Technical Note: Comments on “On the Matt-Shuttleworth approach to estimate crop water requirements”

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Abstract

It is clear from Lhomme et al (2014) that aspects of the explanation of the Matt-Shuttleworth approach can generate confusion. Presumably this is because the description in Shuttleworth (2006) was not sufficiently explicit and simple. This paper explains the logic behind the Matt-Shuttleworth approach clearly, simply and concisely. It shows how the Matt-Shuttleworth can be implemented using a few simple equations and provides access to ancillary calculation resources that can be used for such implementation. If the crop water requirement community decided that it is preferable to use the Penman-Monteith equation to estimate crop water requirements directly for all crops, the United Nations Food and Agriculture Organization could now update Irrigation and Drainage Paper 56 using the Matt-Shuttleworth approach by deriving tabulated values of surface resistance from Table 12 of Allen et al (1998), with the estimation of crop evaporation then being directly made in a one-step calculation using an equation similar to that already recommended by the United Nations Food and Agriculture Organization for calculating reference crop evaporation.

1. Introduction

It is clear from Lhomme et al (2014) that aspects of the explanation of the Matt-Shuttleworth approach can generate confusion. Presumably this is because the description in Shuttleworth (2006) was not sufficiently explicit and simple. I welcome the opportunity to redress this and I am grateful to Lhomme et al (2014) for bringing this need to my attention.

The fundamental premise of the Matt-Shuttleworth approach is that describing evapotranspiration from a crop canopy using the Penman-Montieth Equation (Monteith, 1964) [hereafter referred to as PM] is theoretically superior to describing evapotranspiration using the formula given (for example) as Equation (1) of Lhomme et al (2014) [hereafter referred to as FAO]. Justifications for this premise are as follows.

i. It is now widely accepted In advanced models of surface-atmosphere energy exchanges that the control exerted by vegetation can be represented by a plant-dependent surface resistance, r_{sc}, which is broadly equivalent to the canopy average effect of stomatal resistance, and that the control exerted by turbulent transfer can be represented by an aerodynamic resistance, r_{ac}, dependent on wind speed and aerodynamic properties of the canopy. PM merges these two resistances with the surface energy balance. In such advanced models r_{sc} is either assumed fixed or dependent on in-canopy environmental variables and soil moisture if these affect stomatal resistance.

ii. The capability of PM to describe crop evapotranspiration is now explicitly accepted as being appropriate by the crop water requirement community because the United Nations Food and Agriculture Organization recommends that reference crop evaporation is calculated from PM (Allen et al, 1998).

2. How is surface resistance calculated in the Matt-Shuttleworth approach?

In the Matt-Shuttleworth approach r_{sc} is a crop-dependent, “effective” value for each day during the crop growth cycle. The value of r_{sc} for each day would ideally be determined from a seasonal model that has been calibrated by field experiment (in much the same way that the fixed value 70 s m^{-1} was defined for the reference crop). Shuttleworth (2006) recommends this, but then seeks to make interim estimates of r_{sc} from the seasonal models of K_c in the existing literature. Presumably there must be environmental conditions when there is a definable pairing between the effective values of K_c and r_{sc}, specifically the prevailing meteorological conditions when the field experiment to determine K_c was carried out. If these meteorological conditions were known then the same data used to specify the particular value of K_c relevant in these conditions could alternatively be used to specify the equivalent value of r_{sc} used in the Matt-Shuttleworth approach. But unfortunately the meteorological conditions when tabulated values of K_c were defined are not available, hence assumptions are required.

Allen et al (1998) deem the tabulated values of K_c most reliable in “sub-humid conditions” for wind speed 2 m s^{-1}, and provide an empirical formula to correct K_c for crops with different heights exposed to different wind speeds in different atmospheric aridity conditions. [Note that such an empirical correction is not required when using the Matt-Shuttleworth approach because crop-specific aerodynamics rather than reference crop-specific aerodynamics are used.] In the Matt-Shuttleworth approach the preferred wind speed of 2 m s^{-1} is adopted, but specification of “sub-humid” is also required to establish pairing between the tabulated value of K_c on a particular day and required value of r_{sc}. Allen et al (1998) specify sub-humid conditions as being when average minimum daytime relative humidity is 45%. However, this specification does not recognize the long established fact that both available energy and vapor pressure deficit control evapotranspiration (Penman, 1948), which suggests that characterizing the influence of atmospheric humidity relative to the influence of available energy is arguably a more appropriate way to define the meaning of sub-humid conditions. For this reason, the climatological resistance, r_{clim}, is adopted as a measure of atmospheric aridity in the Matt-Shuttleworth approach, this being calculated from:

$$r_{c\,\mathrm{lim}} = \frac{\rho c_p D_2}{\Delta A} \qquad \text{or} \qquad r_{c\,\mathrm{lim}} \approx \frac{187250\gamma D_2}{\left(275 + T_2^C\right)\Delta A_{mm}} \qquad \mathrm{s\ m^{-1}} \qquad (1)$$

where ρ (kg m^{-3}) is the density of air, c_p (J kg^{-1} K^{-1}) is the specific heat of moist air, D_2 (kPa) is the vapor pressure deficit and T_2^C (°C) the air temperature both at 2m, Δ (kPa K^{-1}) is the rate of change of saturated vapor pressure with temperature, γ (kPa K^{-1}) is the psychrometric constant, and A (W m^{-2}) is the available energy which becomes A_{mm} (mm) when expressed as evaporated water equivalent. The pairing between the tabulated value of K_c and value of r_{sc} is then defined by specifying r_{clim} in sub-humid conditions when the tabulated value of K_c in Equation (1) of Lhomme et al (2014) provides its best estimate of crop evapotranspiration.

Shuttleworth (2006) does not assume that reference crop evapotranspiration rate, E_o, is equal to the Priestley-Taylor estimate of evapotranspiration rate, E_{PT}, every day. This is obviously not an acceptable assumption because, as shown in Equation (23.20) of Shuttleworth (2012), the relationship between E_o and E_{PT} on a particular day can in fact be expressed as a function of r_{clim} on that day. However, the condition $E_o = E_{PT}$ is used to specify the *default* value of r_{clim} considered characteristic of sub-humid conditions (i.e., the ratio of ($\rho c_p D$) to (ΔA) in sub-humid conditions). One argument for selecting this as a default condition is the history behind the calculation of reference crop evaporation. The original approach in Doorenbus and Pruit (1977) - who largely defined the tabulation of K_c - allowed calculation of reference crop evaporation in several different ways, including as E_{PT}. Later Allen et al (1998) added the calculation based on PM which is here referred to as E_o. If either K_cE_o or K_cE_{PT} can be used to calculate crop evapotranspiration optimally in sub-humid conditions, then $E_o = E_{PT}$ can presumably be used to specify the ratio of ($\rho c_p D$) to (ΔA) (i.e., the value of r_{clim}) in these sub-humid conditions. In addition, Shuttleworth (2006) describes modeling studies of feedback interactions between the atmospheric boundary layer and surface exchanges at regional scale which suggest there is a range of surface resistances and atmospheric aridity when the concept of "potential evaporation" applies, and when there is at least approximate equality between E_{PT} and evapotranspiration calculated from PM for a range of surface resistances typical of pastureland and many agricultural crops, including 70 s m^{-1}.

Specifying the default value of r_{clim} in sub-humid conditions from $E_o = E_{PT}$ ultimately leads to the result that r_{sc} should be calculated from K_c for a preferred value of r_{clim} given by:

$$r_{\text{clim}}^{\ pref} = 104\left(1.26\frac{\Delta + 1.67\gamma}{\Delta + \gamma} - 1\right) \qquad \text{s m}^{-1} \qquad (2)$$

To calculate the required value of r_{sc} Shuttleworth (2006) then simply adopts the definition of $K_c = \alpha_a\alpha_s$ given (for example) as Equation (7) of Lhomme et al (2014) and inverts this to express r_{sc} as a function of K_c, but with the ratio of ($\rho c_p D$) to (ΔA) set equal to the (now defined) value of r_{clim}^{pref} in sub-humid conditions. However, in order to allow this calculation for crops with height greater than 2 m the interrelationship between r_{sc} and K_c is first recast to a height of 50 m, and appropriate forms of aerodynamic resistances and vapor pressure used, these being defined using equation (24) with Z = 50m, and equations (27) and (29) in Shuttleworth (2006).

However, equation (2) means the default value of r_{clim} when r_{sc} should be calculated is not yet fully specified because Δ it is a function of temperature. Ideally the temperature selected to calculate Δ would be that when K_c was calibrated, but this temperature is generally not known. For this reason Shuttleworth and Wallace (2010) investigated the sensitivity of the calculation of r_{sc} to temperature and on this basis recommended using 20°C, which implies r_{sc} is optimally calculated from K_c when r_{clim}^{pref} is 55 s m^{-1}. For this value of r_{clim}^{pref} and a 2m wind speed of 2 m s^{-1}, the second "advective" term in the numerator of the equation used to calculate E_o is, for example, around 50% of the first "radiation" term; and for this value of r_{clim}^{pref} the Allen et al (1998) criteria that relative humidity is greater than 45% is met at 20 °C for A_{mm} values in the range zero to 7 mm day^{-1}. By using the preferred values of temperature (20 °C) and wind speed (2 m s^{-1}) and assuming a pressure of 100 k Pa in equations (23.34), (23.35) and (23.36) of Shuttleworth (2012) then simplifying, it can be shown that:

$$r_{sc} = \frac{1.2614R_c^{50} + 168.66}{K_c} - 1.5881R_c^{50} \qquad \text{s m}^{-1} \qquad (3)$$

where

$$R_c^{50} = 3.56\ln\left[\frac{(50 - 0.67h_c)}{0.123h_c}\right]\ln\left[\frac{(50 - 0.67h_c)}{0.0123h_c}\right] \qquad \text{(no units)} \qquad (4)$$

with h_c being the height of the crop. Table 23.5 of Shuttleworth (2012) gives values of r_{sc} calculated from selected values of K_c and h_c representative of tabulated values during Stage 3 of crop growth. For the hypothetical 1m high crop considered in Lhomme et al (2014), *assuming* the crop factor K_c = 1 was calibrated in conditions when the value of preferred climatological resistance had the default value 55 s m^{-1}, the fixed value of surface resistance estimated using Equations (3) and (4) is 111 s m^{-1} (but see below).

It is important readers understand that the use of the value r_{clim}^{pref} = 55 s m^{-1} derived from $E_o = E_{PT}$ is *a default assumption* whose use is recommended when the meteorological conditions prevailing when the values of K_c given in Allen et al (1998) were calibrated *are not known*. In fact the Matt-Shuttleworth approach is easily adapted to fine tune estimates of r_{sc} if additional information on or assumptions about the conditions when values of K_c were calibrated are made. To do this the calculation of r_{sc} is merely made using the value r_{clim}^{pref} relevant during the period of calibration.

If, for example, it is known or if it can be safely assumed that the value $K_c = 1$ on a particular day in the season for the 1m high hypothetical crop considered by Lhomme et al (2014) *had been calibrated in the sub-humid conditions that they specify*, then it is the value of climatological resistance in these specified conditions that should be used as the preferred value when calculating r_{sc} using the Matt-Shuttleworth approach. For the purpose of illustration, assume the clear sky conditions sub-humid conditions adopted by Lhomme et al (2014) prevailed when this calibration was made, that the crop had an albedo of 23% and the temperature was 20 °C and wind speed 2 ms^{-1}. In this case, with net longwave radiation estimated from equation (5.22) in Shuttleworth (2012), the preferred value of climatological resistance to be used when calculating r_{sc} from K_c would be 35.5 s m^{-1} (corresponding to a Priestly-Taylor $\alpha = 1.107$), and the corresponding equation used to calculate r_{sc} from K_c would be:

$$r_{sc} = \frac{1.4349 R_c^{50} + 116.27}{K_c} - 1.5881 R_c^{50} \qquad \text{s m}^{-1}$$

Consequently the value of r_{sc} for this crop on this day would be 89 s m^{-1}.

Similarly if the values of K_c and h_c given by Allen et al (1998) during stage 3 for cassava (in year one 0.8 and 1.0 m; and in year two 1.1 and 1.5m, respectively), banana (in year one 1.1 and 3.0 m, and in year two 1.2 and 4.0 m, respectively), and millet (1.0 and 1.5m, respectively) were assumed to have been calibrated in these same sub-humid conditions, then the equivalent values of r_{sc} would be for cassava 182 s m^{-1} and 61 s m^{-1} in years one and two, respectively; for banana 70 s m^{-1} and 53 s m^{-1} in years one and two, respectively; and for millet 92 s m^{-1}. These values of r_{sc} when applied in equation (5) in the same sub-humid conditions of course give the same estimates of evapotranspiration as FAO estimates in these conditions, as they should since both the value of K_c and (in effect) r_{sc} are assumed calibrated in these conditions. In different conditions the two estimates will differ to some extent, not least because the two approaches make different assumptions regarding crop aerodynamics. A similar approach could be used to derive r_{sc} for crops that can be safely assumed to have had K_c calibrated in semi-arid conditions.

3. How is surface resistance applied in the Matt-Shuttleworth Approach?

Because some crops have a crop height greater than 2m, it is preferable to use the value of r_{sc} in a version of PM which applies for a reference height of 50m. This version of PM can be written in a form similar to that recommended by the United Nations Food and Agriculture Organization for calculating reference crop evaporation, see equation (23.37) of Shuttleworth (2012), thus:

$$ET_C = \left(\frac{\Delta}{\Delta + \gamma^{**}}\right) A_{mm} + \left(\frac{\gamma}{\Delta + \gamma^{**}}\right)\left(\frac{187250}{275 + T_2^C}\right)\left(\frac{D_{50}}{D_2}\right)\frac{u_2 D_2}{R_c^{50}} \qquad \text{mm} \qquad (5)$$

where u_2 (m s^{-1}) and D_2 (k Pa) are the wind speed and vapor pressure deficit at 2m, $\gamma_m^{**} = \gamma(1 + r_{sc}u_2/R_c^{50})$, and

$$\left(\frac{D_{50}}{D_2}\right)=\left(\frac{(\Delta+\gamma)302+70\gamma u_2}{(\Delta+\gamma)208+70\gamma u_2}\right)+\frac{1}{r_{\mathrm{clim}}}\left[\left(\frac{(\Delta+\gamma)302+70\gamma u_2}{(\Delta+\gamma)208+70\gamma u_2}\right)\left(\frac{208}{u_2}\right)-\frac{302}{u_2}\right] \quad \text{(no units)} \qquad (6)$$

Note that in equation (6) r_{clim} is the value calculated by equation (1) *using measured values of weather variables on the day that ET_c is calculated*, not r_{clim}^{pref}.

4. Concluding Comments

If the crop water requirement community decided that it is preferable to use PM to estimate crop water requirements directly for all crops, the United Nations Food and Agriculture Organization could now update Irrigation and Drainage Paper 56 using the Matt-Shuttleworth approach, in default conditions using equation (3) to derive tabulated values of r_{sc} from Table 12 of Allen et al (1998), with a one-step estimate of ET_c then directly made from equations (5). However, if there is a decision to update, arguably the first step should be to define specific values of r_{clim} corresponding to sub-humid and semi-arid conditions by also specifying the available energy and temperature in such conditions, then to attempt to define for which crops it should be assumed the calibration of K_c was made in sub-humid, semi-arid, and default conditions.

To facilitate the application of the Matt-Shuttleworth approach I provide two Excel spreadsheets (amongst other files) at http://www.hwr.arizona.edu/~shuttle/Terrestrial_Hydrometeorology/ which are ancillaries to this paper. The first spreadsheet uses equations (3) and (4) to duplicate the calculations of r_{sc} in Table 23.5 of Shuttleworth (2012): it can be modified to make calculations for other combinations of K_c and h_c. The second spreadsheet is edited from that used to calculate Table 23.6 of Shuttleworth (2012) and makes example calculations of ET_c using the Matt-Shuttleworth approach, i.e. equations (5) and (6), and also using the traditional FAO method for several example crops (hypothetically) growing at three example sites (Oxford, Tucson, and Manaus) on three example days. It can be modified to make (or test) such calculations with alternative data from alternative sites.

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