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3	Technical note:
4	On the Matt-Shuttleworth approach
5	to estimate crop water requirements
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1 Abstract The Matt-Shuttleworth method provides a way to make a one-step estimate of crop water requirements with the Penman-Monteith equation by translating the crop coefficients, 2 3 commonly available in FAO publications, into equivalent surface resistances. The methodology is based upon the theoretical relationship linking crop surface resistance to crop coefficient and 4 involves the simplifying assumption that the reference crop evapotranspiration  $(ET_0)$  is equal to 5 the Priestley-Taylor estimate with a fixed coefficient of 1.26. This assumption, used to eliminate 6 7 the dependence of surface resistance on certain weather variables, is questionable: numerical 8 simulations show that it can lead to substantial differences between the true value of surface 9 resistance and its estimate. Consequently, the basic relationship between surface resistance and crop coefficient, without any assumption, appears to be more appropriate for inferring crop surface 10 resistance, despite the interference of weather variables. 11

Keywords Reference crop evapotranspiration, Crop coefficient, Surface resistance, Penman-Monteith
equation, Matt-Shuttleworth approach, One-step approach.

## 14 **1. Introduction**

The most common way of estimating crop water requirements, as recommended by the United Nations Food and Agriculture Organization (FAO) (Doorenbos and Pruitt, 1977; Allen et al., 1998), consists in the so-called "two-step" approach: first, a reference crop evapotranspiration ( $ET_0$ ), defined under optimal conditions, is calculated from weather data measured at a reference height; second, evapotranspiration from any other well-watered crop ( $ET_c$ ) is obtained by multiplying the reference evapotranspiration by an empirical crop coefficient  $K_c$ . The basic relationship writes

$$ET_c = K_c ET_0 \quad . \tag{1}$$

The effect of weather conditions is supposed to be incorporated into  $ET_0$  and the crop characteristics into  $K_c$ . The estimated values of crop coefficients exist in tabulated form and can be found in many FAO publications. Although the methods used to define and calculate  $ET_0$  have changed along the years (Shuttleworth, 1993), FAO-56 (Allen et al., 1998) presently defines  $ET_0$  as the daily evapotranspiration from "a hypothetical reference crop with an assumed crop height of 0.12 m, a fixed surface resistance  $r_{s,0} = 70$  s m<sup>-1</sup> and an albedo of 0.23", calculated by means of the Penman-Monteith equation (Monteith, 1965)

$$ET_0 = \frac{\Delta A_0 + \rho c_p D_r / r_{a,0}}{\Delta + \gamma (1 + \frac{r_{5,0}}{r_{a,0}})}$$
(2)

3  $A_0 = R_{n,0}$  -  $G_0$  is the available energy of the reference crop ( $R_{n,0}$ : net radiation;  $G_0$ : soil heat flux);  $D_r$  is the water vapour pressure deficit at a reference height  $z_r = 2$  m (screen height for weather data 4 5 measurements);  $r_{a,0}$  is the aerodynamic resistance calculated between the mean canopy source height 6 and the reference height and the other parameters are defined in the nomenclature. It is specified that 7 "the reference surface closely resembles an extensive surface of green grass of uniform height, 8 actively growing, completely shading the ground and with adequate water". The "one-step" approach, 9 as opposed to the "two-step" approach, consists in estimating crop evapotranspiration directly from a Penman-Monteith equation similar to Eq. (2), with the effective surface resistance of the crop used in 10 11 replacement of the crop coefficient. Two main problems arise, however, in using the one-step method. 12 First, several crops having a crop height close to (or greater than) the reference height of 2 m, a means should be designed to infer weather variables at a higher level than the reference height to be 13 14 introduced in the Penman-Monteith equation. Second, the surface resistance is generally unknown for 15 most of crops and should be determined, either experimentally or by calculation.

The Matt-Shuttleworth (M-S) approach (Shuttleworth, 2006, 2012) provides a response to both questions: it infers weather variables at a blending height higher than the screen height and it calculates crop surface resistance from FAO crop coefficient. These two steps are first summarized, stressing that the way the M-S approach infers crop surface resistance relies on a questionable assumption concerning the estimation of  $ET_0$ . Numerical simulations are carried out to prove that this assumption can be partially misleading. As a consequence, some conclusions are drawn on the applicability and reliability of the Matt-Shuttleworth one-step method.

## 23 **2.** Inferring weather variables at a higher level

1 In the Matt-Shuttleworth approach, the evapotranspiration from a given crop under standard 2 conditions (i.e., unstressed vegetation, as defined in FAO-56), is expressed in the form of a Penman-3 Monteith equation, but with air characteristics taken at a blending height arbitrarily set at  $z_b = 50$  m 4 (Shuttleworth, 2006, 2007)

$$ET_c = \frac{\Delta A_c + \rho c_p D_b / r_{a,c}}{\Delta + \gamma (1 + \frac{r_{s,c}}{r_{a,c}})}$$
(3)

5  $A_c$  is the available energy of the crop and  $r_{s,c}$  is the crop surface resistance, which is unknown and 6 should be determined.  $D_b$  is the water vapour pressure deficit at the blending height obtained by 7 expressing  $ET_0$  in two different forms, with weather variables taken respectively at blending height  $z_b$ 8 (= 50 m) and reference height  $z_r$  (= 2 m), and by assuming that there is no significant divergence of 9 mass and energy fluxes between the reference height and the blending height (Shuttleworth, 2006)

$$\frac{\Delta A_0 + \rho c_p D_b / r_{a,0,b}}{\Delta + \gamma (1 + \frac{r_{s,0}}{r_{a,0,b}})} = \frac{\Delta A_0 + \rho c_p D_r / r_{a,0}}{\Delta + \gamma (1 + \frac{r_{s,0}}{r_{a,0}})} \quad .$$
(4)

10 The resistance  $r_{a,0,b}$  is the aerodynamic resistance between the reference crop and the blending height 11 and  $\Delta$  is calculated at the reference temperature  $T_r$ . Some mathematical manipulations of Eq. (4) lead 12 to

$$D_b = \left(D_r + \frac{\Delta A_0 r_{a,0}}{\rho c_p}\right) \left[\frac{(\Delta + \gamma) r_{a,0,b} + \gamma r_{s,0}}{(\Delta + \gamma) r_{a,0} + \gamma r_{s,0}}\right] - \frac{\Delta A_0 r_{a,0,b}}{\rho c_p} \quad .$$

$$\tag{5}$$

The crop aerodynamic resistance  $r_{a,c}$  (see Eq. 16) is calculated from the wind speed at blending height ( $u_b$ ), which is inferred from the one measured at reference height ( $u_r$ ) assuming there is no divergence of momentum flux between these two heights

$$u_{b} = u_{r} \frac{ln\left(\frac{z_{b} - d_{0}}{z_{om,0}}\right)}{ln\left(\frac{z_{r} - d_{0}}{z_{om,0}}\right)} ,$$
(6)

where  $d_0$  is the zero plane displacement height of the reference crop and  $z_{0m,0}$  its roughness length for momentum.

# 3. Inferring crop surface resistance from FAO crop coefficient

2 The evapotranspiration from any given crop ET<sub>c</sub> (Eq. 3) can be expressed as a function of the
3 reference evapotranspiration ET<sub>0</sub> (Eq. 2) in the following way (Pereira et al., 1999, Eq. 25;
4 Shuttleworth, 2006, Eq. 10)

$$ET_c = \alpha_a \alpha_s ET_0 \quad , \tag{7}$$

5 where the coefficients  $\alpha_a$  and  $\alpha_s$  are given by

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$$\alpha_a = \frac{\Delta f_c A_0 r_{a,c} + \rho c_p D_b}{\Delta A_0 r_{a,0} + \rho c_p D_r} \quad , \tag{8}$$

$$\alpha_s = \frac{(1 + \Delta/\gamma)r_{a,0} + r_{s,0}}{(1 + \Delta/\gamma)r_{a,c} + r_{s,c}}.$$
(9)

6 The parameter  $f_c = A_c /A_0$  allows for differences in available energy between the crop  $(A_c)$  and the 7 reference crop  $(A_0)$ . Comparing Eq. (7) with Eq. (1) leads to  $K_c = \alpha_a \alpha_s$ , from which the crop surface 8 resistance can be inferred

$$r_{s,c} = \frac{\alpha_a}{K_c} \left[ \left( 1 + \frac{\Delta}{\gamma} \right) r_{a,0} + r_{s,0} \right] - \left( 1 + \frac{\Delta}{\gamma} \right) r_{a,c} \,. \tag{10}$$

9 The coefficient α<sub>a</sub> can be rewritten in a different way by introducing the "equilibrium" resistance r<sub>s,e</sub>.
10 defined as (Pereira et al., 1999, Eq. 16)

$$r_{s,e} = \frac{\rho c_p}{\gamma} \frac{\Delta + \gamma}{\Delta} \frac{D_r}{A_0} , \qquad (11)$$

11 which is slightly different from the "climatological" resistance ( $r_{clim}$ ) used by Shuttleworth (2006) ( $r_{s,e}$ 12 =  $(1 + \Delta/\gamma)r_{clim}$ ). Taking into account Eq. (5) and expressing  $\alpha_a$  as a function of  $r_{s,e}$  lead to

$$\alpha_a = (1 + \Delta/\gamma) \frac{f_c r_{a,c} - r_{a,0,b}}{r_{s,e} + (1 + \Delta/\gamma) r_{a,0}} + \frac{r_{s,0} + (1 + \Delta/\gamma) r_{a,0,b}}{r_{s,0} + (1 + \Delta/\gamma) r_{a,0}} .$$
(12)

13 The introduction of the equilibrium resistance  $r_{s,e}$  into Eq. (12) allows the weather variables linked to 14 radiation balance  $(A_0)$  and air moisture  $(D_r \text{ and } D_b)$  to be encompassed into a unique parameter. Eq.

1 (10) constitutes the basic relationship linking crop surface resistance to crop coefficient. It shows that  $r_{s,c}$  is not a unique function of  $K_c$ , but also depends on weather data: water vapour pressure deficit  $(D_r)$ , 2 3 net radiation (A<sub>0</sub>), wind speed through the aerodynamic resistances ( $r_{a,0}$ ,  $r_{a,0,b}$  and  $r_{a,c}$ ) and air 4 temperature  $(T_r)$  through  $\Delta$ . It is worthwhile noting that Eq. (10) is only valid under the standard climatic conditions used to derive the value of the crop coefficient. Consequently, the crop surface 5 resistance  $r_{s,c}$  should be first determined under the "fictitious" standard climatic conditions 6 7 corresponding to the determination of crop coefficients and then introduced into Eq. (3) with the actual climatic conditions. The problem, however, is to define these "fictitious" or "preferred" weather 8 9 conditions in order to estimate the most correct value of crop resistance through Eq. (10).

10 Shuttleworth (2006) eliminated the dependence of crop surface resistance on some weather 11 variables by equating reference crop evapotranspiration  $ET_0$  (Eq. 1) with the Priestley-Taylor estimate 12 (Priestley and Taylor, 1972) expressed as

$$ET_{PT} = \alpha_{PT} \frac{\Delta A_0}{\Delta + \gamma} \quad with \quad \alpha_{PT} = 1.26 \quad .$$
 (13)

This assumption is supported by works on modeling experiments dealing with the daytime evolution
of the atmospheric boundary-layer (de Bruin, 1983; McNaughton and Spriggs, 1989). It leads to

$$r_{s,e} = 1.26 r_{s,0} + 0.26 \left(1 + \frac{\Delta}{\gamma}\right) r_{a,0} \quad . \tag{14}$$

15 By putting  $ET_0 = ET_{PT}$  the Matt-Shuttleworth approach makes the equilibrium resistance a simple function of temperature (through  $\Delta$ ) and wind speed (through  $r_{a,0}$ ). In this way, the relationship 16 between crop surface resistance  $r_{s,c}$  and crop coefficient  $K_c$  (Eq. 10) involves only wind speed through 17 18 the three aerodynamic resistances  $(r_{a,0}, r_{a,0,b} \text{ and } r_{a,c})$  and air temperature through  $\Delta$   $(r_{s,0} \text{ being})$ prescribed). The assumption  $(ET_0 = ET_{PT})$  is questionable, however, because the effective value of the 19 Priestley-Taylor coefficient depends upon the atmospheric conditions and can be fairly different from 20 the preferred value of 1.26. For instance, Jensen et al. (1990) note that  $\alpha_{PT}$  can be as high as 1.74 in 21 arid conditions. This point is thoroughly discussed below using numerical simulations. 22

### **4.** Basis of the numerical exploration

We examine hereafter whether the Matt-Shuttleworth assumption really holds and how the relationship between crop surface resistance and  $K_c$  depends on climatic conditions, assessing their impact on the determination of crop surface resistance. For this examination a different writing of the reference crop evapotranspiration is used. After some algebraic manipulations and introducing the equilibrium resistance  $r_{s,e}$  defined by Eq. (11), the Penman-Monteith equation applied to the reference crop can be put in a form comparable to Eq. (13) (Pereira et al., 1999, Eq. 18):

$$ET_0 = \alpha \left(\frac{\Delta A_0}{\Delta + \gamma}\right) \qquad \text{with} \quad \alpha = \frac{1 + \frac{\gamma}{\Delta + \gamma} \frac{r_{s,e}}{r_{a,0}}}{1 + \frac{\gamma}{\Delta + \gamma} \frac{r_{s,0}}{r_{a,0}}}.$$
(15)

8 This form of the Penman-Monteith equation allows exploring the effective value of the coefficient  $\alpha$ 9 compared to the preferred value of 1.26. It shows that the theoretical form of the Priestley-Taylor 10 coefficient ( $\alpha$ ) is a complex function of the surface resistance ( $r_{s,0}$ ) and of some weather variables 11 involved in  $r_{s,e}$  and  $r_{a,0}$  (available energy, air humidity, temperature, wind speed). By setting its value 12 at 1.26, the Matt-Shuttleworth assumption implicitly identifies specific atmosphere conditions, 13 supposed to be the ones used to determine the crop coefficient.

In FAO-56 (Allen et al., 1998, p. 114), it is specified that the values of crop coefficients 14 "represent those for a sub-humid climate with an average daytime minimum relative humidity  $(RH_{n,r})$ 15 of about 45 % and with calm to moderate wind speeds ( $u_r$ ) averaging 2 m s<sup>-1</sup>". When  $RH_{n,r}$  and  $u_r$  differ 16 from 45 % and 2 m s<sup>-1</sup> respectively, FAO-56 proposes an empirical equation (Allen et al., 1998, Eq. 17 62) to adjust the  $K_c$  value to the prevailing conditions. Nothing is said, however, about air temperature 18 19 and incoming radiation. In the Matt-Shuttleworth approach, incoming radiation and air humidity are eliminated due to the assumption that  $ET_0 = ET_{PT}$  with  $\alpha_{PT} = 1.26$ . In Shuttleworth (2006), a typical 20 value of 15°C was arbitrarily chosen for reference air temperature  $(T_r)$  with a wind speed of 2 m s<sup>-1</sup>, 21 22 whereas in a study on irrigated crops in Australia, Shuttleworth and Wallace (2009) selected a value of 23 20°C for air temperature.

1 Our simulation process makes use of the semi-empirical formulae given in FAO-56 (Allen et 2 al., 1998) for the different parameters involved in the theoretical relationships described above. The 3 three aerodynamic resistances ( $r_{a,0}$ ,  $r_{a,0,b}$ ,  $r_{a,c}$ ) are calculated without stability corrections following the 4 generic formula

$$r_a = \frac{\ln\left(\frac{z-d}{z_{om}}\right)\ln\left(\frac{z-d}{z_{oh}}\right)}{k^2 u} , \qquad (16)$$

5 where u is the wind speed a height z ( $z_r$  or  $z_b$ ), d the zero plane displacement height,  $z_{0m}$  the roughness 6 length for momentum and  $z_{0h}$  the roughness length for scalar (heat and water vapour). Aerodynamic 7 parameters (for the reference crop and the given crop) are calculated as simple functions of crop height:  $d = 0.67 z_h$ ,  $z_{0m} = 0.123 z_h$  and  $z_{0h} = z_{0m}/10$ . The slope of the saturated vapour pressure curve 8 ( $\Delta$ ) is a function of air temperature (Allen et al., 1998, Eq. 13). The psychrometric constant ( $\gamma$ ) 9 10 depends on atmospheric pressure and hence on elevation (Allen et al., 1998, Eqs. 8 and 7). Air density  $(\rho)$  is a function of atmospheric pressure and temperature (Allen et al., 1998, Eq. 3.5). Soil heat flux  $G_0$ 11 12 is generally neglected on a 24 h time step, which means that  $A_0 \approx R_{n,0}$ . The daily net radiation of the reference crop  $(R_{n,0})$  is estimated following Allen et al. (1998, Eqs. 37, 38 and 39) from the measured 13 or calculated solar radiation ( $R_s$ ) and from the clear sky solar radiation ( $R_{s,0}$ ), which is approximated by 14  $R_{s,0} = (0.75+2 \ 10^{-5}z) R_a$  (Allen et al., 1998, Eq. 37), z (m) being the elevation above sea level and  $R_a$ 15 the extraterrestrial solar radiation. 16

### 17 5. Results and discussion

Numerical explorations are carried out varying primarily air temperature and exploring different conditions of wind speed, air humidity and radiation. Following FAO-56 (Table 16 and Fig. 32), three types of climate shown in Table 1 are considered: they are defined as a function of their minimum  $(RH_{n,r})$  and mean  $(RH_{m,r})$  relative humidity at the reference height. Solar radiation is taken at sea level and assumed to be at its maximum value  $R_{s,0}$  corresponding to a clear sky day:  $R_s = R_{s,0} = 0.75 R_a$ . In the lower latitudes of both hemispheres (below 40°), where irrigation is most needed, the range of value for the extraterrestrial radiation  $R_a$  is approximately between 30 and 40 MJ m<sup>-2</sup> day<sup>-1</sup> during the 1 growing season, which corresponds to  $R_s$  varying between 22.5 and 30 MJ m<sup>-2</sup> day<sup>-1</sup>. Additionally and 2 for the sake of convenience, the ratio  $f_c = A_c/A_0$  is set at 1 in all the simulations.

3 In Fig. 1 the coefficient  $\alpha$  defined by Eq. (15) is plotted as a function of air temperature for 4 different climatic conditions, extraterrestrial solar radiation ( $R_a$ ) being set at a constant value of 35 MJ m<sup>-2</sup> day<sup>-1</sup> (i.e.,  $R_s = R_{s,0} = 26.25$  MJ m<sup>-2</sup> day<sup>-1</sup>). The value of  $\alpha$  increases with reference temperature, 5 6 moderately for low wind speed and more significantly for higher wind speed. For the sub-humid 7 climate and a moderate wind speed (which correspond to the conditions under which the crop 8 coefficients were supposedly derived), the value of  $\alpha$  is much lower than the preferred value of 1.26 9 used in the Matt-Shuttleworth approach, whereas with the semi-arid climate  $\alpha$  is closer to 1.26 10 (Fig.1a). Fig. 1b shows that for a wide range of wind speed under a sub-humid climate the coefficient  $\alpha$  is always below the 1.26 value. Therefore, the Matt-Shuttleworth assumption should be considered 11 12 with much care: using a fixed value for  $\alpha$  (1.26) is a way of hiding its complex dependence on weather conditions and can be misleading. As a consequence of this fixed value of  $\alpha$ , the Matt-Shuttleworth 13 14 estimate of the equilibrium resistance  $r_{s,e}$  can be significantly greater than the true value for the current 15 range of reference temperature (results not shown).

16 The influence of weather variables on the relationship between crop surface resistance  $r_{s,c}$  and  $K_c$ 17 is investigated hereafter with and without the Matt-Shuttleworth assumption. Two contrasting cases are considered: one representing the initial stage of an annual crop, with  $K_c = 0.5$  and a crop height  $z_h =$ 18 0.5 m, and the other case, with  $K_c = 1.1$  and  $z_h = 1.5$  m, representing the mid-season stage. The 19 adjustment of crop coefficient to differing climate conditions is systematically applied using the 20 21 empirical equation given in Allen et al. (1998, Eq. 62). Fig. 2 shows how the crop surface resistance 22 varies as a function of reference temperature for two different environmental conditions (semi-arid and sub-humid climates). For the initial stage (Fig.2a), the surface resistance is high and there is a fairly 23 good agreement between the two estimates (with and without the M-S assumption): in semi-arid 24 25 conditions the agreement is almost perfect and under sub-humid climate the M-S assumption slightly overestimates the surface resistance by around 30 s m<sup>-1</sup> (6 % on average). For the mid-season stage 26 (Fig. 2b), the surface resistance is lower and the discrepancy is larger in relative value. Under sub-27

4 In Fig. 3a, the surface resistance of a crop with  $K_c = 1.0$  and  $z_h = 1.0$  m is plotted against reference 5 temperature for two different values of extraterrestrial solar radiation  $(R_a)$ , under sub-humid climate 6 and moderate wind. The M-S approach systematically overestimates the true value of surface 7 resistance and the higher the solar radiation, the greater the overestimation. Fig. 3b shows the net 8 impact of the M-S assumption on the estimate of crop evapotranspiration under standard conditions 9  $ET_c$  (Eq. 3). The same crop and the same environmental conditions as in Fig. 3a are used. The effect is 10 clearly mitigated since the M-S assumption results in a relatively low underestimation: only -3 % on average for  $R_a = 30$  MJ m<sup>-2</sup> d<sup>-1</sup> and -8 % for  $R_a = 40$  MJ m<sup>-2</sup> d<sup>-1</sup>. Given that the surface resistance is 11 12 only one component of a more complex equation involving other climatic and surface parameters, the net impact of an overestimated surface resistance is necessarily reduced. 13

14 These results show that there is a complex dependence of surface resistance on weather conditions, partially hidden when the Matt-Shuttleworth assumption is used. In the simulations 15 performed above, the M-S approach appears to work better in the semi-arid conditions than in the sub-16 17 humid conditions described in our Table 1. This can be explained by the fact that the coefficient  $\alpha$  (Eq. 15) is closer to 1.26 (i.e.,  $ET_0$  closer to  $ET_{PT}$ ) in the semi-arid conditions than in the sub-humid 18 conditions, as shown in Fig. 1a. It is well known, indeed, that the coefficient  $\alpha$  can vary from values 19 close to 1 in very humid conditions (high relative humidity, such as in equatorial regions) to values 20 21 greater than 1.7 in arid conditions (very dry air) (Shuttleworth, 2012, Fig. 23.1)). This point has been extensively discussed in the framework of the complementary relationship (Lhomme, 1997). The 22 "semi-arid" conditions, as defined in terms of relative humidity in our Table 1, certainly represents a 23 24 mid-value of air humidity, where the coefficient  $\alpha$  is close to 1.26 and where consequently the M-S 25 assumption better holds.

1 The relationship between crop surface resistance  $(r_{s,c})$  and FAO crop coefficient  $(K_c)$  is not as straightforward as could be expected because of the interference of weather variables such as air 2 3 temperature, solar radiation, wind speed and air humidity. The Matt-Shuttleworth assumption, which 4 to some extent eliminates this interference by equating the reference crop evapotranspiration  $(ET_0)$  to 5 the Priestley-Taylor estimate ( $ET_{PT}$  with  $\alpha_{PT} = 1.26$ ), does not hold in many climatic conditions and 6 can lead to substantial differences between the estimated and true value of surface resistance. We have 7 to recognize, however, that the real impact of the M-S assumption on crop evapotranspiration estimate 8 is relatively minor, given that the generated bias on surface resistance is partially damped when the 9 calculated resistance is introduced into the evaporation formulation.

10 In order to infer the surface resistance of a given crop from its crop coefficient, it is certainly sounder to work directly with the basic relationship linking crop surface resistance to crop coefficient 11 12 (i.e., Eqs. 10 and 12) without any assumption, but with the most plausible weather conditions. Indeed, 13 the weather conditions corresponding to a tropical crop (such as cassava, banana or millet) are surely 14 different from those corresponding to a temperate one (such as winter wheat or potato). Unfortunately, the meteorological conditions corresponding to the tabulated values of FAO crop coefficients are 15 generally not available. Because of that, the transformation of crop coefficients into surface resistances 16 17 is undoubtedly not an easy task.

#### 18 Nomenclature

- **19**  $A_0$  available energy of the reference crop (W m<sup>-2</sup>)
- 20  $A_c$  available energy of a given crop (W m<sup>-2</sup>)
- 21  $c_p$  specific heat of air at constant pressure (J kg<sup>-1</sup> K<sup>-1</sup>)
- 22  $D_r$  water vapour pressure deficit at a reference height of 2 m (Pa)
- 23  $D_b$  water vapour pressure deficit at a blending height of 50 m (Pa)
- 24 d zero plane displacement height of the crop (m)
- 25  $ET_0$  evapotranspiration from the reference crop (W m<sup>-2</sup>)
- 26  $ET_c$  evapotranspiration from a given crop under standard conditions (W m<sup>-2</sup>)

 $ET_{PT}$  evaporation given by the Priestley-Taylor equation (Eq. 13) (W m<sup>-2</sup>)

- $f_c$  ratio between crop available energy and that of the reference crop (dimensionless)
- $K_c$  FAO crop coefficient defined by Eq. (1) (dimensionless)
- *k* von Karman's constant (dimensionless)
- $R_a$  extraterrestrial solar radiation (MJ m<sup>-2</sup> day<sup>-1</sup>)
- $R_{s,0}$  clear sky solar radiation (MJ m<sup>-2</sup> day<sup>-1</sup>)
- $R_s$  incoming solar radiation (MJ m<sup>-2</sup> day<sup>-1</sup>)
- $RH_{n,r}$  minimum relative humidity at reference height (%)
- $RH_{m,r}$  mean relative humidity at reference height (%)
- $r_{a,0}$  aerodynamic resistance of the reference crop calculated up to the reference height  $z_r$  (s m<sup>-1</sup>)
- $r_{a,0,b}$  aerodynamic resistance of the reference crop calculated up to the blending height  $z_b$  (s m<sup>-1</sup>)
- $r_{a,c}$  aerodynamic resistance of a given crop calculated up to the blending height  $z_b$  (s m<sup>-1</sup>)
- $r_{s,0}$  surface resistance of the reference crop = 70 s m<sup>-1</sup>
- $r_{s,c}$  surface resistance of a given crop under standard conditions (s m<sup>-1</sup>)
- $r_{s,e}$  equilibrium resistance defined by Eq. (11) (s m<sup>-1</sup>)
- $T_r$  air temperature at reference height (°C)
- $u_r$  wind speed at reference height (m s<sup>-1</sup>)
- $u_b$  wind speed at blending height (m s<sup>-1</sup>)
- $z_h$  crop height (m)
- $z_r$  reference height = 2 m

 $z_b$  blending height = 50 m

- $z_{0m}$  roughness length for momentum of a given crop (m)
- $z_{0h}$  roughness length for scalar of a given crop (m)
- $c_p$  specific heat of air at constant pressure (J kg<sup>-1</sup> K<sup>-1</sup>)

- 1  $\alpha$  theoretical expression of the Priestley-Taylor coefficient (Eq. 15) (dimensionless)
- 2  $\alpha_{PT}$  value of the Priestley-Taylor coefficient (= 1.26)
- 3  $\Delta$  slope of the saturated vapour pressure curve (Pa K<sup>-1</sup>)
- 4  $\gamma$  psychrometric constant (Pa K<sup>-1</sup>)

5  $\rho$  air density (kg m<sup>-3</sup>)

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# 9 Figures captions

- 10 Table 1- Typical values of daily minimum relative humidity  $(RH_{n,r})$  and its mean value  $(RH_{m,r})$  for 11 three types of climate (from FAO-56, Table 16).
- 12 Fig. 1- Value of the coefficient  $\alpha$  inferred from Eq. (15) as a function of air temperature at reference
- 13 height, the straight dotted line representing the "preferred" value 1.26: (a) for different climatic
- 14 conditions (see Table 1) with  $u_r = 2 \text{ m s}^{-1}$ ; (b) for different values of wind speed under sub-humid 15 conditions (SH).
- 16 Fig. 2- Variation of crop surface resistance as a function of air temperature for two climatic
- 17 environments (SA: semi-arid (thin line), SH: sub-humid (bold line),  $u_r = 2 \text{ m s}^{-1}$ ) and comparison with
- 18 the Matt-Shuttleworth estimate (M-S) (dotted line): (a)  $K_c = 0.5$  and  $z_h = 0.5$  m; (b)  $K_c = 1.1$  and  $z_h$
- 19 =1.5 m.
- Fig. 3- Variation of crop surface resistance  $r_{s,c}$  (a) and daily standard evapotranspiration  $ET_c$  (b) as a function of air temperature for two different values of extraterrestrial solar radiation ( $R_a$ ) expressed in MJ m<sup>-2</sup> d<sup>-1</sup> (30 and 40) and comparison with the Matt-Shuttleworth estimate (M-S) (dotted line) for a crop with  $K_c = 1$  and  $z_h = 1$  m, under a sub-humid climate with  $u_r = 2$  m s<sup>-1</sup>.

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Climatic classification	$RH_{n,r}(\%)$	$RH_{m,r}$ (%)
Semi-arid (SA)	30	55
Sub-humid (SH)	45	70
Humid (H)	70	85

- 7 Table 1- Typical values of daily minimum relative humidity  $(RH_{n,r})$  and its daily mean value  $(RH_{m,r})$
- 8 for three types of climate (from FAO-56, Table 16).



Fig. 1- Value of the coefficient  $\alpha$  inferred from Eq. (15) as a function of air temperature at reference height, the straight dotted line representing the "preferred" value 1.26: (a) for different climatic conditions (see Table 1) with  $u_r = 2 \text{ m s}^{-1}$ ; (b) for different values of wind speed under sub-humid



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T<sub>r</sub> (°C)