

Surface seiches in Flathead Lake

Georgiy Kirillin^{1,2}, Mark S. Lorang², Tom C. Lippmann^{2,3}, Chris C. Gotschalk²,
and Sebastian Schimmelpfennig¹

¹Department of Ecohydrology, Leibniz-Institute of Freshwater Ecology and Inland Fisheries (IGB),
Berlin, Germany

²The University of Montana, Flathead Lake Biological Station, Polson MT, USA

³University of New Hampshire, Dept. of Earth Sciences and Center for Coastal and Ocean

Correspondence to: Georgiy Kirillin (kirillin@igb-berlin.de)

Abstract. Standing surface waves or seiches are inherent hydrodynamic features of enclosed water bodies. Their two-dimensional structure is important for estimating flood risk, coastal erosion and bottom sediment transport and for understanding shoreline habitats and lake ecology in general. In this work, we present analysis of two-dimensional seiche characteristics in Flathead Lake, Montana, USA, a large intermountain lake known to have high seiche amplitudes. To examine spatial characteristics of different seiche modes we used the original procedure of determining the seiche frequencies from the primitive equation model output with subsequent derivation of the spatial seiche structure at fixed frequencies akin the tidal harmonic analysis. The proposed procedure revealed specific seiche oscillation features in Flathead Lake including maximum surface level amplitudes of the first fundamental mode in straights around the largest island; several higher modes appearing locally in the vicinity of the river inflow; the “Helmholz” open harbor mode, with the period approximately twice that of the longest seiche mode, generated by a large shallow bay connected to the main lake basin; and several rotating seiche modes potentially affecting the lake-wide circulation. We discuss the lake management problems related to of the spatial seiche distribution, such as shoreline erosion, floods and transport of sediments and invasive species in Flathead Lake.

Keywords. Harmonic Analysis

Princeton Ocean Model

Helmholtz oscillations

Seiche-driven shoreline erosion

Flood risk

1 Introduction

Since Forel (1893; 1895) first described free surface oscillations in Alpine lakes and coined the term “seiches” the standing basin-scale waves are recognized as distinguishing features of lake hydrodynamics. Seiches exist in any (semi-) enclosed basin, where long surface waves reflecting at the opposite shores interfere to produce a standing wave. The longest possible seiche (the first “fundamental” mode) has the wavelength λ that is twice the distance between the shores, L . The wavelengths of other possible modes decrease as even fractions of the distance between shores. Therefore, for the majority of possible modes, seiche waves are very long compared to the lake depth. This characteristic allows application of shallow water linear wave theory to the analysis of seiche motion (see e.g., Turner, 1973). The dispersion relation for the one-dimensional long linear waves yields the well-known Merian’s formula (von der Mühl, 1886),

$$\omega = c\lambda^{-1} = nc(2L)^{-1} \quad (1)$$

where $\omega = 1/T^{-1}$ is the oscillation frequency, T is the period of oscillation, $c = (gH)^{1/2}$ is the phase speed of the long wave in a basin with mean depth H , and $n = 1, 2, 3 \dots$ is the seiche mode. (1) corresponds to a planar wave propagating in one lateral dimension, and performs well for lower modes (small n) in basins of quasi-rectangular or elongated canal-like shape (Defant, 1961). In lakes of more complex shape, the lateral structure of modes is essentially two-dimensional in the horizontal plane (Rueda and Schladow, 2002; Kirillin et al., 2008). Hutter et al. (2011) presented an overview of the seiche problem with mathematical background, bibliography and examples from different lakes. Interference of multiple modes in basins with a highly indented shoreline, as well as strong variations in depth, may produce seiches with frequencies deviating from those predicted by Eq. (1) (Rueda and Schladow, 2002; Hutter et al., 1982), may strongly increase amplitudes of near-shore water level oscillations (Hollan et al., 1980; Rudnev et al., 1995), or give rise to local rotating modes (Rao and Schwab, 1976). These seiche effects are especially important in large lakes, gulfs, and seas, where a local increase of seiche amplitudes can threaten populated areas with flooding (Hollan et al., 1980; Kulikov and Medvedev, 2013). Coastal erosion, nearshore wetland habitat structure, and sediment transport are also seiche-related issues relevant to water and coastal management. Determining the two-dimensional seiche structure from observations requires large amounts data on water level and current velocity from multiple sites along the shoreline and can have high uncertainty due to contamination of data by frequency aliasing or non-seiche oscillations (Mortimer and Fee, 1976). Numerical modeling is an effective alternative to observational approaches. Among modeling studies on the two-dimensional structure of seiches in large basins with complex morphometry, the “normal modes” approach has been widely used, based on replacement of the system of equations for water motion with their equivalents for harmonic functions of space and time. This approach reduces the numerical problem to a discrete set of eigenvalue solutions, each corresponding to a certain seiche mode (Csanady, 1967; Platzman, 1972; Rao and Schwab, 1976). A more straightforward approach

based on isolation of seiche motions from the results of direct hydrodynamic simulation with primitive equations of motion has a certain advantage owing to the recent improvements in computational techniques (Suursaar et al., 2002; Jönsson et al., 2008). Since circulation modeling with primitive equation models has become routine during the last decades, many model codes are freely available and their application is not overly time-consuming. In addition, existing outputs from circulation models, applied previously to different bays, lakes or estuaries, may be later re-analyzed using appropriate methods for estimation of seiche characteristics. In this study we present an analysis of the modal structure of seiches in Flathead Lake (described below). Instead eigenvalue solutions for a priori periodic velocities and surface elevations, we solved a “backward” problem by applying harmonic analysis to the output of a primitive equations model and deriving a discrete set of seiche eigenfrequencies from a “synthetic” spectrum for all grid points of the calculation domain. The method is an effective way to obtain the detailed information on the possible spatial structure of free surface oscillations in Flathead Lake. We discuss the lake-specific features of these oscillations, mechanisms of their generation, and potential importance for predicting floods, coastal erosion and occurrence and structure of nearshore wetland habitats around the lake.

2 Study site and methods

2.1 Flathead Lake

2.1.1 Lake origin and morphology

Flathead Lake is a natural glacial lake located in the Rocky Mountains of the western US ($47^{\circ}52'N$; $114^{\circ}07'W$ approximate lake center). It is the largest fresh water lake in the western contiguous USA, extending 45 km by 24 km and with 496 km² surface area (Fig. 1). Its origin is attributed to the final retreat of the Flathead glacial lobe, a southern extension of the continental Cordilleran ice sheet, that resulted in melt waters filling the southern end of a north-south oriented fault-block valley (Lorang et al., 1993b). The process of glacial advance and retreat created Polson Bay (PN in Fig. 1), a shallow (approximately 8 m deep) circular bay at the southern outlet which runs through the terminal moraine. Polson Bay is connected to the main deep body of the lake through a relatively narrow bedrock island-filled gap (The Narrows, NR in Fig. 1). A branch of the Flathead glacial lobe also created Big Arm Bay, a narrow medium-deep (25 m average depth) east-west oriented bay that is bounded by Wildhorse Island, the main island in the lake. The main body of the lake is dominated by a north-south oriented 100 m deep trench bordered by a wider and more shallow shelf with a mean depth of 50 m (Fig. 1). The combination of plate tectonics and glacial processes (basin scour during advance and deposition during retreat) resulted in a lake with a range of shoreline types, islands, bays, and bathymetric provinces. Mountain ranges on either side topographically control wind that provides the principal driving force for seiche motion in the lake. Wind-generated surface

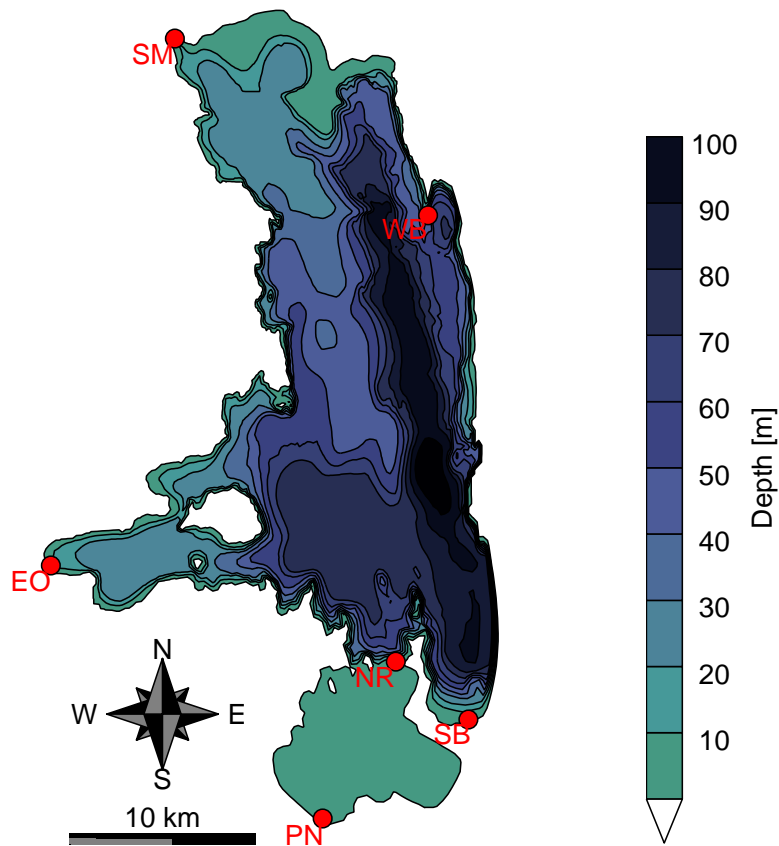


Figure 1. Flathead Lake: bathymetry map and location of pressure and water level measurement sites (red circles), referred in the text.

gravity waves (with periods of 1-3 s) generated in the main body of the lake maintain a sandy-silt sediment deposit in the northern half of Polson Bay (Lorang, 1984), and fine silts and clay elsewhere in the bay and most of the lake bottom. Sediments delivered by the Flathead River (which drains a 18,372 km² catchment) formed a large delta where the river enters the lake. This sand-silt delta extends several kilometers out into the lake (Fig. 1) and results in a very dissipative, low slope, nearshore environment for the north shore (Lorang et al., 1993b). Deposition of glacial till along much of the east and west shores combined with exposed bedrock resulted in reflective nearshore systems characterized by gravel beaches and steep inshore shelves armored by wave-washed cobble (Lorang et al., 1993b).

100 2.1.2 Winds and lake level fluctuations

Winds on Flathead Lake are thermally driven in the summer months due to differential heating and cooling on the lake surface and the adjacent mountain ranges. This thermal regime is characterized by distinct westerly thermal winds draining from mountains out on to the lake in the evening and

northerly winds driven by air masses that drain north to south in the valley early in the morning.

105 These winds typically have a maximum speed of out around 5 m s^{-1} with durations of two to six hours and cover 30-50% of the lake. Passing frontal systems produce north-northwest and south-southwest wind events in which wind speeds often reach $10\text{-}15 \text{ m s}^{-1}$ with durations of 2-6 hours. These events often spin up and down repeatedly over periods of days and cover the entire lake. The strongest winds come out of the east and are driven by high pressure systems that sit off to the east of

110 the Rocky Mountains producing local topographically channeled winds along the east shore. These east winds have been observed to reach $30\text{-}50 \text{ m s}^{-1}$, and can last for hours and extending several kilometers out into the main body of the lake, being however limited to the central portion of the lake. During these events it can be completely calm on the west shore. Flathead Lake levels fluctuate seasonally with the spring freshet from snow melt. However, Kerr Dam built in 1938 and located

115 6 km below the lake outlet now holds water in the lake for long durations as compared to pre-dam conditions (Lorang et al., 1993a; Lorang and Stanford, 1993). Kerr Dam operations attempt to hold the lake level at a “full-pool” elevation of 881.78 m, without exceeding that limit for approximately 4 months then slowly lowers the lake over late fall and winter months (Lorang et al., 1993a). These regulated fluctuations have concentrated wave energy at the full-pool operational lake elevation that

120 has reconfigured the near-shore zone lake-wide, causing variable shoreline retreat ranging from tens of meters to over a kilometer (Lorang et al., 1993b, a; Lorang and Stanford, 1993). However, large differences in shoreline loss have been shown to result from small changes in lake level of 30 cm or less (Lorang et al., 1993a; Lorang and Stanford, 1993). Some of the variations are seasonal while others have diurnal or hourly periods hence water level change occurs across a range of time scales

125 that spans five orders of magnitude (Morrison et al., 2005). Mode 1 seiche period along the long axis of the lake can be approximated from Eq. (1); however, actual seiche motions are influenced by bathymetry, the shape of the shoreline, and other factors. For a 50 km-long lake with an average depth of 50 m the seiche will have a period of approximately 2 hrs while, the cross-lake seiche will have a period of approximately 30 minutes as would Polson Bay. Morrison et al. (Morrison et al., 2005)

130 measuring at a single location in Polson Bay observed a strong spectral peak in surface oscillations at a period of 2 hrs and a less energetic peak at a period of 35 min.

2.2 Circulation model, configuration of model runs

The Princeton Ocean Model (POM, Blumberg and Mellor, 1987) was run on a curvilinear grid domain of 100 by 200 cells approximating the bathymetry of Flathead Lake. The model was run

135 in barotropic mode (no baroclinic density currents). Horizontal eddy viscosity was modeled by the Smagorinsky diffusivity (Smagorinsky, 1963) with a non-dimensional coefficient $C = 0.2$. We adopted the simple bottom stress parameterization based on the law-of-the-wall and the thickness of the logarithmic layer dependent on the lake depth from (Schimmelpfennig et al., 2012). The circulation was forced by an initial linear tilt of the lake surface of $0.5 \cdot 10^{-6}$, corresponding to the surface

140 slope produced by surface wind stress of approximately $3 \cdot 10^{-4} \text{ m}^2 \text{ s}^{-2}$, corresponding to winds of about $8\text{-}10 \text{ m s}^{-1}$. Several model runs with duration of 4 days model time were performed with varying directions of the initial tilt. Comparison of the model outputs demonstrated essentially the same spectral composition of the seiche oscillations, so that only a single run with the south-north initial slope is analyzed below, roughly coinciding with the main axis of the lake.

145 2.3 Harmonic analysis

Harmonic analysis relies on a prescribed set of frequencies persisting in the analyzed signal. Therefore, it became to a state-of-the-art method in the analysis of tidal motions, where the prevailing frequencies are determined by the external forcing and are fixed independently from the characteristics of the water body. Harmonic analysis is rarely, if ever, applied to the seiches, as their harmonics cannot be known *a priori*. Being defined by the basin morphometry, the contribution of different harmonics into the total variance of surface oscillations varies spatially over the lake. Therefore, neither single point data nor the spectrum averaged over the lake area are representative for the entire basin. In order to uncover the frequencies set of seiche oscillations relevant to the whole lake, the following procedure was developed and applied. In order to retain only the oscillatory motions in the surface elevation time series, the mean values were removed from velocity and surface elevation records. A spectrum was taken of the free surface oscillations at *every* grid point of the model domain. Then, a “maximum spectrum estimation” was constructed by superimposing the significant spectral peaks among all grid points. The significance level of the spectral peaks was defined as the upper 99% significance level for red noise signal containing the same variance as the spectrum estimation (Kirillin et al., 2008). This approach conserved the “local modes” with relatively high amplitudes but confined to small areas, like bays or straits. In contrast, using an average spectrum over the entire lake would smooth out local modes. Frequencies with significant spectral peaks provided a basis for subsequent harmonic analysis. The procedure of the harmonic analysis followed that described in (Thomson and Emery, 2001): Time series of velocities and water levels at every grid point were approximated with least-squares by a Fourier series containing only the selected frequencies. By this approach, the spatial distribution was established with amplitudes and phases for every mode over the lake.

For analysis of spatial structure of vector velocity fields, the distribution of the rotary coefficients, R , over the lake surface for each seiche mode was estimated. The rotary spectra were defined as $R = (S^+ - S^-)(S^+ + S^-)$, where S^+ and S^- are the counterclockwise and clockwise rotary spectra of velocity vectors respectively. The rotary spectra were calculated by taking Fourier transform of the complex velocity vector $u + iv$, where u and v are the orthogonal cartesian components of the 2-dimensional velocity, (see Thomson and Emery, 2001, for details) resulting in a two-sided spectral power density estimation, with positive (right-side) range of frequencies corresponding to the anticlockwise rotation and negative (left-side) range of frequencies corresponding to the clockwise

rotation (Gonella, 1972; Hayashi, 1979). As such, the rotary coefficient ranges from -1 for purely clockwise rotation to +1 for counterclockwise rotation, and is zero for unidirectional motion (see Thomson and Emery, 2001, for details).

2.4 Measurements

180 In the following analysis, we use data on underwater pressure oscillations at collected during 2011-2012 at five measurement sites around the lake (points EO, SM, WB, NR, SB in Fig. 1) and lake level data collected by the United States Geologic Service (USGS) at Polson (point PN in Fig. 1). The pressure variations at site EO were measured by onset Pressure-Temperature (PT) sensors taking instantaneous measures every 2 minutes. Other pressure measurements were collected with experi-
185 mental PT wave recorders (msi) that sampled at 4 Hz and then smoothed the data and re-sampled at 2 minute intervals to match the onset PT loggers. The USGS gauging station was located in a stilling well connected by a pipe to the lake and recorded lake level values at 1 hour interval.

3 Results

The model was driven by the initial surface slope without any additional external forcing, so that
190 the resulting spectrum of the surface level oscillations has a line shape in contrast to continuous spectra of natural water level oscillations. The seiche frequencies and periods were therefore easily identifiable from the line spectral peaks (Fig. 2): 16 frequencies were identified with the weakest peak containing about 1% of the spectral energy of the strongest one. The sum of spectral energies residing at the 16 frequencies contained more than 99% of the total variance (Fig. 2, Table 1).

195 An additional support to the results of the harmonic analysis was provided by the comparison of the time series restored from the 16 Fourier components against the original modeled time series: The restored time series captured all major features of the original variability (3), suggesting that the 16 frequencies contain the bulk of the free barotropic motions of the lake.

For further analysis, we conditionally divided the seiche modes in three groups, based on the
200 spatial distribution of the elevation amplitudes and phases: 6 “lake-wide modes” (Fig. 4), 6 “strong bay modes” (Fig. 5), and 4 short-period seiche modes with the smallest amplitudes. The latter group is characterized by very small energy of oscillations and is therefore excluded from the detailed analysis below. For completeness, the spatial characteristics of the four least energetic short-period modes are provided in Fig. S1, *Supplementary Material I*.

205 The amount and distribution of the zero-phase of the lake-wide modes with periods of 63.0, 32.4, 21.6, and 14.2 min (red lines in Fig. 4B, D-F) allows suggesting these modes to be one- to four-node analogues of one-dimensional channel modes respectively. Comparison of their periods to those following from the Merian formula (1) supports this suggestion: Substitution of the maximum lake length $L = 44$ km and of the mean lake depth $H = 50$ m into Eq. (1) yields similar values for the

Table 1. Periods of free oscillation modes in Flathead Lake

Significant seiche periods sorted by frequency		Significant seiche periods sorted by spectral energy	
mode	Period (min)	Period (min)	mode
0	117.0	63.00	1
1	63.00	48.48	2
2	48.48	24.18	5
3	32.40	32.40	3
4	29.58	117.00	0
5	24.18	17.16	9
6	21.60	29.58	4
7	20.70	20.70	7
8	17.76	16.02	10
9	17.16	17.76	8
10	16.02	21.60	6
11	14.20	13.80	12
12	13.80	11.76	13
13	11.76	10.38	14
14	10.38	14.20	11
15	10.00	10.00	15

210 periods of the first four longitudinal modes: 66, 33, 22, and 16.5 min. Their spatial distribution is
however modified by the irregular shape of the lake and differs significantly from that of plane lon-
gitudinal oscillations (color plots in Fig. 4, see also animated figures 1-4 in *Supplementary Material*
II). The highest water level amplitudes of the 1st mode (Fig. 4B) are concentrated in Big Arm Bay at
the western side of the lake. These high amplitudes are apparently conditioned by narrowing of the
215 lake cross-section towards the bay, which is separated from the main basin by two narrow straights
around the Wild Horse Island. Eq. (1) fails to predict 117 and 48.5 min modes (Fig. 4AC). The lat-
ter oscillation has maximum amplitudes in the Big Arm Bay on the west side of the lake and one
nodal line in the main basin of the lake. Hence, it can be associated with a one-node seiche along a
shorter length scale than the maximum lake length of 44 km-an analogue of a transverse seiche in a
220 rectangular basin.

The slowest oscillation mode has the period of 117 min, which exceeds significantly the longest
possible period of a standing wave in an enclosed basin. The oscillations contain about 6% of the
energy of the largest “1st longitudinal” mode making it the fifth strongest oscillation mode. The
oscillation has the greatest amplitudes concentrated in Polson Bay (Fig. 6). However, the amplitudes
225 in the main basin are appreciable and take place in the counter-phase to the oscillations in the Polson

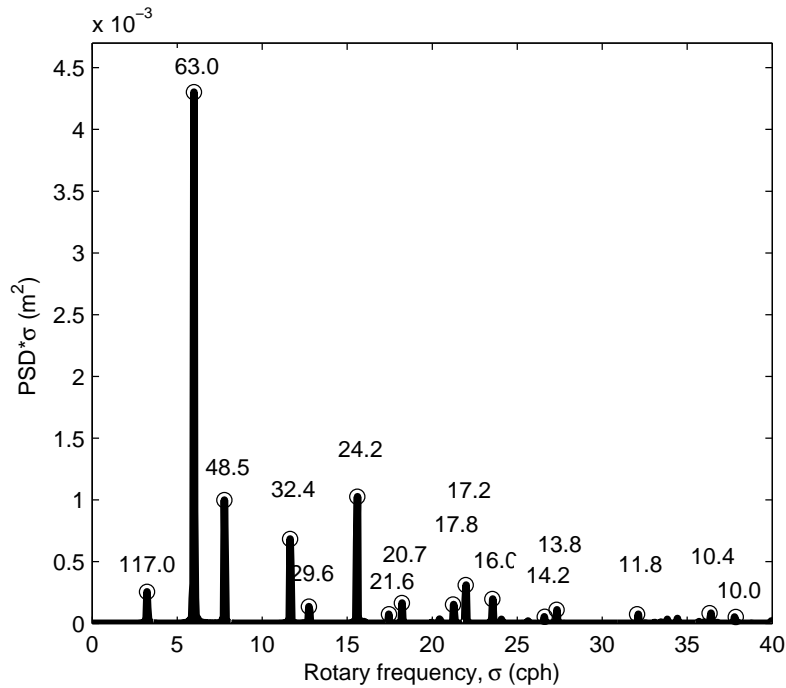


Figure 2. The synthetic maximum spectrum of modeled lake surface oscillations. Significant peaks are marked with circles subscribed with corresponding seiche periods in min.

Bay. The zero-phase line is located in the narrow straight separating the Polson Bay from the lake. These characteristics suggest the 117 min mode to be a kind of the resonant Helmholtz or “open channel” mode (Miles and Munk, 1961; Miles, 1974). In particular, this suggestion legitimizes the long period of the oscillation. In an ideal case of a half-open channel of characteristic length L connected to an infinitely large basin, the Helmholtz mode has the wavelength $4L$, which is twice the wavelength of the longest seiche in an enclosed lake. The corresponding oscillation period is also two times longer than the fundamental seiche, as provided by Eq. (1). The 117 min oscillation is indeed almost two times slower than that of the one-node 63 min seiche. However, in semi-enclosed basins with narrow and shallow openings, the characteristics of the Helmholtz oscillations deviate strongly from those given by idealized models (Platzman, 1972; Miles, 1974) and are determined by the width and depth of the mouth. In Flathead Lake, this role is played by the narrow straight between the main basin and Polson Bay. According to (Lee, 1971; Miles and Lee, 1975) the water level amplitude of the Helmholtz mode is approximately uniform over the lake and the maximum current velocities are localized in the mouth. The ratio of the Helmholtz mode period T to that of the fundamental seiche T_0 is proportional to the square root of the ratio of lake depth H to the depth of the straight h_l : $T/T_0 = (H/h_l)^{1/2}$, i.e., the Helmholtz mode is slower than the 1st mode seiche. The shape of the 117 min mode in the main basin agrees well with these estimations.

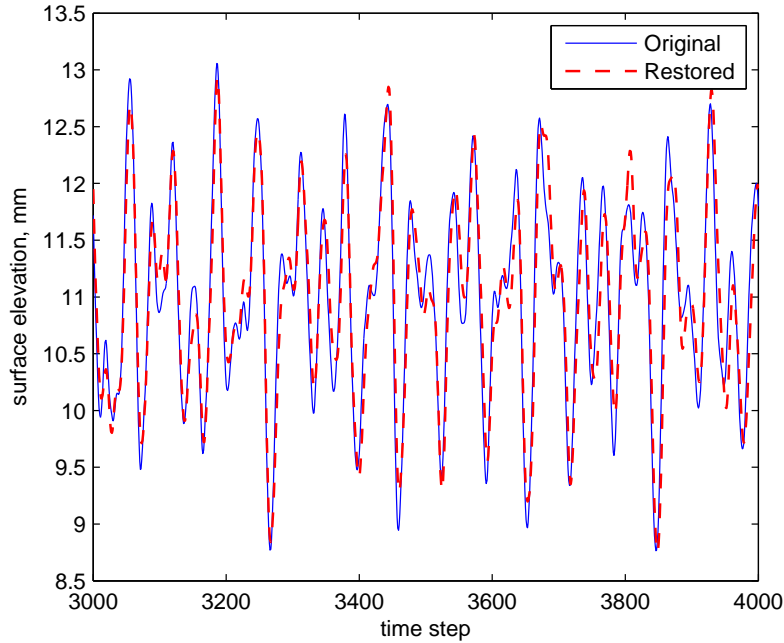


Figure 3. Lake surface oscillations at a single point in the middle of the lake produced by the model (blue solid line) and restored from the 12 harmonics (modes) of the Fourier analysis (red dashed line).

Six local modes do not produce any appreciable water level oscillation in the open part of Flathead Lake (Fig. 5). Among them, the 29.6 min mode (Fig. 5A) is the fundamental one-node mode of the Polson Bay with a single node line and water level oscillations confined to the bay with small response in the main lake basin. The 17.16 min mode (Fig. 5D) is an analogous oscillation in the Big Arm Bay. Both the 24.2 min and 20.7 min modes have pronounced transversal components: the former in the southern and the latter in the northern part of the lake (Fig. 5BC). This “splitting” of the transverse mode in two results from the irregular width of the lake along the main axis: the slower 24.2 min mode propagates along the widest transverse axis in the southern part of the lake. The 20.7 min mode is confined to the partially isolated northern part of the lake. The isolated morphometry of the northern basin gives rise to several local oscillation modes confined to this area (Fig. 5E,F and Fig. S2, *Supplementary Material 1*).

The distribution of the seiche current velocities is characterized by local spots of strong current intensification, confined to the shallow near-shore areas or narrow straights connecting different parts of the lake (Figs. 7 and 8). Importantly, the areas of high current velocities of many seiche modes are concentrated in the northern part of Lake Flathead, where the two major tributaries—the Flathead River and the Swan River—enter the lake. This suggests an appreciable interaction of the seiche currents with the river inflows and sediment transport.

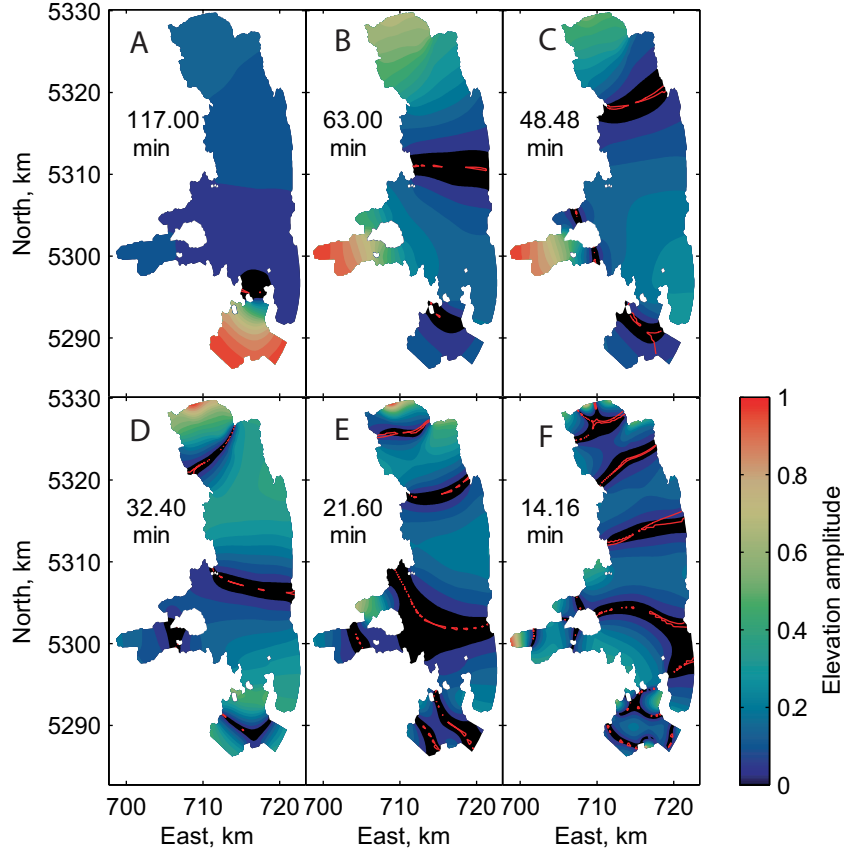


Figure 4. Relative amplitudes of the 6 lake-wide free oscillation modes. Zero-amplitude nodal lines are shown in red.

260 Currents produced by four seiche modes have rotational features revealed by the spatial distribution of the rotary coefficients (Fig. 8). The 2nd longitudinal mode with period of 48.5 min produces rotation in Polson Bay, while the transverse mode of 24.2 min has a single gyre in the mid part of the lake (Fig. 8A, C). Both gyres may be tentatively attributed to the effect of lateral friction at the lake shores. A more interesting pattern is revealed in oscillations with 29.6 min and 20.7 min periods:

265 both produce counter-rotating gyres in the center of the lake (Fig. 8B, D). While the direction of the rotational movement is changing with the corresponding seiche frequency, the patterns suggest formation of horizontal fronts- areas of strong lateral shear -developing away from the solid boundaries. The rotary coefficients of the other seiche modes are provided in Fig. S3 (*Supplementary Material I*). Among them, the most energetic long-period modes do not reveal rotational movements, while

270 the short-period modes have a patchy pattern, which, assuming the high frequency of these modes, does not produce any appreciable long-lasting effects.

The modal structure of seiches derived from the modeling results shows good qualitative agreement with water level records from Flathead Lake (Fig. 9). The spatial structure of different seiche

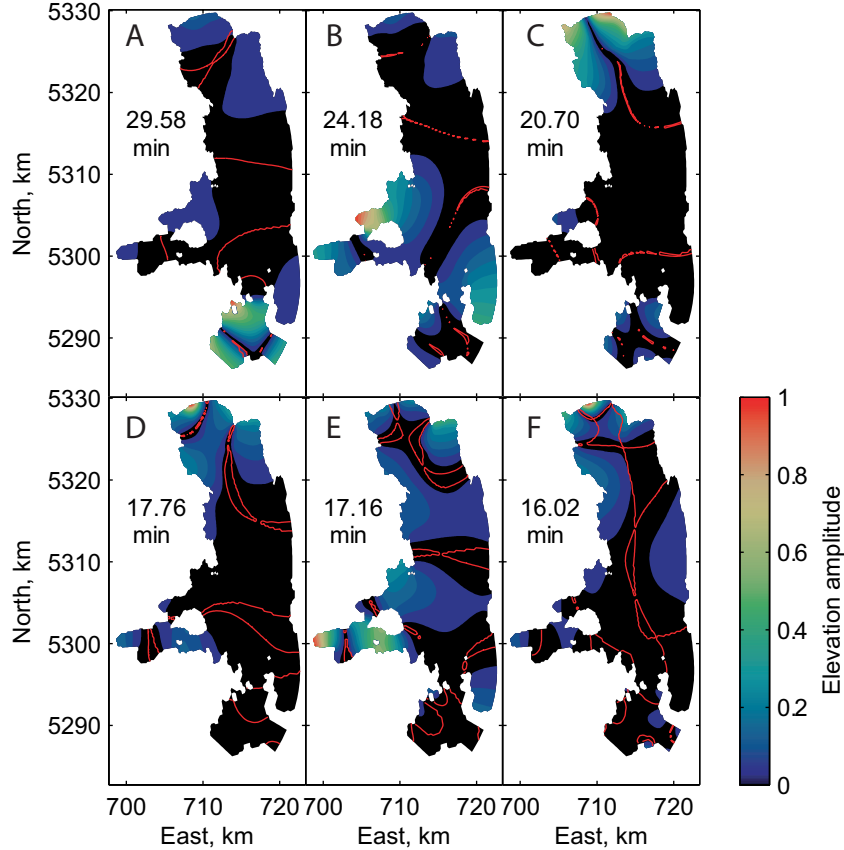


Figure 5. The same as Fig. 4 for the 6 bay modes of free oscillations.

modes as revealed by modeling, explains general features of the measured water level spectra variability over the lake: As predicted by the model, the 117 min Helmholtz mode has the strongest peak in the Polson Bay and is practically absent on other locations in the lake (Fig. 9B); the 1st and 2nd longitudinal modes with periods 63.0 and 48.5 min show strong peaks in the main lake basin (Fig. 9A, D); at the measurement sites WB and NR, located close to the nodal lines of modes 1 and 2, the 3rd mode of 32.4 min has a strong contribution to the oscillations (Fig. 9C, D).

Several important discrepancies between the modeled and observed spectra provide an insight into the features not captured by the model, but potentially significant for the hydrodynamics of Flathead Lake. The high frequency modes with periods of 10-15 min are generally stronger in the observations than in the model. Observations at measurement sites WB and NR (Fig. 9C, D) show strong oscillations at periods about 10 min, which are not pronounced on the modeled single point spectra from the closest grid points. However, these periods are found in the results of the harmonic analysis: three local modes have periods of 9.96, 10.38, and 11.76 min; among them the 10.38-min mode is predicted to produce appreciable water level amplitudes in the vicinity of the measurement sites WB and NR (see Fig. S1 in *Supplementary Material I*). The likely reasons for amplification of

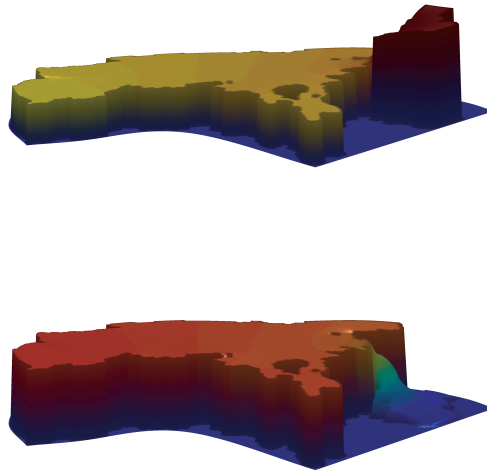


Figure 6. The shape of the lake surface corresponding to the maximum slope due to the slowest open-channel Helmholtz mode (see also animated surface oscillations in Supplementary Material II).

these modes are local interactions between wind and lake topography, which cannot be reproduced
 290 in the model driven by the linear initial surface slope.

4 Discussion and conclusions

Seiches in Flathead Lake have been measured in past studies (Morrison et al., 2005). However, the true complexity of their spatial variability was not revealed with limited observational series. Our model-based approach proved to be a useful tool to assess an array of possible complex seiche
 295 motions and their rotational characteristics.

4.1 Effectiveness of the method.

Fourier analysis is an established method for studying tidal oscillations, but is usually considered inapplicable to seiche analysis because seiche frequencies are basin-conditioned and are not a priori known. We demonstrated that isolation of the seiche frequencies by constructing a maximum
 300 spectrum over the basin under investigation is an effective way to isolate the eigenfrequencies from the output of primitive equation models, at the expense of estimation of the spectral density at each model grid point. The latter suggests 10^3 - 10^4 spectrum calculations that is computationally more expensive than eigenvalue analysis (e.g., Rao and Schwab, 1976). However, modern computers make the computations approachable and the results more accurate given that the model accurately takes

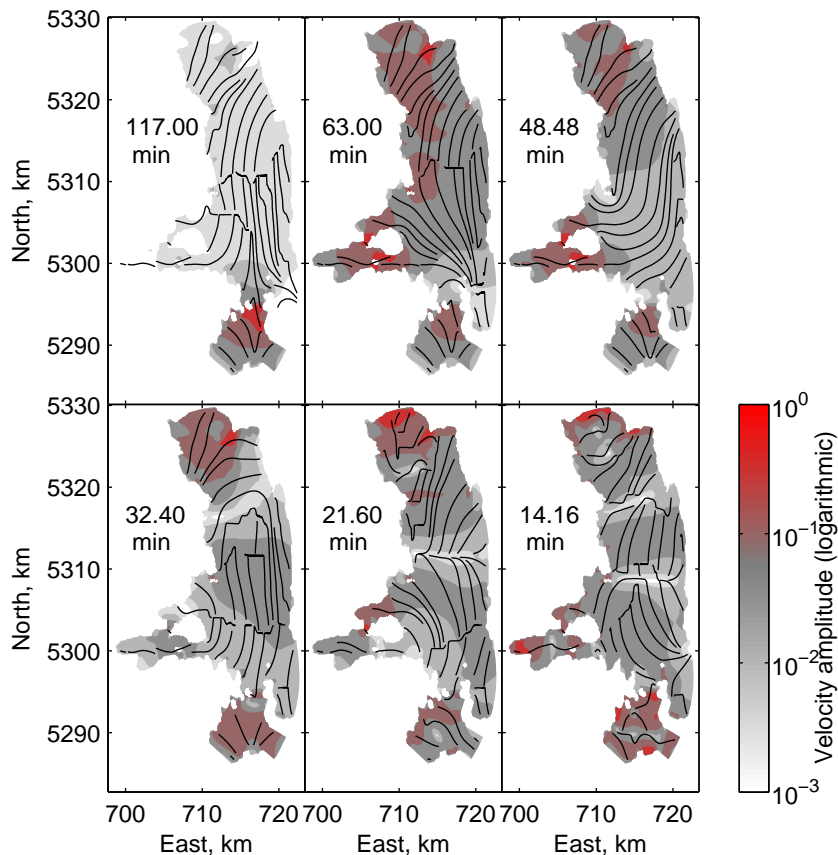


Figure 7. Normalized amplitudes of the velocity oscillations for the 6 modes with strong intensification of seiche currents in the nearshore areas (red color). Note the logarithmic color scale.

305 into account effects on seiche motion like shoreline morphometry, basin bathymetry, Coriolis force, and bottom friction. Moreover other effects, like variable wind forcing or variable water level, can be easily added. We used specially designed model runs with initial slope of the surface being the only driving force for the circulation rendering isolation of the seiche frequencies straightforward. The model outputs driven by realistic forcing contain continuous spectra of motions. Significant seiche peaks can be identified by comparison of the model spectra to a background red noise spectrum containing the same integral spectral energy (Gilman et al., 1963; Bernhardt and Kirillin, 2013).

Our method allowed identification of several specific features of the basin-scale oscillations in FLathead Lake, indistinguishable by the simple channel-like approximation (1), the most crucial being the existence of the Helmholtz mode strongly affecting the dynamics of the small Polson Bay 315 connected to the main lake basin by a narrow straight. Another remarkable feature of the lake-wide modes revealed by the method is the deviation of their periods from those following from Eq. (1). The deviation is stronger for higher modes of seiches with shorter wavelengths, which are apparently stronger affected by the irregular lake morphometry: The 4th longitudinal mode has the period of

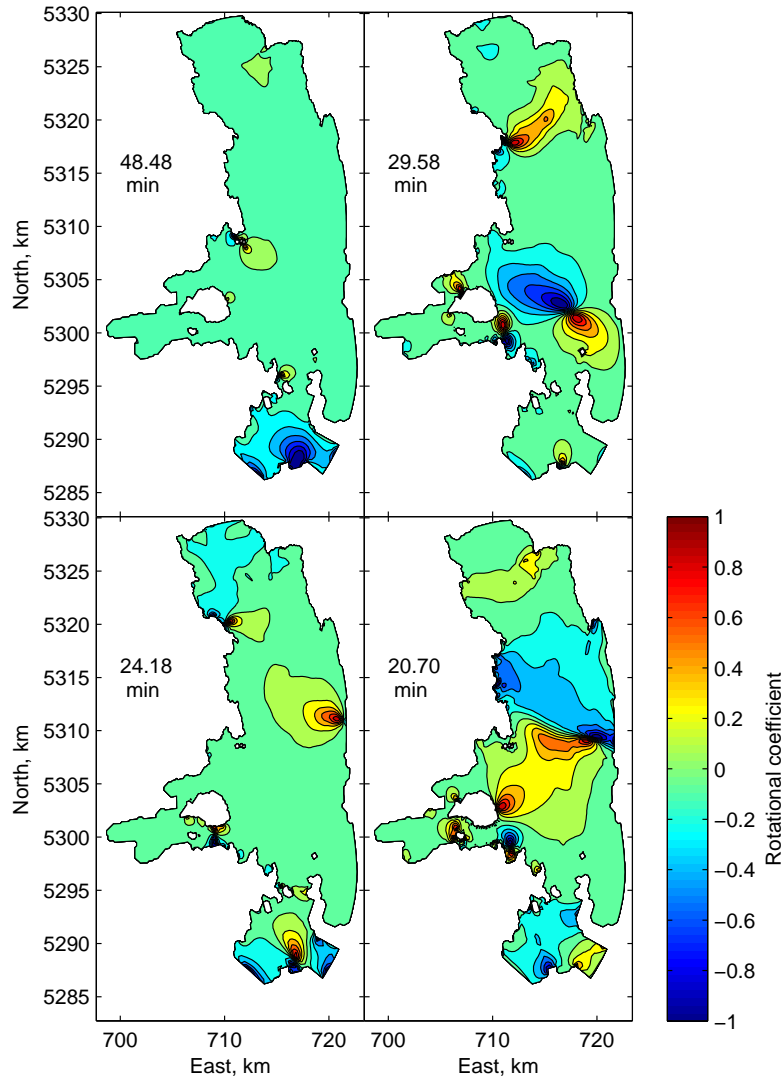


Figure 8. Rotary coefficients for the four free oscillation modes with rotational character. See Fig. S3 in Supplementary Material I for spatial distribution of rotary coefficients for other modes.

21.6 min (Table 1), which is remarkably longer than the period of the 4th channel mode of 16.5 min
 320 (Eq. 1). Hence, the simple comparison of the oscillation periods with Eq. (1) would result in a wrong
 association of the 4th mode with the period of 16.02 min: the potential source of confusion when
 applying the channel approximation to seiche analysis.

4.2 The role of surface seiches in the dynamics of Flathead Lake.

The essence of seiche analysis presented above consists of establishing the two-dimensional picture
 325 of seiche oscillations and designation of areas with maximum water level and current speed amplitudes.
 Among the applied aspects of these results are the lakeshore management issues such as

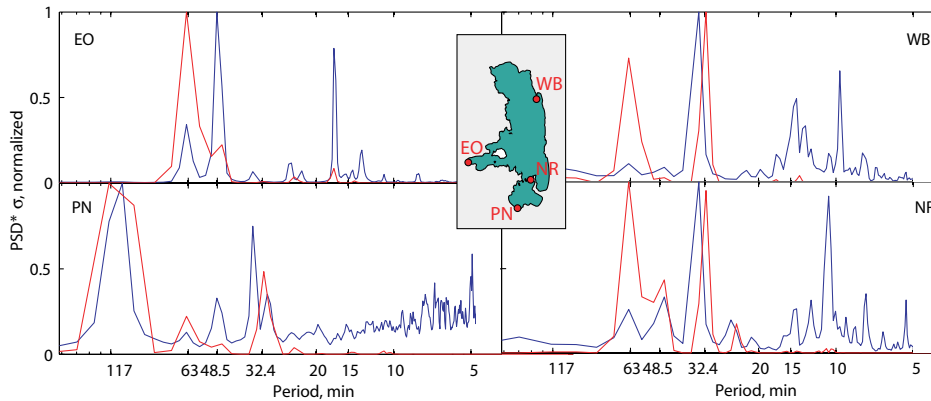


Figure 9. Spectra of measured (blue) vs modeled (red) free surface oscillations at four different locations in Flathead Lake. The inset shows the positions of the corresponding measurement points.

shoreline erosion and estimation of the flood risks. The North Shore and Polson Bay are two areas of the lake that this modeling effort has shown to have both high amplitude and high current velocities associated with seiche motions. Both of these areas have experienced very high levels of shoreline retreat (1-14 m yr⁻¹, see Fig. S4 in *Supplemental Material I*) due mainly to lake level regulation concentrating wave energy at a single "full pool" elevation (Lorang and Stanford, 1993). Erosion processes, and hence retreat rates, are also tightly correlated to overwash events (Lorang and Stanford, 1993), which are directly related to seiche action in the lake. For marsh shorelines in Polson Bay, along the North Shore and in Big Arm Bay seiche motions control overwash and inundation patterns that ultimately control the pattern of sedimentation and subsequent vegetative colonization. Hence, seiche motions can affect shoreline erosion processes, rates of shoreline retreat, and sedimentation patterns, as well as impart a shift in the mosaic of habitat (Stanford et al., 2005) composing the riparian transition from lake to terrestrial environment.

Seiches have also caused significant flooding to shoreline development activities that have built too close to the lakeshore and at too low elevations. We found through our modeling exercise that the low-frequency Helmholtz mode had high amplitudes in Polson Bay, Big Arm Bay and areas of the North Shore. These areas would be particularly in danger of potential flooding especially in Polson Bay given the shorter frequencies and possibility of developing resonance conditions. High velocity amplitudes were also found along Wildhorse Island and Polson Bay. The high-frequency modes concentrated at the North Shore could affect sediment transport distribution from the river inflow as well as the drift of aquatic invasive species. Results from this study will be valuable to local planning departments and developers alike to limit future problems associated with seiche induced flooding

(Fig. S4, *Supplemental Material I*) by providing maps of spatially distributed zones of expected seiche impacts.

350 In addition, the maps of rotational coefficients for expected dominant modes of oscillation (Fig. 8) provide valuable information about open-water mixing between gyre systems (Kirillin et al., 2008). These mixing zones are potentially important zones or "hot spots" of ecological activity from phytoplankton production to predation by zooplankton and fish. Hence, the structure and pattern of the seiche motions and associated currents and mixing patterns provide a physical template for complex
355 food-web interaction. In addition knowing the patterns of mixing zones also provides a template or map to help guide plankton tow efforts aimed at early detection of aquatic invasive species.

4.3 Combining observations and model efforts in seiche studies.

As discussed above, the method applied in the present study provides an effective way to gain an information on the precise seiche temporal characteristics and, more important, on the two-
360 dimensional lateral distribution of the seiche amplitudes and currents. The latter are difficult to reveal from direct field observations constrained to irregular point measurements at the lake surface, are however crucial for understanding the seiche contribution to the transport of suspended matter and lake-wide mixing. Moreover, knowledge on relative distribution of seiche intensity along the lake shores is of key importance for the shoreline management. With regard to estimation of
365 seiche effects on the littoral zone, our model effectively complements the observation data on the near-shore water level variability, as well as provides guidelines for design of the water level monitoring. Our results do not include information on the absolute magnitudes of water level oscillations and currents. The latter can vary in a wide range, depending on wind forcing, wind-seiche resonance, or being produced by other disturbances, such as earthquakes (which are particularly relevant
370 to the Flathead Lake area; Qamar et al., 1982). Variations in wind speed and direction are the major forcing for seiches, posing a number of relevant questions, among them the effects of seasonal variability in wind speeds and direction over the lake on the seiche-produced lateral mixing patterns. The potential consequences of this variability for the seiche oscillations would consist in seasonally varying typical seiche amplitudes, seasonal intensification of certain seiche modes, and seiche inter-
375 action with seasonally varying drift currents. Investigation of these seiche-driven processes would be most efficient in the general context of seasonally variable transport within the lake including, along with seiches, the temporally and spatially variable wind drift and the seasonal variability of inflows and outflows. To override the inevitable deficiencies of numerical modeling approaches (such as reproduction of the bottom friction, non-linear wave transport and turbulence in stratified interior),
380 the model simulations should be combined in these complex investigations with spatially-resolved measurements at seasonal time scales.

Wind, dam operations, and seiche oscillations in lakes can play a significant role in shoreline erosion by exposing fragile and otherwise protected backshore environments to the action of wind

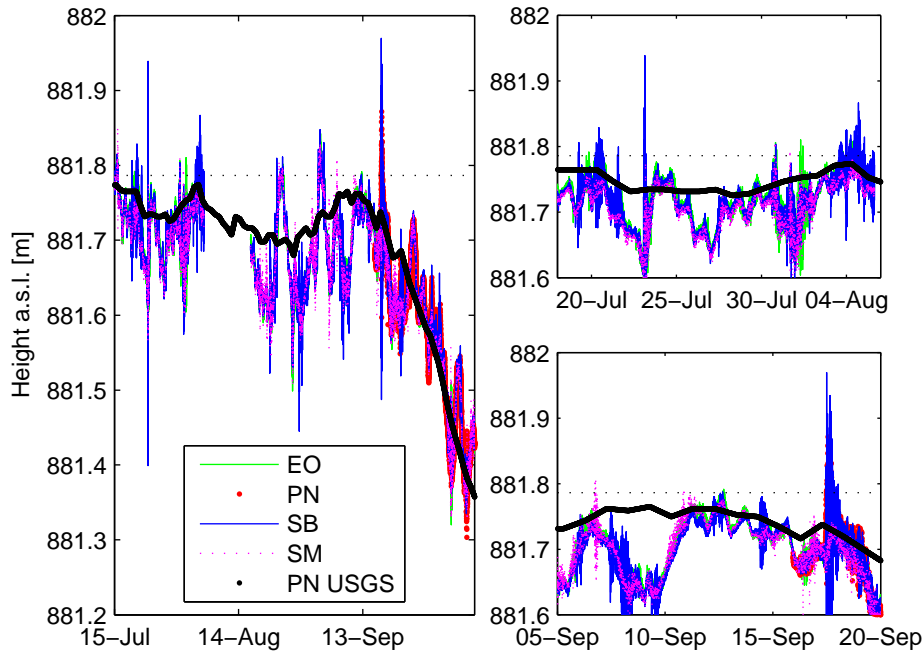


Figure 10. (A) A 3-month outtake of the lake surface oscillations in Flathead Lake. (B) and (C) 15-day zoom of the panel A for periods of strong seiche oscillations. Thick solid line is the record from the USGS gauging station in Polson Bay used for regular lake level monitoring. Thin lines are derived from the pressure records at different positions in the lake (see Fig. 1 for measurement locations). The dotted line marks the prescribed maximum regulated elevation of 881.78 m a.s.l.

waves. An alteration of hydrologic connection with marsh shorelines through seiche motions ultimately impacts the mosaic of riparian habitat. Therefore, understanding lake food-web dynamics in the lake, management of lake level regulation with dams to early detection of aquatic invasive species and shoreline development planning can all benefit from a more quantitative understanding seiche motions based on combination of our model results and water level/currents monitoring.

Comparison of pressure records from transducers from different locations around the lake show distinct seiche oscillations (Fig. 10) with amplitudes exceeding up to 15 cm the regulated maximum pool elevation of 881.8 m occurred in Skidoo Bay (blue line in Fig. 10, see Fig. 1 for the site location). Filtered measurements by the USGS gauge used for water level regulation do not capture these oscillations. The modeling efforts in this paper indicate that this regulated level would be exceeded in other bays along the North Shore including much of the East and West shores of the north half of the lake, bays and shorelines west of Wildhorse Island and all of Polson Bay. Hence, if lake level regulation is aimed at controlling the shoreline erosion at lake-wide scales then an array of real-time lake level sensors situated around the lake has to be used or the maximum elevation threshold has to be reduced by the largest seiche amplitude.

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References

- Bernhardt, J. and Kirillin, G.: Seasonal pattern of rotation-affected internal seiches in a small temperate lake, *Limnology and Oceanography*, 58, 1, 1, 2013.
- Blumberg, A. F. and Mellor, G. L.: A description of a three-dimensional coastal ocean circulation model, *Three-dimensional coastal ocean models*, pp. 1–16, 1987.
- 405 Csanady, G.: Large-scale motion in the Great Lakes, *Journal of Geophysical Research*, 72, 4151–4162, 1967.
- Defant, A.: *Physical oceanography*, Pergamon Press, 1961.
- Forel, F.: *Le Léman: Monographie Limnologique. Tome 2: mécanique, chimie, thermique, optique, acoustique*, F. Rouge, Lausanne, 1895.
- 410 Forel, F. and von Zeppelin, E.: *Die Schwankungen des Bodensees*, Stettner, 1893.
- Gilman, D., Fuglister, F., and Mitchell Jr, J.: On the power spectrum of “red noise”, *Journal of the Atmospheric Sciences*, 20, 182–184, 1963.
- Gonella, J.: A rotary-component method for analysing meteorological and oceanographic vector time series, *Deep Sea Research and Oceanographic Abstracts*, 19, 833–846, 1972.
- 415 Hayashi, Y.: Space-time spectral analysis of rotary vector series, *Journal of the atmospheric sciences*, 36, 757–766, 1979.
- Hollan, E., Rao, D., and Bäuerle, E.: Free surface oscillations in Lake Constance with an interpretation of the “Wonder of the Rising Water” at Konstanz in 1549, *Archiv für Meteorologie, Geophysik und Bioklimatologie, Serie A*, 29, 301–325, 1980.
- 420 Hutter, K., Raggio, G., Bucher, C., and Salvadè, G.: The surface seiches of Lake of Zürich, *Schweizerische Zeitschrift für Hydrologie*, 44, 423–454, doi:10.1007/BF02502301, 1982.
- Hutter, K., Wang, Y., and Chubarenko, I.: *Physics of Lakes Vol. II — Lakes as Oscillators*, 2011.
- Jönsson, B., Döös, K., Nycander, J., and Lundberg, P.: Standing waves in the Gulf of Finland and their relationship to the basin-wide Baltic seiches, *Journal of Geophysical Research: Oceans (1978–2012)*, 113, 2008.
- 425 Kirillin, G., Engelhardt, C., and Golosov, S.: A mesoscale vortex in a small stratified lake, *Environmental fluid mechanics*, 8, 349–366, 2008.
- Kulikov, E. and Medvedev, I.: Variability of the Baltic Sea level and floods in the Gulf of Finland, *Oceanology*, 53, 145–151, 2013.
- Lee, J.-J.: Wave-induced oscillations in harbours of arbitrary geometry, *Journal of Fluid Mechanics*, 45, 375–394, 1971.
- 430 Lorang, M. S.: The use of sediment analysis, sedimentary structures and water wave measurements to quantify wave energy in a shallow fresh water bay, Flathead Lake, Montana, *Northwest Geology*, 13, 5–14, 1984.
- Lorang, M. S. and Stanford, J. A.: Variability of shoreline erosion and accretion within a beach compartment of Flathead Lake, Montana, *Limnology and Oceanography*, 38, 1783–1795, 1993.
- 435 Lorang, M. S., Komar, P. D., and Stanford, J. A.: Lake level regulation and shoreline erosion on Flathead Lake, Montana: A response to the redistribution of annual wave energy, *Journal of Coastal Research*, pp. 494–508, 1993a.
- Lorang, M. S., Stanford, J. A., Hauer, F. R., and Jourdonnais, J. H.: Dissipative and reflective beaches in a large lake and the physical effects of lake level regulation, *Ocean & coastal management*, 19, 263–287, 1993b.
- 440 Miles, J.: Harbor seiching, *Annual Review of Fluid Mechanics*, 6, 17–33, 1974.

- Miles, J. and Munk, W.: Harbor paradox, *Journal of the Waterways and Harbors Division, ASCE*, 87, 111–130, 1961.
- Miles, J. W. and Lee, Y.: Helmholtz resonance of harbours, *Journal of Fluid Mechanics*, 67, 445–464, 1975.
- Morrison, A., Williams, A., and Lorang, M.: Waves and seiches in Flathead Lake, Montana: measurements of a
 445 quiet lake by differential travel-time current measurements, in: *OCEANS, 2005. Proceedings of MTS/IEEE*, pp. 1421–1426, IEEE, 2005.
- Mortimer, C. H. and Fee, E. J.: Free surface oscillations and tides of Lakes Michigan and Superior, *Philosophical Transactions for the Royal Society of London. Series A, Mathematical and Physical Sciences*, pp. 1–61, 1976.
- Platzman, G. W.: Two-dimensional free oscillations in natural basins, *Journal of Physical Oceanography*, 2,
 450 117–138, 1972.
- Qamar, A., Kogan, J., and Stickney, M. C.: Tectonics and recent seismicity near Flathead Lake, Montana, *Bulletin of the Seismological Society of America*, 72, 1591–1599, 1982.
- Rao, D. B. and Schwab, D. J.: Two dimensional normal modes in arbitrary enclosed basins on a rotating earth: Application to Lakes Ontario and Superior, *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 281, 63–96, 1976.
 455
- Rudnev, S., Salvadè, G., Hutter, K., and Zamboni, F.: External gravity oscillations in Lake Onega, *Annales Geophysicae*, 13, 893–906, 1995.
- Rueda, F. J. and Schladow, S. G.: Surface seiches in lakes of complex geometry, *Limnology and oceanography*, 47, 906–910, 2002.
- 460 Schimmelpfennig, S., Kirillin, G., Engelhardt, C., and Nützmann, G.: Effects of wind-driven circulation on river intrusion in Lake Tegel: modeling study with projection on transport of pollutants, *Environmental fluid mechanics*, 12, 321–339, 2012.
- Smagorinsky, J.: General circulation experiments with the primitive equations: I. the basic experiment*, *Monthly weather review*, 91, 99–164, 1963.
- 465 Stanford, J. A., Lorang, M. S., and Hauer, F. R.: The shifting habitat mosaic of river ecosystems, *Internationale Vereinigung für Theoretische und Angewandte Limnologie Verhandlungen*, 29, 123–136, 2005.
- Suursaar, Ü., Kullas, T., and Otsmann, M.: A model study of the sea level variations in the Gulf of Riga and the Väinameri Sea, *Continental shelf research*, 22, 2001–2019, 2002.
- Thomson, R. E. and Emery, W. J.: *Data analysis methods in physical oceanography*, Elsevier, 2001.
- 470 Turner, J.: *Buoyancy effects in fluids*, Cambridge Univ Press, 1973.
- von der Mühl, K.: Ueber die Bewegung tropfbarer Flüssigkeiten in Gefässen. Nach Johann Rudolf Merian, *Mathematische Annalen*, 27, 575–600, 1886.