A methodological approach of estimating of resistance to flow under unsteady flow conditions

Magdalena M. Mrokowska[1], Paweł M. Rowiński[1], and Monika B. Kalinowska[1]

[1]Institute of Geophysics Polish Academy of Sciences, Ks. Janusza 64, 01-452 Warsaw, Poland

Correspondence to: M.M. Mrokowska (m.mrokowska@igf.edu.pl)

Abstract.

This paper presents evaluation and analysis of resistance parameters: friction slope, friction velocity and Manning coefficient in unsteady flow. The methodology to enhance the evaluation of resistance by relations derived from flow equations is proposed. The main points of the methodology are: (1) to choose a resistance relation with regard to a shape of a channel and (2) type of wave, (3) to choose appropriate method to evaluate slope of water depth, (4) to assess the uncertainty of result. Beside critical analysis of existing methods, new approaches are presented: formulae for resistance parameters for trapezoidal channel, and translation method instead of Jones formula to evaluate the gradient of flow depth. Measurements obtained from artificial dam-break flood waves in a small lowland watercourse have made it possible to apply the method and to analyse to what extent resistance parameters vary in unsteady flow. The study demonstrates that results of friction slope and friction velocity are more sensitive to applying simplified formulae than Manning coefficient. Manning n, adequate as flood routing parameter, may appear to be misleading when information on trend of resistance with flow rate is crucial. Then friction slope or friction velocity seems to be better choice.

1 Introduction

Resistance is one of the most important factors affecting the flow in open channels. In simple terms it is the effect of water viscosity and the roughness of the channel boundary which result in friction forces that retard the flow. The largest input into the resistance is attributed to water-bed interactions.

Resistance to flow is expressed by friction slope S which is dimensionless variable or boundary shear stress τ which refers directly to the shearing force acting on the channel boundary, with the

dimension of Pascal [Pa]. Alternatively, shear stress is expressed in velocity units $[\mathrm{m\,s^{-1}}]$ by friction (shear) velocity u_*, which is related to the shear stress and friction slope by the equation:

$$u_* = \sqrt{\frac{\tau}{\rho}} = \sqrt{gRS}, \tag{1}$$

25   where g – gravity acceleration $[\mathrm{m\,s^{-2}}]$, ρ – density of water $[\mathrm{kg\,m^{-3}}]$. Shear stress and friction velocity are crutial in research on hydrodynamics problems such as bed load transport (Dey, 2014), rate of erosion (Garcia, 2007), contaminants transport (Kalinowska and Rowiński, 2012; Kalinowska et al., 2012), turbulence characteristics of flow (Dey et al., 2011).

On the other hand, in engineering practice the resistance is traditionally characterised by Man-
30   ning n, Chezy or Darcy-Weisbach coefficients. The flow resistance equation (Eq. 2) relating flow parameters through Manning n was originally derived for steady uniform flow conditions:

$$n = \frac{R^{2/3}}{U} S^{1/2}, \tag{2}$$

where R – hydraulic radius [m], U – mean cross-sectional velocity $[\mathrm{m\,s^{-1}}]$. Its application is accepted for gradually varied flows for which friction slope can be approximated by bed slope I.
35   Manning n was supposed to be invariant with the water stage; however, research has shown that the resistance coefficient very often varies (Ferguson, 2010; Fread, 1985; Julien et al., 2002). Furthermore, the trend of n versus flow rate Q may be falling or rising depending on the geometry of wetted area. Fread (1985) reported, based on computations of n from extensive data of flood waves in American rivers, that the trend is falling when inundation area is relatively small compared to
40   inbank flow area; in reverse case the trend is rising.

In unsteady flow additional factors affect flow resistance compared to steady flow. As Yen (2002) presents after Rouse (1965), besides water flow-channel boundary interactions represented by skin friction and form drag, resistance has two more components: wave resistance from free surface distortion and resistance due to local acceleration or flow unsteadiness. Consequently, in order to
45   evaluate resistance in unsteady flow it might be not sufficient to approximate friction slope S by bed slope I.

Large variety of methods of bed shear stress and friction velocity evaluation have been devised in order to study the flow resistance experimentally. The majority of methods measure bed shear stress indirectly, e.g. using hot wire and hot film anemometry (Albayrak and Lemmin, 2011), a Preston
50   tube (Mohajeri et al., 2012), methods that take advantage of theoretical relations between shear stress and the horizontal velocity distribution (Khiadani et al., 2005), methods based on Reynolds shear stress (Czernuszenko and Rowiński, 2008; Nikora and Goring, 2000) or turbulent kinetic energy (Pope et al., 2006), or methods that incorporate double-averaged momentum equation (Pokrajac et al., 2006). These methods are impractical or even impossible to be applied during flood wave propaga-
55   tion. Instead, a number of authors recommend formulae derived from flow equations (Ghimire and Deng, 2011; Graf and Song, 1995; Guney et al., 2013; Rowiński et al., 2000); nonetheless, this method

needs further development because scarce measurement data very often restrict the relationships on resistance to simplified forms which provide uncertain results. Among simplifications applied in literature there are simplifications of momentum balance equation terms and simplifications that refer to the evaluation of the gradient of flow depth. This method requires flow velocity and flow depth as input variables and for this reason its practical application is restricted. However, it is a good choice for research purposes.

In this study we apply formulae derived from flow equations to obtain values of friction slope, Manning n and friction velocity given data on flow parameters. The objectives of this paper are twofold: (1) to enhance the evaluation of resistance to flow by relations derived from flow equations and by providing relevant methodology, (2) to analyse to what extent friction slope, friction velocity and Manning n vary in unsteady flow. The first objective could be valuable for those who would like to apply relations derived from flow equations to evaluate resistance and its impact on hydrodynamic processes, e.g. sediment transport, while the other could be of interest to those who use resistance coefficients in modelling practice.

The paper is structured as follows: Section 2 presents settings of dam-break field experiment and measurement data. Methodology of evaluation of friction slope, friction velocity and Manning n in unsteady flow with focus on detailed aspects of application of formulae derived from flow equations is outlined in Sect 3. In Sect 4 results of computations of friction slope, friction velocity and Manning n are presented for field experiment. In Sect 5 conclusions are provided. The problem presented herein has been partially considered in the unpublished Ph.D. thesis of the first author of this paper (Mrokowska, 2013).

2   Experimental data

The data originate from an experiment carried out in the Olszanka which is a small lowland watercourse in central Poland (see upper panel of Fig. 1) convenient for experimental studies. The aim of the experiment was to conduct measurements of hydraulic properties during artificial flood wave propagation. To achieve this goal, a wooden dam was constructed across the channel, then the dam was removed in order to initiate a wave. Then, measurements were carried out at downstream cross-sections. Two variables were monitored: the velocity and the water stage. Velocities were measured by propeller current meter in three verticals of a cross-section at two water depths. Water stage was measured manually by staff gage readings. Geodetic measurements of cross-sections were performed prior to the experiment. An in-depth description of the experimental settings in the Olszanka watercourse may be found in (Szkutnicki, 1996; Kadłubowski and Szkutnicki, 1992), and a description of similar experiments in the same catchment is presented in (Rowiński and Czernuszenko, 1998; Rowiński et al., 2000).

In the study, two cross-sections, denoted in Fig. 1 as CS1 and CS2, are considered. Cross-section CS1 was located about 200 m from the dam, and cross-section CS2 about 1600 m from it. The shape of the cross-sections is presented in the bottom panel of Fig. 1. Both were of trapezoidal shape with side slopes of $m_1 = 1.52$, $m_2 = 1.26$ and $m_1 = 1.54$, $m_2 = 1.36$ for CS1 and CS2, respectively (Fig. 2). The bed slope I was 0.0004 for CS1 and 0.0012 for CS2.

Two data sets are used in this study, denoted as follows: Ol-1, Ol-2. Other data sets provided qualitatively similar results and therefore, for simplicity, are not presented herein. The first set was collected in cross-section CS1 and the other in cross-section CS2 during the passage of the same wave on 26 April 1990 at the beginning of vegetation season when banks were slightly vegetated (Fig. 3). The bed was composed of sand and silt with no significant bed forms. Figure 4 illustrates the results of the measurements – the temporal variability of mean velocity (U) and flow depth (h). Mean velocity has been evaluated by the velocity-area method from propeller current meter readings and flow depth has been calculated from geodetic data and measurements of water stage. Please note the time lag between maximum values of U and h, which indicates the non-kinematic character of the waves. Consider that waves represent a one-dimensional subcritical flow, with a Froude number (Fr $= U/\sqrt{gh}$) smaller than 0.33. The loop-shaped relationship between flow rate (Q) and water stage (H) may be observed in Fig. 5. From the figure it can be seen that the rating curves are not closed, which is probably caused by too short series of measurement data.

3 Methods

The methodology of evaluating resistance to flow from flow equations is proposed. It comprises four questions that need to be answered to obtain reliable values of resistance:

1. What is the shape of the channel – is simplification of the channel geometry applicable?

2. Is it admissible to apply simplified formula with regard to the type of wave?

3. What methods of evaluating input variables, especially the gradient of flow depth, are feasible in the case under study?

4. What is the uncertainty of the input variables, and which of them are most significant?

In proceeding sections a thorough review of each questioned issue is given. Methods used in the literature are facilitated with critical analysis, and some new approaches are proposed by the authors.

3.1 Relations for resistance in unsteady non-uniform flow derived from flow equations

In this study, resistance to flow is evaluated by formulae derived from flow equations – the momentum conservation equation and the continuity equation. Here we propose to evaluate resistance to flow for dynamic wave from the relations derived from the St. Venant model for trapezoidal channel

(Mrokowska et al., 2013):

$$U(b+mh)\frac{\partial h}{\partial x} + \left(b+\frac{m}{2}h\right)h\frac{\partial U}{\partial x} + (b+mh)\frac{\partial h}{\partial t} = 0, \tag{3}$$

$$\frac{\partial h}{\partial x} + \frac{U}{g}\frac{\partial U}{\partial x} + \frac{1}{g}\frac{\partial U}{\partial t} + S - I = 0, \tag{4}$$

where t – time [s], x – longitudinal coordinate [m], b – width of river bed [m], h – here: the maximum flow depth in the channel section (trapezoidal height) [m], $m = m_1 + m_2$, m_1 and m_2 – side slopes [-] defined as $m_1 = l_1/h$ and $m_2 = l_2/h$. The cross sectional shape with symbols is depicted in Fig. 2. Equation (3) is the continuity equation and Eq. (4) is the momentum balance equation which the terms represent as follows: the gradient of flow depth (hydrostatic pressure term), advective acceleration, local acceleration, friction slope and bed slope. Further on, derivatives will be denoted by Greek letters to stress that they are treated as variables, namely $\zeta = \frac{\partial U}{\partial t}$ [m s^{-2}], $\eta = \frac{\partial h}{\partial t}$ [m s^{-1}], $\vartheta = \frac{\partial h}{\partial x}$ [-].

The friction slope derived analytically from the set of equations is represented by the following formula:

$$S = I + \left(\frac{U^2}{g}\frac{b+mh}{bh+m\frac{h^2}{2}} - 1\right)\vartheta + \frac{U}{g}\frac{b+mh}{bh+m\frac{h^2}{2}}\eta - \frac{1}{g}\zeta \tag{5}$$

To evaluate friction velocity and Manning n Eq. (5) is incorporated into Eq. (1) and Eq. (2), respectively:

$$u_* = \left[gR\left(I + \left(\frac{U^2}{g}\frac{b+mh}{bh+m\frac{h^2}{2}} - 1\right)\vartheta + \frac{U}{g}\frac{b+mh}{bh+m\frac{h^2}{2}}\eta - \frac{1}{g}\zeta\right)\right]^{\frac{1}{2}}. \tag{6}$$

$$n = \frac{R^{2/3}}{U}\left(I + \left(\frac{U^2}{g}\frac{b+mh}{bh+m\frac{h^2}{2}} - 1\right)\vartheta + \frac{U}{g}\frac{b+mh}{bh+m\frac{h^2}{2}}\eta - \frac{1}{g}\zeta\right)^{\frac{1}{2}}. \tag{7}$$

Equations (5, 6 and 7) are considered in this study, as Olszanka watercourse has nearly trapezoidal cross-section.

Flow equations for rectangular channel or unit width are the most frequently used mathematical models to derive formulae on resistance. A number of formulae for friction velocity has been presented in the literature, e.g.:

– Graf and Song (1995) derived the formula from the 2D momentum balance equation:

$$u_* = \left[ghI + \left(-gh\vartheta(1 - (\mathrm{Fr})^2)\right) + (\eta - h\zeta)\right]^{\frac{1}{2}}, \tag{8}$$

– Rowiński et al. (2000), and next Shen and Diplas (2010) applied the formula derived from the St. Venant set of equations:

$$u_* = \left[gh\left(I + \left(\frac{U^2}{gh} - 1\right)\vartheta + \frac{U}{gh}\eta - \frac{1}{g}\zeta\right)\right]^{\frac{1}{2}}. \tag{9}$$

5

– Tu and Graf (1993) derived the equation from the St. Venant momentum balance equation:

$$u_* = \left[gh \left(I + \frac{1}{C}\eta - \frac{1}{g}\zeta \left(1 - \frac{U}{C} \right) \right) \right]^{\frac{1}{2}}, \tag{10}$$

where C – wave celerity $[\mathrm{m\,s^{-1}}]$.

3.2 Simplifications of relations with regard to type of flow

If the acceleration terms of momentum balance equation for dynamic wave (Eq. 4) are negligible, they may be eliminated, and the model for a diffusive wave is obtained. Further omission of the hydrostatic pressure term leads to the kinematic wave model, in which only the term responsible for gravitational force is kept. According to Gosh (2014); Dooge and Napiórkowski (1987); Julien (2002), in the case of upland rivers, i.e. for average bed slopes, it could be necessary to apply the full set of St. Venant equations. Aricó et al. (2009) have pointed that this may be the case for mild and small bed slopes. Moreover, artificial flood waves, such as dam-break-like waves (Mrokowska et al., 2013), and waves due to hydro-peaking (Shen and Diplas, 2010), are of a dynamic character. On the other hand, when the bed slope is large, then the gravity force dominates and the wave is kinematic (Aricó et al., 2009). Because of the vague recommendations in the literature, we suggest analysing whether simplifications are admissible separately in each studied case.

Below we provide simplified relations for diffusive wave which are applied in this study:

$$S = I - \vartheta, \tag{11}$$

which is equivalent to water surface slope,

$$u_* = \left[gR\left(I - \vartheta \right) \right]^{\frac{1}{2}}, \tag{12}$$

$$n = \frac{R^{2/3}}{U}\left(I - \vartheta \right)^{\frac{1}{2}}. \tag{13}$$

Relations for steady flow are as follows:

$$S = I, \tag{14}$$

$$u_* = \left(gRI \right)^{1/2}. \tag{15}$$

$$n = \frac{R^{2/3}}{U} I^{1/2}, \tag{16}$$

3.3 Evaluation of the gradient of flow depth ϑ

The evaluation of ϑ is widely discussed in hydrological studies on flow modelling and rating curve assessment (Dottori et al., 2009; Perumal et al., 2004). The gradient of flow depth is evaluated based on flow depth measurements at one or a few gauging stations. Due to the practical problems with performing the measurements, usually only one or two cross-sections are used.

3.3.1 Kinematic wave concept

Paradoxically, kinematic wave approximation is widely applied in cases of non-kinematic waves where $\frac{\partial h}{\partial x} \approx 0$, e.g. in friction velocity assessment studies (Graf and Song, 1995; Ghimire and Deng, 2011). As Perumal et al. (2004) presented, Jones introduced this concept in 1915 in order to overcome the problem of $\frac{\partial h}{\partial x}$ evaluation in reference to non-kinematic waves. According to the concept, the gradient of flow depth is evaluated implicitly based on measurements in one cross-section:

$$\vartheta_{\text{kin}} = \frac{\partial h}{\partial x} = -\frac{1}{C}\frac{\partial h}{\partial t}, \tag{17}$$

$$\vartheta_{\text{kin}} = \frac{\partial h}{\partial x} = -\frac{1}{BC^2}\frac{\partial Q}{\partial t}. \tag{18}$$

The application of this method has been challenged in rating-curve studies (Dottori et al., 2009; Perumal et al., 2004) due to its theoretical inconsistency, as it neglects attenuation and subsidence of a flood wave (Henderson, 1963). The kinematic wave has a one-to-one relationship between the water stage and flow rate, which is equivalent to a steady flow rating curve, while non-kinematic wave is loop-shaped (upper panel of Fig. 6). As shown the figure, in the case of a non-kinematic subsiding wave, the peak of the flow rate $\frac{\partial Q}{\partial t} = 0$ in a considered cross-section is followed by the temporal peak of the flow depth $\frac{\partial h}{\partial t} = 0$, while the spatial peak of the flow depth $\frac{\partial h}{\partial x} = 0$ is the final one. For the purposes of this study the true arrival time of $\frac{\partial h}{\partial x} = 0$ is analysed. The bottom panel of Fig. 6 presents schematically the true arrival time of $\frac{\partial h}{\partial x} = 0$ for the non-kinematic wave, and the arrival time approximated by the kinematic wave assumption in the form of Eq. (17) and Eq. (18). Both formulae underestimate the time instant at which $\frac{\partial h}{\partial x} = 0$. As a matter of fact, from the practical point of view, the evaluation of the friction velocity is exceptionally important in this region, as intensified transport processes may occur just before the wave peak (Bombar et al., 2011; De Sutter et al., 2001).

In order to apply the kinematic wave approximation, the wave celerity must be evaluated. Celerity can be assessed by the formula derived from the Chezy equation (Eq. 19) (Henderson, 1963), and it is applied in this study.

$$C = \frac{3}{2}U. \tag{19}$$

7

Tu and Graf (1993) proposed another method for evaluating C:

$$C = U + h \frac{\partial U}{\partial t} \Big/ \frac{\partial h}{\partial t}. \tag{20}$$

However, we would like to highlight the fact that in Eq. (20) $\frac{\partial h}{\partial t}$ is in the denominator, which con-
strains the application of the method. As a result, a discontinuity occurs for the time instant at which
$\frac{\partial h}{\partial t} = 0$. When the results of Eq. (20) are applied in Eq. (17), the discontinuity of ϑ as a function
of time occurs at the time instant at which $C = 0$, which is between $t(\frac{\partial U}{\partial t} = 0)$ and $t(\frac{\partial h}{\partial t} = 0)$. This
effect is illustrated in the section on field data application (Sect 4.1).

We propose another approach for evaluation of ϑ, which is compatible with the kinematic wave
concept, but does not require the evaluation of temporal derivatives, and for this reason may appear to
be easier to be used in some cases. Let us assume a reference cross-section P0 and two cross-sections
P1 and P2 located at a small distance Δs downstream and upstream of P0, respectively. Knowing
the $h(t)$ relationship, let us shift this function to P1 and to P2 by $\Delta t = \frac{\Delta s}{C}$ in the following way:
$h_1(t) = h_0(t - \Delta t)$, and $h_2(t) = h_0(t + \Delta t)$. The spatial derivative $\frac{\partial h}{\partial x}$ is next evaluated as follows:

$$\vartheta_{\mathrm{wt}} = \frac{\partial h}{\partial x} = \frac{h_2(t) - h_1(t)}{2\Delta s}. \tag{21}$$

The method is denominated as wave translation method, and is applied in this study.

3.3.2 Linear approximation based on two cross-sections

Because of the drawbacks of kinematic wave approximation, it is recommended to evaluate the gra-
dient of the flow depth based on data from two cross-sections (Aricó et al., 2008; Dottori et al., 2009;
Julien, 2002), which is, in fact, a two-point difference quotient (backward or forward). Nonetheless,
a number of problematic aspects of this approach have been pointed out. Firstly, Koussis (2010) has
stressed the fact that flow depth is highly affected by local geometry. Moreover, Aricó et al. (2008)
have pointed that lateral inflow may affect the evaluation of the gradient of flow depth, and for
this reason the cross-sections should be located close enough to each other to allow the assumption
of negligible lateral inflow. On the other hand, the authors have claimed that the distance between
cross-sections should be large enough to perform a robust evaluation of the flow depth gradient.
The impact of distance between cross-sections on the gradient of flow depth has been studied in
(Mrokowska et al., 2015) with reference to dynamic waves generated in a laboratory flume. The
results have shown that with a too long distance, the gradient in the region of the wave peak is mis-
estimated due to the linear character of approximation. On the other hand, with a too short distance,
the results may be affected by fluctuations of the water surface which in such case are large relative
to the distance between cross-sections.

Another drawback of the method is the availability of data. Very often, data originate from mea-
surements which have been performed for some other purpose. Consequently, the location of gauging
stations and data frequency acquisition do not meet the requirements of the evaluation of the gradient
of flow depth (Aricó et al., 2009). The latter problem applies to the case studied in this paper.

8

Due to the linear character of a two-point (backward and forward) difference quotient, it is not able to represent properly the peak region of a flood wave. In (Mrokowska et al., 2015) it has been stated that for better representation of ϑ the central difference quotient should be applied. Due to not enough measurement cross-sections for Olszanka watercourse, in this study only two-point difference quotient is applied.

3.4 Uncertainty of resistance evaluation

The results of resistance evaluation should be given alongside the level of uncertainty. In the case of unrepeatable experiments Mrokowska et al. (2013) have suggested applying deterministic approach – the law of propagation of uncertainty (Holman, 2001; Fornasini, 2008). Let us denote dependent variable as Y (here: S, n or u_*), and independent variables as x_i. Then maximum deterministic uncertainty of Y is assessed as:

$$\Delta Y_{\max} \simeq \sum_{i=1}^{n} \left| \frac{\partial Y}{\partial x_i} \right| \Delta x_i. \tag{22}$$

The method is valid under the assumption that the functional relationship describes correctly the dependent variable. In this method the highest possible values of uncertainty of input variables are assessed based on the knowledge of measurement techniques and experimental settings. Hence, it provides maximum uncertainty of a result.

4 Results

4.1 Evaluation of the gradient of flow depth

As presented in Sect 2 a number of measurements were performed in Olszanka watercourse. Nonetheless, the location and the number of cross-sections constrain the evaluation of spatial derivative ϑ. It is feasible to use the data from only two subsequent cross-sections: for data set Ol-1, ϑ could be evaluated based on cross-sections CS1 and CS1a located 107 m downstream of CS1, and for Ol-2 based on CS2 and CS2a located 315 m upstream of CS2 (upper panel of Fig. 1).

The following methods of evaluating ϑ are examined and compared:

- Linear approximation denoted as ϑ_{lin}

- Kinematic wave approximation in the form of the Jones formula (Eq. 17), denoted as ϑ_{kin} with C evaluated from Eq. (19)

- Wave translation (Eq. 21) denoted as ϑ_{wt} proposed in this paper with $\Delta s = 10$ m, and C evaluated from Eq. (19)

- Kinematic wave approximation (Eq. 17) with C evaluated from Eq. (20) which is denoted as $\vartheta_{\mathrm{Tu\&Graf}}$.

9

As can be seen from Fig. 7, ϑ_{kin} and ϑ_{wt} provide compatible results. Nonetheless, huge discrepancies in the ϑ_{lin} values are evident compared to ϑ_{kin} and ϑ_{wt}. The reason for this is that the linear method is applied to data from two cross-sections, which are located at a considerable distance apart. Moreover, due to the linear character of this method, ϑ_{lin} is unsuitable to express the variability of the flood wave shape. As a result, it overestimates the time instant at which $\vartheta = 0$ when the downstream cross-section is taken into account (as in Ol-1), and underestimates the time instant when the upstream cross-section is used (as in Ol-2). Next, the lateral inflows might have an effect on the flow, and thus the estimation of ϑ by the linear method. When it comes to $\vartheta_{\text{Tu\&Graf}}$, the results are in line with ϑ_{kin} and ϑ_{wt} except for the region near the peak of the wave where discontinuity occurs. This occurs due to the form of Eq. (20), which cannot be applied if $\frac{\partial h}{\partial t} = 0$, as was theoretically analysed in Sect 3.3.1. Consequently, the method must not be applied in the region of a rising limb in the vicinity of the wave peak and in the peak of the wave itself.

4.2 Evaluation of resistance to flow

Friction slope S, friction velocity u_* and Manning n are evaluated by formulae for dynamic, diffusive waves and steady flow. Wave translation method is used to assess ϑ. Results evaluated by formulae for dynamic wave are presented with uncertainty bounds, which allow to assess if the results obtained by simplified methods lie within the acceptable bounds or not. Uncertainty bounds are evaluated by the law of propagation of uncertainty. The uncertainties of the input variables are assessed based on knowledge of measurement techniques and experimental settings as follows: $\Delta h = 0.01$ m, $\Delta U = 10\% \ U$ (measurement performed by a propeller current meter), $\Delta R = 0.01$ m, $\Delta\zeta = 0.0001$ [m s^{-2}], $\Delta\eta = 0.0001$ [m s^{-1}], $\Delta\vartheta - 0.00001$ [-], $\Delta I = 0.0001$, $\Delta m = 0.001$, $\Delta b = 0.01$.

4.2.1 Evaluation of friction slope

In order to assess to which category of flood wave (dynamic, diffusive or kinematic) the case under study should be assigned, the terms of the momentum balance equation are compared. The results are shown in Fig. 8. All terms are evaluated analytically from measurement data. For data set Ol-1, the bed slope and the maximum flow depth gradient are of magnitude 10^{-4}, and the acceleration terms reach the magnitude of 10^{-4} along the rising limb. For Ol-2 bed slope is of magnitude 10^{-3}, the maximum flow depth gradient is of magnitude 10^{-4}, and other terms are negligible. However, the acceleration terms are of opposite signs, and the overall impact of flow acceleration on the results might not be so pronounced. The comparison between Ol-1 and Ol-2 shows that in cross-section CS1, which is closer to the dam, more terms of the momentum balance equation are significant. From the results for CS2 it may be concluded that the significance of the temporal variability of flow parameters decreases along the channel. In the case of data set Ol-1, along the rising limb local acceleration term is slightly bigger than the advective one, which may indicate dynamic character of the wave. On the other hand, it may be concluded that the wave for Ol-2 is of a diffusive character.

10

Fig. 9 presents comparison between the results of friction slope evaluated by formulae for dynamic wave S_{dyn} (Eq. 5), diffusive wave S_{dif} (Eq. 11) and approximated by bed slope I (Eq. 14). Values of S_{dyn} range in the following intervals: [0.00027, 0.00085] for Ol-1 and [0.0013, 0.0015] for Ol-2 with the maximum before the peak of wave. Difference between values of S_{dyn} for Ol-1 and Ol-2 is affected to large extent by difference of bed slope between cross-sections CS1 and CS2.

In the case of data set Ol-1 S_{dif} slightly differs from S_{dyn} along the rising limb of the wave. There are regions in which the results for diffusive wave lie outside the uncertainty bounds of friction slope evaluated by formula for dynamic wave. This is another argument for choosing the formula for dynamic wave along the rising limb of the wave in Ol-1. For the falling limb formula for diffusive wave may be applied. Steady flow approximation is not recommended in this case as the values of bed slope fall outside the uncertainty bounds in both rising and falling limb. In the case of Ol-2 results of friction slope for both approximations – diffusive wave and steady flow are within uncertainty bounds. However, the formula for diffusive wave is recommended, as it reflects the temporal variability of friction slope. With steady flow formula the information about friction slope variability during the propagation of wave is not provided. Before the peak of wave $S_{\mathrm{dyn}} > I$ and after the peak $S_{\mathrm{dyn}} < I$.

4.2.2 Evaluation of friction velocity

Figure 10 presents the comparison of the results of friction velocity evaluated by dynamic $u_{*\mathrm{dyn}}$ (Eq. 6), diffusive $u_{*\mathrm{dif}}$ (Eq. 12) and steady flow $u_{*\mathrm{st}}$ (Eq. 15) formulae. The results for friction velocity are in line with the results of friction slope. Values of $u_{*\mathrm{dyn}}$ range in the following intervals: [0.031, 0.052] for Ol-1 and [0.057, 0.061] for Ol-2 with the maximum before the peak of wave.

As can be seen from Fig. 10, the results for friction velocity in Ol-1 obtained by the formula for dynamic wave and the formula for diffusive wave agree well with each other along the falling limb. The slight difference along the rising limb of the wave between the results occurs, as $u_{*\mathrm{dif}}$ falls outside uncertainty bounds. This is caused by the acceleration terms, which appear to be significant in Ol-1 along the leading edge (Fig. 8). Consequently, in this region, the application of formula for dynamic wave may be considered, while for falling limb formula for diffusive wave may be applied. In the case of Ol-1 $u_{*\mathrm{dyn}}$ and $u_{*\mathrm{st}}$ differ from each other. The results for steady flow formula fall outside the uncertainty bounds along the substantial part of the wave, which indicates that the application of steady flow approximation is incorrect. In the case of Ol-2 diffusive wave formula may be applied, as $u_{*\mathrm{dyn}}$ and $u_{*\mathrm{dif}}$ agree well with each other. Moreover, discrepancy between results for dynamic wave and steady flow is smaller, and steady flow approximation might be considered in friction velocity evaluation. However, then the information on maximum value of resistance along rising limb is missing.

4.2.3 Evaluation of the Manning coefficient

Figure 11 presents the comparison of the results of Manning n evaluated by dynamic n_{dyn} (Eq. 7), diffusive n_{dif} (Eq. 13) and steady flow n_{st} (Eq. 16) formulae.

345   Values of n_{dyn} range in the following intervals: [0.015, 0.039] for Ol-1 and [0.024, 0.032] for Ol-2. The values of Manning n correspond with the values assigned to natural minor streams in the tables presented in (Chow, 1959). The minimum values of Ol-2 correspond with "clean straight, full stage, no rifts or deep pools", while the minimum value of Ol-1 does not match n for natural streams presented in the tables. The maximum values may be assigned to "same as above, but more stones

350   and weeds". The Manning n coefficients have been evaluated in a completely different way for the measurement data from this field site by Szkutnicki (1996); Kadłubowski and Szkutnicki (1992). In that study, n was treated as a constant parameter in the St. Venant model, and its value was assessed by optimising the model performance. The authors have reported that for spring conditions, $n \in [0.04, 0.09]$. In this analysis, the results are smaller.

355   Results for n_{dyn}, n_{dif} and n_{st} follow the same trend achieving minimum values for time instant of U_{max}. The results for Manning n obtained by the formula for dynamic wave and the formula for diffusive wave agree well with each other in both cases: Ol-1 and Ol-2. Results obtained by formula for steady flow differ slightly from n_{dyn} along the rising limb of Ol-1, and lie on the edge of uncertainty bounds, while n_{st} agree well with n_{dyn} in the case of Ol-2. Consequently, Manning n

360   may be approximated by formula for diffusive wave along the rising limb of Ol-1, while along the falling limb of Ol-1 and for Ol-2 steady flow approximation may be applied.

4.3 The variability of resistance to flow during flood wave propagation

The variability of resistance in unsteady flow is very often analysed in terms of flow rate Q, and Manning n is considered as a reference variable (Fread, 1985; Julien et al., 2002). It seems reason-

365   able to compare Manning n and friction velocity vs. flow rate Q. The comparison is illustrated in Fig. 12. As can be seen from the figure, Manning n decreases with increasing flow rate. This trend is characteristic of the majority of streams with inbank flow (Chow, 1959), which has been observed by Fread (1985) when the inundation area was relatively small compared to inbank flow area. This is the case considered herein, as the experiment was performed under inbank flow conditions. The re-

370   verse trend has been observed by Julien et al. (2002) for flood waves in the River Rhine. The authors discussed extensively impact of bed forms on Manning n. However, we would like to emphasise another aspect – the shape of inundation area which determines the reverse trend. In (Julien, 2002) interpretation of rising n as rising resistance is qualitatively correct, while in the case of Olszanka watercourse false conclusions may be drawn from the analysis of Manning n, that the bulk resistance

375   decreases with flow rate. As the results for friction velocity show, the maximum values of resistance are in the rising limb of the waves, before the maximum flow rate Q.

5 Concluding remarks

Proper determination of resistance parameters: friction slope, friction velocity and Manning coefficient in unsteady flow is very often hampered by scarcity or high uncertainty of input data. However, when resistance relations are applied with an awareness of their constraints, and proper effort is made to minimise the uncertainty of the input data, it is likely to obtain reliable results. To facilitate the evaluation of resistance parameters, we have proposed the methodology which provides means to enhance reliability of results obtained by relations derived from flow equations. The methodology comprises four questions which help to judge if simplifications with regard to shape of a channel and type of wave are admissible, to decide which method of $\frac{\partial h}{\partial x}$ evaluation is the best in the case under study, and to evaluate the uncertainty of results. Besides critical analysis of existing methods we have proposed some new approaches: the formulae for resistance parameters for trapezoidal channel and wave translation method instead of Jones formula to evaluate $\frac{\partial h}{\partial x}$. The analysis of $\frac{\partial h}{\partial x}$ evaluation has shown that it is constrained by the spatial data, and this is the weakest point of application of relations for resistance. Hence, this element needs particular attention when resistance parameters are evaluated.

The paper has demonstrated the application of proposed methodology to experimental data; hence, the detailed conclusions drawn in the study apply to similar cases. The methodology has been applied to assess if the simplified formulae are admissible. The analysis of terms of momentum balance equation has provided identification of the type of waves. In the first case – Ol-1, which is closer to the dam, the wave has dynamic character along rising limb and diffusive character along falling limb. In the second case - Ol-2, the wave is of diffusive character with relatively small difference between water slope and bed slope. Thanks to the uncertainty analysis the reliability of the results of resistance parameters obtained by simplified formulae has been assessed.

The analysis revealed that for S and u_* steady state formula is unacceptable, while for Manning n the steady flow approximation is admissible when the wave is of diffusive character. Hence, Manning n is less sensitive to simplifications of formulae than S and u_*. It is an asset when n is considered as a parameter in flood routing practice, because reliability of results is less dependent on quality and quantity of data used. The study has demonstrated that S and u_* are better choice than Manning n when information on variability and trend of resistance to flow during flood wave propagation is required.

Flood wave phenomena are so complex that it is currently impossible to provide a comprehensive analysis, and the problem of resistance to flow in unsteady non-uniform conditions still poses a challenge. For this reason, more research on resistance in unsteady non-uniform conditions is necessary.

415 **References**

Albayrak, I. and Lemmin, U.: Secondary currents and corresponding surface velocity patterns in a turbulent open-channel flow over a rough bed, J. Hydraul. Eng.-ASCE, 137(11), 1318–1334, doi:10.1061/(ASCE)HY.1943-7900.0000438, 2011.

Aricó, C., Tuciarelli, T., Dottori, F., Martina, M., and E.Todini: Peak flow measurement in the Arno river by

420   means of unsteady-state water level data analysis, in: Proc. International Conference on Fluvial Hydraulics, Cesme-Izmir, Turkey, 3-5 September 2008, River Flow 2008, 2008.

Aricó, C., Nasello, C., and Tucciarelli, T.: Using unsteady-state water level data to estimate chanel roughness and discharge hydrograph, Adv. Water Resour., 32(8), 1223–1240, doi:10.1016/j.advwatres.2009.05.001, 2009.

425 Bombar, G., Elci, S., Tayfur, G., Guney, M. S., and Bor, A.: Experimental and numerical investigation of bed load transport under unsteady flows, J. Hydraul. Eng.-ASCE, 137(10), 1276–1282, doi:10.1061/(ASCE)HY.1943-7900.0000412, 2011.

Chow, V.: Open-channel hydraulics, The Blackburn Press, New Jersey, USA, 1959.

Czernuszenko, W. and Rowiński, P. M.: Shear stress statistics in a compound channel flow, Archives of Hydro-

430   Engineering and Environmental Mechanics, 55(1-2), 3–27, 2008.

De Sutter, R., Verhoeven, R., and Krein, A.: Simulation of sediment transport during flood events: Laboratory work and field experiments, Hydrolog. Sci. J., 46(4), 599–610, doi:10.1080/02626660109492853, 2001.

Dey, S.: Fluvial Hydrodynamics. Hydrodynamic and Sediment Transport Phenomena, in: GeoPlanet: Earth and Planetary Sciences, Springer, 2014.

435 Dey, S., Sarkar, S., and Solari, L.: Near-bed turbulence characteristics at the entrainment threshold of sediment beds, J. Hydraul. Eng.-ASCE, 137(9), 945–958, doi:10.1061/(ASCE)HY.1943-7900.0000396, 2011.

Dooge, J. and Napiórkowski, J.: Applicability of diffusion analogy in flood routing, Acta Geophysica Polonica, 35(1), 65–75, 1987.

Dottori, F., Martina, M., and Todini, E.: A dynamic rating curve approach to indirect dis-

440   charge measurements, Hydrol. Earth Syst. Sc., 13, 847–863, doi:10.5194/hess-13-847-2009, www.hydrol-earth-syst-sci.net/13/847/2009/, 2009.

Ferguson, R.: Time to abandon the Manning equation?, Earth Surf. Proc. Land., 35(15), 1873–1876, doi:10.1002/esp.2091, 2010.

Fornasini, P.: The uncertainty in physical measurements. An introduction to data analysis in the physics labora-

445   tory, Springer, 2008.

Fread, D. L.: Hydrological forecasting, chap. Channel routing, pp. 437–498, John Wiley & Sons Ltd., Chichester, United Kingdom, 1985.

Garcia, M.: Sedimentation engineering. Processes, measurements, modeling and practice, chap. Sediment transport and morphodynamics, pp. 21–146, American Society of Civil Engineers, Virginia, USA, 2007.

450 Ghimire, B. and Deng, Z.: Event flow hydrograph-based method for shear velocity estimation, J. Hydraul. Res., 49(2), 272–275, doi:10.1080/00221686.2011.552463, 2011.

Gosh, S.: Flood control and drainage engineering, CRC Press, 4th edn., 2014.

Graf, W. and Song, T.: Bed shear stress in non uniform and unsteady open-channel flows, J .Hydraul. Res., 33(5), 699–704, doi:10.1080/00221689509498565, 1995.

15

455   Guney, M., Bombar, G., and Aksoy, A.: Experimental study of the coarse surface developement effect on the bimodal bed-load transport under unsteady flow conditions, J. Hydraul. Eng.-ASCE, 139(1), 12–21, doi:10.1061/(ASCE)HY.1943-7900.0000640, 2013.

Henderson, F. M.: Flood waves in prismatic channels, Journal of the Hydraulics Division, 89(HY4), 39–67, 1963.

460   Holman, J.: Experimental methods for engineers, McGraw-Hill, USA, 7th edn., 2001.

Julien, P.: River mechanics, Cambridge University Press, Cambridge, UK, 2002.

Julien, P. Y., Klaassen, G. J., Brinke, W. B. M. T., and Wilbers, A. W. E.: Case study: bed resistance of Rhine River during 1998 flood, J. Hydraul. Eng.-ASCE, 128(12), 1042–1050, doi:10.1061/(ASCE)0733-9429(2002)128:12(1042), 2002.

465   Kadłubowski, A. and Szkutnicki, J.: Assessment of roughness of a small river-channel influenced by vegetation, Geophysical Review, 3-4, 161–176, (in Polish), 1992.

Kalinowska, M. B. and Rowiński, P. M.: Uncertainty in computations of the spread of warm water in a river - lessons from Environmental Impact Assessment case study, Hydrol. Earth Syst. Sc., 16, 4177–4190, doi:10.5194/hess-16-4177-2012, http://www.hydrol-earth-syst-sci.net/16/4177/2012/, 2012.

470   Kalinowska, M. B., Rowiński, P. M., Kubrak, J., and Świątek, D. M.: Scenarios of the spread of a waste heat discharge in a river Vistula River case study, Acta Geophysica, 60, 214–231, 2012.

Khiadani, M. H., Beecham, S., Kandasamy, J., and Sivakumar, S.: Boundary shear stress in spatially varied flow with increasing discharge, J. Hydraul. Eng.-ASCE, 131(8), 705–714, doi:10.1061/(ASCE)0733-9429(2005)131:8(705), 2005.

475   Koussis, A.: Comment on "A praxis-oriented perspective of streamflow inference from stage observations the method of Dottori et al. (2009) and the alternative of the Jones Formula, with the kinematic wave celerity computed on the looped rating curve" by Koussis (2009), Hydrol. Earth Syst. Sc., 14, 1093–1097, doi:10.5194/hess-14-1093-2010, www.hydrol-earth-syst-sci.net/14/1093/2010/, 2010.

Mohajeri, S. H., Safarzadeh, A., and Neyshabouri, S. A. A. S.: A three tube pressure instrument for measuring
480   the local bed shear stress in smooth and rough beds, in: Water - infinitely deformable but still limited - Proceedings of the 2nd IAHR Europe Congress, 27 - 29 June 2012, Technische Universität München, Germany, p. USB stick, 2012.

Mrokowska, M. M.: Evaluation of resistance to flow during flood wave propagation, Ph.D. thesis, Institute of Geophysics Polish Academy of Sciences, Warsaw, Poland, (in Polish), 2013.

485   Mrokowska, M. M., Rowiński, P. M., and Kalinowska, M. B.: Experimental and computational solutions of hydraulic problems, 32nd International School of Hydraulics, chap. The uncertainty of measurements in river hydraulics - evaluation of friction velocity based on an unrepeatable experiment, pp. 195–206, GeoPlanet: Earth and Planetary Sciences, Springer-Verlag Berlin Heidelberg, Germany, doi:10.1007/978-3-642-30209-1_13, 2013.

490   Mrokowska, M. M., Rowiński, P. M., and Kalinowska, M. B.: Evaluation of friction velocity in unsteady flow experiments, J. Hydraul. Res., doi:10.1080/00221686.2015.1072853, 2015.

Nikora, V. and Goring, D.: Flow turbulence over fixed and weakly mobile gravel beds, J. Hydraul. Eng.-ASCE, 126(9), 679–690, doi:10.1061/(ASCE)0733-9429(2000)126:9(679), 2000.

Perumal, M., Sherestha, K. B., and Chaube, U. C.: Reproduction of hysteresis in rating curves, J. Hydraul. Eng.-ASCE, 130(9), 870 – 878, doi:10.1061/(ASCE)0733-9429(2004)130:9(870), 2004.

Pokrajac, D., Finnigan, J., Manes, C., McEwan, I., and Nikora, V.: On the definition of the shear velocity in rough bed open channel flows, in: Proc. of the International Conference on Fluvial Hydraulics, Lisbon, Portugal, 6-8 September 2006, River Flow 2006, edited by Ferreira, R., Alves, E., Leal, J., and Cardoso, A., vol. 1, pp. 89–98, 2006.

Pope, N., Widdows, J., and Brinsley, M.: Estimation of bed shear stress using the turbulent kinetic energy approach - a comparison of annular flume and field data, Continental Shelf Research, 26(8), 959–970, doi:10.1016/j.csr.2006.02.010, 2006.

Rowiński, P. M. and Czernuszenko, W.: Experimental study of river turbulence under unsteady conditions, Acta Geophysica Polonica, 46(4), 462–480, 1998.

Rowiński, P. M., Czernuszenko, W., and Pretre, J.-M.: Time-dependent shear velocities in channel routing, Hydrolog. Sci. J., 45(6), 881–895, 2000.

Shen, Y. and Diplas, P.: Modeling unsteady flow characteristics for hydropeaking operations and their implications on fish habitat, J. Hydraul. Eng.-ASCE, 136(12), 1053–1066, doi:10.1061/(ASCE)HY.1943-7900.0000112, 2010.

Szkutnicki, J.: An experimental estimation of a roughness of rivers beds, in: Hydrology and Oceanology Series, 19, Institute of Meteorology and Water Management, Warsaw, (in Polish), 1996.

Tu, H. and Graf, W.: Friction in unsteady open-channel flow over gravel beds, J. Hydraul. Res., 31(1), 99–110, 1993.

Yen, B.: Open channel flow resistance, J. Hydraul. Eng.-ASCE, 128(1), 20–39, doi:10.1061/(ASCE)0733-9429(2002)128:1(20), 2002.
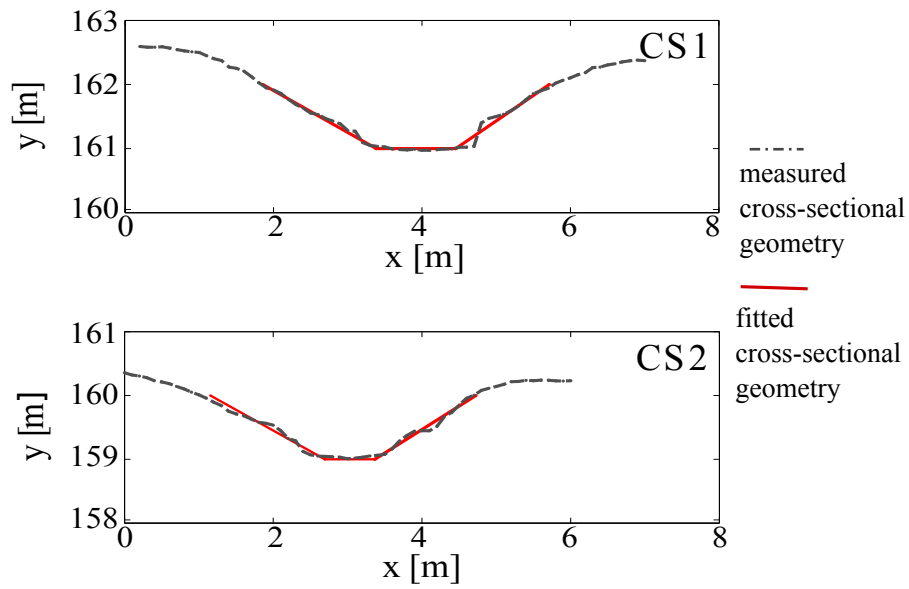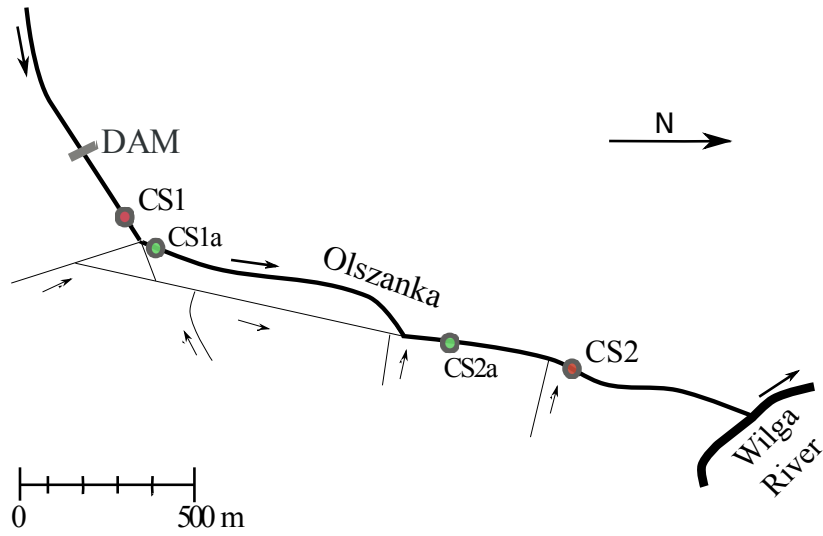
Figure 1. The site of the experiment in Olszanka watercourse (upper panel), and the shape of measurement cross-sections CS1 and CS2 (lower panel).
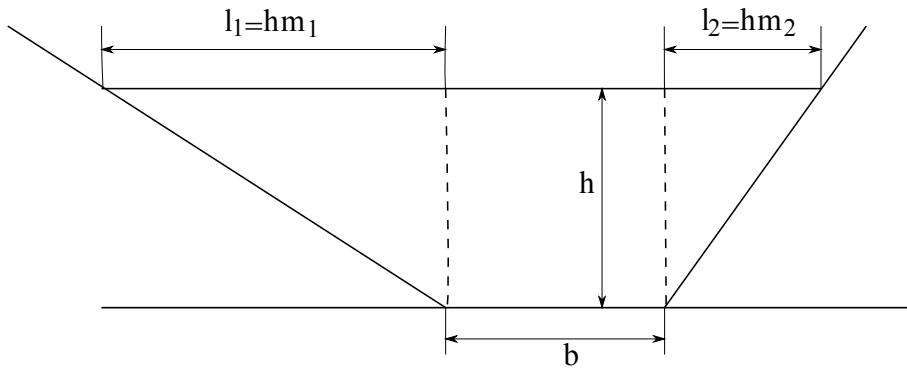
Figure 2. Trapezoidal cross-section of a channel with definitions of symbols used in the text.

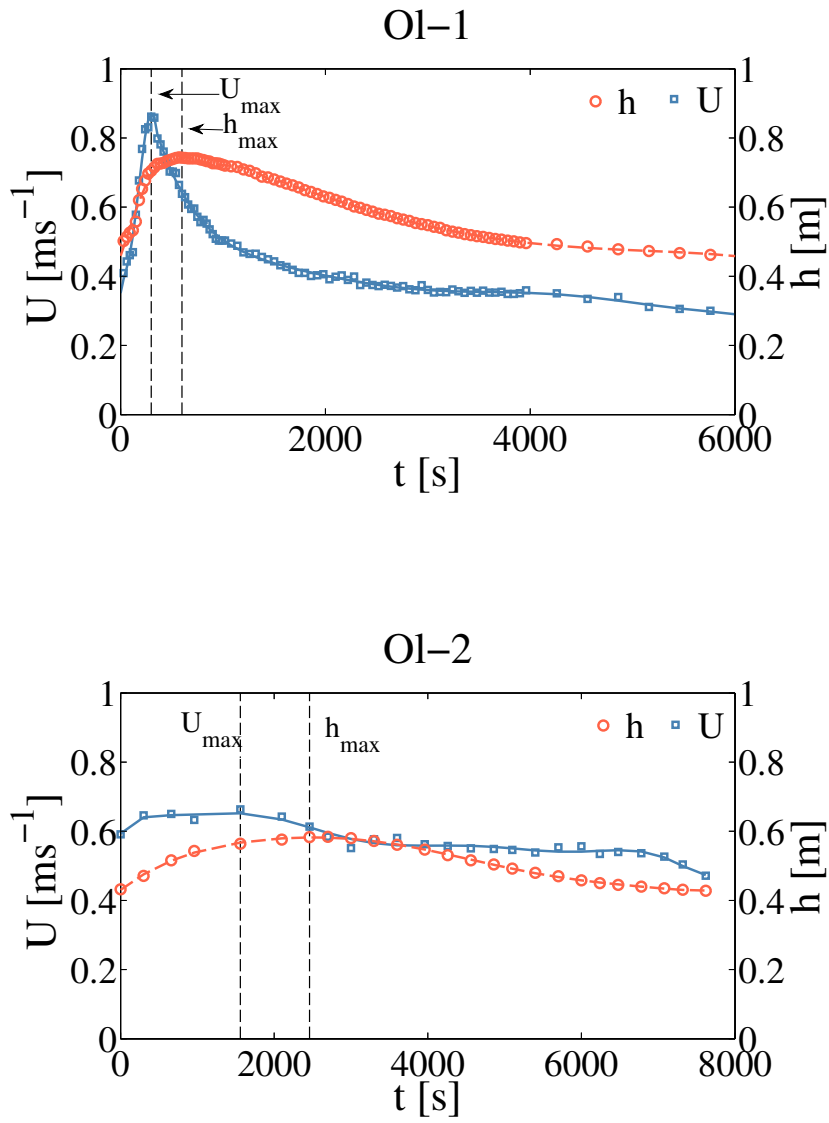**Figure 3.** Experimental reach of Olszanka watercourse. (courtesy of Jerzy Szkutnicki)

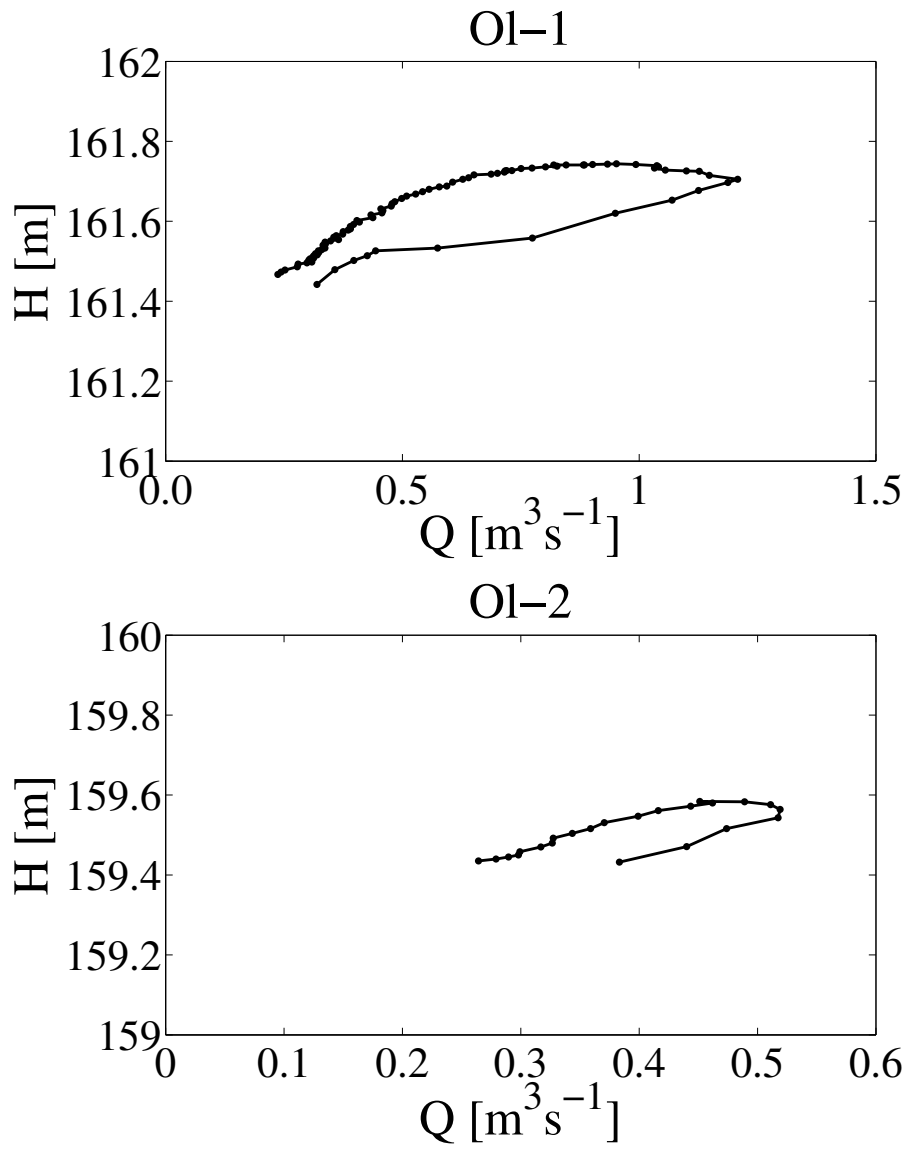**Figure 4.** Temporal variability of flow depth h and mean velocity U for experimental flood waves in Olszanka watercourse.

Figure 5. Rating curves of experimental flood waves in Olszanka watercourse.
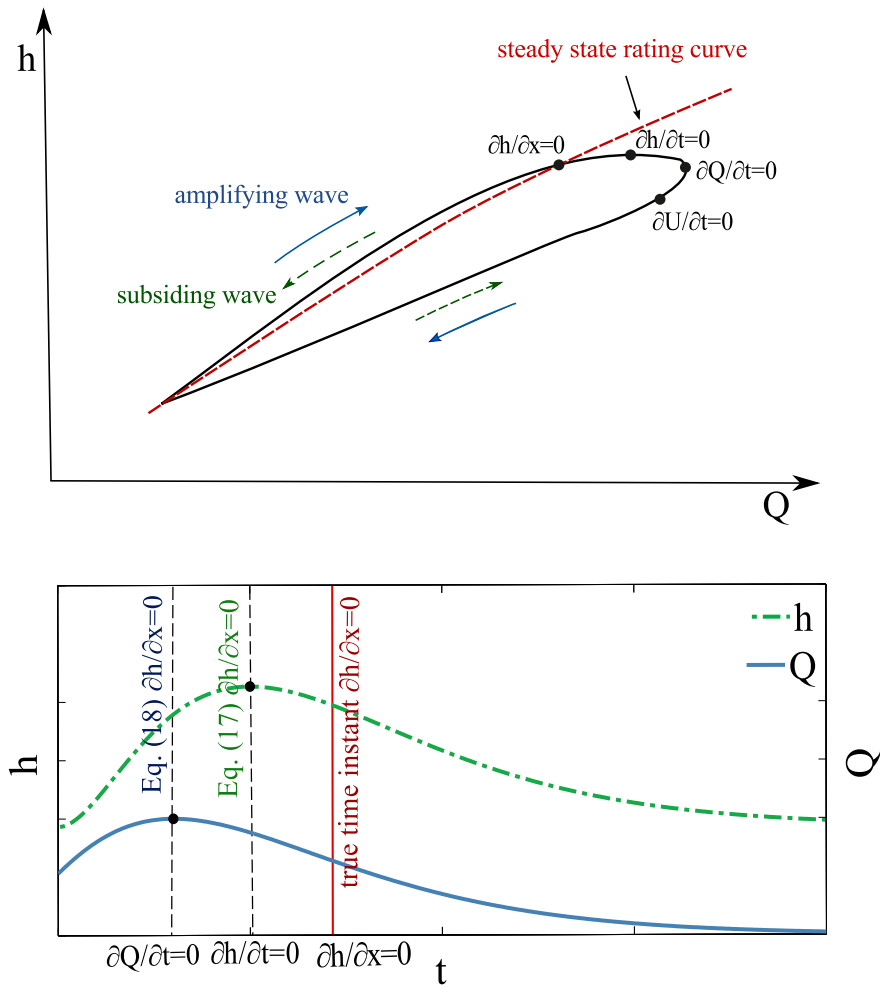
Figure 6. Comparison between rating curve for flood wave and steady flow with characteristic points, based on (Henderson, 1963) (upper panel), and impact of kinematic wave approximation (Eqs. (17) and (18)) on the assessment of time instant at which $\frac{\partial h}{\partial x} = 0$ (lower panel).
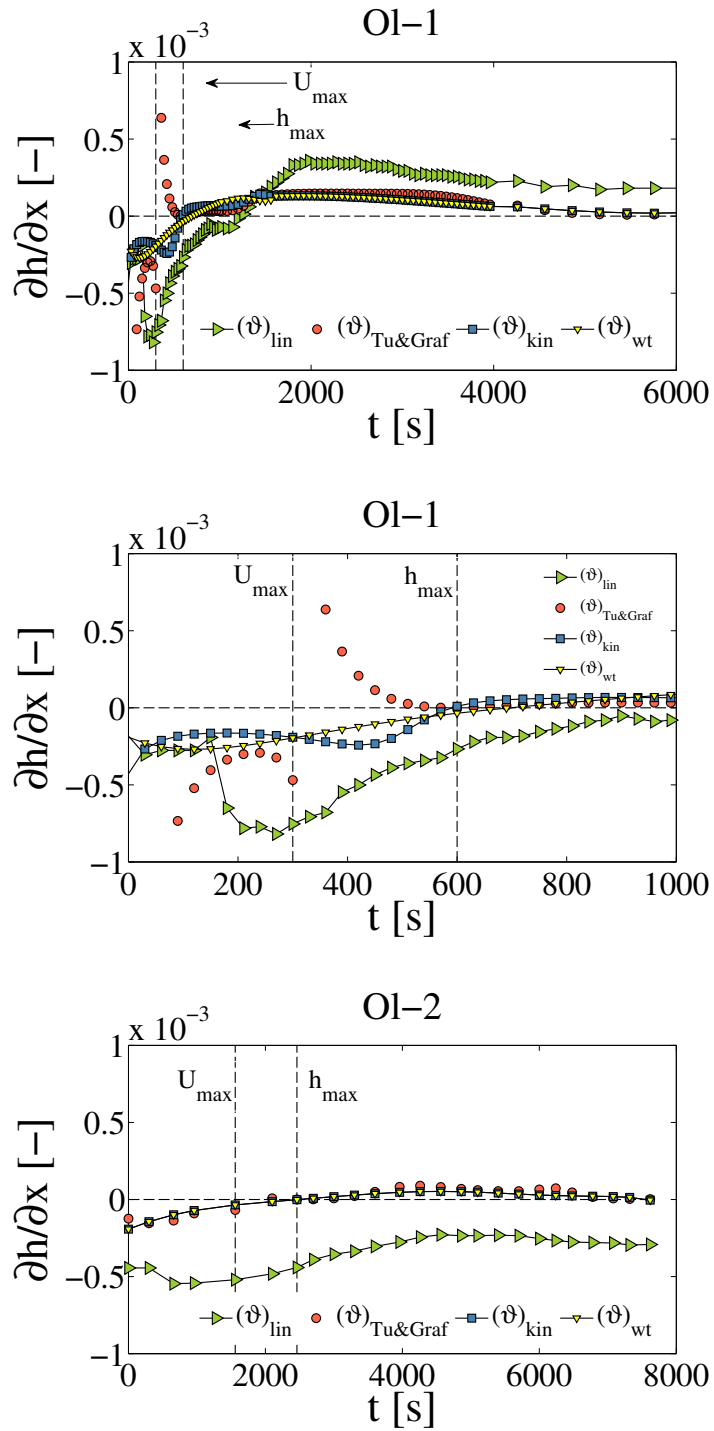
Figure 7. Temporal variability of gradient of flow depth $\vartheta = \frac{\partial h}{\partial x}$ for experimental flood waves in Olszanka watercourse. Middle panel shows a zoom of the rising limb of the wave for Ol-1.
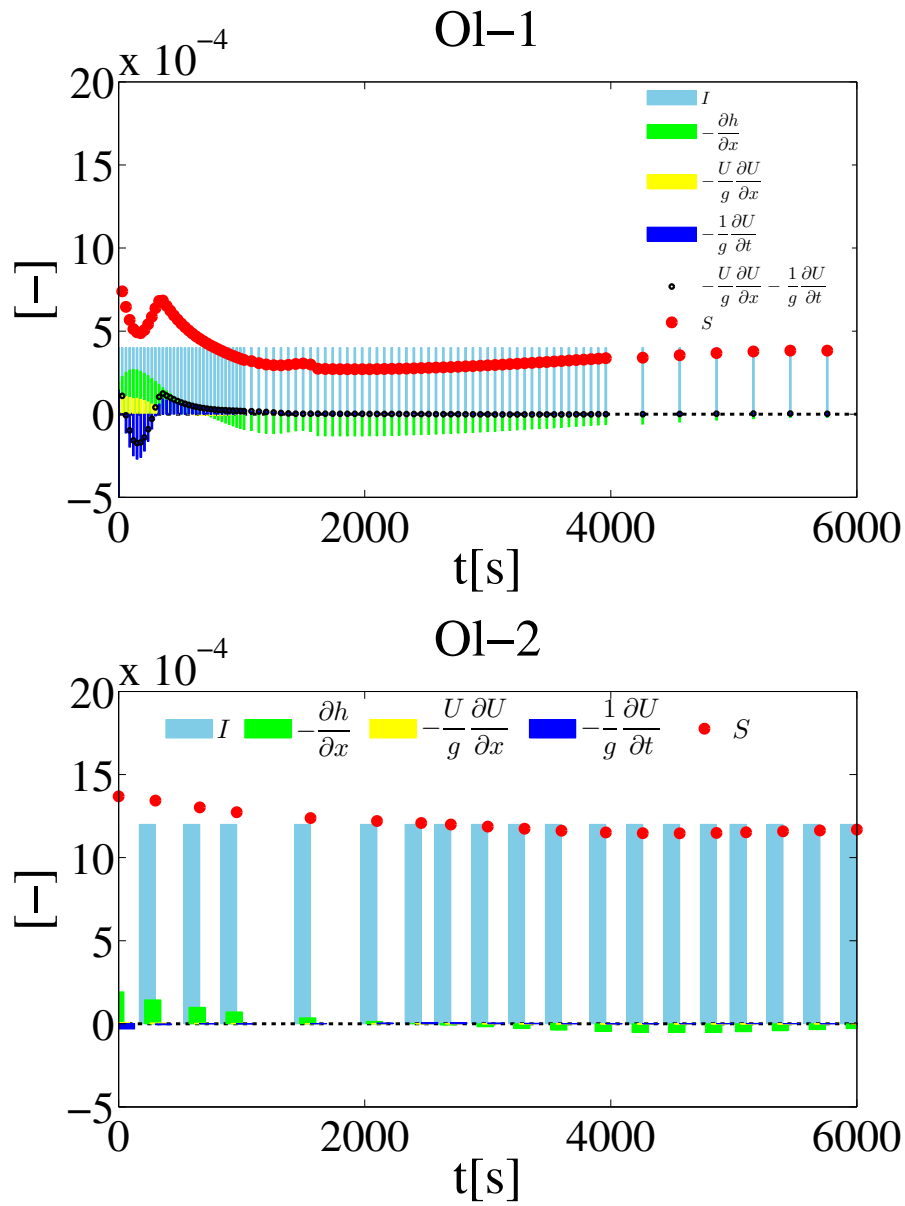
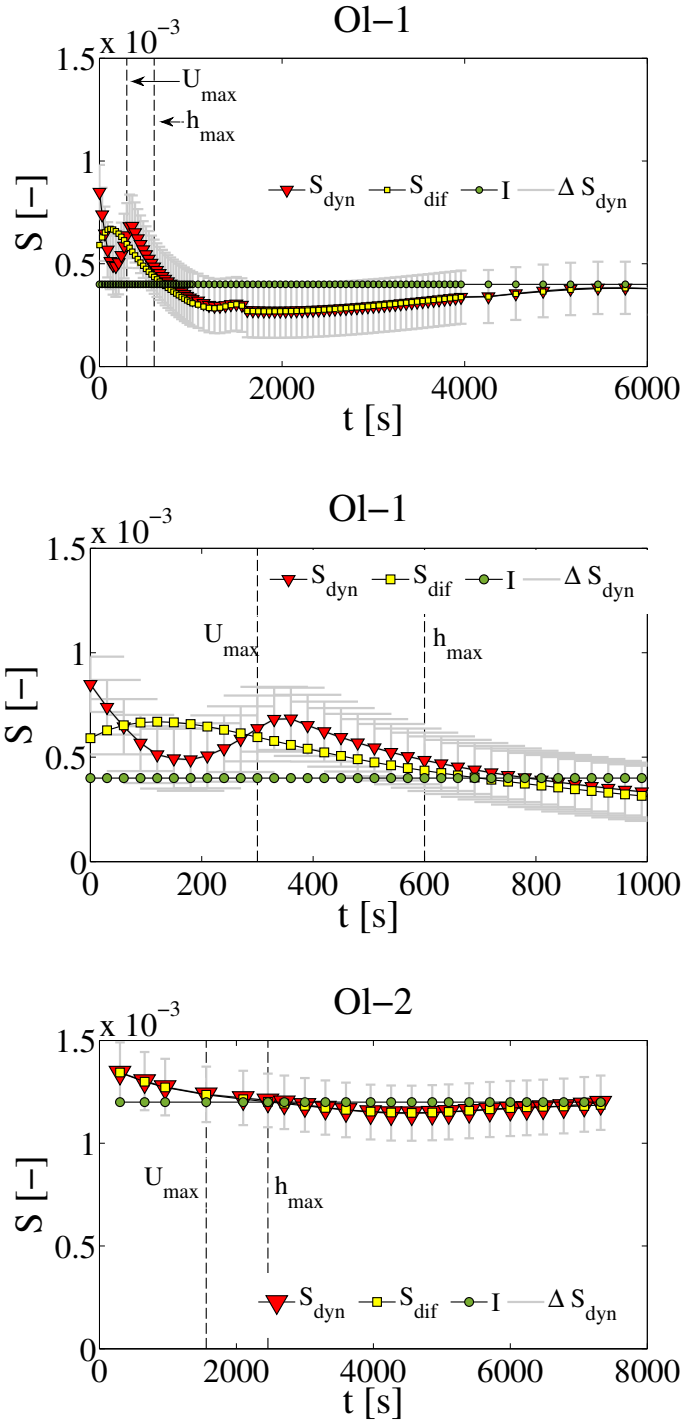**Figure 8.** Comparison of terms of the momentum balance equation for experimental flood waves in Olszanka watercourse.

Figure 9. Comparison of friction slope evaluated by formulae for dynamic S_{dyn}, diffusive wave S_{dif} and steady flow I with uncertainty bounds ΔS_{dyn} for experimental flood waves in Olszanka watercourse. Middle panel shows a zoom of the rising limb of the wave for Ol-1.
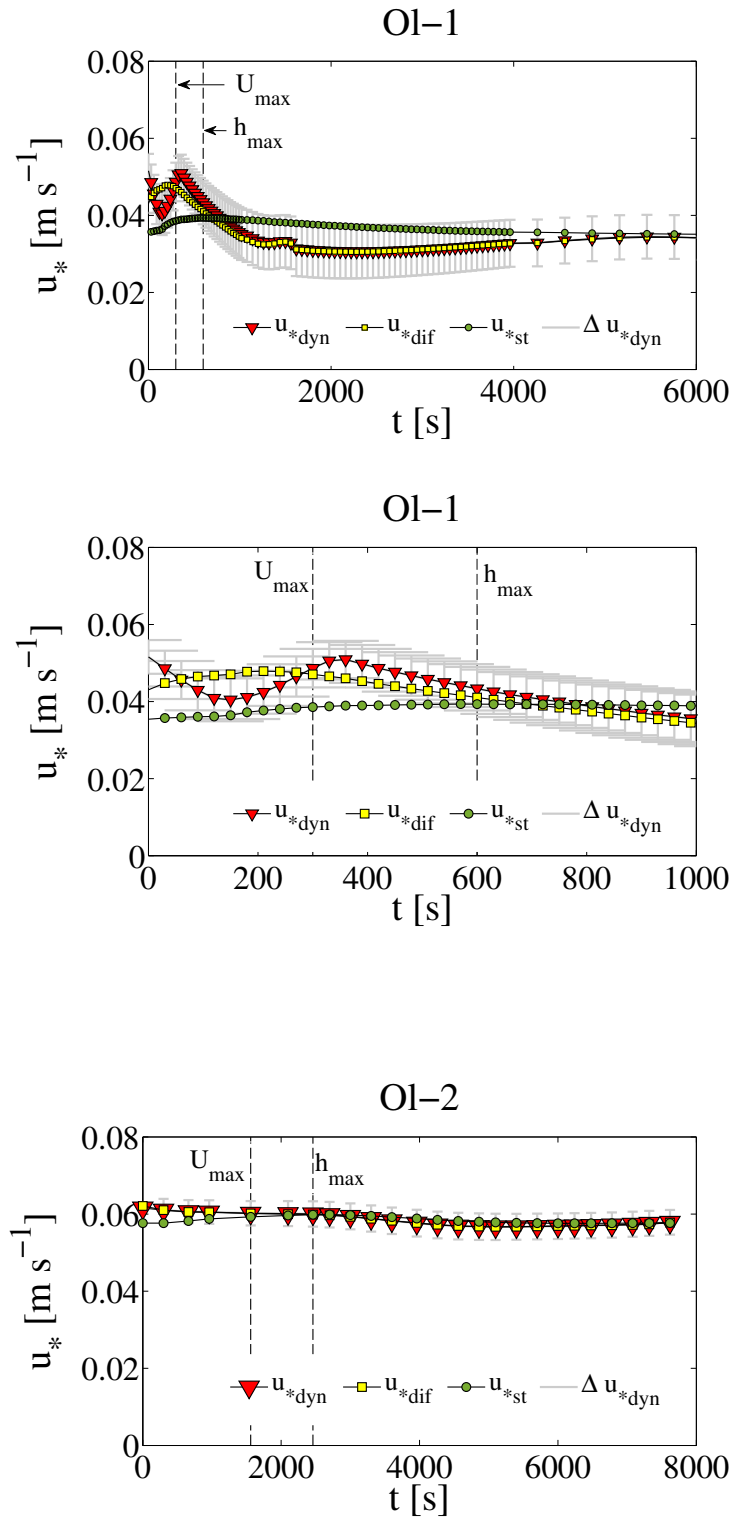
Figure 10. Comparison of friction velocity evaluated by formulae for dynamic u_{*dyn}, diffusive wave u_{*dif} and steady uniform flow u_{*st} with uncertainty bounds Δu_{*dyn} for experimental flood waves in Olszanka watercourse. Middle panel shows a zoom of the rising limb of the wave for Ol-1.
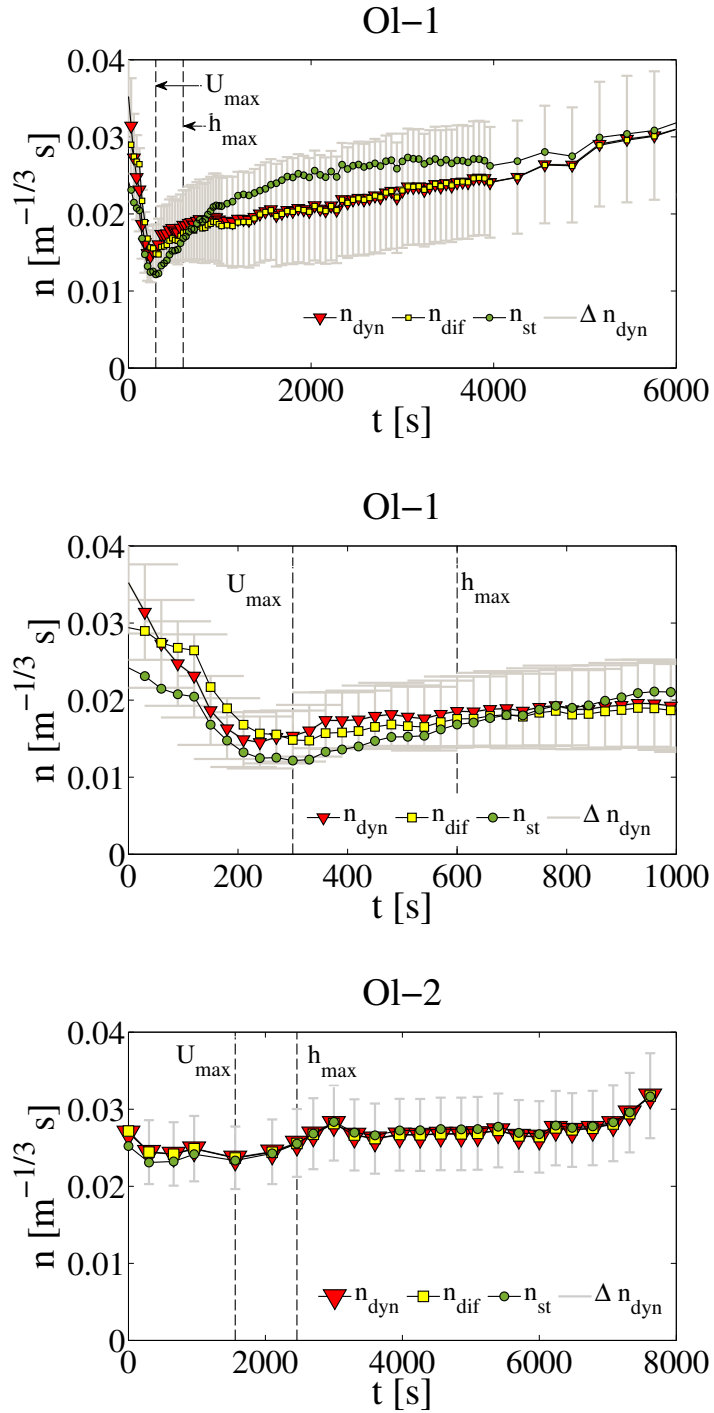
Figure 11. Comparison of Manning n evaluated by formulae for dynamic n_{dyn}, diffusive wave n_{dif} and steady uniform flow n_{st} with uncertainty bounds Δn_{dyn} for experimental flood waves in Olszanka watercourse. Middle panel shows a zoom of the rising limb of the wave for Ol-1.
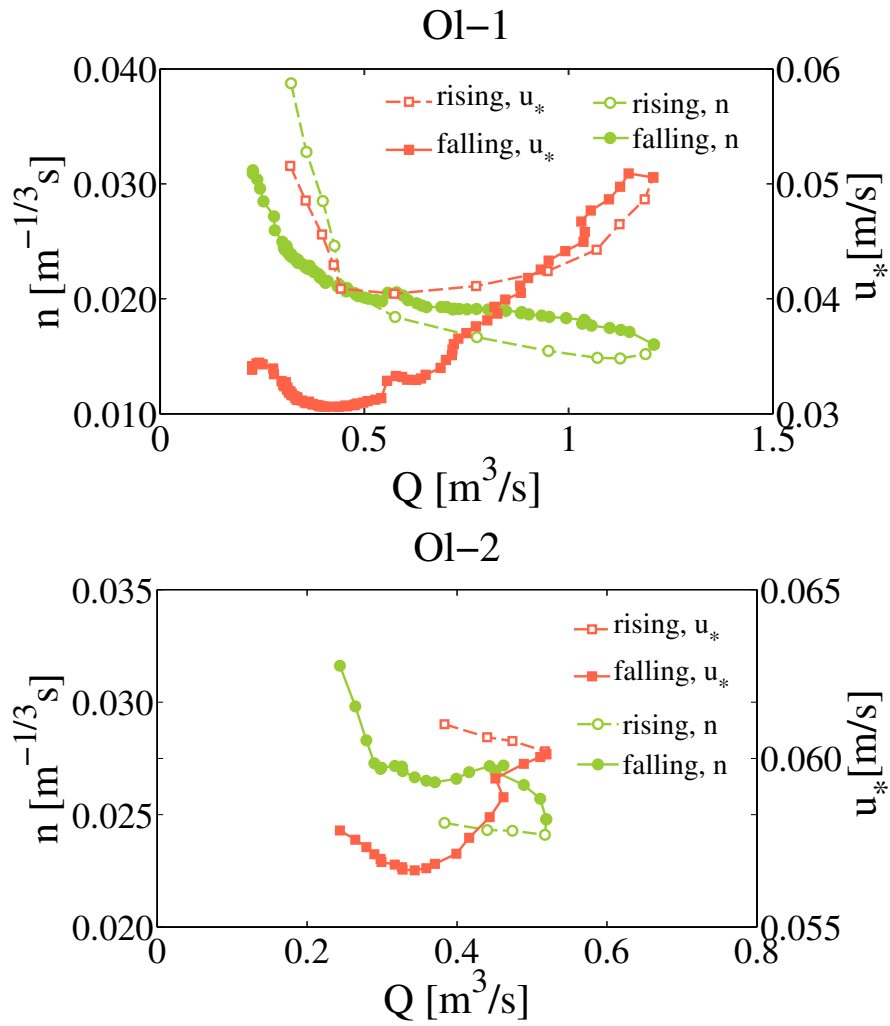
Figure 12. Comparison of the relation of Manning n vs flow rate Q and friction velocity u_* vs Q along rising and falling limbs of waves for experimental flood waves in the Olszanka watercourse.