

A comparison of interpolation methods on the basis of data obtained from a bathymetric survey of lake Vrana, Croatia

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Abstract

The bathymetric survey of Lake Vrana included a wide range of activities that were performed in several different stages, in accordance with the standards set by the International Hydrographic Organization. The survey was conducted using an integrated measuring system which consisted of three main parts: a single-beam sonar *Hydrostar 4300*, GPS devices *Ashtech Promark 500* – base, and a *Thales Z-Max* – rover. A total of 12 851 points were gathered.

In order to find continuous surfaces necessary for analysing the morphology of the bed of Lake Vrana, it was necessary to approximate values in certain areas that were not directly measured, by using an appropriate interpolation method. The main aims of this research were as follows: a) to compare the efficiency of 14 different interpolation methods and discover the most appropriate interpolators for the development of a raster model; b) to calculate the surface area and volume of Lake Vrana, and c) to compare the differences in calculations between separate raster models. The best deterministic method of interpolation was RBF multiquadratic, and the best geostatistical ordinary cokriging. The mean quadratic error in both methods measured less than 0.3 metres.

The quality of the interpolation methods was analysed in 2 phases. The first phase used only points gathered by bathymetric measurement, while the second phase also included points gathered by photogrammetric restitution.

The first bathymetric map of Lake Vrana in Croatia was produced, as well as scenarios of minimum and maximum water levels. The calculation also included the percentage of flooded areas and cadastre plots in the case of a 2-metre increase in the water level. The research

1 presented new scientific and methodological data related to the bathymetric features, surface
2 area and volume of Lake Vrana.

3 **Keywords:** bathymetric survey, single beam sonar, interpolation methods, RTK-GPS, Lake
4 Vrana

5
6 **1 Introduction**

7 The methodology ~~of~~ bathymetric ~~research~~ survey has undergone many conceptual changes in the last few
8 decades, especially since the mid 20th century ~~and~~ due to the availability of the single-beam echo
9 sounder. Rapid advances continued with the development of multi-beam sounders and laser
10 systems (airborne laser sounding systems) which can gather high-density data samples and
11 enable the development of a realistic underwater bottom model (Finkl et al., 2004; Ernsten et
12 al., 2006).

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13 The process of hydrographic measurement includes measurement and ~~researching~~ establishing the
14 configuration of the bottom of an ocean, sea, river, lake or any other water-related object on
15 Earth (NOAA, 1976). The main goal of most such hydrographic surveys is to gain data
16 necessary to develop nautical charts featuring special details of types of navigational hazards.

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17 Other goals include gaining information crucial to the management and protection of coastal
18 areas, exploitation of resources, ~~scientific practices,~~ national spatial data infrastructure, tourism purposes, (IHO,
19 2005). Contemporary bathymetry, as a field within hydrography, is the science of measuring
20 depths and determining the physical properties of the underwater ~~etc~~ features, on the basis of
21 analysing data gained from recorded profiles. There are several different methods and
22 techniques ~~of~~ for bathymetric measurement, which depend on the complexity of the project. The

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23 success of bathymetric measurement depends mostly on a detailed planning process, which in
24 turn enables the organization and tracking of the measurement process from start to finish
25 (IHO, 2005). During this particular research, the measurement plan included a wide range of
26 activities and was performed in several phases according to the standards of the International
27 Hydrographic Organization. The area surveyed included the whole of Lake Vrana, with a total
28 surface area of 29.865 km² (Šiljeg, 2013). Lake Vrana is the largest, natural, freshwater lake
29 in the Republic of Croatia. This cryptodepression is an ecologically sensitive area, located in
30 the Mediterranean part of Croatia (Zadar County) (Romić et al., 2003). The lake is an
31 important economic resource for the local community, but also provides a natural habitat for

1 many bird species (Šikić et al., 2013). The Lake Vrana is complex body, which also affected
2 the bathymetric survey.

3 ~~Because the earth's landforms can be extremely complex, most scientists opt for research via~~
4 ~~the development and analysis of digital elevation models (DEM),~~ (Dikau et al., 1995; Bishop
5 and Shroder, 2000; Millaresis and Argialas, 2000; Wilson and Gallant, 2000; Tucker et al.,
6 2001; Shary et al., 2002; Chaplot et al., 2006; Wilson, 2011). ~~Most gathered data in elevation~~
7 ~~sets are,~~ point-related, regardless of rapid developments in technology. ~~(Wilson and Gallant,~~
8 ~~2000; Li et al. 2005; Fisher et al., 2006; Wilson, 2011).~~ In order to find continuous surfaces,
9 which are necessary for the process of ~~understanding our environment,~~ some values need to
10 be approximated for ~~areas~~ which are not measured directly. This is done using various
11 methods of interpolation (Collins and Bolstad, 1996; Hartkamp et al., 1999; Hu et al., 2004;
12 Naoum et al., 2004; Li and Heap, 2008, Erdogan, 2009). The final result ~~of the interpolation is~~
13 the model that approximates or simplifies the Earth's surface. Each method produces a
14 different result, so the main challenge is to determine the characteristics of errors and
15 variability of approximated values by comparing and testing different interpolation methods.

16 The bathymetric survey of Lake Vrana was performed in order to enable optimal management
17 of the water level, to classify the lake's bottom, to create a model and bathymetric map, and to
18 enable better management and protection of the lake's flora ~~and~~ fauna. ~~This process includes~~
19 ~~all the hydro-technical measures, which can determine changes of the time-and-space~~
20 ~~distribution of affected water, enabling a more efficient management of natural water~~
21 ~~resources. The water regime includes the entire dynamics of constant change: quantitative and~~
22 ~~qualitative water characteristics and the dynamics of the interchange between water and the~~
23 ~~environment (Ožanić, 2002; Kuspilić, 2008). A lack of systematic water regime management~~
24 ~~has led to extreme fluctuations in the water level, salinity, temperature, and oxygen levels.~~
25 ~~This has resulted in lakes being under-exploited and under-protected in many ways for~~
26 ~~example in terms of tourism, water resource use, biodiversity and ecology. The unsatisfactory~~
27 ~~protection~~ culminated in a series of negative consequences in 2012, when a record number of
28 fish died (URL 1).

29 An optimal water regime can only be achieved if the amount of water in the lake is known at
30 any moment, and if Prosika drainage canal has a regulatory water infrastructure, as well as an
31 efficient drainage ditch used to regulate the water level, depending on the season. ~~The canal~~
32 ~~(dimensions: 875 m length, 8 m width and 4-5 m depth) which connect lake Vrana and~~

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Deleted: This process implies all the hydro-technical measures and infrastructure that facilitate a deliberate change in water distribution, which in turn enables the more efficient management of natural water resources, protection from water hazards and prevention of water pollution. The water regime includes the entire dynamics of constant change, both the quantitative and qualitative aspects of water, and the dynamics between the water and surrounding area (Kuspilić, 2008). Inconsistent management of the water regime has caused some extreme changes in the water level, salinity, temperature, oxygen levels etc. As a result, the lake has been poorly exploited for other purposes: tourism, water resource management, biodiversity, ecological activities etc.

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1 Adriatic sea was dug through in 1770 to attain new agricultural areas in Vrana field and
2 protect them from seasonal floodings.

3 The main aims of this research are as follows: 1) to compare the efficiency of 14 methods of
4 interpolation and determine the most appropriate interpolators for the development of a raster
5 model of the lake (on the basis of data gained by bathymetry and by using the cross-validation
6 method); 2) to calculate the surface area and volume of the lake and to compare the results
7 between the raster models; 3) to develop the first bathymetric map of Lake Vrana, which will
8 enable calculation of the percentage of flooded areas in the Nature Park and the flooded plots,
9 in the case of a 2-metre rise in the water level. This will serve as tool for developing a
10 scenario for future changes in water level.

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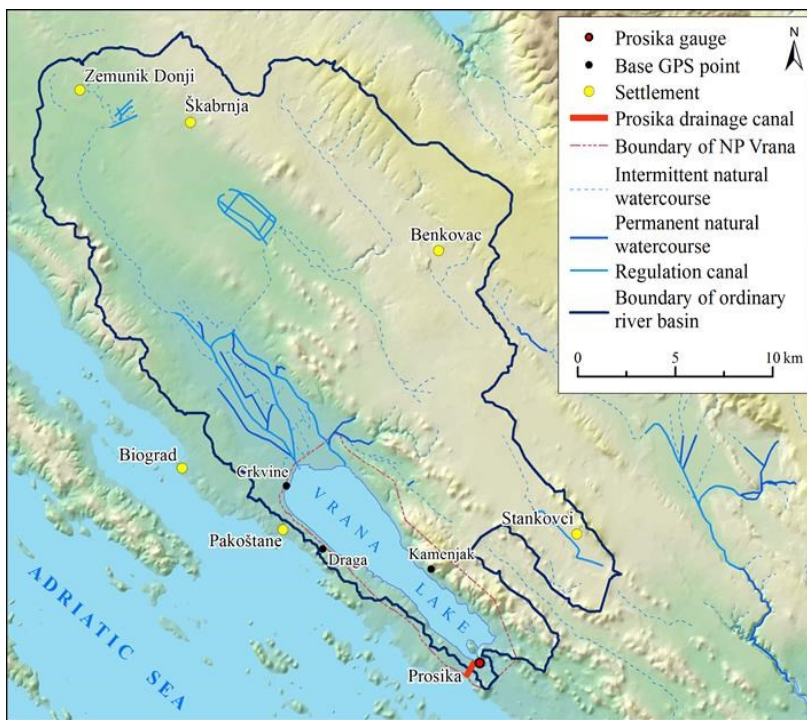
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12 2 Study Area

13 Lake Vrana in Dalmatia is the largest natural lake in Croatia by surface area of 30.2 km², with
14 the length of 13.6 km and width of 1.4 – 3.5 km (JUPPV, 2010).

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Deleted: However, none of the written sources mention the process used to calculate the surface area, the water level included, what year, month, or methods and techniques used. ¶



15

16 Figure 1. Study area with ordinary river basin

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1 The surface area of the lake changes frequently. In the period from 1948 to 2008, the lowest
2 water level was 12 cm (measured in reference to the Prosika gauge) or 0,03 meters above sea
3 level, measured in 1990 and 2008. Prosika gauge is located in the north part of Prosika
4 drainage canal (Fig. 1). The highest water level was 236 cm, measured in reference to the
5 Prosika gauge (2.24 m a.s.l., measured in 1974 and 1994.). The mean value was 0.81 m a.s.l.
6 (JUPPVJ, 2010). The water level is influenced by factors such as inflow, drainage and
7 evaporation, but also by complex hydrological and hydraulic effects such as water balance,
8 salt and fresh water content, sea tides and other factors influencing changes in sea level
9 (JUPPVJ, 2010).

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10 The characteristics of Lake Vrana water system, affected the selection of methods for the
11 bathymetric survey. The area included Lake Vrana in its entirety, with a surface area of
12 29.865 km² (in relation to the water level of +0.42 m, measured in reference to the Prosika
13 gauge) (Šiljeg, 2013).

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14 The lake is characterized by following characteristics:

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15 1) A high percentage of shallow water – over 65% of the lake's surface features a water depth
16 of -1.76 m, while the deepest is -3.73 m (in relation to the water level of +0.42 m, measured
17 in reference to the Prosika gauge or 0.3 m a.s.l.).

Deleted: Prosika gauge is located near Prosika drainage canal which connect Vrana lake and Adriatic sea (Fig. 1). The canal (dimensions: 875 m length, 8 m width and 4-5 m depth) was dug through in 1770 to attain new agricultural areas in Vrana field and protect them from seasonal floodings. ¶

18 2) Low vertical dissection – the absolute vertical difference over the entire area of the lake
19 bottom is only 3.46 m. More than 90% of the lake's bottom features a slope inclination of 2°

20 3) Low water transparency and high turbidity, especially during even the slightest winds

21 4) Lush vegetation (grass) on the lake's bottom and the surrounding shoreline
22 (*Phragmitetalia*)

23 5) Significant seasonal oscillations in the lake's water level

24 6) Coverage of parts of the lake's bottom by unconsolidate sediments

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3.1 Plan for the Bathymetric Survey ¶
In order to perform a bathymetric survey, it is necessary to have a detailed plan, which enables tracking research development and the organization of the research from start to finish. The plan included a wide range of activities and was structured in a number of phases: 1) determining the exact research area, 2) determining the purpose of the bathymetric survey, 3) application of the survey method (techniques, accuracy, referential horizontal and vertical geodesic system, equipment, etc.), 4) determining the time frame (short or long), 5) gathering various secondary data (aerial photos, data from the cadastre, information on the water level, salinity, temperature, etc.), 6) considering limiting factors (budget, logistics, etc.) and 7) data processing (conversion, filtering, interpolation methods, etc.) ¶

26 **3 Research Materials and Methods,**

27 **3.1 Equipment used**

28 Based on the characteristics of the lake, more efficient techniques, such as measuring using a
29 multi-beam echo sonar, or laser sonar, would have been inappropriate, considering the
30 morphology of the bottom. The percentage of the recorded bottom would increase greatly in
31 relation to recordings from a single-beam sonar, but the cost of the survey and amount of data
32 acquired would significantly increase as well. After consideration, it was clear that the most

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1 efficient solution was bathymetric measurement and the use of a single-beam ultrasound
2 device.

3 In order to avoid frontal waves (proposed by IHO, 2005), an inflatable *Hondawave* boat was
4 used (Fig. 2a). The boat was the optimal vehicle due to its small dimensions (3.85 m) and
5 economical engine, and because it was easy to install the surveying equipment on it.

6 The bathymetric measurement was performed using an integrated measuring system (Fig. 3)
7 Installed equipment included three main components: a *Hydrostar 4300* sonar, GPS devices
8 *Ashtech Promark 500* and a *Thales Z-Max*. These were connected via the RTK controller
9 *Juniper System-Allegro*, which enabled real-time connection and data registration in the
10 *FastSurvey* programme. This enabled recording of the sonar coordinates and corresponding
11 depth. The programme automatically recalculated the coordinates from the GPS into the local
12 projection coordinates. The selected projection was the universal transversal Mercator, Gauss-
13 Krüger shape with a central meridian of 15, a factor of scale change of 0.9999 and a false
14 easting of 5 500,000. The Bessel 1841 ellipsoid was used.

15 Two GPS devices were also used: a base or referential device (Fig. 2b), which was positioned
16 according to precisely determined coordinates, and a rover device (Fig. 2c), which was used
17 in the work area. A data-exchanging connection was established between them via a UHF
18 radio transmitter, which would also have been possible via various GSM devices.



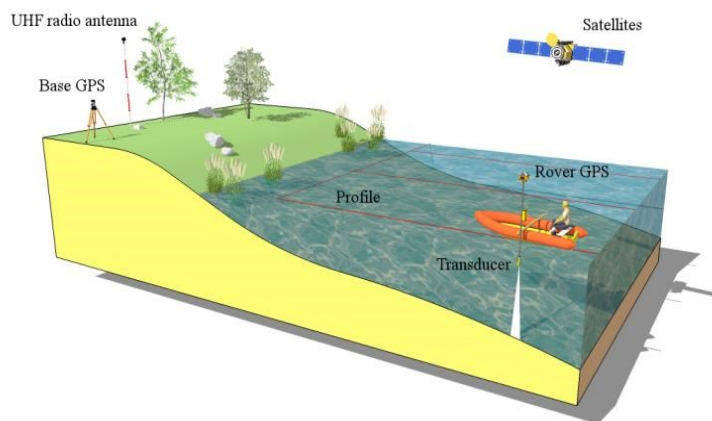
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21 Figure 2. (a) *Hondawave* inflatable boat with wooden support. (b) Base GPS and UHF
22 antenna. (c) Rover GPS and dual-frequency probe

23

24 The distance between the base and referential devices had to be determined in advance, in
25 order to achieve an adequate degree of precision. This was named the *base line* and its

1 maximum value was 50 kilometres. The distance between the base GPS and the UHF
2 transmitter had to be a minimum of 10 m.



3
4 Figure 3. Integrated measuring system – combination of GPS-RTK and a sonar

5
6 Since the UHF signal was rather weak throughout the lake, three base points were determined
7 using the Ashtech Promark 500 and CROPOS system: 1) coordinates $\lambda=5\ 541\ 365.709$, $\varphi=4\ 865\ 017.188$ m – 2.02 m above sea level in the northeast section of the Nature Park (Crkvine),
8 2) coordinates $\lambda=5\ 543\ 197.353$, $\varphi=4\ 861\ 981.863$ m – 36.69 m above sea level in the western
9 parts of the Nature Park (Draga), 3) coordinates $\lambda=5\ 548\ 694.214$, $\varphi=4\ 860\ 958.663$ m – 62 m
10 above sea level in the eastern part of the Nature Park (Kamenjak) (Fig. 1). They were
11 connected by a benchmark and measuring gauge at the Prosika location. A base GPS device
12 was set at those points, depending on the phase of the survey, and connected to a UHF
13 transmitter (with all components) in order to achieve a connection (signal) with the mobile
14 GPS installed on the inflatable boat.
15

16 A dual-frequency probe was fixed to this support with a rover GPS device submerged 20 cm
17 below the water level (Fig. 2c). This arrangement was necessary due to the shallow water of
18 the lake and low water level at the northwest end. Since the *Hydrostar 4300* sonar supports
19 depth recording simultaneously at two frequencies, the survey was conducted at two
20 frequencies: low – 30 kHz and high – 200 kHz.

21 The bathymetric survey was performed according to the previously established profiles, on a
22 geo-referential cartographic surface (Croatian Base Map and digital orthophoto to the scale
23 1:5000). The basic measuring profiles were planned perpendicular to the slope of the terrain,
24 in a northeast-southwest direction. The planned profiles of the survey (basic bathymetric

1 profiles) ensured good coverage and high resolution in the research area. The survey also
2 included four transversal profiles which intersected with the main profiles, enabling the
3 comparison and control of the measured depths.

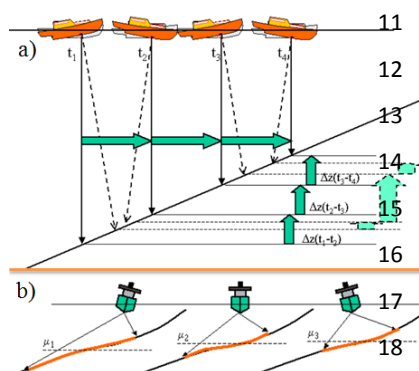
4 Within the borders of the shoreline of Lake Vrana, 375 basic profiles were achieved. The
5 distance between adjacent profiles was set at 200 metres, which corresponds to the desired
6 mapping resolution to the scale 1:30 000.

7

8 **3.2 Time Frame**

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9 The time frame, and the first day of the survey were determined by the water level. The water
10 level is important since it is impossible to register a depth of more than 0.5 metres by



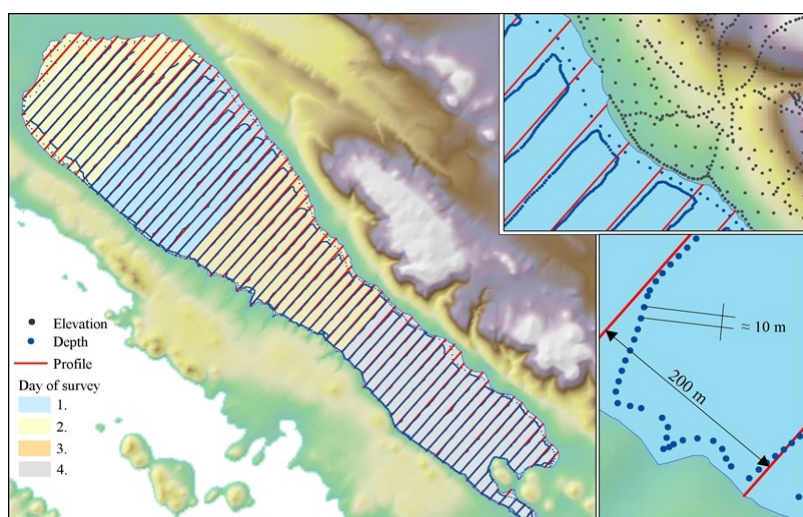
11 transducer. Weather conditions are important for
12 navigation and the quality of data registration (Fig. 4).
13 Wind, rain, waves and cold, for example, are usually
14 limiting factors. Weather reports and water level
15 oscillations were continuously observed from the
16 production of preliminary plans in November 2010
17 until the beginning of the survey.

19 Figure 4. The effect of frontal (a) and dorsal (b) waves on data registration (Clarke, 2003).

20 The measurement process was conducted in two phases (Fig. 5): 1) from 10-12 May 2012,
21 and 2) from 7-9 June 2012.

22 The first phase took two days, and included a survey of 14.351 km² of the northern part of
23 Lake Vrana. The total length of the measured profiles was 71.3 km, and the total amount of
24 points gathered was 5643. In the first phase of investigation the water level measured at the
25 Prosika station was 0.42 m. The limiting factors for the survey in this part of the lake were the
26 dense grassy vegetation on the bottom, the shallow water and the lush surface-level vegetation
27 which hindered navigation. Measurement was cancelled in these parts, based on previously
28 established profiles, while the shallow water was measured using a plumb-line. As a result,
29 this survey cannot be classified as systematic. It is nevertheless very important in relation to
30 the part of the lake that was measured, since the terrain there is flat or minimally inclined. An
31 acceptable level of interpolation is possible in areas featuring an irregular layout of profiles.

1 The second phase featured negligible limiting factors, so the survey was conducted according
2 to plan. The water level at the Prosika station was 0.37 m. A total area of 15.514 km² was
3 surveyed in the southern part of the lake. The total length of the measured profiles was 82.5
4 km, and the total amount of points gathered was 7208.



7 Figure 5. The phases and plan of the bathymetric survey

9 **3.3 Processing the Bathymetric Data**

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10 The data obtained from measurement was transferred to a PC via the *Juniper System-Allegro*
11 controller and the *Fast Survey* programme package for further processing and interpolation.
12 During measurement, the controller creates a separate file with information regarding the
13 point coordinates, time obtained, and depth recorded. Data processing included filtering out
14 noise, calibrating the checked depths to a common referential level, and interpolation. The
15 filtering process was implemented according to a programme which enabled the removal of
16 errors in the data registry (Fabulić, 2012). Records of water depth were calibrated in relation
17 to the Prosika benchmark and measuring gauge.

18 Since parts of Lake Vrana are quite difficult to survey, measurements taken by ultrasound
19 showed some background noise. In simple terms, the ultrasound beam bounces off the first
20 obstacle it encounters, so the echo sounder calculates the distance to that obstacle and
21 represents it as a depth measurement. However, such obstacles are not always on the bottom
22 of the lake, and indeed, random noise may be generated by floating matter, plankton, fish, or

1 vegetation (Pribičević et al., 2007). These sounds need to be filtered and reduced in order to
2 obtain correct, usable data. An additional caution is necessary when filtering such data. Low
3 frequencies (30 kHz) cannot penetrate the dense, complex, “sedimentary” vegetation which
4 forms the new bottom. As a result, low frequency measurement did not yield adequate results,
5 since it could not properly determine the density of the silt or vegetation, or the boundary
6 between the rocky and muddy bottom. Therefore it was used only during the first day of the
7 survey. Another deficiency recorded using the low frequency was significant leaps in profiles,
8 especially in places where the frequency penetrated the vegetation and muddy deposits. This
9 also indicated significant differences in the levels of muddy deposits. In order to perform a
10 more detailed analysis, a sediment profiler should be used, featuring a frequency of up to 15
11 kHz, which could be used to gain detailed information regarding the lake’s bottom (Lafferty
12 et al., 2005; Pribičević et al., 2007). Since the lake is shallow, and water transparency during
13 the survey was relatively good, it was relatively easy to determine the features of the lake’s
14 bottom and differentiate vegetated from non-vegetated areas.

15

16 **3.4 Interpolation Methods**

17 The spatial interpolation methods have been applied to many disciplines, where the most
18 prominent, are environmental sciences (Burrough and McDonnell, 1998; Webster and Oliver,
19 2001; Zhou et al., 2007). In comprehensive published studies, many authors (Aguilar et al.,
20 2005; Weng, 2006; Zhou et al., 2007; Li and Heap, 2008; Heritage et al., 2009; Guarneri and
21 Weih, 2012; Tan and Xiao, 2014) compared the performance of the spatial interpolation
22 methods. Some studies indicate that among the many existing interpolation techniques,
23 geostatistical ones perform better than the others and vice versa (Li and Heap, 2008).
24 Although there have been many studies on the accuracy of interpolation techniques for the
25 generation of digital elevation models (DEMs) there is still a need to evaluate the
26 performance of these techniques (Chaplot et al., 2006) because there are still no consistent
27 findings about the performances of the spatial interpolators (Li and Heap, 2008; Tan and
28 Xiao, 2014). In this paper we compared the 14 interpolation method implemented in
29 Geostatistical Analyst extension.

30 The most appropriate methods have been chosen, based on seven statistical parameters:
31 minimum value, maximum value, range, sum value, mean value, variance and standard
32 deviation. Of these, standard deviation, or mean quadratic error, is especially worth
33 mentioning, since it is the most used method world-wide for determining the precision of

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1 digital elevation models (Yang and Hodler, 2000; Aguilar et al., 2005). In addition to
2 analyzing parameters, interpolation methods were compared on the basis of high-fidelity,
3 two-dimensional and three-dimensional graphic representations of data sets. Volume
4 comparison methods were also used, by employing various algorithms, as well as methods for
5 calculating and comparing profiles (Pribičević et al., 2007; Medved et al., 2010).

6 In order to compare the accuracy of the interpolation methods, the method of cross-validation
7 was used. Most authors suggest using this method in order to achieve a successful evaluation
8 of accuracy (Cressie, 1993; Smith et al., 2003; Webster and Oliver, 2007; Hofierka et al.,
9 2007).

Deleted: The main aims of this research were as follows: to compare the efficiency of 14 different interpolation methods and discover the most appropriate interpolators for the development of a raster model

10 The fourteen interpolation methods were used as follows (with abbreviations):

11 Deterministic methods: Inverse distance weighting (IDW), Local polynomial function (LP),
12 RBF (radial basis function) - Completely regularized spline (CRS), RBF - Spline with tension
13 (SWT), RBF - Multiquadric function (MQ) and RBF - Inverse multiquadric (IMQ).

14 Geostatistical methods: Ordinary kriging (OK), Simple kriging (SK), Universal kriging
15 (UK), Disjunctive kriging (DK), Ordinary cokriging (OCK), Simple cokriging (SCK),
16 Universal cokriging (UCK) and Disjunctive cokriging (DCK).

18 **4 Research Results**

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19 **4.1 Interpolation of data gathered from the bathymetric survey**

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20 In order to generate continuous areas necessary for research and knowledge of the bottom of
21 Lake Vrana, it was necessary to approximate values in areas that were not sampled directly.

22 This was done using various interpolation methods. The effectiveness (quality) of
23 interpolation methods was analyzed in two phases. In the first phase, 12 851 points were used
24 to develop a model of the lake and compare interpolation methods. The second phase covered
25 30 233 points. Using the *ArcGIS* extension within the *Geostatistical Analyst* programme,
26 interpolation parameters were automatically optimized for each interpolation methods (Table
27 1).

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The main aims of this part are as follows:¶
1. To compare the effectiveness of fourteen interpolation methods ¶
2. To determine the most appropriate interpolators for the purpose of developing a raster model of the lake, on the basis of bathymetric data, by using the cross-validation method¶
3. To calculate the surface area and volume of the lake, and to compare differences in the calculation between raster models.¶

28 Four parameters influenced the quality of the output deterministic methods results: distance
29 exponent, number of neighbours, distance, and number of sectors. The number of neighbours
30 which influenced an approximated point was set at 15. The criteria for distance used a circular
31 search zone with a defined distance radius. All methods, except local polynomial methods,
32 featured a radius of 3619.9 m (Table 1).

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1 Table 1. Parameters of interpolation methods calculated only on point data gathered by
 2 bathymetric measurement

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IM*	Power	Model	Range	Sill	Nugget	Lag	Distance	NL*	NS*
IDW	2						3619.90		1
LP	1						228.20		1
CRS	12.3						3619.90		1
SWT	17.7						3619.9		1
MQ	0						3619.90		1
IMQ	0						3619.90		1
OK		Spherical	8496.40	0.591	0.227	886.11	10 633.32	12	4
SK		Spherical	2453.10	0.496	0.088	394.96	4739.52	12	4
UK		Spherical	10 058.80	0.000	0.031	886.11	10 633.32	12	4
DK		Spherical	2395.60	0.767	0.223	388.72	4664.64	12	4
OCK		Spherical	6461.03	0.560	0.191	886.11	10 633.32	12	4
SCK		Spherical	2451.89	0.496	0.087	394.88	4738.56	12	4
UCK		Spherical	8496.35	0.000	0.030	886.11	10 633.32	12	4
DCK		Spherical	2394.07	0.768	0.221	388.57	4662.84	12	4

3 *IM – interpolation method, NL – number of lags, NS – number of sectors

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4
 5 Geostatistical methods are more demanding to process, since they require semi-variogram
 6 modeling and the appertaining defining parameters.

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7 Table 2. Cross-validation of method results calculated only on point data gathered by
 8 bathymetric measurement

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IM	Number of points measured	Minimum value (m)	Maximum value (m)	Range (m)	Value sum (m)	Mean value (m)	Variance (m ²)	Standard deviation (m)
IDW	12 851	-1.748	2.265	4.013	-67.424	-0.005	0.062	0.249
LP	12 851	-1.702	2.100	3.802	79.836	0.006	0.049	0.222
CRS	12 851	-1.702	2.239	3.941	-48.410	-0.004	0.052	0.229
SWT	12 851	-1.707	2.234	3.941	-49.528	-0.004	0.052	0.228
MQ	12 851	-1.736	2.273	4.009	-23.102	-0.002	0.065	0.255
IMQ	12 851	-1.743	2.159	3.902	-68.307	-0.005	0.055	0.234
OK	12 851	-1.737	2.030	3.767	19.950	0.002	0.054	0.232
SK	12 851	-1.701	2.177	3.877	-8.482	-0.001	0.050	0.223
UK	12 851	-1.827	1.948	3.775	51.824	0.004	0.057	0.239
DK	12 851	-1.664	2.143	3.807	-3.060	0.000	0.051	0.225
OCK	12 851	-1.660	2.060	3.720	11.443	0.000	0.051	0.226
SCK	12 851	-1.526	2.007	3.533	-6.873	-0.000	0.038	0.197
UCK	12 851	-1.827	1.949	3.776	51.825	0.004	0.057	0.239
DCK	12 851	-1.535	2.022	3.557	-6.678	-0.000	0.041	0.203

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1 The first phase showed that all used methods of interpolation showed satisfying results based
2 on the ranges of values of the standard deviation (0.058 m), and were adequate for developing
3 digital elevation models of the lake, since they had similar parameter values (Table 2). The
4 main reason for this is the slight difference in depth values, low vertical dissection of the
5 lake's bottom and minimal percentage of elements with sudden leaps in height. The range of
6 value for standard deviation, considering the automatically optimized parameters, was
7 between 0.197 and 0.255 m. According to all parameters, the best method was simple
8 cokriging (0.197 m). The reasons for that were the principle of the method's process ($\mu =$
9 known stationary mean value, taken as a constant for the entire research area and calculated
10 from the median data value) and the maximum range between the depth values (only -3.46
11 m). The mean value for the entire area was -1.763 m.

12 Since most authors point out that the quality of stochastic methods depend on the choice of
13 criteria regarding semi-variograms, a comparison was made between the criteria automatically
14 determined by software and those manually determined for the ordinary cokriging method.
15 The two most common theoretical models were tested: spherical and Gauss (Table 3). The
16 purpose of manually assigning criteria is to find out the minimum deviation and minimum
17 value for standard deviation. In the case of the spherical model, the minimum value of
18 standard deviation was the distance of 1800 m (0.221 m). Unlike the automated software
19 process, finding the minimum value of standard deviation manually is more difficult and
20 time-consuming (it requires inputting the parameters of interpolation repeatedly until the
21 minimum value is found).

22

23 Table 3. Comparison of manually and automatically determined parameters of the
24 interpolation method (example for OCK).

Model	Range	Sill	Nugget	Lags	Distance	NL*	NB*	SD*	MPE*
Spherical (CAD*)	8496.4	0.591	0.227	886.11	10 633.32	12	4	0.232	2.030
Spherical (MD*)	1777.9	0.418	0.027	150.00	1800.00	12	4	0.221	2.238
Spherical (CAD*)	6337.5	0.477	0.302	886.11	10 633.32	12	4	0.238	1.948
Gauss (MD*)	133.8	0.042	0.048	20.00	240.00	12	4	0.220	2.235

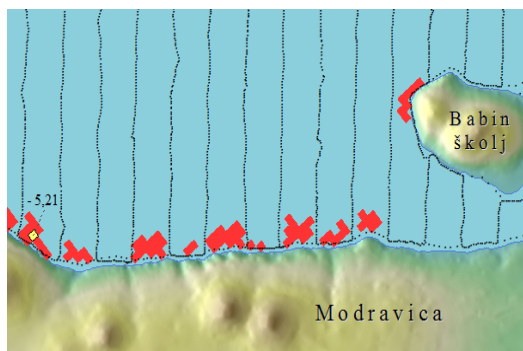
25 *IM – interpolation method, NL – number of lags, NS – number of sectors, SD – standard deviation, MPP –
26 maximum prediction error, CAD – criteria automatically determined, MD – manually determined

27

28 Table 3 shows that the output results regarding the standard deviation do not reveal significant
29 differences. For example, the difference between automatic and manually found standard

1 deviation in the case of the spherical model for 12 851 points is 0.011 m. However, it is
2 notable that the maximum error in the approximation for the same model is 0.208 metres
3 greater (2.238 m).

4 According to Malvić (2008), a decrease in distance also decreases the deviation, since the
5 values of closer points are more similar than the values of more distant ones. The decrease in
6 deviation should decrease the standard deviation calculated from the differences in the
7 measured and the approximated values. However, the quality of approximation in other parts
8 of the model might be questioned. By testing using ordinary kriging, the conclusion was that
9 the decrease in distance affected the standard deviation positively, and negatively in areas that
10 were not included in the direct measurement. The values obtained in such areas greatly
11 surpassed the values of the surrounding measured points. For example, a semi-variogram for
12 Lake Vrana was made, which was used to compare 30 233 points. The determined distance
13 was 1200 m, and the standard deviation for 12 851 points was 0.298 m. For the distance of 12
14 000 m, the standard deviation was 0.471 m. In the case of the first distance (1200 metres) the
15 lowest value of depth for the entire model was -5.21 m (the lowest measured depth was -3.73
16 m). As much as 0.246 km² of the model's surface fell within the category of -3.73 m to -5.21
17 m (Fig. 6). This result implies a serious error that would create an increase in the volume of



the lake. The second distance (12 000 m) did not feature any values above -3.578 m. This example shows that standard deviation can be an unreliable parameter when taking the values of the entire model into account.

Figure 6. Areas that were not directly measured during the survey (red squares)

26
27 Points gathered by the bathymetric survey did not include the entire surface of the lake, since
28 the echo sounder could not gather data in areas above -0.5 m. Since that resulted in a lack of
29 data at the edges of the lake, the modeling toolset poorly extrapolated the surfaces (Fig. 6).

30 Visually compared, the methods generally show the greatest differences in the smoothness of
31 isobaths, which is logical since the differences between the chosen parameters are essentially
32 negligible. A more detailed analysis indicates the results of certain methods (appearance of
33 continuous surfaces at micro levels).

1 In order to develop a digital model of the lake that would enable various simulations, such as
 2 changes in the water level, it is necessary to consider the data that refers to the surrounding
 3 terrain (height data, gathered by aero-photogrammetry). The combination of precisely
 4 obtained data on heights and depths enables the interpolation for the areas that were not
 5 directly included in the survey. The output results turned out well, since the lake features
 6 mostly low, flattened shores.

7 Due to curious output results in the first phase, the comparison of methods of interpolation
 8 was repeated for 30 233 points within the Lake Vrana Nature Park (Table 4). Of those points,
 9 12 851 were depths (bathymetrically measured points), and 17 832 were elevations (points
 10 with x, y and z values gathered by aero-photogrammetry). Statistic indicators were calculated
 11 only for the bathymetrically gathered points. The output results were quite different. The use
 12 of elevation points, which are necessary to develop a good digital elevation model of the lake
 13 and its surroundings, showed deficiencies in seven statistical parameters (Table 4) and
 14 graphic representations of data sets (Figure 7).

15
 16 Table 4. Cross-validation method results calculated for two data set elevation data, but
 17 statistic indicators is only for points gathered by bathymetric measurement.

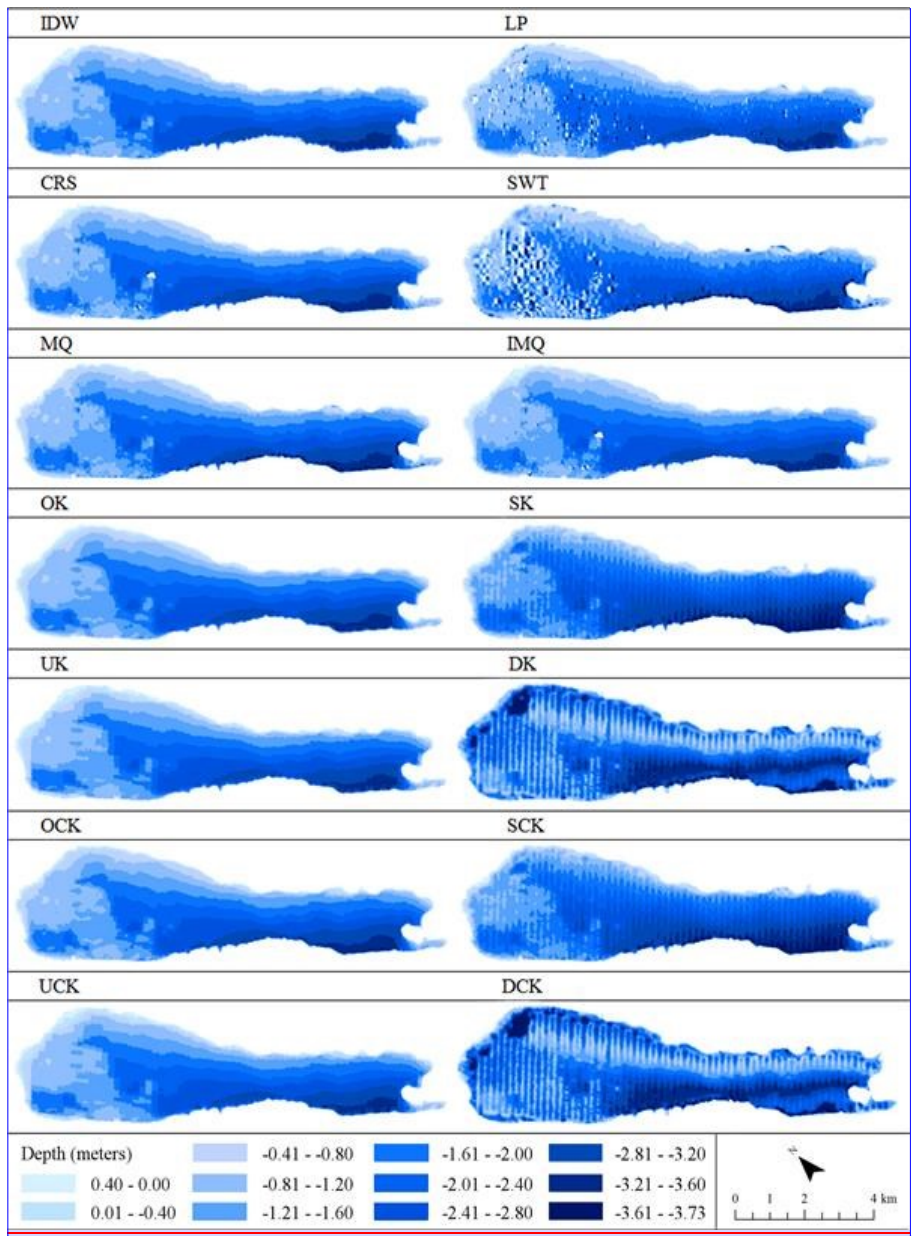
IM	Number of measured points	Minimum value (m)	Maximum value (m)	Range (m)	Value sum (m)	Mean value (m)	Variance (m ²)	Standard deviation (m)
IDW	30 233	-1.748	4.372	6.120	1169.497	0.091	0.199	0.446
LP	30 233	-2.142	4.809	6.951	1793.793	0.140	0.234	0.484
CRS	30 233	-117.351	46.197	163.548	487.438	0.038	1.825	1.351
SWT	30 233	-4.134	2.881	7.016	60.581	0.005	0.107	0.327
MQ	30 233	-1.925	2.618	4.544	360.547	0.028	0.087	0.294
IMQ	30 233	-87.722	40.884	128.607	464.898	0.036	1.298	1.139
OK	30 233	-1.700	5.551	7.250	1738.313	0.135	0.228	0.478
SK	30 233	-1.740	2.363	4.103	186.282	0.014	0.085	0.291
UK	30 233	-1.662	10.137	11.799	2329.834	0.181	0.343	0.586
DK	30 233	-5.977	4.267	10.245	1828.414	0.142	0.562	0.750
OCK	30 233	-1.314	2.280	3.594	543.563	0.042	0.057	0.239
SCK	30 233	-1.656	2.338	3.995	211.185	0.016	0.066	0.258
UCK	30 233	-1.665	10.136	11.802	2331.259	0.181	0.343	0.586
DCK	30 233	-8.972	4.976	13.949	1944.773	0.151	0.570	0.755

18
 19 Ordinary cokriging turned out to be the best method of interpolation according to all relevant
 20 parameters (Table 4). Figure 7 clearly shows the characteristic of the simple kriging method,
 21 when the range of elevation dataset in study area is 307.23 metres, in which case the mean

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1 value for the entire area is 38.02 m. Along with the ordinary cokriging method, satisfactory
 2 results were obtained from the inverse distance weighting method, RBF – multiquadric and
 3 ordinary kriging. The standard deviation according to all three methods was less than 0.5 m.
 4



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5
 6 Figure 7. Digital elevation models generated from surveyed data, showing differences
 7 between deterministic and geostatistical interpolation methods.

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The differences between the four best methods of interpolation are visible in the two-dimensional (Fig. 10a-b-c-d) and three-dimensional graphic representations. Figures 8a and 8b show the more vertically dissected part of the lake, with an AB profile, and a length of 1500 m, which was used as a further testing sample for the four best interpolation methods. The profile line was drawn so as to cover 6 bathymetrically measured points.

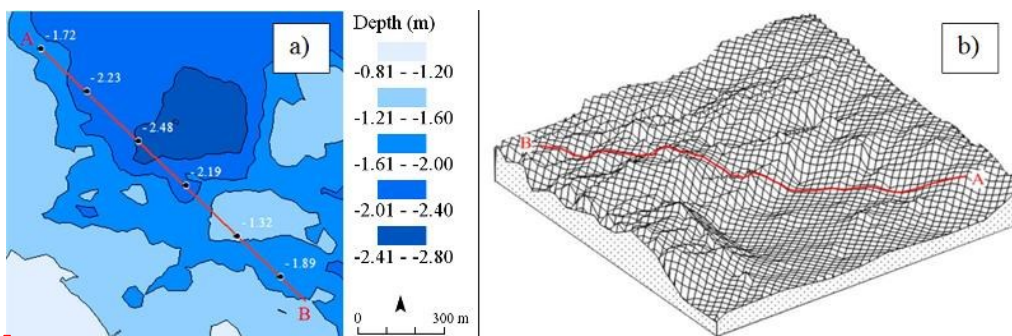


Figure 8. (a) Profile display, contour map. (b) Profile display, three-dimensional model

After drawing the profile line, it was necessary to calculate the intersection for the defined profiles based on the regular network generated by the interpolation, i.e. to convert the two-dimensional profiles into 3D lines which feature x, y and z values.

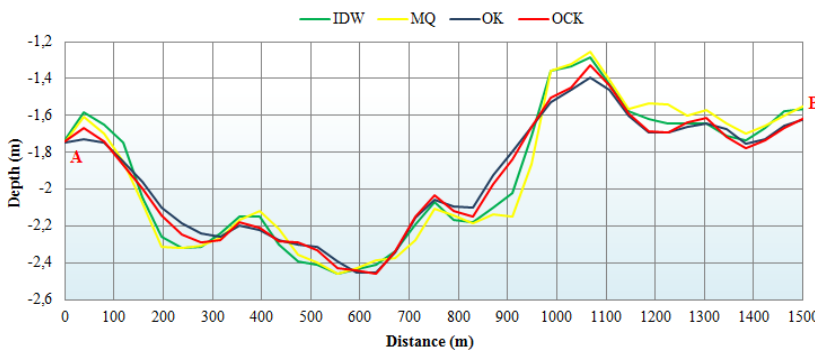


Figure 9. Differences in profile for the four best interpolation methods.

This approach enabled comparison of the profiles, a clear representation of the interpolated lake's bottom and the detection of deviation between the bathymetrically measured points and those approximated by the model. Figure 9 shows a difference in the interpolation method of

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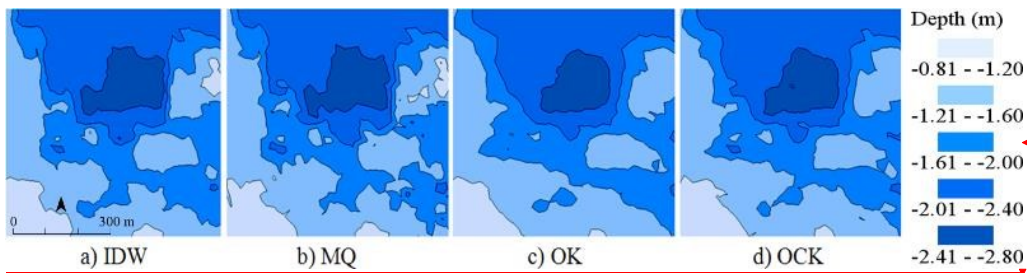
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1 deterministic (inverse distance weighting, RBF – multiquadratic) and geostatistical methods
2 (ordinary kriging, ordinary cokriging).

3 The final result of comparing methods of interpolation using *ArcGIS* extension *Geostatistical*
4 *Analyst* is to obtain a regular spatial network or grid. Usually, the greatest problem is deciding
5 between greater spatial resolution or pixel size (Hengel, 2006). In this case, the software
6 optimized the pixel size at 40 metres. The spatial resolution corresponds to McCullagh's
7 (1988) method of determining pixel size. The size was calculated using a grid calculator and
8 the method of point sample analysis (Hengel, 2006). On the basis of 12 851 points and an area
9 of 29.865 km², a spatial resolution of 24.2 m was generated. This method (McCullagh, 1988)
10 was not chosen due to a disproportionate ratio between the distance of the profiles and the
11 points measured in them. Due to the high density of the sampling within a profile (10 m), but
12 also due to variability in the elevation of the neighboring points, a problem known as the
13 "Prussian helmet" occurs (Šiljeg, 2013). The grid was later used as input data for the purpose
14 of developing a three-dimensional representation. In addition, it can be used to develop
15 various maps to show contours, lake terrain, grid models, slope, etc.



17 Figure 10. Representation of contours in part of the lake (difference between interpolation
18 methods)

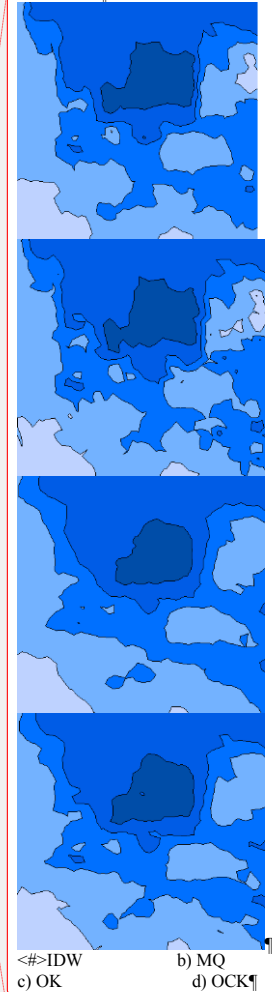
20 4.2 Surface area and volume of the lake

21 The final phase of bathymetric research involved calculating the lake's surface area and
22 volume (Diolaiuti et al., 2005; Ahmed, 2010). The output results of a certain analysis depend
23 on the method of data gathering, dissection of the lake bottom of the lake density and
24 distribution of points, spatial resolution (pixel size), algorithms and the interpolation method
25 used.

26 The volume of a lake can be efficiently calculated by a regular grid obtained by using a
27 certain interpolation method. The calculation process was relatively simple, since the number

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1 of pixels was known (18 714), as well as the surface (40 m x 40 m = 1600 m²) and the height
 2 (z) within the coordinate system. A pixel in this case represents a three-dimensional object
 3 (cube or a quadratic prism) based on which the volume can be calculated.

4 In order to compare it with other algorithms, the volume was calculated for the regular spatial
 5 grid, obtained by the ordinary cokriging interpolation method. The volume amounted to 49
 6 783 536 m³. This method yielded good results, since the difference between the result and the
 7 arithmetic mean for three rules (trapezoidal, Simpson's and Simpson's 3/8) was 293 143 m³
 8 (Table 5). The output results of volume calculation depend primarily on the spatial resolution;
 9 the lower the resolution, the more precise the calculation, because the leaps in values between
 10 pixels become less.

11 In order to calculate the volume, three more complex Newton-Cotes formulae were used: 1)
 12 the extended trapezoidal rule, 2) the extended Simpson's 1/3 rule and 3) the extended
 13 Simpson's 3/8 rule (Press et al., 1988). Newton-Cotes formulae are very useful and provide a
 14 direct technique for approximately calculating an integral by numerical methods and
 15 algorithms (their use results in various degrees of errors in the final calculation) (Medved et
 16 al., 2010). They are used to calculate the surface area and volume of various shapes.
 17 Simpson's rule approximates an integral by the Lagrange polynomial which passes through
 18 three points, while the trapezoidal rule approximates by the Lagrange polynomial passing
 19 through two points (Palata, 2003).

21 Table 5. Volume, surface and perimeter of Lake Vrana at 0.4 metre water level in reference to
 22 the Prosika gauge

Water level (0.4 m)	Interpolation method					
	IDW	MQ	OK	OCK	NaN	TIN
Trapezoid rule (m ³)	49 512 560	50 839 235	48 904 436	50 077 481	50 007 961	50 108 329
Simpson's rule (m ³)	49 523 461	50 822 602	48 902 952	50 070 506	50 008 506	50 107 823
Simpson's 3/8 rule (m ³)	49 516 428	50 821 012	48 906 375	50 082 051	50 011 883	50 105 204
Arithmetic mean (m ³)	49 517 483	50 827 616	48 904 587	50 076 679	50 009 450	50 107 119
Surface (km ²)	29.521	30.009	29.493	29.865	29.897	29.857
Perimeter (km)	36.619	36.703	34.290	35.851	35.918	36.118

23 Table 5 shows calculated values for the volume derived from Newton-Cotes formulae,
 24 applied to five different methods of interpolation. Since every method displays a certain level

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1 of error in the approximation of the volume, arithmetical means for the three methods were
2 also calculated.

3 The border of the lake for all the models was an isobath at 0.4 metres, obtained by
4 interpolating bathymetrically measured depth data and terrain elevation data obtained by aero-
5 photogrammetry. The isobath was converted into a polygon, which was used to determine a
6 raster model within the borders of the polygon. Results of the measurements of surface,
7 perimeter and volume of the lake, regardless of the formula used, greatly depend on the model
8 developed by interpolation (Table 5).

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9
10 Table 6. Perimeter and surface area of Lake Vrana at various water levels, for the most
11 suitable (OCK) interpolation method

Water level (in reference to the Prosika gauge)		Perimeter (km)	Surface area (km ²)
Maximum*	2.36	38.541	33.064
Mean	0.93	38.338	30.815
Minimum	0.15	34.974	29.177

12 *Within Lake Vrana Nature Park

13

14 The surface area of Lake Vrana, in relation to its water level (which annually oscillates by
15 1.93 m) varies by almost 4 km² (Table 6). It can be obtained by manual vectorisation based on
16 a geo-referential digital orthophoto (29.412 km²). The process is relatively simple, and the
17 contour of the lake is represented by the border between the water and land, defined by
18 subjective visual approximation. However, 4.6% of the lake's surface area is covered in dense
19 vegetation (*Phragmitetalia*), which makes determining the surface area a more complex task.
20 Considering the limitations of the aforementioned method, the research employed previously
21 stated interpolation methods for determining the lake's surface area.

22 The total surface area of the lake is 30.815 km², calculated based on the 0.93 m isobath (mean
23 water level in the observed period from 1947 to 2008) obtained by interpolating data on an
24 elevation of the surrounding terrain and depth of the lake. The interpolation method provided
25 good results, in relation to the subsequent testing of the model, because most of the lake's
26 shore is flattened featuring mild slopes and almost no anomalies in data values obtained by
27 bathymetric survey and aero-photogrammetry. The method was also tested by field work,
28 using a precise GPS. The device was used to record information on the most distant borders

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1 of the lake at six randomly chosen locations. Since the interpolated border of the lake was
 2 transferred into GPS, it was easy to determine the deviation.

3 The average width of the lake is 2201.4 metres (minimum 262.26 and maximum 3469.31
 4 metres). The average length of the longitudinal profiles is 8765.43 metres (minimum 1843.55
 5 and maximum 13 245.34 m). These values were obtained by analyzing 68 transverse
 6 (northeast-southwest) and 17 longitudinal (southeast-northwest) profiles at 200-metre
 7 intervals (at the water level of 0.4 metres).

8

9 **5 Discussion and Conclusion**

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10

11 The last phase of digital terrain modeling refers to the application of a model (Weibel and
 12 Heller, 1991; Hutchinson and Gallant, 2000; Hengel et al., 2003; Oksanen, 2006), therefore
 13 this research visualized the annual water level oscillation. A scenario was made for the
 14 northwestern Jasen inundation area, outside the Nature Park (Fig. 11). A section of the
 15 flooded habitats and cadastre plots within the Nature Park were also determined (Table 7), at
 16 the water level of 2 metres (Fig. 12).

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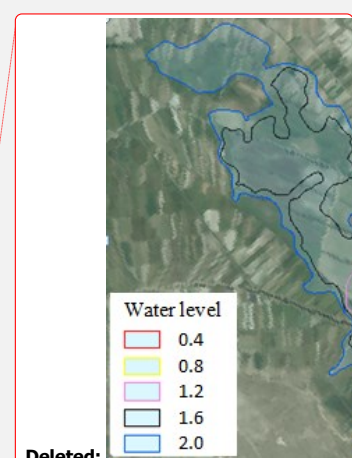
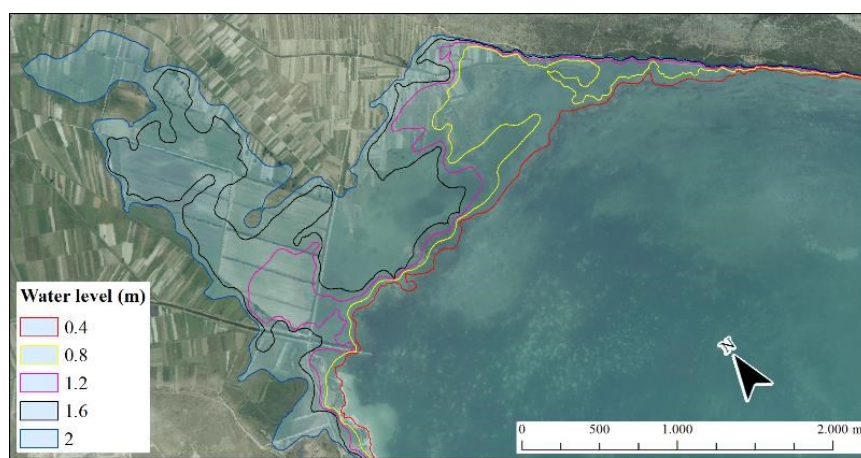
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18 Table 7. Percentage of flooded habitats at the water level of 2 m in reference to the Prosika
 19 gauge

NKS_DESCRIPTION	Flooded area (ha)	Total area of the habitat in the NP (ha)	Percentage (%)
Complex mosaic of crops	37.8	206.3	18.3
Illyrian-Sub-Mediterranean river valley meadows / Mediterranean halophytic <i>Juncus</i> species	32.6	34.9	93.4
Mixed evergreen forests and holm oak maquis	15.6	696.3	2.2
Brambles	6.6	685.9	1.0
Shore uncovered or rarely covered by vegetation	4.4	6.3	70.9
Illyrian-Sub-Mediterranean river valley meadows	2.6	2.6	100.0
Tree lines at the edges of cultivated areas	2.1	7.2	29.5
Brambles / Thermophile flooded underbrush	1.2	3.5	34.9
Thermophile flooded underbrush	0.6	1.2	50.0
Aleppo pine plantations	0.6	65.6	0.9
Tyrhenian-Adriatic limestone	0.6	1.0	60.3
Consolidated arable land with monoculture crops (cereals)	0.6	1.1	52.7
Man-made or industrial habitats	0.5	11.2	4.6

20

1 The water level map at 2 m was overlaid with the map of habitats for Lake Vrana Nature Park
2 to the scale of 1:5000. The map was made in accordance with the rules of National Croatian
3 Habitat Classification and comprises 30 classes of habitats (Jelaska, 2010). A sudden change
4 in the water level can change the ecological features of a particular habitat, affecting the flora
5 and fauna of Lake Vrana Nature Park. The analysis concluded that almost half the habitats are
6 endangered if the water level rises to 2 metres. The highest level of threat (100%) relates to
7 Illyrian-Sub-Mediterranean river valley meadows and the lowest level (1%) relates to
8 brambles. It is worth noting that 52.7% of the endangered areas are consolidated arable lands
9 with monoculture crops (cereals), while 18.3% (37.8 ha) are complex mosaics of crops.



12 Figure 11. Annual water level oscillation in the northern part of Lake Vrana Nature Park
13 (probable scenario in case the Jasen water pump stops working)

14
15 Most field parcels in the Park are used for intensive agricultural purposes. The northern part
16 of the Park features horticultural plants with multiannual crop rotations. Plants include mostly
17 hybrid species. Various agro-technical methods are used in order to produce a better level of
18 crop success, as well as fertilizers and chemical components for plant protection (JUPPVJ,
19 2010). In the northwestern lake area, there is a mixed culture of olive fields, vineyards,
20 horticulture and some cereal crops (JUPPVJ, 2010). Should the water level rise by 2 metres, it
21 would partially or completely threaten 45.94% of the cadastre plots. In the northern part of the
22 Park (a flatter area), flooding would threaten the entire area. In the northwestern part, flooding
23 would mostly threaten areas at a lower elevation. These areas have been more susceptible to
24 flooding in the past, as is evident from the specific shape of the field parcels (especially in the

1 northwestern part). The parcels there are narrow (10 metres on average) and extremely
2 elongated (150 metres). The inclination of these parcels (2-5°) is perpendicular to the lake.



3
4

5 Figure 12. Flooded agricultural parcels in the Pakošćane cadastre at a water level of 2 metres
6 (northwestern section of the Park)

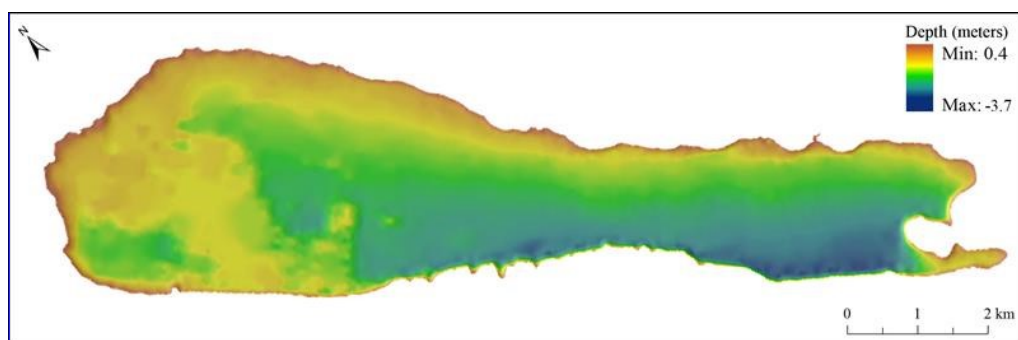
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8 The results of this research show that the output results of the digital terrain modelling and
9 corresponding analyses depend on the data gathering methods, density of samples,
10 interpolation methods, terrain features (mostly vertical dissection), pixel size and algorithms
11 applied. In the research, 14 methods of interpolation were compared; 6 deterministic and 8
12 geostatistical. Of the five most common methods for gathering elevation data and comparing
13 interpolation methods, two sets of data were used (depth and elevation). They were obtained
14 by various methods, techniques and procedures: bathymetry and aero-photogrammetry. The
15 conclusion is that there is no universal method of interpolation which shows the best results in
16 both sets of data, since the output results depend on the data gathering method. For example,
17 an optimal method for developing a DEM of the lake's shore was developed, but it turned out
18 to be inadequate for developing a DEM of the lake's bottom. In addition, regardless of the
19 fact that certain authors point out either deterministic or geostatistical methods as more
20 advantageous, it is important to note that there is no single best interpolation method, since
21 they are all conditioned by spatial and temporal components. This means that the result of the
22 comparison and selection of the best method are in fact provisional and dependent on time
23 and space components, the technology used to gather and process data, and the area of
24 research.

1 The fact that geostatistical methods of interpolation employ mathematical functions and the
2 probability theory was one of the reasons for hypothesizing that geostatistical methods would
3 be better interpolators. This was proven, but the research also showed that the differences
4 between geostatistical and deterministic methods were negligible. The multi-quadratic
5 function, as the globally most commonly accepted method, was proven to be the best radial
6 basic function, but also one of the best deterministic interpolation methods in general.

7 In order to develop a digital model from the bathymetrically gathered data, 14 interpolation
8 methods were compared in two phases. In the first phase (which used 12 851 bathymetrically
9 measured points), all the methods compared showed good results, due to the low vertical
10 dissection of the terrain. By using the method of cross-validation and analyzing statistical
11 parameters, the conclusion was that the best results were yielded by the simple cokriging
12 method (the standard deviation was 0.197 m).

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13 Figure 13. Bathymetric map of Lake Vrana (ordinary cokriging)

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16 The range of the standard deviation for all 14 methods was between 0.197 and 0.255 m. Due
17 to characteristic issues with output results and the problem of extrapolating data in the first
18 phase, the process of comparing interpolation methods was repeated for the sample of 30 233
19 points within Lake Vrana Nature Park. The output results in the second phase were notably
20 different, and the majority of methods applied showed imperfections. According to all the
21 statistical parameters, the best method of interpolation was ordinary cokriging (Fig 13). Along
22 with ordinary cokriging, good results were shown by the inverse distance weighting method,
23 RBF – multiquadratic method and ordinary kriging. The standard deviation for all three
24 methods was less than 0.5 m. These methods were compared by graphic representation,
25 calculation and comparison of the profiles, surface area and volume of the lake. The

1 conclusion was that there were no significant differences between the statistical indicators in
2 deterministic or geostatistical methods, whether the parameters were determined
3 automatically or manually. However, by testing the ordinary kriging method, the conclusion
4 was that the reduction in the distance positively affected standard deviation, but negatively
5 affected approximation in the areas that were not included in the direct survey. The
6 interpolated values in those areas turned out to be much greater than the values actually
7 measured at the surrounding points.

8 Based on the optimal method of interpolation, the lake's surface area, perimeter and volume
9 were calculated at the water level of 0.4 meters in reference to the Prosika gauge. The surface
10 area of the lake is 29.865 km², the perimeter is 35.851 km and the volume is 50 076 679 m³.
11 During the bathymetric survey, the conclusion was that a low frequency (30 kHz) could not
12 penetrate the very thick, intertwined "sediment" vegetation which formed the new bottom of
13 the lake. Another problem with low frequency is occasionally significant leaps in profiles,
14 especially in places where the frequency managed to penetrate the vegetation or mud. In order
15 to perform a detailed analysis, a sediment profiler with a frequency of up to 15 kHz should be
16 used to gain detailed information about the layers on the lake's bottom (Lafferty et al. ,2005;
17 Pribičević et al., 2007).

18 All the analyses and conclusions derived can be used for further research on data gathering
19 methods, interpolation methods, methods of spatial resolution selection and methods of digital
20 terrain analysis. In any future research of Lake Vrana, it would be useful to extend the profiles
21 during the survey, if a single beam sounder is used, so that the distance between the profiles is
22 no greater than 50 metres. In that case the relation between the profiles and the data gathered
23 from the profiles (every 10 metres) would be much more proportional. In addition, it would
24 be useful to compare the results of the development of the lake's bottom model using single
25 beam, multi-beam and laser sounder techniques. It is important to note that the more efficient
26 techniques, such as multi-beam ultrasound or laser measurement, might not yield significantly
27 better results due to the morphology of the bottom and the relatively high percentage of dense,
28 native vegetation. The portion of the bottom surveyed would increase in relation to the
29 portion surveyed with the single beam sounder, but the costs of such research would
30 drastically increase, as well as the amount of data yielded for the processing. In that case,
31 processing stations would have to be employed as well. A frequency of under 15 kHz is
32 recommended for future research, in order to determine the density and volume of sediments.
33 Since 4.6% of the lake's surface is covered in dense vegetation, it was difficult to determine

1 the exact borders. The dense vegetation prevents sounders from effectively reaching the
2 surface. In order to avoid extrapolation in the bordering areas, the research employed
3 elevation data obtained by aero-photogrammetry and stereo-restitution, where the average
4 distance between the elevation points was 90 metres. If future interpolation projects aim at
5 higher level of precision in the bordering areas, it will be necessary to reduce the distance
6 between the elevation points. Recommended methods include aerolaser or aero-
7 photogrammetry. In this case, the distance between the points should be less during stereo-
8 restitution (maximum 10 metres).

9 The data measured and evaluated enable the development of hydrological and hydro-
10 technical studies which would result in an optimal water level, ensure a biological minimum
11 and economical water minimum, and optimize the water system. In order to determine the
12 volume of the lake, it was necessary to map the lake's bottom, gather data required for the
13 development of the digital elevation model, and make a topographic map of the lake's bottom
14 and shoreline relative to its optimal water level, thus creating a sound basis for future
15 activities.

16

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