is bottom elevation, ρ is the water density, n is Manning's coefficient, and R is the hydraulic radius.

For uniform steady flow in a prismatic channel, Eq. (R1) can be simplified as the well-known Manning equation by neglecting the first, the second and the fourth terms:

$$V = \frac{1}{n} R^{2/3} S^{1/2} \tag{R2}$$

where $S = -\partial z_b / \partial x$ is the slope of the channel.

Hence the expression for river discharge is given by:

$$Q_0 = AV = \frac{1}{n} AR^{2/3} S^{1/2} \tag{R3}$$

where A is the cross-sectional area.

For steady flow when depth may vary along a short section of the channel (e.g., during a flood), the residual water level slope ($\partial h / \partial x$) should be taken into account and Eq. (R1) reduces to:

$$\frac{\partial h}{\partial x} + \frac{\partial z_b}{\partial x} + n^2 \frac{V|V|}{R^{4/3}} = 0 \tag{R4}$$

Consequently, the Manning's equation (R2) is modified as:

$$V = \frac{1}{n} R^{2/3} \left(S - \frac{\partial h}{\partial x} \right)^{1/2} \tag{R5}$$

while the river discharge becomes:

$$Q_1 = Q_0 \left(1 - \frac{\partial h}{\partial x} \frac{1}{S} \right)^{1/2} \tag{R6}$$

In the Lagrangean reference frame, the continuity equation can be written as (see Savenije, 2005, 2012):

$$\frac{\mathrm{d}V}{\mathrm{d}t} = r_S \frac{cV}{h} \frac{\mathrm{d}h}{\mathrm{d}x} - cV \left(\frac{1}{b} - \frac{1}{\eta} \frac{\mathrm{d}\eta}{\mathrm{d}x} \right) \tag{R7}$$

where r_S is the storage width ratio, b is the convergence of width, c is the wave celerity.

In a tidal region, it is noted that both depth and discharge change along the channel axis (i.e., varied unsteady flow). Thus, Eq. (R1) when combined with (R7) becomes (see Savenije, 2005, 2012):

$$r_S \frac{cV}{h} \frac{\mathrm{d}h}{\mathrm{d}x} - cV \left(\frac{1}{b} - \frac{1}{\eta} \frac{\mathrm{d}\eta}{\mathrm{d}x} \right) + g \frac{\partial h}{\partial x} + g \frac{\partial z_b}{\partial x} + g \frac{h}{2\rho} \frac{\partial \rho}{\partial x} + g n^2 \frac{V|V|}{R^{4/3}} = 0 \tag{R8}$$

An analytical expression for the tidal damping can be obtained by subtracting high water (HW) and low water (LW) envelopes while accounting for the effect of river discharge (Cai et al., 2014):

in the downstream tide-dominated zone, where $U_r < \upsilon \sin(\varepsilon)$,

$$\frac{1}{\eta} \frac{\mathrm{d}\eta}{\mathrm{d}x} \left(\theta - r_s \frac{\varphi}{\sin(\varepsilon)} \zeta + \frac{g\eta}{c\upsilon \sin(\varepsilon)} \right) = \frac{\theta}{a} - f \frac{\upsilon}{\bar{h}c} \left(\frac{2}{3} \sin(\varepsilon) + \frac{16}{9} \varphi\zeta + \frac{2}{3} \frac{\varphi^2}{\sin(\varepsilon)} + \frac{L_1}{6} - \frac{L_0}{9} \frac{\zeta}{\sin(\varepsilon)} \right) \tag{R9}$$

in the upstream river discharge-dominated zone, where $U_r \geq \upsilon \sin(\varepsilon)$,

$$\frac{1}{\eta} \frac{\mathrm{d}\eta}{\mathrm{d}x} \left(\theta - r_s \frac{\varphi}{\sin(\varepsilon)} \zeta + \frac{g\eta}{c\upsilon \sin(\varepsilon)} \right) = \frac{\theta}{a} - f \frac{\upsilon}{\bar{h}c} \left(\frac{8}{9} \zeta \sin(\varepsilon) + \frac{4}{3} \varphi + \frac{8}{9} \frac{\varphi^2}{\sin(\varepsilon)} \zeta + \frac{L_1}{6} - \frac{L_0}{9} \frac{\zeta}{\sin(\varepsilon)} \right) \tag{R10}$$

When river discharge dominates over tide ($\varphi \geq 1$), it is noted that the coefficients L_0 and L_1 can be calculated according to Eq. (12). Substituting Eq. (12) into Eq. (R10) then yields a quadratic equation for the dimensionless river discharge φ:

$$\sigma_1 \varphi^2 + \sigma_2 \varphi + \sigma_3 = 0 \tag{R11}$$

with

$$\sigma_1 = -\frac{4}{3}\frac{f\upsilon a\zeta}{\bar{h}c\sin(\varepsilon)} \tag{R12}$$

$$\sigma_2 = \frac{1}{\eta}\frac{\mathrm{d}\eta}{\mathrm{d}x}\frac{r_S a\zeta}{\sin(\varepsilon)} - 2\frac{f\upsilon a}{\bar{h}c} + \left(\frac{1}{\eta}\frac{\mathrm{d}\eta}{\mathrm{d}x}a - 1\right)\frac{\sqrt{1+\zeta}-1}{\sin(\varepsilon)} \tag{R13}$$

$$\sigma_3 = -\frac{f\upsilon a}{\bar{h}c}\left[\frac{8}{9}\zeta\sin(\varepsilon) + \frac{2}{9}\frac{\zeta}{\sin(\varepsilon)}\right] - \frac{1}{\eta}\frac{\mathrm{d}\eta}{\mathrm{d}x}a\left[1 + \frac{g\eta}{c\upsilon\sin(\varepsilon)}\right] \tag{R14}$$

where the unknown variables ε, c, υ can be calculated with the explicit equations (i.e., the phase lag equation, the celerity equation and the scaling equation in Table 2) for given water level observations.

Eq. (R11) gives two solutions:

$$\varphi_1 = \frac{-\sigma_2 + \sqrt{\sigma_2^2 - 4\sigma_1\sigma_3}}{2\sigma_1}, \quad \varphi_2 = \frac{-\sigma_2 - \sqrt{\sigma_2^2 - 4\sigma_1\sigma_3}}{2\sigma_1} \tag{R15}$$

in which the first root is always negative since both σ_1 and σ_2 are always negative. Hence the positive solution for φ can only be given by the second root, which can be rewritten as:

$$U_r = \upsilon\frac{-\sigma_2 - \sqrt{\sigma_2^2 - 4\sigma_1\sigma_3}}{2\sigma_1} \tag{R16}$$

We can see that Eq. (R16) is actually a modified Manning equation, accounting for friction and the effects of residual water level slope (i.e., $\mathrm{d}\bar{h}/\mathrm{d}x$ implicitly included in the parameter of the cross-sectional area convergence a since $\frac{1}{a} = \frac{1}{b} + \frac{1}{d} = -\frac{1}{\bar{\bar{B}}}\frac{\mathrm{d}\bar{B}}{\mathrm{d}x} - \frac{1}{\bar{\bar{h}}}\frac{\mathrm{d}\bar{h}}{\mathrm{d}x}$) and tidal damping (i.e., $\mathrm{d}\eta/\mathrm{d}x$).

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