

Vienna, September 9th 2015

Dear Dr. Moussa,

hereby we would like to submit the revised research manuscript “From runoff to rainfall: Inverse rainfall-runoff modelling in a high temporal resolution” (MS No.: hess-2014-487) for publication in HESS.

Our understanding of reviewer #1 and #2 is that the comments were responded in a satisfactory manor after the first round of review and that they proposed a minor revision. They state that “the authors provided a detailed reply and deeply revised their manuscript to answer the comments made by the reviewers” and “found that the manuscript is greatly improved”, since “key clarifications were added by the authors along with interesting additional modelling experiments”.

The authors would like to express their appreciation to the additional recommendations made by the reviewers and have significantly improved the former manuscript thanks to the reviewer’s pertinent comments and valuable suggestions. We believe that the manuscript carries substantial new and novel content for the hydrological community.

A number of changes were made in the revised manuscript, based on the comments of the reviewers. We have added a simulation experiment, where we investigate the effects of short-term errors in runoff on the inferred rainfall. Data was re-analysed and tables were changed to contain information requested. Additionally, all the reviewer’s comments have been addressed.

Sincerely,

Dr. Mathew Herrnegger

Point-by-point responses to the reviewer's comments. Indicated line numbers refer to the final manuscript without mark-ups.

Report #1

The authors provided a detailed reply and deeply revised their manuscript to answer the comments made by the reviewers. I found that the revised version is now clearer. However it became a bit long, especially section 4 which sometimes goes too much into the details. Maybe the authors could try to remove some unnecessary details there. I have only a few suggestions of minor modifications and think the article is almost ready for publication.

The authors appreciate and thank reviewer #1 for his valuable comments and suggestions which have improved the manuscript. The authors are aware that the manuscript and especially section 4 is long and shows many details. This is however mainly a result of incorporating the comments and suggestions from the first round of review. The authors would prefer to maintain the provided details, as the current version is based on the comments and suggestions of the reviewers. The authors appreciate the reviewer's understanding for this preference.

1. The introduction is long and a few sub-sections could be introduced to better structure this important part.

The authors have revised the introduction and have added sub-sections, namely "Uncertainties in catchment precipitation", "Uncertainties in runoff observations" and "Catchment precipitation from runoff observations through inverse modelling". The authors appreciate the reviewer's comment, that the sub-sections make the introduction clearer.

2. L47: "rainfall-runoff"

Comment acknowledged and corrected.

3. L87: "considerably"

Comment acknowledged and corrected.

4. Section 2.1 (and also later in the text): All symbols from equations explained in the text should appear in italic.

Symbols from equations have been changed to italic.

5. L180: Why an exclamation mark at the end of the line?

The exclamation mark was inserted to stress, that only one single Input I_t , which results in an output O_t , exists. From the reviewer comments we had the impression, that this fact was not clear. The authors have however changed the exclamation mark to a full stop, since it is more appropriate.

6. L188: "QOBSt are the simulated and observed runoff respectively."

Updated accordingly.

7. L232-237: There are existing models which conceptually include leakage functions, and do not necessarily fail on leaky catchments.

We do not deny, that, given a quantification of the leakage process, hydrological models can be applied to leaky catchments. We however stress, that an additional uncertainty is introduced, which is difficult to quantify and may results in wrong estimates of the water balance components. Therefore, in the novel application of the inverse model presented in the manuscript, it makes sense to exclude

this possible source of error and therefore to exclude leaky catchments. This is now more clearly explained in the updated section of the manuscript (L234-242).

8. L252-261: I do not agree with this point. The knowledge of areal catchment rainfall is generally better known on large catchments than on small ones. On small catchments, it is often difficult to find raingauges on or close to the catchment,(as it is the case for the catchments studied here) and given the rainfall variability, catchment rainfall is difficult to estimate. On large catchments, there are more raingauges available, and the sampling of the rainfall field is statistically better. Therefore, it is generally more common to get poor model performance on small catchments than on large ones. This has been clearly shown by some authors, e.g. Merz et al. (2009).

The section L252-261 does not claim, that areal rainfall estimates based on observations for larger catchments are poorer, compared to smaller catchments. However, the application of a *lumped model* to a larger catchment may fail, since spatial variability in rainfall cannot be considered in the lumped model setup due to the lumped input used. If it only rains in the headwaters of large catchment, the lumped input into a model for a time step or rainfall event will be much lower, since it will be spatially aggregated. This input is simply not applicable to the whole catchment and the simulations will show deficits. In this case, an inversion will be highly flawed. We agree with the reviewer's perspective and have added a sentence to clarify this issue (L266-268).

9. L315 and L330: "three different"

Acknowledged and corrected.

10. L440: "used in the"

Acknowledged and corrected.

11. L460: "would result"

Updated accordingly.

12. L528: "ranges"

Changed.

13. Fig 2: Indicate units on the y axes

Comment acknowledged and added the information to the caption.

14. L987: "two study"

Comment acknowledged and corrected.

Cited reference

Merz, R., Parajka, J. & Blöschl, G. 2009. Scale effects in conceptual hydrological modeling. Water Resour. Res., 45, W09405.

Report #2

1 General comments

The paper is a re-submission of a paper submitted by the same authors for publication in HESS. The paper describes an innovative approach to inverse rainfall-runoff simulations in order to infer catchment rainfall from runoff observations. We participated in the review of the prior version of the paper and found that the manuscript is greatly improved compared to the first submission. Key clarifications were added by the authors along with interesting additional modelling experiments.

We thank reviewer #2 for participating in the review process and for the valuable comments from the first round, which helped to substantially improve the manuscript. We highly acknowledge that reviewer #2 values the additional work and effort, which was put into the present manuscript.

We believe that the paper can be accepted for publication with minor revisions. However, without requiring another round of review, further clarifications need to be added related to two points:

- The authors must clearly indicate when and why they infer catchment rainfall P_{inv} using a rainfall-runoff model that was calibrated with observed catchment rainfall as an input. This experiment is theoretically interesting but practically useless because if rainfall is the unknown, it cannot be used as an input of the forward model. In that regard, we believe that the experiment exp5 is critical because it shows that the method developed by the authors provides reasonable results in the case where the forward model is calibrated with rainfall data that are independent from the observed catchment rainfall. We were very interested by the comparison of results between exp5 and exp3: the forward model exhibits significantly lower NSE in exp5 compared to exp3, which is expected because the forward model is driven with the lower quality INCA rainfall in exp5. As a result, we expected the model from exp5 to be less representative of the catchment dynamic than exp3. However, this does not prevent the correlation between Pobs and P_{inv} to be higher for exp5 than exp3. This is a counter intuitive result and may call for a few comments.

Our approach presented here is different from the “doing hydrology backward approach” of Kirchner (2009), where precipitation is directly inferred from time series of runoff (except that he needs observations on rain-free periods to estimate his $\ln Q/dt - Q$ relationships). We apply a rainfall runoff model that has to be calibrated with data. We however also show results for independent validation periods and single years, which were not used for calibration. It is clear that the application of the inverse model is not possible, if the catchment is completely ungauged (Given runoff data and applying methods developed within PUB for parameter estimation in ungauged basins an application of the inverse model would be, at least, theoretically possible). This issue is comparable to the application of conventional rainfall-runoff models in gauged and ungauged catchments. As long as a rainfall-runoff model shows reasonable results for the calibration and validation period, the model can be used for different practical applications, e.g. environmental change impact studies, design flood estimations or flood-forecasting (and given the “success” in the calibration and validation period, we even have some belief in the function of that model). This is also conceivable for the inverse model, since additional information on the catchment rainfall is made available. Some potential applications of the inverse model include gaining additional information on catchment rainfall, flood forecasting or the estimation of snow melt contribution. We have added this information to manuscript in order to address the issue raised by the reviewer (L321-323; L 791-801).

The comparison of Exp3 and Exp5 is indeed interesting, since it shows, that the inverse model provides reasonable results in the case where the forward model is calibrated with rainfall data that are independent from the observed rainfall in the proximity of the catchment. This is however also the case for independent validation periods and years, which were not used in the calibration. The

correlation values between PObs and PInv for Exp5 are always lower, compared to Exp3 (see Table 8 and Fig. 10 in the revised manuscript). This is the result which could be expected and it is unclear, why reviewer #2 states, that the correlation between Pobs and Pinv is higher for Exp5 than Exp3. We have added a short paragraph to include the comment of the reviewer (L630-636).

- It is not clear in the text, especially in section 2.2.1, that there exist situations where the inversion method proposed by the authors is theoretically impossible. We are aware of at least one such situation, i.e. when actual evapotranspiration is greater than rainfall within a single time step. In this case, the rainfall evaporates back to the atmosphere completely and we believe there is no way to obtain an inversed estimate from runoff data. If the authors do not agree with this view, they should demonstrate the possibility of inversion by applying their method to a set of water limited catchments with an aridity index (mean ratio of rainfall over potential evapotranspiration) far lower than 1. We insist on the fact that a method being theoretically inapplicable is different from limitations introduced by model structures (e.g. thresholds) or data errors (e.g. uncertainty in streamflow data).

Evapotranspiration in a time step generally depletes the system states of the model, having some effect on the runoff. In the forward model, if given rainfall input of a single time step evaporates, this will affect the system states and runoff: In case potential evapotranspiration is larger than rainfall, the remaining “evapotranspiration potential” for the depletion of system states will be the difference between the potential evapotranspiration and rainfall. In case rainfall is larger than potential evapotranspiration no additional water from the system states will evaporate back to the atmosphere. In any case both situations have an effect on the system states and in consequence runoff. The same mechanisms will be true when applying the inverse model for the same situation, since basically the forward model is run within the root finding algorithm. If the conditions of the catchment leading to runoff are appropriately captured by the model, then the effects of evapotranspiration on the inverse rainfall, e.g. by reducing the rainfall quantity during a time step, will be accounted for, as outlined above.

Despite these theoretical consideration, we agree with the reviewer’s perspective, that the comment addressed provides room for further investigations. Because the method is novel and is not a standard practice in the hydrological community, the authors appreciate the reviewer’s understanding that not all details can be answered or proofed with additional modelling exercises in a single manuscript. The authors have however highlighted the comment by adding the information that the application to water limited catchments is an important task in the near future (L815-817).

We also regret that the authors did not consider an application of their method to a larger sample of catchments, which could have facilitated the response to the reviewers. However, we understand that this is not a standard practice in the hydrological research community yet. As a result, we do not want to penalise this paper for a more general comment on research methods in hydrology. However, we hope that the greater availability of hydrological data will change this situation in the near future.

We acknowledge the reviewers understanding that the available sample of catchment data is limited. Apart from the difficulties finding catchments with appropriate data (also considering restrictions due to geological conditions (e.g. leaky catchments due to Karst), catchment size, temporal resolution of data, station observations in the proximity of the catchment), the focus on only two catchments enables a more detailed assessment of the results, which is necessary, since the method is, as stated by reviewer #2, not a standard practice in the hydrological community. In this context it can be (however not as an excuse) mentioned that in the previously quoted study by Kirchner (2009), the method was initially only tested for two catchments in Wales. Only later was the method extended to other areas by

different authors. We have clearly stated the importance of the application and analysis of the proposed method to a wider range of catchments, including water limited catchments (L813-817).

Overall, we believe that the clarifications we requested can be addressed by adding several sentences without requiring any more modelling work. Additional detailed comments are provided in the following section.

We have added the requested clarifications to the manuscript.

2 Specific comments

1. Page 1 Line 15, 'The only additional information available concerning the precipitation of a catchment is the runoff observation': we suggest replacing this sentence by 'Runoff observations constitute a good proxy to precipitation observations with a considerably lower level of associated uncertainty.'

Updated accordingly.

2. Page 1 Line 18, 'a simulated runoff value that corresponds to the observation': We suggest replacing this statement by 'a simulated runoff value closely matching the observed runoff'.

Updated accordingly.

3. Page 1 Line 19, 'also evaluating different model parameter sets': we suggest removing this statement.

Updated accordingly.

4. Page 3 Line 84, 'at the 95 % confidence level': please remove this statement. There is no way to attach such a precise probability estimate to generic confidence intervals.

We have removed the statement.

5. Page 4 Line 93, 'Two inverse problems can be identified with the forward problem': we suggest replacing this sentence by 'Two inverse problems related to this forward problem can be identified'.

Updated accordingly.

6. Page 4 Line 105, 'integral of rainfall over a certain period, considering evapotranspiration losses and water storage characteristics': we suggest replacing this statement by 'integral of rainfall minus evapotranspiration losses and change in water storage over a certain period of time'.

Updated accordingly.

7. Page 5 Line 125, 'wet catchments are more likely to react as simple dynamical systems': please clarify this statement. What do you mean?

We have removed the statement. Interested readers can refer to the citation given.

8. Page 6 Line 172 'These functions have a time component, which is indicated by the index t.': Please clarify this statement. First we suggest introducing two different names for the state and output functions (e.g. f and g). Second, if the functions had a time component, we would expect Equations (2) and (3) to be :

$$S_t = f(S_{t-1}, I_t, t | Q_t) \quad (1)$$

$$O_t = g(S_{t-1}, I_t, t | Q_t) \quad (2)$$

In other words, the functions f and g would be dependent off the time variable t , which is the case for non-stationary catchments. We suspect that this is not the intent of the authors. We suggest removing this statement.

We introduced two different names for the two functions and have also removed the statement concerning the time component. It is clear, that this was not our intention.

9. Page 6 Line 180: Please add at the beginning of the line the statement: „If the function f is invertible, `'. The invertibility of f is a critical assumption to apply the inversion method described in this paper as indicated in lines 274 to 276 , Page 10, and in line 388, Page 13. It must be stated here unambiguously because the structure of many rainfall-runoff models may not satisfy this requirement.

The statement has been added to the manuscript.

10. Page 8 Line 219, „This is in principally possible": Please change this statement to „In principle, this is possible if the rainfall-runoff equation is invertible". See previous comment.

We have added the information to the manuscript and edited the section.

11. Page 8, Line 244, „For the forward model used here, the differential equations of the linear reservoirs are solved analytically. An internal time step discretization is included in the model code to guarantee, that the transition between system states above and below the threshold value are solved exactly. This is not possible in the analytical solution.": This statement is not clear. Why would a numerical solution provide a better solution than an analytical one? Numerical solutions remain an approximation that always introduce a lack of precision (even if it remains negligible when the numerical scheme is design correctly).

The “analytical solution” we address to in the text refers to the analytically inverted rainfall-runoff model presented in Herrnegger (2013). In that approach no internal time step discretization can be implemented, leading to the violation of the precondition that the rainfall-runoff model is at all times invertible. The reason lies in discontinuities introduced by threshold values. Note, that if the threshold values are set to 0, the inversion is possible. However, in the model presented in the manuscript, an internal time step discretization is included in the model code to guarantee, that the transition between system states above and below the threshold value within a time step are solved exactly. Therefore the model is invertible. We have edited the section to clarify the addressed points (L221-232).

12. Page 15 Line 315, „In a first step 3 different periods are used for calibration of the model parameters": Please indicate that observed catchment rainfall may be used to calibrate the forward model for testing purposes, even if this configuration is not of practical interest (see general comment).

We have added this information to manuscript in order to address the issue raised by the (L321-323; L 791-801).

Report #3

1. The authors have made substantial changes to their paper in response to many of the points raised in the reviews, and overall the manuscript is much clearer than the original version.

However, the authors have simply side-stepped some of the reviewers' most important points, with the result that some fundamental problems identified in the reviews have not been sufficiently addressed (or maybe even recognized).

We appreciate the reviewer's comments and suggestions.

2. Here is perhaps the most significant example of this issue. The major motivation of the paper is that precipitation data are often fraught with both random and systematic errors. The paper proposes that inferring rainfall from runoff could give better whole-catchment precipitation estimates than the instrumental measurements themselves. In principle this is a promising idea, and the same concept has also motivated previous attempts at inverse hydrology. The central problem here, however, is that the proposed model is calibrated to match the instrumental precipitation record, including its errors. Then how can the inverse model give a substantially better (and therefore substantially different) precipitation estimate than the (presumably erroneous) instrumental record that it was already calibrated to?

This is a potentially fatal issue, which Reviewer #3 raises rather directly in his item 2. The author's entire response is only "See above answer to referee #3." The "above answer" simply states that hydrological models are inevitably calibrated, and therefore require rainfall and runoff data. That entirely misses the point.

Consider a possible scenario, in which the rain gauge is sited in an unrepresentative location, such that the precipitation measurements overestimate whole-catchment rainfall by 20%. When the model is calibrated, it will presumably adjust the evapotranspiration parameters so that these erroneously high precipitation rates are made consistent with the measured discharge (by making ET correspondingly high). Then when the model is run in its inverse mode, it will presumably predict precipitation rates that are consistent with the precipitation measurements that it was calibrated to – that is, it will match the measurements, and therefore will match their 20% bias in relation to the true whole-catchment precipitation. Thus it seems that the proposed approach will not meet its stated objective of overcoming the "major errors" in precipitation measurements that are mentioned in the abstract.

The authors cannot just dance around this issue. They either need to prove that their method gives better precipitation estimates than the measurements it is calibrated to, or they need to remove any claims – explicit or implicit – that the proposed method estimates mean areal rainfall better than instrumental measurements do... or even that it estimates mean areal rainfall at all (since in fact it is just matching the instrumental measurements, which are often not representative of catchment-averaged precipitation). Since that is the main rationale for the paper, this is a fundamental challenge that the authors cannot and should not dodge.

Since it is not possible to observe the mean catchment precipitation, it is not possible to state or prove, that the proposed method gives "better" areal precipitation estimates. There is simply no observational evidence available to make this statement.

The manuscript *never* claims, as indicated by reviewer #3, that the proposed method estimates mean areal rainfall better than estimates derived from measurements do. The method can however provide an additional information source on areal rainfall. The runoff simulation in the beginning of June 2008 in the Schlieffau catchment, for example, clearly shows deficits, since the flood peak is underestimated (Fig. 6 in the revised manuscript; lower left). From the inverse model simulations it is evident, that P_{Inv} is higher than P_{Obs} or P_{Inca} in this period (Fig. 8 in the revised manuscript; lower left). It can therefore be concluded, that P_{Obs} and P_{Inca} show deficits in this period and that P_{Inv} gives additional quantitative information on the rainfall during this event. This additional information is not limited to

the simulated hourly data, but also includes the aggregated daily rainfall rates, which show a significant higher correlation to the observed values.

The application of the inverse model is based on the assumption that the forward model can represent the catchment responses to rainfall. The forward model is therefore calibrated against runoff observations, using observed rainfall values. If the input used to calibrate the forward model is highly flawed and the parameters cannot compensate the errors, then the model simulations will also not be able to represent runoff, excluding the application of the inverse model. Given the scenario outlined above by reviewer #3 (assuming a rainfall bias of +20 %) rainfall input would be increased by about 120 to 140 mm (based on mean observed rainfall in the catchments). A compensation of these quantities is unrealistic, since the ETa in the model would have to increase by this range or about 40%. The ETa in this case would be approximately the same magnitude of ETp, what is not plausible considering the hydrology of the catchments. If the input used to calibrate the forward model is only slightly flawed and the parameters compensate the errors, then the rainfall simulations will also show an unknown bias.

In this context, the comparison of Exp3 and Exp5 presented in the manuscript is critical, since it shows, that the inverse model provides reasonable results in the case where the forward model is calibrated with rainfall data that are independent from the observed rainfall in the proximity of the catchment. Additionally, we also show that the inverse model performs reasonable in independent validation periods and single years, which were not used for calibration.

3. Because evapotranspiration is crucial to the precipitation inversions, the manuscript must be absolutely clear about how ET is estimated and implemented in the model. The authors claim that two key parameters, ETVEGCOR and INTMAX, were not calibrated but instead "estimated a priori". But these parameters are shown in Table 2 as having a significant range of possible values (meaning that they are apparently NOT fixed), and the manuscript never explains how they are "estimated a priori". This undermines the technical credibility of the manuscript.

(As far as I can tell the function $f()$ is not specified anywhere either, but maybe I missed it).

The interception storage is represented by the model parameter INTMAX, which is estimated as a function of the land use and month of year to consider changes of interception within the annual cycle. ETVEGCOR, comparable to the widely used crop coefficient (Allen et al., 1998), is also estimated depending on the month of year and land use. Values for INTMAX and ETVEGCOR can be found in Herrnegger et al. (2012). For the application, monthly INTMAX- and ETVEGCOR-values were calculated as area weighted mean values, depending on the land uses in the catchments, since a lumped setup is used. For the implementation of the evapotranspiration calculations in the model the reader is also referred to Kling et al. (2015). This information has been added to the manuscript (L352-361).

4. Reviewer #3 pointed out that the inverse model is guaranteed to do well over long periods of time, simply because it conserves mass and thus (average) precipitation must equal (average) streamflow, plus evapotranspiration. In response, the authors say only that this is not correct, because ET is significant and "ETa from the model reflects the complex interplay and temporal dynamics of the system states of the different parts of the model." But one must remember that whatever these "complex interplay and temporal dynamics of the system states" are, ET in the model has been calibrated so that inputs and outputs are forced to match. Thus the cumulative rainfall curves shown in Figures 9 and 10 represent exceptionally weak tests of the model (see more on this in point 6 below).

Cumulative curves are a frequently used method, not only in the hydrological community, to show or analyse model results. Accordingly, Fig. 8 and 9 show the cumulative sums of the inverse rainfall of the different experiments. The experiments differ in the calibration periods. Therefore the cumulative curves also include independent validation periods and years, in which the simulations are not “forced” to match the observation. In Exp5 the independent INCA rainfall was used for calibration. Following the argument of reviewer #3, the sums of inverse rainfall from Exp5 should follow the cumulative rainfall curves of PInca very closely. This is not the case. The modelled cumulative sums differ from the observations after some flood events, suggesting, that the observed rainfall shows deficits during the extreme events.

5. Two reviewers pointed out that the inverse model will be very sensitive to errors in the streamflow data, potentially magnifying them by orders of magnitude in the precipitation estimates. In response, the authors have introduced the "Exp4" simulation, in which (apparently) all the measured discharges are increased by 10 percent. This, however, does not address the issue that was posed. The problem is that the inferred rate of precipitation will strongly depend on the time derivative of discharge, and thus will be particularly sensitive to short-term errors (such as random noise) in discharge measurements. Re-scaling all the discharges by a constant does not provide a meaningful test of this issue.

The authors have added additional investigations to the manuscript to address the issue raised by reviewer #3. Thereby, virtual experiments were performed, in which random noise drawn from a zero-mean normal distribution and rescaled to represent a range of measurement errors was added to a runoff simulation of the forward model. These time series are then used as input into the inverse model to test the sensitivity of the inferred precipitation rates to short-term errors in the discharge measurements (see L298-308 in section 2.3.1 and L423-441 in section 4.1).

6. The reviewers pointed out that the tests of the method were very weak. In response, the revised manuscript adds a second catchment, and several new "experiments". Skeptical readers will notice that the second catchment is similar to the first, and exhibits very similar behavior. This is not the kind of comparison that the reviewers were asking for. The reviewers were specifically asking for evidence that the model can correctly simulate behavior that is clearly different from the calibration data (for example, different seasons of the year). Instead we have just two very similar catchments, simulated for multiple summers, but each with about 600-800 mm of precipitation.

We agree, that at least for the periods shown in the manuscript, the precipitation sums in both catchments are quite similar. The catchments however differ concerning size, topography and geology. Additionally, the runoff coefficients are different in the catchments. Additionally we kindly highlight, that the other reviewers were satisfied with the additional catchment added after the first revision.

Apart from the difficulties finding catchments with appropriate data (also considering restrictions due to geological conditions (e.g. leaky catchments due to Karst), catchment size, temporal resolution of data, station observations in the proximity of the catchment), the focus on only two catchments enables a more detailed assessment of the results, which is necessary, since the method is, as stated by reviewer #2, not a standard practice in the hydrological community (see also the extensive study by Kirchner (2009), who initially tested his method in only 2 catchments with typical Welsh climate) We therefore do not see any possibility to add additional catchments to the analysis, but expect that the greater availability of hydrological data will change this situation in the near future. The authors have clearly stated the importance of the application and analysis of the proposed method to a wider range of catchments, including water limited catchments (L813-817).

Furthermore, tables 5 and 6 now reveal that the Nash-Sutcliffe and bias statistics are calculated for the entire period 2006-2009, which includes both the "calibration" and "validation" years! This violates the fundamental distinction between validation and calibration which underlies all model testing. Of the 24 cells in table 5 (6 experiments times 4 years), 15 are calibrations. Thus nearly two-thirds of the data used to "validate" the approach actually consist of calibration data. And for four of the six "experiments", that fraction rises to three-fourths.

This is not the way that model testing normally goes; you cannot (or at least you should not) test a model against data that it has already been calibrated with. The approach should be much more rigorously tested, for example by calibrating to only one year at a time, and validating against all three of the other years (and, of course, excluding all calibration data from the validation statistics!). Skeptical readers will wonder why more rigorous testing has not been done.

We thank reviewer #3 for this remark and have updated our statistics accordingly. Table 5 and 6 included the entire period 2006-2009 as an overview of the model performances of the single experiments. However, we want to point out, that the model performances for the single years were and are included (Fig. 5 and 10 in the revised manuscript), since these figures potentially contain more information compared to a distinction in calibration and validation periods in a table. Since the length and periods of the calibration and validation periods of the experiments differ, a comparison is also not straightforward. We have however updated the original Table 5, 6 and 7 to include the performance metrics as suggested by reviewer #3 (see Table 7, 8, 9 in the revised manuscript and the explanations in sections 4.2 and 4.3.2.)

The simulation experiments were added on the basis of the comments of the reviewers in the first round, to evaluate the influences of different calibration periods and lengths, also analysing independent years and validation periods. We did not do this exactly, as reviewer #3 had in mind (and also did not suggest in the review). The authors have however used a setup, which is very frequently used in the hydrological community and are therefore confident, that the analysis of the influence of calibration periods on the results are valid.

(A small technical note: exp3 and exp4 in Table 6 show that the model generates the same NSE when the correct discharge is used and when discharge that is wrong by 10% is used. This suggests that perhaps r^2 has been calculated rather than NSE.)

NSE-values have been calculated and are, as stated by reviewer #3, of similar magnitude. The mean bias is however larger, showing the influence of the "wrong" discharge data.

7. The reviewers pointed out that the "virtual experiment" in 2.3.1 presents an exceptionally weak test, in which the model is run as a forward simulation to generate runoff, and then this same simulated runoff forms the basis of an inverse simulation (with the same model, and exactly the same parameter values) to reproduce the original rainfall input. Reviewer #3 pointed out that this does not even demonstrate numerical stability, in any sense that really matters. But the revised manuscript not only retains this "virtual experiment", it even adds a figure showing the mathematically inevitable 1:1 relationship between the original precipitation input and the one obtained through this forward-backward procedure.

The analysis that Reviewer #3 suggested, which involved perturbing the streamflow time series, the model parameters, or the model structure, has not been carried out, and no satisfactory explanation has been given. Instead the manuscript says that the virtual experiments "enable a rigorous evaluation of the inverse calculations, neglecting uncertainties concerning measurement errors in runoff, model structure or model parameters". These are precisely the uncertainties and errors that the reviewers say should not be neglected.

The proposed method will only be applicable, if the rainfall-runoff model is invertible. This is an essential prerequisite which is also highlighted in the comments of reviewer #2. Thresholds in the model structure, numerical errors in the calculations or possibly unknown issues may lead to the violation of this. The authors therefore performed the virtual experiments to test that the invertibility is guaranteed. To test the invertibility for the whole parameter range, the mentioned Monte Carlo simulations were performed.

Since the invertibility is not guaranteed in the first place (Reviewer #3 in the first round stated: *“This result is somewhat surprising, because mathematically speaking one would expect the inversion of a multi-compartment model to be ill-posed (because different rainfall inputs at different times, and different combinations of storage levels in the different compartments, should lead to the same discharge), and possibly also mathematically unstable. In that respect the results claimed here are intriguing.”*), showing that the inverse model performs as desired is mathematically not necessarily inevitable. We have nevertheless removed the addressed figure, since the information content does not justify a separate figure and we also refer to results from a different publication.

We have added virtual experiments to the manuscript, in which streamflow time series are perturbed, as suggested by reviewer #3 (see point 5. above and L298-308 in section 2.3.1 and L423-441 in section 4.1). The development and testing of different model structures in the context of the presented method is time and work intensive and not a straightforward procedure. The same is true for the uncertainty analysis of the parameters. Such additional analyses will certainly provide interesting information, but we feel that after having already included additional analysis on the effect of runoff data perturbations, such an extension would be beyond the scope of this manuscript. It is however clearly stated in section 5 (L818-823) that the influences and uncertainties from different model structures and parameters must be analysed systematically in a next step. We have removed “enable a rigorous evaluation of the inverse calculations, neglecting uncertainties concerning measurement errors in runoff, model structure or model parameters” from the manuscript.

8. Section 4.3.4. says that up to 9 months is needed for the effects of the startup “cold system state” to be forgotten. But the simulations presented here are for only three or four months! How is this supposed to work, in practice?

The cold states in the dry scenario were reduced by the factor 0.5 and increased by the factor 1.5 for the wet scenario, based on a reference scenario. These are extreme assumptions, especially when considering the long-term memory of the ground water storage, which also explains the long warm-up period in the presented results. The intention of Exp6 was to evaluate the general influences of the cold states and spin-up time on the inferred rainfall. In practice it will work, when reasonable cold states are defined at start-up. This is however not a specific issue of the inverse model and the method presented. All models formulated in a state space approach need an appropriate estimation of system states. We have clarified this issue in the manuscript (L724-730).

9. One would have thought that in response to the reviewers' comments, the revised manuscript would be more careful about the claims that it makes for the inverse modeling method. Instead, the opposite has happened; the claims have become even bolder (but without more substantial evidence to support them). For example, section 5 now says that the model can be used to “update system states” in real-time flood forecasting. No clear evidence is presented to show that this works as intended, or that it improves flood forecasts; instead the reader is simply told that the system states in the inverse model will always guarantee that the simulated runoff is identical to the observations. This may be true, but it

does not demonstrate that those system states are the right ones, particularly because the system states are generated by the entire time series of (presumably flawed) precipitation and discharge measurements. So even if the simulated runoff is identical to the observations, this result could arise not because the system states are correct, but instead because the model can adjust the assumed rainfall rate to compensate for the system states being wrong.

The manuscript says, that applications of the inverse model are in the context of flood warning systems are *conceivable*. This does not imply the provision of clear evidence, especially when it is meant as an outlook or potential application.

At least during driven periods, which are relevant for flood warning systems, the system states from the inverse model will guarantee that the simulated runoff is close or identical to the observations (Fig. 11 in the revised manuscript). This fact may be used as a basis for updating system states in flood forecasting models. If the simulated runoff is identical to the observed runoff at $t=0h$, then the forward runoff simulation for the forecasting period (using precipitation forecasts) will be of better quality, compared to forecasts using biased system states. We never claim, that the modelled system states are the “right” ones, since it is not possible to observe and prove the “right” ones. We only claim, that the system states from the inverse model lead to a very good agreement between simulated and observed runoff. And this is most relevant in flood forecasting.

The reader is also told that the method could be used to generate "nowcasting fields" of rainfall for short-term flood forecasting. Never mind the rather clear circularity involved (one needs measurements of discharge in order to estimate rainfall, in order to predict discharge, which has already been measured anyway). In any case the reader is not shown any evidence that this works, in any way that would be useful for forecasting. Thus what has been presented is simply speculation, but appears in the manuscript's "summary and conclusions".

Indeed, Figure 12 shows that estimated rainfall rates during extreme events can be wrong by a factor of two or more. This result would seem to argue rather clearly against the claims that are advanced starting on line 712.

The intention is not “to predict discharge, which has already been measured anyway”, but to use the additional information on rainfall quantities from the inverse model to enhance rainfall fields of $t=0h$. An extrapolation of the improved rainfall fields could improve the nowcasting fields. It is conceivable, that adding additional information into a forecasting system can improve the forecasts. Data assimilation methods in numerical weather models are implemented for that reason.

We have changed the name of section 5 into “Summary and outlook”, because this seems more appropriate and have removed the section addressed above by the reviewer.

10. The manuscript completely side-steps the issue of parameter uncertainty and equifinality, saying that the issue is important but is outside the scope of the paper. If the issue is important, why not make space for it? One can understand that a full exploration of parameter uncertainty in a 12-parameter model would be a substantial undertaking, but there is no good reason for avoiding the topic entirely, and not even presenting some illustrative results.

We have incorporated most suggestions and comments of the reviewers to the revised manuscript, adding several new simulations experiments and a catchment to the analysis. The manuscript was substantially revised, which was valued by the other reviewers. Parameter uncertainty analysis is time and work intensive and we intend to combine such an uncertainty analysis with the extension to different locations as a follow up paper.

It is however clearly stated (L818-824) that the influences and uncertainties from model parameters must be analysed systematically in a next step.

11. In summary: the general idea presented here is interesting, but it should be rigorously tested. The results of those tests should be openly and fairly presented, and only claims that can be rigorously supported should be made. If the paper is published, the source code, data sets, and all numerical results should be made available as online supplementary information, so that other researchers can verify the findings.

We thank reviewer #3 for participating in the review process and for his critical, but very helpful and valuable thoughts, comments and suggestions, which helped to substantially improve the manuscript.

Data sets and numerical results will be made available, if the paper is published. It must however be mentioned, that the data providing institutions must give their approval (what should not be a problem). The source code cannot be made available. The inverse model is embedded and uses code from the also commercially used COSERO-model, which is not open source.

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From runoff to rainfall: inverse rainfall-runoff modelling in a high temporal resolution

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Abstract

~~This paper presents a novel technique to calculate mean areal rainfall in a high temporal resolution of 60 min on the basis of an inverse conceptual rainfall-runoff model and runoff observations.~~

Rainfall exhibits a large spatio-temporal variability, especially in complex alpine terrain. Additionally, the density of the monitoring network in mountainous regions is low and measurements are subjected to major errors, which lead to significant uncertainties in areal rainfall estimates. In contrast, the most reliable hydrological information available refers to runoff, which in the presented work is used as input for an inverted HBV-type rainfall-runoff model that. Thereby a conceptual, HBV-type model is embedded in a root finding algorithm. For every time step a rainfall value is determined, which results in a simulated runoff value closely matching the observed runoff. ~~simulated runoff value that corresponds to the observation. The inverse model, also evaluating different model parameter sets, is applied and tested to the Schliefaue and Krems catchments, situated in the northern Austrian Alpine foothills. The correlations between inferred rainfall and Generally, no substantial differences between the catchments are found. Compared to station observations in the proximity of the catchments are of similar magnitude compared to the correlations between station observations and independent INCA rainfall analysis provided by the Austrian Central Institute for Meteorology and Geodynamics (ZAMG), the inverse rainfall sums and time series have a similar goodness of fit, as the independent INCA rainfall analysis of Austrian Central Institute for Meteorology and Geodynamics (ZAMG). The cumulative precipitation sums also show similar dynamics. The application of the inverse model is a promising approach to obtain improved additional information estimates of mean areal rainfall. This additional information is not solely limited to the simulated hourly data, but also includes the aggregated daily rainfall rates, which show a~~

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significant higher correlation to the observed values. ~~Some~~ Potential applications of the inverse model include gaining additional information on catchment rainfall for interpolation purposes, flood forecasting or the estimation of snow melt contribution. ~~These can be used to enhance interpolated rainfall fields, e.g. for the estimation of rainfall correction factors, the parameterisation of elevation dependency or the application in real time flood forecasting systems.~~ The application is limited to (smaller) catchments, which can be represented with a lumped model setup and to the estimation of liquid rainfall.

1 Introduction

The motivation for the concept presented in this paper comes from practical hydrological problems. Some years back we set up rainfall-runoff models for different alpine rivers (e.g. Stanzel et al., 2008; Nachtnebel et al., 2009a, 2009b, 2010a or 2010b). In the course of these projects, we were confronted with massive errors in the precipitation input fields. This is a known problem, especially in alpine environments. Although the temporal dynamics in the runoff simulations were captured quite well, significant mass balance errors between observed and simulated runoff were found. It could be excluded, that erroneous evapotranspiration calculations were biasing the results (Herrnegger et al., 2012). In the HYDROCAST project (Bica et al., 2011) we tested different precipitation interpolation and parameterisation schemes by using the ensemble of generated inputs for driving a rainfall-runoff model and comparing the simulated runoff time series with observations. In essence, the results showed, that no significant improvements could be made in the runoff simulations and that the information on the precipitation fields is strongly determined and limited by the available station time series. Runoff observations as an additional information source constitute a good proxy to precipitation observations with a considerably lower level of associated uncertainty. ~~The only additional information available concerning the precipitation of a catchment is the runoff observation.~~ The main aim is therefore to present a proof-of-concept for the inversion of a conceptual rainfall-runoff model. That is to show, that it is possible to use a widely applied model concept to calculate mean areal rainfall from runoff observations.

Uncertainties in catchment precipitation

Areal or catchment ~~rainfall-precipitation~~ estimates are fundamental, as they represent an essential input for modelling hydrological systems. They are however subject to manifold uncertainties, since it is not possible to observe the mean catchment rainfall itself (Sugawara, 1992; Valéry et al., 2009). Catchment rainfall values are therefore generally estimated by interpolation of point measurements, sometimes incorporating information on the spatial rainfall structure from remote sensing, e.g. radar (e.g. Haiden et al., 2011). Measurement, sample and model errors can be identified as sources of uncertainty. Point observations of rainfall, which are the basis for the calculation of mean areal rainfall values, are error inflicted (Sevruk, 1981, 1986; Goodison et al, 1998; Sevruk and Nespor, 1998; Seibert and Moren, 1999; Wood et al., 2000; Fekete et al., 2004). Occult precipitation forms like fog or dew are frequently ignored. Although not generally relevant, this precipitation form can be a

significant contribution to the water budget of a catchment (Elias et al., 1993; Jacobs et al., 2006; Klemm and Wrzesinsky, 2007). The highest systematic measurement errors of over 50 % are found during snowfall in strong wind conditions. Other sources of systematic measurement errors and their magnitudes are listed in Table 1.

➔ Approximate location of Tab. 1

In complex terrain the rainfall process is characterised by a high temporal and spatial variability. Especially in these areas the density of the measurement network tends to be low, not capturing the high variability and leading to sample errors (Wood et al., 2000; Simoni et al., 2011; de Jong et al., 2002). Further uncertainties arise in the interpolation of catchment scale rainfall from point observations. Systematic and stochastic model errors of the regionalisation methods can be identified. Systematic model errors can arise during the regionalisation of rainfall in alpine areas, when e.g. the elevation dependency is not considered (Haiden and Pistotnik, 2009). Quantitative areal rainfall estimates from radar products are, although they contain precious information on the rainfall structure, still afflicted with significant uncertainties (Krajewski et al., 2010; Krajewski and Smith, 2002). A general magnitude of overall uncertainty, which arises during the generation of areal rainfall fields, is difficult to assess, as different factors, e.g. topography, network density or regionalisation method, play a role.

Uncertainties in runoff observations

Errors in runoff measurements are far from negligible (Di Baldassarre and Montanari, 2009; McMillan et al., 2010; Pappenberger et al., 2006; Pelletier, 1987). When applying the rating-curve method for estimation of river discharge the uncertainties are a function of the quality of the rating curve and the water level measurements. The quality of the rating curve depends on (i) the quality and stability of the measured cross-section over time, (ii) the representativeness of the velocity measurements and (iii) the influence of steady and unsteady flow conditions. According to literature the overall uncertainty, ~~at the 95 % confidence level,~~ can vary in the range of 5% - 20% (Di Baldassarre and Montanari, 2009; Pelletier, 1987). Although it can be expected, that the measurement error will certainly be large during flood events due to its dynamic features, the errors are considerably lower compared to rainfall measurements and to the uncertainties introduced, when calculating mean areal rainfall. It must however be assumed, that transboundary flows and groundwater flows around the gauging station are negligible.

Catchment precipitation from runoff observations through inverse modelling

A classical application of hydrology, the problem of reproducing observed runoff with meteorological forcings as input through a formalised representation of reality, is a forward or direct problem. Two inverse problems related to this forward problem can be identified~~Two inverse problems can be identified with the forward problem~~ (Groetsch, 1993):

1. Causation problem: Determination of input I (=cause), with given output O (=effect) and given model K, including model parameters θ (=process)

2. Model identification problem: Determination of model K, given input I and output O

The model identification problem can be divided into (i) the problem of identifying the model structure itself and (ii) the determination of model parameters that characterise the system (Tarantola, 2005). The focus in this contribution lies in solving the causation problem, i.e. in the determination of rainfall input from runoff, with a given model structure and parameters. In the following, the model, which calculates mean catchment rainfall values from runoff, will be called *inverse model*. The conventional model, which uses rainfall and potential evapotranspiration as input to calculate runoff, will be called *forward model*.

Runoff from a closed catchment is the integral of rainfall minus evapotranspiration losses and change in water storage over a certain period of time~~integral of rainfall over a certain period, considering evapotranspiration losses and water storage characteristics within the catchment.~~

Therefore, runoff observations can be used to derive information on rainfall. This has been done in several studies, e.g. Bica et al., 2011; Valéry et al., 2009, 2010; Ahrens et al., 2003; Jasper and Kaufmann, 2003; Kunstmann and Stadler, 2005 or Jasper et al., 2002. The common basis of these studies was to indirectly gain information on catchment rainfall by comparing simulated runoff results with observations. Hino and Hasabe (1981) fitted an AR (autoregressive) model to daily runoff data, while assuming rainfall to be white noise. By inverting the AR model they directly generated time series of rainfall from runoff. Vrugt et al. (2008) and Kuczera et al. (2006) derived rainfall multipliers or correction factors from stream flow with the DREAM- and BATEA-methods, these methods however being computationally intensive. In a well-received study, Kirchner (2009) analytically inverted a single-equation rainfall-runoff model to directly infer time series of catchment rainfall values from runoff. The Kirchner model (when deriving the storage-discharge relationship directly from runoff

data) only has a single parameter and does not [explicitly](#) need rainfall as driving input for calibration. Rainfall data is however needed for the determination of rainless periods for the estimation of the sensitivity function. Krier et al. (2012) applied the model of Kirchner (2009) to 24 small and mesoscale catchments in Luxembourg to generate areal rainfall. No systematic differences in the quality of the rainfall estimates are found between different catchment sizes. In periods with higher soil moisture the rainfall simulations [however](#) show a higher performance, ~~which is explained by the fact, that wet catchments are more likely to react as simple dynamical systems.~~ The parsimonious approach of Kirchner (2009) is ~~however~~ limited to catchments, where discharge is determined by the volume of water in a single storage and which can be characterized as simple first-order nonlinear dynamical systems. Also due to the larger number of model parameters describing several linked storages, accounting for a variety of different runoff components, HBV-type conceptual models offer higher degrees of freedom and flexibility in the calibration procedure. They can, in consequence, be applied to catchments with a wider range of runoff characteristics (Bergström, 1995; Kling et al., 2015; Kling, 2006; Perrin et al., 2001). Therefore, in this study, the conceptual rainfall-runoff model COSERO (Nachtnebel et al., 1993; Eder et al., 2005; Kling and Nachtnebel, 2009, Herrnegger et al., 2012; Kling et al., 2015, among others), which in its structure is similar to the HBV-model, is used as a basis for the inverse model. The COSERO model has been frequently applied in research studies, but also engineering projects (see Kling et al., 2015 for details).

This paper is organized as follows: Following this introduction the methods-section describes the conventional conceptual rainfall-runoff model (forward model) and the inverse model, including the preconditions and limitations of its application. The concept of virtual experiments to test the [invertibility of the](#) inverse model and to analyse ~~the-existence, uniqueness and stability~~ [the effects of errors in the discharge measurements on](#) the inverse rainfall simulations are presented. Additionally, the setup of different simulation experiments, e.g. to evaluate the influence of differing calibration periods or possible runoff measurement errors on the simulations, are explained. The inverse model is applied to two headwater catchments in the foothills of the northern Austrian Alps, with differing hydro-climatic and physical conditions. The catchments and the data base, including the calibration periods for the simulation experiments, is presented. The runoff simulations of the forward model and the rainfall simulations of the inverse model are described in detail in the results and discussion section. Finally the paper ends with a summary and [conclusionsoutlook](#).

168 **2 Methods**

169 **2.1 Forward model (Rainfall-runoff model)**

170 In the state space formulated forward model, the unknown runoff Q_t is a function f of known
171 variables rainfall input R_t , potential evapotranspiration ETp_t , system states S_{t-1} and a set of
172 model parameters θ_i , whereas the index t denotes time:

173
$$Q_t = f(R_t, ETp_t, S_{t-1} | \theta_i) \quad (1)$$

174 The rainfall-runoff model is based on the COSERO model (see introduction for references),
175 but has a simpler model structure. It includes an interception and soil module and three
176 reservoirs for interflow, base flow and routing. The model structure is shown in Fig. 1, model
177 parameters are summarized in Table 2 and fluxes and system states in Table 3.

178 ➔ Approximate location of Fig. 1

179 ➔ Approximate location of Tab. 2

180 ➔ Approximate location of Tab. 3

181 The COSERO-model is formulated in a state space approach, with state transition functions

182
$$S_t = f(S_{t-1}, I_t | \theta_i) \quad (2)$$

183 and output functions

184
$$O_t = g(S_{t-1}, I_t | \theta_i) \quad (3)$$

185 with

186 I_t Input, e.g. rainfall

187 O_t Output, e.g. total runoff

188 S_t System states, e.g. water stored in soil module

189 θ_i Model parameters.

190 ~~These functions have a time component, which is indicated by the index “t”.~~ So, the model
191 state and the output at time t depend only and exclusively on the previous state S_{t-1} , the inputs
192 I_t and parameters θ_i . The simplified model formulation can be found in the appendix.

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2.2 Inverse model (Runoff-rainfall model)

In the inverse model the unknown rainfall R_t is a function of runoff Q_t , potential evapotranspiration ETp_t , system states S_{t-1} and a given set of model parameters θ_t , where again the index t denotes time:

$$R_t = f^{-1}(Q_t, ETp_t, S_{t-1} | \theta_t) \quad (4)$$

If the above eq. (4) is invertible and given ETp_t , S_{t-1} and θ_t , there is only one single input I_t , which results in an output Q_t (eq. (3)). To calculate the inverse rainfall rate the forward model is therefore embedded in a search algorithm, to find, for every time step t , the rainfall rate R_t that best fits the observed runoff:

$$f(R_t) = QSIM_t(R_t, ETp_t, S_{t-1} | \theta_t) - QOBS_t \leq \varepsilon \quad (5)$$

with

$$R_{t,min} \leq R_t < R_{t,max} \quad (6)$$

The upper and lower brackets of rainfall ($R_{t,min}$ and $R_{t,max}$) is set to 0 and 50 mm/h. The value of the upper bound is an arbitrary value, but any reasonable bounds can be applied. $QSIM_t$ and $QOBS_t$ is are the simulated and observed runoff respectively. ε denotes a small value, which is ideally zero.

Solving eq. (5), which reflects the objective function used in the search algorithm, is basically a root finding problem. Different root finding algorithms were tested, with the Van Wijngaarden–Dekker–Brent Method (Brent, 1973; Press et al., 1992) being the method of choice, as this method exhibited the fastest results. The Brent's method combines root bracketing, bisection and inverse quadratic interpolation to converge from the neighbourhood of a zero crossing and will always converge, as long as the function can be evaluated within the initial defined interval (in our case $R_{t,min}$ and $R_{t,max}$) known to contain a root (Press et al., 1992). The iteration progress for one model time step is illustrated in Fig. 2. The left y-axis shows the objective function values, the right y-axis (in logarithmic scale) the associated rainfall values estimated during the iteration procedure.

➔ Approximate location of Fig. 2

The state space approach of the model is a first order Markov process: The system states S_t and outputs Q_t of the calculation time step depend only on the preceding states S_{t-1} and some

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222 inputs I_t and not on the sequences of system states, that preceded it, e.g. $S_{t-2}, S_{t-3}, \dots, S_{t-n}$ (see
223 eq. (2) and eq. (3)). All information of the sequence of the preceding inputs $(I_{t-1}, I_{t-2}, \dots, I_{t-n})$ is
224 implicitly included in the last relevant system state S_{t-1} . No hysteretic effects are considered in
225 the model and it does not include a parameter, which introduces a lag effect between inputs
226 and outputs.

227 Given the model structure, parameters and potential evapotranspiration as input, the inverse
228 rainfall and resulting runoff are solely a function of the initial cold system states. The
229 influence of the initial cold system states on the inverse rainfall calculation are analysed in the
230 results section.

231 The determined rainfall value R_t represents the “best” simulated rainfall of the catchment and
232 is also used as input into the forward model to simulate runoff. Therefore, for every time step
233 the inverse model simulates a rainfall and corresponding runoff value and also resulting
234 system states. The simulated runoff value should ideally be identical to the observed value.
235 This is however not always the case, as will be shown later.

236 A more elegant method to calculate rainfall from runoff is by analytically inverting the
237 equations of a given model, i.e. bringing the rainfall term onto the right side of the equation.
238 In (Herrnegger, (2013)- ~~this method was presented, but is principally possible, but has~~
239 ~~showed~~ some disadvantages. The model structure, which was used in Herrnegger (2013) and
240 which can be inverted analytically, differs from the model presented here. It does not include
241 interception and routing. Additionally the inversion is not possible in certain periods, since
242 the discontinuities introduced by threshold values lead to non-~~inversibility-invertibility~~ in the
243 analytical solution. ~~(Herrnegger, 2013)- The precondition that the rainfall-runoff model is~~
244 ~~invertible is violated in certain periods.~~ For the forward model used here, ~~the differential~~
245 ~~equations of the linear reservoirs are solved analytically. An~~ internal time step discretization
246 is included in the model code to guarantee, that the transition between system states above
247 and below the threshold value within a time step are solved exactly. This is not possible in the
248 analytical solution presented in Herrnegger (2013), since no internal time step discretization
249 can be implemented.

250 2.2.1 Preconditions and limitations of the application of the inverse model

251 It ~~must be~~ assumed that runoff from the catchment passes through the measurement cross-
252 section of the gauging station and that subsurface and transboundary flows are negligible. It

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~~does not make sense~~is difficult to apply the inverse model to leaky catchments or catchments, where a significant part of the runoff is not observed at the gauging site. Even with a given quantification of the leakage process, the application of ~~a the inverse hydrological~~ model would lead to an additional uncertainty difficult to quantify. ~~Since a novel approach is presented, it is also reasonable to exclude this possible source of error at this point.~~ This is however not necessarily a limitation of the inverse model. Also the application of a forward hydrological model, which needs to be calibrated against runoff observations, will fail or will result in wrong estimates of water balance components.

The inverse model is based on a lumped model setup and the resulting inverse rainfall value corresponds to the mean areal rainfall. Applying a spatially distributed model is not possible, since the origin of outputs of different zones or cells of a distributed model setup cannot be reproduced by the inverse model in a deterministic way without additional assumptions. The information of origin gets lost as soon as cell values are summed and routed to a catchment runoff value. It is however conceivable to spatially disaggregate the mean areal rainfall from the inverse model using additional information, e.g. assuming an elevation dependency of rainfall.

Solid precipitation is accumulated without any direct signal on the hydrograph. It is therefore impossible to use the inverse model to estimate solid precipitation. The inverse model can ~~therefore~~ only be used to calculate rainfall in snow-free catchments, or, as in our case, periods, in which runoff is not influenced by snow melt (i.e. summer months). However, in rainless periods, where it is clear, that snow melt is dominating runoff (e.g. in spring), the inverse model can be used to quantify snow melt rates from a catchment.

The applicability of the inverse model is limited to catchments, which are representable with a lumped model setup and the proposed model structure. If a catchment is too large, ~~one it~~ will ~~be~~ generally ~~have problems~~ difficult modelling to simulate that system with a lumped model setup. ~~Not~~ necessarily because of neglecting spatial heterogeneity in the model parameters (although this may also be an issue) or ignoring a lag between the rainfall and runoff signal, but simply because the lumped rainfall input used is “wrong” and is not representable for the whole catchment. If it only rains in the headwaters of large catchment, the lumped input into the forward model for this time step or rainfall event will be much lower, since it will be spatially aggregated. This input is ~~simply~~ not applicable to the whole catchment and the simulations will show deficits. In this case, an inversion will be highly flawed. [This](#)

[consideration is independent of the fact that the sampling of rainfall field in larger catchments tends to be statistically better, compared to smaller catchments, where observations are rarer.](#)

It is also clear, that catchments, independent of size, exist, where the application of this particular model structure will fail (e.g. flatland catchments dominated by groundwater). If hydro-meteorological conditions of the catchment change or are different from the calibration period and the forward model (e.g. due to poor parameter estimation, inadequate model structure, wrong representation of the real world prototype etc.) is not able to capture these changes, then again the calculation of rainfall from runoff will fail (as they do for the forward case).

However, being able to fit the forward model to observed runoff data and as long as the forward model is able to represent the catchment responses to rainfall, an inversion will be possible.

2.3 Simulation setups

2.3.1 Virtual experiments

In a first step the inverse model is evaluated and tested with virtual experiments, [in order to guarantee, that the model equations are invertible, in which the preconditions of existence, uniqueness and stability of the inverse rainfall values are evaluated.](#) Runoff simulations are performed with the forward model driven by observed rainfall as input. The simulated runoff time series of the forward models are then used as input into the inverse model, with the aim to reproduce the observed rainfall. Simulated runoff from the forward model is dependent on the model parameters. Therefore, to test the inversion procedure for the whole parameter range, synthetic hydrographs are produced with Monte Carlo simulations. 20 000 different parameter combinations are chosen randomly from the parameter space, with the same number of model runs to evaluate the inverse model. The sampled parameters and associated range are shown in Table 2. The schematic setup of the virtual experiment and the evaluation of the inverse model is shown in Fig. 3. Note, that the setup and the evaluation is performed for every individual Monte Carlo run, as the simulated runoff from the forward model varies, depending on selected model parameters.

➔ Approximate location of Fig. 3

~~The virtual experiments enable a rigorous evaluation of the inverse calculations, neglecting uncertainties concerning measurement errors in runoff, model structure or model parameters.~~

All system states and fluxes of the forward model are perfectly known at every time step. This information is used to evaluate the inverse models. Only after a successful evaluation of the inverse model with the virtual experiments, can observations of runoff be used as input into the inverse models.

~~Additionally, virtual experiments are performed, in which random noise drawn from a zero-mean normal distribution and rescaled to represent a range of measurement errors is added to a runoff simulation of the forward model. These time series are then used as input into the inverse model to test the sensitivity of the inferred precipitation rates to short-term errors in the discharge measurements:~~

$$Q_{FN_{i,t}} = Q_{F_t} + Q_{F_t} * N(\mu, \sigma^2) * \alpha_i \quad (7)$$

~~with~~

~~$Q_{FN_{i,t}}$ Noisy input into inverse model~~

~~Q_{F_t} Forward simulated runoff based on observed precipitation~~

~~$N(\mu, \sigma^2)$ Normal distribution with mean $\mu=0$ and standard deviation $\sigma^2=1$~~

~~α_i Noise scaling factor: 0%, 1%, 2%, 5% and 10%~~

2.3.2 -Model calibration and simulations experiments with observed data

The application of the inverse model is based on the assumption that the forward model can represent the catchment responses to rainfall. ~~The forward model is therefore but needs to be calibrated against runoff observations, using observed rainfall values. Depending on the calibration setup, different model parameters will be estimated.~~ The calibration setup and in consequence model parameters (for a given model structure) can depend on (i) the calibration period and length and (ii) the driving input used. The inverse rainfall is also a function of the observed runoff, which may also exhibit possible measurement errors. Finally, the initial conditions of the system states at the beginning of the simulations also influence the results of the forward, but also inverse model. To evaluate these influences, i.e. different model parameters due to different calibration periods and lengths, different runoff observations, different parameter optimisation data basis and different initial conditions, several simulation

experiments are performed. An overview table of the simulation experiments can be found in section 3.3 (Table 5) after the presentation of the available data. Apart from the calibration period all simulation experiments include independent validation periods, which allow to test the inverse model in periods, in which no observed rainfall was used.

In a first step three different periods are used for calibration of the model parameters. In a further simulation experiment, the runoff observation is increased by a constant offset of 10% to evaluate the influence of possible systematic streamflow errors on the simulations and the inverse rainfall. A fifth experiment is performed, in which an independent rainfall realisation is used as driving input for model calibration, in order to test the conditioning of the model parameters and in consequence the simulations to the driving input. Given the model structure, the inverse rainfall is a function of observed runoff, potential evapotranspiration, system states and model parameters (eq. (4)). Extending eq. (4) explicitly with all relevant system states leads to

$$R_t = f^{-1}(Q_t, ETp_t, BWI_{t-1}, BW0_{t-1}, BW2_{t-1}, BW3_{t-1}, BW4_{t-1} | \theta_i) \quad (8)$$

The forward and inverse models are run as a continuous simulation in time. The preceding system states are therefore an integral part of the simulation and are determined intrinsically within the simulation. However, the initial system states at the beginning of the simulation period (cold states) will influence the results of the simulation, but should, after an adequate spin-up time, not influence the runoff but also inverse rainfall simulations. Therefore, a sixth experiment was set up, in which three strongly differing cold start scenarios are defined:

- Reference scenario
- Dry system states scenario
- Wet system states scenario

For the reference scenario the system states from the continuous simulation were used. For the cold states in the dry scenario the states from the reference scenario were reduced by the factor 0.5 and increased by the factor 1.5 for the wet scenario.

The simulation experiments do not allow a systematic analysis of parameter uncertainty, since this is not the aim of this paper. The simulation experiments however enable a first assessment of the robustness of the results. That is to show the forward and inverse model

performance, when the conditions are different from the conditions the model has been calibrated against (i.e. validation period) or if different driving inputs are used.

The model structure applied includes 12 parameters, of which 10 have to be calibrated. Two parameters (INTMAX and ETVEGCOR) are estimated a priori (see Table 2). The interception storage is represented by the model parameter INTMAX, which is estimated as a function of the land use and month of year to consider changes of interception within the annual cycle. ETVEGCOR, comparable to the widely used crop coefficient (Allen et al., 1998), is also estimated depending on the month of year and land use. Values for INTMAX and ETVEGCOR can be found in Herrnegger et al. (2012). For the application, monthly INTMAX- and ETVEGCOR-values were calculated as area weighted mean values, depending on the land uses in the catchments, since a lumped setup is used. For the implementation of the evapotranspiration calculations in the model the reader is also referred to Kling et al. (2015).

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In a first step 3 different periods are used for calibration of the model parameters. In a further simulation experiment, the runoff observation is increased by a constant offset of 10% to evaluate the influence of possible streamflow errors on the simulations and the inverse rainfall. A fifth experiment is performed, in which a differing rainfall realisation is used as driving input for model calibration, in order to test the conditioning of the model parameters and in consequence the simulations to the driving input. Given the model structure, the inverse rainfall is a function of observed runoff, potential evapotranspiration, system states and model parameters (eq. (4)). Extending eq. (4) explicitly with all relevant system states leads to

$$R_t = f^{-1}(Q_t, ETp_t, BW1_{t-1}, BW2_{t-1}, BW3_{t-1}, BW4_{t-1} | \theta_t) \quad (7)$$

The forward and inverse models are run as a continuous simulation in time. The preceding system states are therefore an integral part of the simulation and are determined intrinsically

within the simulation. However, the initial system states at the beginning of the simulation period (cold states) will influence the results of the simulation, but should, after an adequate spin-up time, not influence the runoff but also inverse rainfall simulations. Therefore, a sixth experiment was set up, in which 3 three different cold start scenarios are defined:

- Reference scenario
- Dry system states scenario
- Wet system states scenario

~~For the reference scenario the system states from the continuous simulation were used. For the cold states in the dry scenario the states from the reference scenario where reduced by the factor 0.5 and increased by the factor 1.5 for the wet scenario.~~

Generally only June, July, August and September are used, since it can be guaranteed, that no snow melt influences runoff in these months (see section 2.2.1). Parameter calibration in the simulation experiments is performed for the forward model, using the Shuffled Complex Evolution Algorithm (Duan et al., 1992). As an optimisation criterion the widely used Nash-Sutcliffe-Efficiency (NSE, Nash and Sutcliffe, 1970) was chosen.

3 Materials

3.1 Study areas

The inverse model is applied to two catchments with different size, geology and land use located at the foothills of the Northern Alps. The Schlieffau catchment is located about 110 km south-west of the Austrian capital of Vienna and covers an area of 17.9 km² with a mean elevation of 608 m.a.s.l.. About 55% of the area is covered by grassland and meadows, 40% by coniferous forest and 5% by mixed forest. The underlying geology is dominated by marl and sandstone. The Krems catchment is located about 170 km south-west of ~~the Austrian capital of~~ Vienna and covers an area of 38.4 km² with a mean elevation of 598 m.a.s.l.. The topography is more heterogeneous, with an elevation range of 413 to 1511 m.a.s.l., compared to 390 to 818 m.a.s.l. in the Schlieffau catchment. Approximately 46% of the area is covered by grassland and meadows, 48 % by mixed forest, 4 % by settlements and 2 % by coniferous forest. On a long term basis, in both catchments, the highest runoff can be expected during snow melt in spring, the lowest runoff in summer and autumn until October. Fig. 4 shows a map of the catchments and Table 4 summarizes important characteristics of the study areas.

→ Approximate location of Fig.4

→ Approximate location of Tab.4

3.2 Meteorological database

Generally, two different rainfall time series are used. Ground observations of rainfall are available from the station St. Leonhard im Walde (Schliefau catchment) and Kirchdorf (Krems catchment), both located in the proximity of the catchments (Fig. 4). Additionally, areal rainfall data from the INCA system (Integrated Nowcasting through Comprehensive Analysis; Haiden et al., 2011) is used. INCA is the operational nowcasting and analysis application developed and run by the Central Institute for Meteorology and Geodynamics of Austria (ZAMG), which is also used for the majority of real-time flood forecasting systems in Austria (Stanzel et al., 2008). For the presented study analysis fields derived from observations, but no nowcasting fields, are used. Rainfall in INCA is determined by a nonlinear spatial interpolation of rain-gauge values, in which the radar field is used as a spatial structure function. In addition an elevation correction is applied (Haiden and Pistotnik, 2009). The stations used for the interpolation of the INCA-rainfall fields are shown as triangles in Fig. 4. Note, that the stations St. Leonhard im Walde and Kirchdorf are not included in the INCA analysis, since they are operated by a different institution. The rainfall fields from the INCA system cover the test basins in a spatial resolution of 1 km². From the spatial data set mean catchment rainfall values are obtained by calculating area-weighted means from the intersecting grid cells.

Potential evapotranspiration input is calculated with a temperature and potential radiation method (Hargreaves and Samani, 1982).

3.3 Simulation periods

Runoff and rainfall data is available for the period 2006 to 2009 in a temporal resolution of 60 minutes, which is also the modelling time step. The virtual experiments are performed for a period of 4.5 months (15.5.2006 – 30.09.2006) resulting in 3336 time steps being evaluated. As described in section 2.3.2 different model calibration and simulation experiments are performed. An overview of these experiments is given in Table 5.

→ Approximate location of Tab.5

4 Results and discussions

4.1 Virtual experiments

In the virtual experiments it could be shown, that the ~~precondition of existence, uniqueness and stability of the inverse model results is given~~ invertibility of the model equations is given. Using all 20 000 simulated hydrographs from the Monte Carlo runs, where the parameters were varied stochastically, the observed rainfall time series could be identically reproduced by the inverse model. Apart from the rainfall also all fluxes and system states were identical in the forward and inverse model runs. The comprehensive results from these virtual experiments are documented in Herrnegger (2013). ~~Fig. 5 shows as an example of a virtual experiment the identical (i) observed rainfall and simulated inverse rainfall and (ii) system state of soil water content from the forward and inverse model.~~

~~For the second set of virtual experiments~~ station data from the Schlieffau catchment with model parameters of Exp3 (see Table 5) were used as driving input in the forward model and the resulting runoff simulation in succession as input into the inverse model. ~~To these resulting runoff simulations, however, noise with different magnitudes was added beforehand.~~ Depending on the magnitude of noise added to the runoff input time series, the inferred precipitation rates differ from the observed values, as is shown in Table 6. Without any noise the observed rainfall is reproduced exactly. With increasing noise a deterioration of the model performance is evident. Temporal aggregation leads to an increase in the correlation values, since the resulting noise in the inferred precipitation rates are smoothed out. The mean observed precipitation rate for the evaluated period in these virtual experiments is 0.21 mm for hourly precipitation, 1.26 mm for the 6h-sums and 5.03 mm for the daily precipitation rates. Based on these values, the mean quantitative bias ranges between -0.6% and -6.3% relative to the mean observed rainfall, depending on added noise scaling factor of 1% to 10%. The inferred precipitation totals are higher, compared to the observed values, since the noise also leads to a quantitative bias between the runoff simulation of the inverse model and the runoff used as input. From the results it is clear that the inferred precipitation rates are sensitive to potential short-term errors in discharge measurements. Especially for the case, in which the noise scaling factor was set to 10%, assuming large short-term errors, the inverse model is not able to reproduce the disturbed input time series. This is also evident from the mean squared error values. The noise with a scaling factor of 10% however leads to a strongly

[perturbed runoff time series. Also in the forward case it would not be able to reproduce this runoff time series with the given precipitation in a reasonable manner.](#)

→ Approximate location of [FigTable_56](#)

4.2 Forward model: Parameter calibration and validation of the different simulation experiments

A precondition for the application of the inverse model is that the observed runoff characteristics of the catchment are reproduced reasonably by the forward model, since these parameters are also used in the inverse model. The following section therefore presents the runoff simulations of the forward model, based on the different simulation experiments Exp1 to Exp5.

The model performance for ~~the period 2006 to 2009~~[different periods](#) of the forward model, expressed by Nash-Sutcliffe-Efficiency (NSE) and the mean bias between simulated and observed runoff in percent of observed runoff is shown in Table [76](#). As mentioned before, only the months June, July, August and September of the single years are used.

→ Approximate location of Tab.[67](#)

[With the exception of Exp5 the NSE-values of the calibration periods are larger than 0.8 in both catchments. The highest NSE-values of 0.87 \(Schliefau\) and 0.88 \(Krems\) are found for Exp1. The short calibration period used in this experiment \(only June to September 2006 are used; see Table 5\) enables a good fitting of the model parameters to the runoff observations. In consequence the largest deterioration of the model performance in the validation period is evident for Exp1 for both catchment, since ~~the runoff conditions~~ differ from ~~the calibration~~. For the other experiments the differences in the NSE-values between calibration and validation period are less pronounced, with some experiments showing higher model performance in the validation period. In Exp5 INCA rainfall data is used as driving input for the simulations. The main intention of Exp5 is to evaluate the influence of a different rainfall input on the calibration of the model parameters and in consequence also on the inverse rainfall. For both catchments, the NSE-values of the forward model are mostly significantly lower, also compared to Exp3, which has the same calibration and validation periods. Although INCA uses a complex interpolation scheme, also incorporating radar data and a rainfall intensity depending elevation correction \(Haiden et al., 2011; Haiden and Pistotnik, 2009\), it seems that the data set has deficits representing catchment rainfall compared to the](#)

[station observations in the proximity of the catchments. This can be explained by the larger distance of about 10 to 35 km of the INCA stations from the catchment \(see Fig. 4\). Note, that the ground observations in the proximity of the catchments are not used in the interpolation process for the INCA-rainfall fields, as they belong to a monitoring network operated by a different institution.](#)

For Exp1 to Exp3, the NSE-values for the period 2006 to 2009 show, that the overall model performance is fairly stable and comparable, independent of the calibration length. The NSE-values are larger than 0.82, with the exception of Exp1 in the Krems catchment. Although the calibration lengths and periods in Exp2 and Exp3 differ, identical model parameters were found for the Krems catchment in the optimisation for both simulation experiments. As a consequence the model performance is identical in these two experiments for the period 2006 to 2009.

The mean bias does not show a clear pattern and seems to be independent from the calibration period and length. In the Schlieffau catchment observed runoff is overestimated by 7.8 to 0.9 % and underestimated by -1.4 to -4.8% in the Krems catchment for the period 2006-2009, depending on the simulation Exp1 to Exp3. Overall the calculated bias between observed and simulated runoff is in reasonable bounds.

In Exp4 the observed runoff is increased by 10%, mainly to evaluate the influence of possible streamflow errors on the simulations and the inverse rainfall. The same calibration periods were used as in Exp3, with station observations as driving input into the model. The NSE of Exp4 is comparable to Exp1, Exp2 and Exp3. The mean bias in Exp4 however becomes larger [in both catchments](#). The observed runoff is now also underestimated in the Schlieffau catchment, what is not surprising, since observed runoff was increased.

~~The mean bias in Exp4 for the Krems catchment is also larger, compared to Exp1 to Exp3. This is also explained by the increased observed runoff.~~

~~In Exp5 INCA rainfall data is used as driving input for the simulations. The main intention of Exp5 is to evaluate the influence of a different rainfall input on the calibration of the model parameters and in consequence also on the inverse rainfall. For both catchments, the NSE values of the forward model are significantly lower, also compared to Exp3, which has the same calibration and validation periods. Although INCA uses a complex interpolation scheme, also incorporating radar data (Haiden et al., 2011), it seems that the data set has deficits representing catchment rainfall compared to the station observations in the proximity~~

of the catchments. This can be explained by the larger distance of about 10 to 35 km of the INCA stations from the catchment (see Fig. 4). Note, that the ground observations in the proximity of the catchments are not used in the interpolation process for the INCA rainfall fields, as they belong to a monitoring network operated by a different institution.

Fig. 6-5 shows the NSE-values of the forward model for the calibration periods of every simulation experiment versus single years performance for the 2 study areas.

→ Approximate location of Fig. 6-5

For Exp1 a significant larger spread in the model performance within the single years is evident. In Exp1 only 2006 was used for calibration. As a consequence, especially for the Krems catchment, the model performance is lower in the years 2007 to 2009, compared to Exp2 and Exp3. In the short calibration period of 2006 the model parameters are overfitted to the observations. If the conditions in the catchment are different from the calibration period, the model performance can be expected to deteriorate, as has been shown before (e.g. Kling, 2015; Seibert, 2003) and explains the findings. For Exp2 to Exp4 the model performance is however stable for the single years, also for 2009, which was not used for calibration in any simulation experiment. In contrary to the Krems area, a large spread in the model performance of the single years for Exp5 is visible in the Schlieffau catchment. The reason is not clear and may be explained by changing availability of station data for the INCA rainfall in the single years. We can however not verify this hypothesis, since we do not have access to the data sets. In the Schlieffau catchment low NSE-values are calculated for the year 2008 for all simulation experiments. In the beginning of June a flood was observed (Fig. 7-6), which is not simulated in the model runs and explains the lower NSE values in this year. Excluding this event in the performance calculations would result in a significantly higher NSE of 0.84 for Exp1 for the year 2008, compared to 0.63 when the flood event is included in the calculation.

Fig. 7-6 (Schlieffau) and Fig. 8-7 (Krems) exemplarily show the runoff simulations based on the results of Exp2. For both catchments, the dynamics and variability of the runoff observations are mostly reproduced in a satisfactory manner. However, a tendency is visible, that larger floods are underestimated in the simulations.

→ Approximate location of Fig. 7-6

→ Approximate location of Fig. 8-7

All simulations are performed with a lumped model setup. Consequently heterogeneity in geology and land use within the catchment are not considered in the parameter estimation. Also taking this into consideration, it can be concluded that the general responses of the catchment to rainfall input are captured appropriately by the forward model. Only for Exp1 with the very short calibration period, a larger [deterioration of the model performance in the validation period and a larger spread in the model performance is evident](#) in independent years [is evident](#). It is therefore justified to calculate areal rainfall from runoff using the inverted forward model, including the optimised parameters.

4.3 Inverse model

For the evaluation of the simulated rainfall from the inverse model (PInv) we will compare the calculated values with observed station data (PObs) of St. Leonhard (Schliefau catchment) and Kirchdorf (Krems catchment) and the rainfall values from the INCA-system (PInca). In the following cumulative rainfall sums and the correlation and bias between simulated and observed rainfall are presented. Additionally the rainfall and runoff simulations of a flood event and the influence of cold system states on the simulations are shown.

4.3.1 Cumulative rainfall sums

Fig. [9-8](#) and [10-9](#) show the cumulative curves of the observed rainfall (PObs), INCA rainfall (PInca) and the inverse rainfall (PInv) of the simulation experiments Exp1 to Exp5 for the Schliefau and Krems catchment. Additionally the cumulative observed runoff (Qobs) is shown as a dashed line. Note that for the Krems catchment (Fig. [10-9](#)) the rainfall curves of Exp2 and Exp3 are identical, since the model parameters are also identical in these simulation experiments.

→ Approximate location of Fig. [9-8](#)

→ Approximate location of Fig. [10-9](#)

The cumulative sums of the inverse rainfall and the observation based rainfall realisations PObs and PInca mostly show very similar temporal dynamics. Although large deviations are sometimes evident for both catchments, the deviations of the cumulative curves of PInca and the different inverse rainfalls (PInv) from the cumulative curves of the ground observation (PObs) are mostly of similar magnitude.

The inverse rainfall curves of Exp1 to Exp5 of the two catchments do not exhibit substantial differences, although different calibration periods and setups were used. At the beginning of June 2008 a flood was observed in the Schlieffau catchment, which was underestimated in the forward simulation, presumably due to inadequate representation of the storm event in the rainfall observations (see runoff simulation in Fig. 76, lower left). Larger rainfall intensities are therefore calculated by the inverse for this period, leading to the larger deviations between the cumulative sums of PObs and PInv of Exp1 to Exp5 as shown in Fig. 98 (lower left). In the Schlieffau catchments larger differences between Exp1 to Exp5 occur in the year 2009 (Fig. 98, lower right). Here, in the second half of June, a period of strong rainfall is evident, which also led to a series of floods in the catchment (see also the hydrographs in Fig. 76). The rainfall sums originating from these high flows were calculated differently in the inverse models, depending on the simulation experiment. In consequence, the inverse rainfall curves differ from July onwards. In 2009, which was the wettest summer in both catchments, the highest inverse rainfall sums are found for Exp4. This is what could be expected, since the observed runoff was increased by 10% in this simulation experiment. However, in the other years Exp4 does not necessarily show the largest inverse rainfall sums. The optimised model parameters in Exp4, that control evapotranspiration, were limiting actual evapotranspiration from the model to fulfil the water balance, since PObs was not changed. In the second half of June 2009, during the flood events with low evapotranspiration, the higher runoff values used as input however show a clearer signal in the inverse rainfall sums.

The large difference between cumulative rainfall and runoff curves highlight the importance of actual evapotranspiration (ETa) in the catchments. For the Schlieffau catchment the mean observed rainfall for the summer months of 2006-2009 is 678 mm. 266 mm are observed in the mean for runoff. Neglecting storage effects, a mean actual evapotranspiration of 412 mm can be calculated from the water balance. Over 60 % of rainfall are therefore lost to evapotranspiration. The mean actual evapotranspiration from the inverse model, depending on the simulation experiment, ranges from 352 mm to 362 mm, and are lower compared to the ETa calculated from the water balance. In the Krems catchment a mean runoff of 334 mm and rainfall of 600 mm, resulting in an actual evapotranspiration of 266 mm, is calculated. Although lower compared to Schlieffau, nearly 45 % of rainfall are here lost to the atmosphere. The mean actual evapotranspiration from the inverse model, again depending on the simulation experiment, range from 276 mm to 310 mm. ~~ETa from the model reflects the complex interplay and temporal dynamics of the system states of the different parts of the~~

~~model.~~ If the model would not capture ETa adequately, the cumulative rainfall curves would not follow the observations so closely.

On the basis of the different cumulative rainfall sums it can be concluded, that on a longer temporal basis, the inverse model is capable of simulating the catchment rainfall from runoff observations. [This is also the case for independent validation periods and years, which were not used in the calibration.](#) The results from the different simulation experiments do not differ substantially and show close correspondence to the observed data, except for a single summer in the Schlieffau catchment.

4.3.2 Correlation and bias between simulated and observed rainfall

The performance of the inverse model expressed by the correlation coefficient is used to measure the model's ability to reproduce timing and shape of observed rainfall values. It is independent of a possible quantitative bias. In the introduction the difficulties involved in the quantitative measurement of rainfall were discussed. It can however be assumed that a qualitative measurement, e.g. if it rains or not, will be more reliable. Table [7-8](#) shows the correlation values ~~for 2006 to 2009~~ between ground observations and the different inverse rainfall realisations (PObs – PInv) and ground observations and INCA rainfall (PObs – PInca) for different [periods and](#) temporal aggregation lengths.

➔ Approximate location of Tab. [7.8](#)

[The highest correlation values between PObs and PInv for the 1h-sums and calibration period are found for Exp1 with 0.71 \(Schlieffau\) and 0.62 \(Krems\). For the other experiments the correlation values in the calibration period are lower \(0.51 to 0.57 in the Schlieffau area and 0.44 to 0.49 in the Krems catchment\). For the validation period the correlation between PObs and PInv deteriorates in Exp1. For the remaining experiments, however, the correlation in the validation period is mostly higher, compared to calibration. This agrees with the finding from the forward simulation results, since better model performance in the validation period of the forward model also leads to a higher correlation between PObs and PInv. For the temporally aggregated 24-h sums the correlation values generally increase for the calibration and validation periods.](#)

For ~~the period 2006 to 2009 and the~~ 1h-sums, the lowest correlation values between PObs and PInv are found for the simulation results of Exp1 in both catchments. The highest correlation values are found for Exp2 in the Schlieffau catchment and Exp2 to Exp4 in the Krems

catchment. This agrees with the performance of the forward model presented in section 4.2.. The correlation of the 1h-sums between PObs and PInv is rather weak. However, the correlation between PObs and PInv is higher for all simulation experiments and 1h-sums compared to the correlation between PObs and PInca. This is interesting, since PInca is based on station rainfall observations and PInv is indirectly derived from runoff through simulations. With temporal aggregation the correlation values generally increase significantly for all combinations. Small differences or timing errors in the 1h-sums are eliminated with temporal aggregation. This is also the case ~~in-for of~~ the INCA data.

For Exp1 to Exp4, the model parameters used for the forward and inverse model were automatically calibrated using the ground observation PObs as input. It could therefore be concluded that the model parameters are conditioned by PObs and that in consequence the fairly good agreement between PObs and PInv originates from this conditioning. Based on this hypothesis, calibrating the model with INCA data should lead to a better agreement between the INCA data and the corresponding inverse rainfall and a deterioration of the correlation between station data and inverse rainfall. For Exp5, the forward model was therefore calibrated with INCA data and the resulting parameters set was then used to calculate the inverse rainfall. The correlation between PInca and PInv for Exp5 is however not higher, compared to the other simulation experiments and Exp3, which had the same calibration period. This excludes that the parameters are conditioned (at least for the rainfall simulations) by the input used for calibration. The comparison of Exp3 and Exp5 is critical and shows, that the inverse model provides reasonable results in the case where the forward model is calibrated with rainfall data that are independent from the observed catchment rainfall: The forward model exhibits significantly lower NSE in Exp5 compared to Exp3, which is expected because the forward model is driven with the lower quality INCA rainfall in Exp5 (see Tab. 7). The correlation between PObs and PInv however suggests that Exp5 is comparably representative of the rainfall dynamics as Exp3.

The correlations between PInca and PInv are generally very weak, with values ranging from 0.25 to 0.29 for the Schlieffau and 0.39 to 0.445 for the Krems catchment. ~~This corresponds to the performance of the forward model in Exp5. Here lower model performance of the forward model is found for the Schlieffau catchment.~~

For the period 2006 to 2009, the correlation between PObs and PInv for the 1-h sums ranges between 0.48 and 0.55, but is higher, compared to the correlation between PObs and PInca. In

contrast Kirchner (2009) shows correlation values between simulated and observed rainfall of 0.81 and 0.88 for his two sites. The Schlieffau and Krems catchments differ substantially in size, hydrological characteristics, land use or geology. The NSE values of the runoff simulations in Kirchner (2009) are higher, compared to the values presented here for the forward model. As a consequence the better performance in the rainfall simulations may be explained with the fact, that the Kirchner (2009) model better reflects the catchment conditions leading to runoff.

For the 24-h sums and the period 2006 to 2009 we calculate a correlation of 0.87 to 0.92, depending on the catchment and simulation experiment. Here Kirchner (2009) shows correlation of 0.96 and 0.97. Krier et al. (2012) present correlations between simulated and observed rainfall of 0.81 to 0.98, with a mean value of 0.91 for a total of 24 catchments, however only on the basis of data of a single year. The correlation in our results is therefore in the range of other studies. Unfortunately Krier et al. (2012) do not present ~~NSE-NSE~~-values of the runoff simulations. It is therefore not possible to check the link between the performance of the forward model and rainfall simulations in their study.

Fig. ~~4+10~~ shows the correlation between PObs and PInv for the calibration periods of the simulation experiments Exp1 to Exp5 versus the correlation in single years for the two study areas. For the Schlieffau catchment the largest spread in the correlation values of the single years is found for Exp1, which also corresponds to the performance of the runoff simulations of the forward model. For Exp2 to Exp5 a spread is also visible between the single years, but differences are smaller. For the years 2006, 2008 and 2009 the correlation values in the Krems catchment do not differ substantially. Here however the correlation for the year 2007 is very low, independent of the simulation experiment. This may be explained by the comparatively dry summer of 2007. Also in the Schlieffau catchment the correlation values are mostly lower in 2007, compared to the other years.

➔ Approximate location of Fig. ~~4+10~~

Tab. ~~8-9~~ summarizes the mean ~~daily~~ bias in mmh^{-1} ~~and~~ mmd^{-1} between different rainfall realisations for the summer months in 2006 to 2009, evaluated for different periods and for 1h- and 24-h-sums between different rainfall realisations. Except for Exp1 the bias is larger in the validation compared to the calibration periods.

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For the period 2006 to 2009 ~~For and~~ the Schlieffau catchment, the bias between PInv and PObs is mostly ~~significantly~~ higher, compared to the bias between PInca and PObs. Only Exp2, with a mean bias of 0.07 mm d^{-1} , is comparable to the bias between PInca and PObs of 0.02 mm d^{-1} . Exp2 also showed the highest performance in the runoff simulations concerning the NSE. In contrary, for the Krems catchment, the bias is lower between PInv and PObs for Exp1 to Exp3, compared to PInca- ~~and~~ PObs. For Exp1 to Exp3 ~~and the period 2006-2009~~ a mean bias of 0.14 mm d^{-1} (Schlieffau) and 0.36 mm d^{-1} (Krems) is calculated. As a comparison, Krier et al. (2014) published mean bias values between simulated and observed rainfall of -3.3 to 1.5 mm d^{-1} (mean -0.35 mm d^{-1}) for 24 catchments on the basis of a single year. From all simulation experiments, Exp4 shows the largest bias, which is explained by the fact, that runoff was increased in this experiment. Here the increased runoff clearly shows a signal in the inverse rainfall, ~~in contrast to the correlation and cumulative sums shown above~~.

→ Approximate location of Tab. ~~89~~

4.3.3 Rainfall and runoff simulations for a flood event

Fig. ~~42-11~~ exemplarily illustrates the temporal development of the different rainfall realisations and runoff simulations for the highest flood event in the Krems catchment. Results from Exp3 are shown. Compared to PObs and PInca the inverse rainfall PInv exhibits higher variability and higher intensities. The higher variability and oscillating nature of the inverse rainfall is explainable with the reaction of the inverse model to small fluctuations in runoff observations: In case of rising runoff observations, rainfall will be estimated by the inverse model. If the observed runoff decreases and the simulated runoff of the inverse model is larger than observed runoff, no inverse rainfall will be calculated, leading to the visible oscillations. Fig. ~~42-11~~ (b) shows, that the forward model, driven with PObs as input, underestimates both flood peaks. The forward model, driven with the inverse rainfall, simulates the driven periods very well (Inverse QSim). However, especially the falling limb after the second flood peak on the 07.09.2007 is overestimated by the inverse model. In this period it is also visible, that in consequence no rainfall is calculated by the inverse model, since simulated runoff is higher than observed runoff.

→ Approximate location of Fig. ~~4211~~

For a given time interval, the inverse model will yield an exact agreement between observed and simulated runoff, as long as there is a positive rainfall value R_t to solve eq. (5). This will

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be the case in periods of rising limbs of observed runoff (driven periods), as a rainfall value can be estimated, which raises the simulated runoff value to match observation. On the contrary, in periods of observed falling limbs (non-driven periods) the simulated runoff will solely be a function of the model structure, its parameters and the antecedent system states, as negative rainfall values are ruled out beforehand. This explains, why in periods, in which the simulated runoff is higher than the observed value, no rainfall is calculated by the inverse model.

4.3.4 Influence of cold system states on the inverse rainfall (Exp6)

To test the influence of cold states on the inverse rainfall simulations the simulation experiment Exp6 was performed. Three different cold states (Reference, dry and wet system states) were thereby defined (see section 2.3.2). Fig. 13 exemplarily shows the results of Exp6 for the Krems catchment.

→ Approximate location of Fig.13

From the monthly rainfall sums of the different model runs it is evident, that the inverse rainfall calculations differ significantly at the beginning of the simulation. In the first month the reference scenario results in a monthly rainfall sum of 30 mm, the dry scenario in 111 mm and the wet scenario in only 9 mm. Generally the model will always strive towards an equilibrium in its system states, which are a function of the model structure and parameters. In the scenario “wet” a lot of water is stored in the states of the model at the beginning, with the result, that little inverse rainfall is calculated. In the dry scenario on the other hand a higher amount of rainfall is estimated, since less water is stored in the states at the beginning. With time, however, the different system states converge. In consequence also the inverse rainfall values converge and after 9 months no differences are evident.

5 As in forward models formulated in a state space approach, it is evident that cold states have a noteworthy influence on the simulation results. After an adequate spin-up time the system states however converge, leading to deterministic and unique inverse rainfall estimates.

4.3.4 Influence of cold system states on the inverse rainfall (Exp6)

To test the influence of cold states on the inverse rainfall simulations the simulation experiment Exp6 was performed. Three different cold states (Reference, dry and wet system

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→ Approximate location of Fig.12

From the monthly rainfall sums of the different model runs it is evident, that the inverse rainfall calculations differ significantly at the beginning of the simulation. In the first month the reference scenario results in a monthly rainfall sum of 30 mm, the dry scenario in 111 mm and the wet scenario in only 9 mm. Generally the model will always strive towards an equilibrium in its system states, which are a function of the model structure and parameters. In the scenario “wet” a lot of water is stored in the states of the model at the beginning, with the result, that little inverse rainfall is calculated. In the dry scenario on the other hand a higher amount of rainfall is estimated, since less water is stored in the states at the beginning. With time, however, the different system states converge. In consequence also the inverse rainfall values converge and after 9 months no differences are visible.

Extreme assumptions were made concerning the dry and wet scenarios, since the intention of Exp6 is to evaluate the general influences of the cold states and spin-up time on the inferred rainfall. Especially the long memory of the ground water storage explains the long warm-up period in the presented results. In practice reasonable cold states must therefore be defined at start-up, as is the case for forward models formulated in a state-space approach. After an adequate spin-up time the system states will however converge, leading to deterministic and unique inverse rainfall estimates.

65 Summary and conclusionsoutlook

A calibrated rainfall-runoff model (forward model) reflects the catchment processes leading to runoff generation. Thus, inverting the model, i.e. calculating rainfall from runoff, yields the temporally disintegrated rainfall. In this paper we applied a conceptual rainfall-runoff model, which is inverted in an iterative approach, to simulate catchment rainfall from observed runoff. The precondition of invertibility of the model equations is successfully tested with virtual experiments, in which simulated runoff time series are used as input into the inverse model to derive rainfall. Additional virtual experiments are performed, in which noise is added to the runoff input time series to analyse the effects of possible short-term errors in runoff on the inferred precipitation rates.

~~The estimated inverse rainfall is compared with two different rainfall realisations: Apart of ground observations, areal rainfall fields of the INCA-system are used.~~ The approach is applied ~~and tested in~~ two study areas in Austria. ~~The estimated inverse rainfall is compared with two different rainfall realisations: Apart of ground observations, areal rainfall fields of the INCA-system are used.~~ Hourly data is available for the years 2006 to 2009. Only the months of June to September are used, as the inverse model can only be applied to simulate rainfall in periods, in which runoff is not influenced by snow melt (i.e. summer months).

In a first step, the forward model is calibrated against runoff observations. To evaluate the influences of (i) different model parameters due to different calibration periods and lengths, (ii) different runoff observations and (iii) different parameter optimisation data basis on the runoff and rainfall calculations, several simulation experiments are performed. Additionally the influence of different initial conditions on the rainfall simulations are evaluated.

The forward model mostly shows stable results in both catchments and reproduces the dynamics and variability of the catchment responses to rainfall in a satisfactory manner. Only the simulation experiment, in which a single summer was used for parameter calibration, shows a larger deterioration of the model performance in validation period and the independent years. The model parameters are then used for deriving catchment rainfall from runoff observations.

The cumulative rainfall curves of the different rainfall realisations (ground observation (PObs), INCA (PInca) and inverse rainfall from the different simulation experiments (PInv)) are very similar, suggesting, that the inverse model is capable of representing the long-term quantitative rainfall conditions of the catchment. About 60 % (Schliefau) and 45% (Krems) of rainfall is lost to the atmosphere due to actual evapotranspiration (ETa). If the model would not capture ETa adequately, the cumulative rainfall curves would not follow the observations so closely. This is also the case for independent validation periods and years, which were not used in the calibration.

The correlation between PInv and PObs, although rather low, is higher or of the same magnitude compared to the correlation between PObs and PInca, suggesting that the inverse model also reflects the timing of rainfall in equal quality of INCA. This is especially the case for the aggregated daily rainfall values. The correlation between PInv and PObs is mostly stable between calibration, validation and in the single years, independent of the simulation experiment. However, again for the simulation experiment with only a single summer for

parameter calibration, a larger spread in the correlation for the single years is visible. An increase in observed runoff (Exp4) does not show negative effects on the inverse rainfall measured by the correlation coefficient. A larger bias between observed and modelled rainfall is however visible in Exp4. Generally, the simulation experiment with the highest performance in the runoff simulation also shows the highest correlation values in the rainfall simulations.

To test, if the inverse rainfall is conditioned by observed rainfall used as calibration input, additional model calibration is conducted using ~~the independent~~ INCA data as driving rainfall input for the forward model calibration. The simulation of inverse rainfall on the basis of this model parameters set show similar results as before, suggesting, that the inverse rainfall is not conditioned to the rainfall input used for model calibration. This result is interesting, since it shows, that the inverse model provides reasonable results in the case where the forward model is calibrated with rainfall data that are independent from the observed rainfall in the proximity of the catchment. Generally, the results do not differ substantially between the two test catchments.

Since the inverse model is formulated in a state-space approach additional simulations are performed with differing cold states at the beginning of the simulations. Here the results show, that the ~~resulting inverse~~inferred rainfall values converge to identical values after an adequate spin-up time.

Like with most environmental models, a calibration of the forward model is necessary. It is clear that the application of the inverse model is therefore not possible, if the catchment is completely ungauged. However, this issue is comparable to the application of conventional rainfall-runoff models in gauged and ungauged catchments. As long as a rainfall-runoff model shows reasonable results for the calibration and validation period, the model can be used for different practical applications, e.g. environmental change impact studies, design flood estimations or flood-forecasting. This is also conceivable for the inverse model, since additional information on the catchment rainfall is made available for potential practical applications mentioned above. This additional information is not solely limited to the simulated hourly data, but also includes the aggregated daily rainfall rates, which show a significant higher correlation to the observed values.

898 ~~Generally, the results do not differ substantially between the two test catchments.~~ It can be
899 concluded that the application of the inverse model is a feasible approach to ~~estimate gain~~
900 ~~additional information on the~~ mean areal rainfall values. The mean areal rainfall values ~~can~~
901 ~~may~~ be used to enhance interpolated rainfall fields, e.g. for the estimation of rainfall
902 correction factors or the parameterisation of elevation dependency. With the inverse model, it
903 is not possible to calculate solid rainfall. In rainless periods, where it is clear, that snow melt
904 is dominating runoff (e.g. in spring), the inverse model can however be used to quantify the
905 snow melt contribution.

906 ~~The estimation of a~~real rainfall ~~estimates~~ leading to extreme flood events ~~is~~are afflicted with
907 major uncertainties. ~~This is underlined by the results where the largest deviations between~~
908 ~~observed and modelled rainfall is found during flood events.~~ Here the inverse modelling
909 approach can be used as an additional information source concerning the rainfall conditions
910 during extreme events.

911 ~~Like with most environmental models, a calibration of the model is necessary. It is clear that~~
912 ~~the application of the inverse model is therefore not possible, if the catchment is completely~~
913 ~~ungauged. However, this issue is comparable to the application of conventional rainfall-runoff~~
914 ~~models in gauged and ungauged catchments. As long as a rainfall-runoff model shows~~
915 ~~reasonable results for the calibration and validation period, the model can be used for different~~
916 ~~practical applications, e.g. environmental change impact studies, design flood estimations or~~
917 ~~flood forecasting. This is also conceivable for the inverse model, since additional information~~
918 ~~on the catchment rainfall is made available for potential practical applications mentioned~~
919 ~~above. This additional information is not solely limited to the simulated hourly data, but also~~
920 ~~includes the aggregated daily rainfall rates, which show a significant higher correlation to the~~
921 ~~observed values. In this context, it is conceivable to use the inverse model in real-time flood~~
922 ~~forecasting systems. Here two different applications of the inverse model are conceivable:~~

923 ~~1. A frequent problem observed in real-time flood forecasting models with state-space~~
924 ~~formulations is that the system states in the models are biased in such a way that the simulated~~
925 ~~and observed runoff differ systematically. Methods exist to cope with this problem and to~~
926 ~~update the system states (e.g. Liu et al, 2012; McLaughlin, 2002). The system states in the~~
927 ~~inverse model will, at least during driven periods, always guarantee, that the simulated runoff~~
928 ~~is identical to observations. This fact may be used as a basis for updating system states in the~~
929 ~~flood forecasting models.~~

2. At least in Austria, 2 different types of precipitation forecasts are used as input in flood or runoff forecasting models – nowcasting fields (used for forecasts of $t=+1h$ to $t=+6h$) and fields from numerical weather forecasting models (used for $t \geq +6h$). The nowcasting fields strongly depend on the quality of station observations ($t=0h$), as they are the basis for extrapolation into the future (Haiden et al., 2011). By assimilating the inverse rainfall into the nowcasting system, i.e. to gain additional information on rainfall quantities, it is conceivable that the rainfall estimates of $t=0h$ can be improved. An extrapolation of the improved rainfall fields could therefore improve the nowcasting fields and in consequence the runoff forecasts.

There are however several methodological issues to be solved, before an application in this context is possible. These include the spatial disaggregation of the inverse rainfall and system states in case the flood forecasting models are set up as distributed models or the limitation of the inverse model, when used to calculate rainfall, to snow free periods. Additionally, the application presented here focused on headwater basins. In this context, the estimation of rainfall from intermediate catchments is also a future challenge.

The inverse model was applied to two catchments. The application and analysis of the proposed method to a wider range of catchments with differing hydrological characteristics is therefore an important task in the near future. Further investigations should include water limited catchments with an aridity index far lower than 1, where the influences of high evapotranspiration on the inferred rainfall must be investigated.

In the presented work several different model parameter sets were used as a basis to calculate inverse rainfall. In further works the influences and uncertainties in the inverse rainfall, which arise from different model parameters should be analysed systematically. Additionally, a comparison of inverse rainfall estimates from a different model structure for the two catchments with our results would be of interest, in order to check the links between the performance of the forward model and the results obtained by the inversion method.

956 **Appendix**

957 The forward model is formulated as follows, considering parameters and variables in Table 2
958 and Table 3:

$$959 \quad BWI_t = \max(\min(INTMAX, BWI_{t-1} + 0.5 * R_t - ETI_t), 0) = \max(\min(INTMAX, BWI_{t-1} + 0.5 * R_t - f(ETp_t, INTMAX)), 0) \quad (A1)$$

$$960 \quad R_Soil_t = 0.5 * R_t + \max(BWI_{t-1} + 0.5 * R_t - ETI_t - INTMAX, 0) \quad (A2)$$

$$961 \quad \begin{aligned} BW0_t &= BW0_{t-1} + R_Soil_t - ETG_t - Q1_t - Q2_t = \\ BW0_{t-1} + R_Soil_t - \min\left(\frac{BW0_{t-1}}{FKFAK * M}, 1\right) * (ETp_t - ETI_t) * ETVEGCOR - \\ R_Soil_t * \left(\frac{BW0_{t-1}}{M}\right)^{BETA} - f(PEX2) * BW0_{t-1} \end{aligned} \quad (A3)$$

$$962 \quad \begin{aligned} BW2_t &= BW2_{t-1} + Q2_t - QAB2_t - QVS2_t = \\ BW2_{t-1} + f(PEX2) * BW0_{t-1} - \alpha_2 * \max(BW2_{t-1} - H2, 0) - \beta_2 * BW2_{t-1} \end{aligned} \quad (A4)$$

$$963 \quad BW3_t = BW3_{t-1} + QVS2_t - QAB3_t = BW3_{t-1} + \beta_2 * BW2_{t-1} - \alpha_3 * BW3_{t-1} \quad (A5)$$

$$964 \quad \begin{aligned} BW4_t &= BW4_{t-1} + Q1_t + QAB2_t + QAB3_t - QSIM_t = \\ BW4_{t-1} + R_Soil_t * \left(\frac{BW0_{t-1}}{M}\right)^{BETA} + \alpha_2 * \max(BW2_{t-1} - H2, 0) + \alpha_3 * BW3_{t-1} - \alpha_4 * BW4_{t-1} \end{aligned} \quad (A6)$$

965 with

$$966 \quad \alpha_i = \frac{\Delta t}{TAB_i} \quad \text{and} \quad (A7)$$

$$967 \quad \beta_i = \frac{\Delta t}{TVS_i} \quad (A8)$$

968 TAB_i / TVS_i = recession coefficients. Δt = modelling time step in units of hours. α and β vary
969 with modelling time step and represent smoothing functions of the linear reservoirs

970 Eq. A1 to A8 are simplified representations of the model algorithm. Min/max operators,
971 which, by introducing discontinuities, can lead to non-inversibilityinvertibility. Eq. A4 and
972 A6 do not include a threshold function in the actual model code. The differential equations of
973 the linear reservoirs are solved analytically. An internal time step discretization is included in
974 the code, to guarantee, that the transition between system states above and below the

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975 threshold value is solved exactly. A3, representing the soil layer, does include a min()
976 operator for estimating the ratio between actual and potential evapotranspiration as a function
977 of soil water content. This is however not a limiting factor for the inversion, since this factor
978 is a function of the preceding soil state $BW0_{t-1}$, which is known. Only 50% of rainfall is used
979 as input into the interception storage BWI . By assuming that the other 50% are always
980 throughfall, eq. A1 and A2 also does not limit the inversion, since a continuous signal through
981 the whole model cascade is guaranteed. The recession coefficient representing percolation
982 processes in the soil layer exhibits a nonlinear characteristic and is calculated as a function of
983 actual soil water content and as a function of the form parameter PEX2 [-]. This model
984 concept reflects the fact, that higher soil moisture levels lead to higher soil permeability
985 values. These induce higher percolation rates which are reflected by lower recession
986 coefficients.
987

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1154 **Tables**

1155 Table 1: Magnitude of different systematic errors in precipitation measurements (Sevruk,
1156 1981, 1986; Goodison et al, 1998; Elias et al., 1993; Jacobs et al., 2006; Klemm and
1157 Wrzesinsky, 2007).

Systematic error	Magnitude
Wind-induced errors	2 - 10 % (liquid precipitation) 10 - >50 % (snow)
Wetting losses	2 - 10 %
Evaporation losses	0 - 4 %
Splash-out and splash-in	1 - 2 %
Fog and dew	4 - 10 %

1158

1159 Table 2: Model parameters θ_i . Parameters in *italics* are calibrated.

Parameter	Units	Range	Description
INTMAX	mm	0.5 - 2.5	Interception storage capacity
<i>M</i>	mm	80 - 250	Soil storage capacity
<i>FKFAK</i>	-	0.5 - 1	Critical soil moisture for actual evapotranspiration
ETVEGCOR	-	0.4 - 1.1	Vegetation correction factor for actual evapotranspiration from soil
<i>BETA</i>	-	0.1 - 10	Exponent for computing fast runoff generation
<i>KBF</i>	h	4000 - 12000	Recession coefficient for percolation from soil module
<i>PEX2</i>	-	5 - 25	Parameter for non-linear percolation
<i>TAB2</i>	h	50 - 500	Recession coefficient for interflow
<i>TVS2</i>	h	50 - 500	Recession coefficient for percolation from interflow reservoir
<i>H2</i>	mm	0 - 25	Outlet height for interflow
<i>TAB3</i>	h	1000 - 5000	Recession coefficient for base flow
<i>TAB4</i>	h	0.05 - 10	Recession coefficient for routing

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1162 Table 3: Model fluxes and system states S_i . Fluxes represent sums over the time step.

Variable	Units	Type	Description
R	mm	Input	Rainfall
ETp	mm	Input	Potential evapotranspiration
ETI	mm	Output	Actual Evapotranspiration - evapotranspiration from interception module
ETG	mm	Output	Actual Evapotranspiration - evapotranspiration from soil module
BWI	mm	State	Water stored in interception module
BW0	mm	State	Water stored in soil module
BW2	mm	State	Water stored in interflow reservoir
BW3	mm	State	Water stored in base flow reservoir
BW4	mm	State	Water stored in routing reservoir
R_Soil	mm	Internal flux	Input into soil module
Q1	mm	Internal flux	Fast runoff from soil module
Q2	mm	Internal flux	Percolation from soil module
QAB2	mm	Internal flux	Interflow
QVS2	mm	Internal flux	Percolation from interflow reservoir
QAB3	mm	Internal flux	Base flow
QSIM	mm	Output	Total runoff

1163
1164 Table 4: Characteristics of the study catchments (BMLFUW, 2007; BMLFUW, 2009).

	Schlieffau	Krems
Basin area [km ²]	17.9	38.4
Mean elevation [m]	608	598
Elevation range [m]	390 - 818	413 - 1511
Mean annual precipitation [mm]	1390	1345
Mean annual runoff [m ³ /s]	0.38	1.12

Table 5: Overview of the model calibration and simulations experiments with observed input data. PObs and Plnca refer to the rainfall from the station observations and the INCA system.

	Jun. to Sept. in year				Driving input (Forward / inverse model)	Purpose
	2006	2007	2008	2009		
Exp1	calib.	valid.	valid.	valid.	PObs / Q	Influence of different calibration periods on simulations
Exp2	calib.	calib.	valid.	valid.	PObs / Q	
Exp3	calib.	calib.	calib.	valid.	PObs / Q	
Exp4	calib.	calib.	calib.	valid.	PObs / Q+10%	Influence of different runoff Q on simulations
Exp5	calib.	calib.	calib.	valid.	Plnca / Q	Influence of different rainfall input on simulations
Exp6	Parameters from Exp3, but different initial conditions				PObs / Q	Influence of cold states on simulations

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Table 6: Correlation (CORR), mean bias and mean squared error (MSE) for different temporal aggregation lengths between observed and inferred precipitation of the virtual experiments, in which different magnitudes of noise was added to the input runoff data. These are indicated with the “Noise scaling factor”.

Noise scaling factor	CORR [-]			Mean BIAS [mm]			MSE [mm²]		
	1h-sums	6h-sums	24h-sums	1h-sums	6h-sums	24h-sums	1h-sums	6h-sums	24h-sums
0%	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
1%	0.994	0.999	1.000	-0.001	-0.007	-0.028	0.011	0.016	0.015
2%	0.982	0.998	1.000	-0.003	-0.015	-0.060	0.034	0.051	0.043
5%	0.921	0.991	0.999	-0.007	-0.040	-0.160	0.154	0.300	0.230
10%	0.819	0.977	0.998	-0.013	-0.079	-0.316	0.408	0.770	0.556

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1176 Table 67: Model performance for the different simulation experiments and the two
1177 catchments of the forward model, expressed by Nash-Sutcliffe-Efficiency (NSE) and the
1178 mean bias between simulated and observed runoff in percent of observed runoff for the period
1179 2006 to 2009 different periods. Only the months June to September are evaluated.

-	-	NSE [-]	mean BIAS [%]				
Schliefau	Exp1	0.822	7.8				
	Exp2	0.832	3.9				
	Exp3	0.828	0.9				
	Exp4	0.830	-5.9				
	Exp5	0.728	-0.6				
Krems	Exp1	0.763	-1.4				
	Exp2	0.851	-4.8				
	Exp3	0.851	-4.8				
	Exp4	0.854	-7.9				
	Exp5	0.787	1.5				
-	-	NSE [-]			mean Bias [%]		
		Calib.	Valid.	2006- 2009	Calib.	Valid.	2006- 2009
Schliefau	Exp1	0.872	0.814	0.822	4.8	8.7	7.8
	Exp2	0.858	0.819	0.832	11.4	-0.8	3.9
	Exp3	0.812	0.837	0.828	1.5	0.1	0.9
	Exp4	0.814	0.840	0.830	-4.4	-8.3	-5.9
	Exp5	0.738	0.715	0.728	2.1	-4.9	-0.6
Krems	Exp1	0.879	0.740	0.763	-9.4	1.2	-1.4
	Exp2	0.849	0.851	0.851	-0.3	-8.6	-4.8
	Exp3	0.842	0.855	0.851	-3.2	-8.0	-4.8
	Exp4	0.845	0.859	0.854	-6.1	-11.5	-7.9
	Exp5	0.748	0.815	0.787	3.7	-2.8	1.5

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Table 78: Correlation between different rainfall realisations, evaluated for different periods and for 1h- and 24-h-sums. (PObs: Ground observation, PInv: Inverse rainfall from Exp1 to Exp5, PInca: INCA rainfall). Correlation for 2006 to 2009

		CORR: 1h-sums					CORR: 24h-sums				
		PObs - PInv		PInca- PInv	PObs - PInca		PObs - PInv		PInca- PInv	PObs - PInca	
		Calib.	Valid.	2006- 2009	2006- 2009	2006- 2009	Calib.	Valid.	2006- 2009	2006- 2009	2006- 2009
Schliefau	Exp1	0.706	0.460	0.504	0.251		0.935	0.857	0.871	0.802	
	Exp2	0.572	0.540	0.549	0.290		0.939	0.895	0.914	0.840	
	Exp3	0.515	0.567	0.534	0.284	0.463	0.913	0.929	0.918	0.845	0.928
	Exp4	0.515	0.558	0.530	0.283		0.910	0.928	0.917	0.843	
	Exp5	0.514	0.545	0.524	0.276		0.916	0.927	0.920	0.842	
Krems	Exp1	0.622	0.430	0.478	0.394		0.880	0.871	0.871	0.847	
	Exp2	0.437	0.602	0.517	0.445		0.907	0.910	0.909	0.889	
	Exp3	0.493	0.581	0.517	0.445	0.469	0.896	0.936	0.909	0.889	0.931
	Exp4	0.494	0.577	0.517	0.445		0.896	0.936	0.909	0.892	
	Exp5	0.473	0.593	0.503	0.445		0.884	0.936	0.901	0.888	

		CORR: 1h-sums			CORR: 6h-sums			CORR: 24h-sums		
		PObs- PInv	PInca- PInv	PObs- PInca	PObs- PInv	PInca- PInv	PObs- PInca	PObs- PInv	PInca- PInv	PObs- PInca
Schliefau	Exp1	0.504	0.251		0.800	0.671		0.871	0.802	
	Exp2	0.549	0.290		0.828	0.703		0.914	0.840	
	Exp3	0.534	0.284	0.463	0.824	0.699	0.849	0.918	0.845	0.928
	Exp4	0.530	0.283		0.818	0.695		0.917	0.843	
	Exp5	0.524	0.276		0.824	0.697		0.920	0.842	
Krems	Exp1	0.478	0.394		0.794	0.771		0.871	0.847	
	Exp2	0.517	0.445		0.831	0.807		0.909	0.889	
	Exp3	0.517	0.445	0.469	0.831	0.807	0.864	0.909	0.889	0.931
	Exp4	0.517	0.445		0.833	0.809		0.909	0.892	
	Exp5	0.503	0.445		0.820	0.805		0.901	0.888	

1190

1191 Table 89: Mean Bias in mm for between different rainfall realisations, evaluated for different
1192 periods and aggregations lengths between 1h- and 24h-sums. different rainfall realisations.

		Mean Bias: 1h-sums [mmh ⁻¹]					Mean Bias: 24h-sums [mmd ⁻¹]				
		PInv - PObs		PInv - PInca	PInca - PObs		PInv - PObs		PInv - PInca	PInca - PObs	
		Calib.	Valid.	2006-2009	2006-2009		Calib.	Valid.	2006-2009	2006-2009	
Schliefau	Exp1	0.001	0.007	0.006	0.005		0.019	0.179	0.139	0.118	
	Exp2	-0.008	0.014	0.003	0.002		-0.204	0.339	0.067	0.046	
	Exp3	0.003	0.027	0.009	0.008	0.001	0.075	0.639	0.216	0.195	0.021
	Exp4	0.009	0.041	0.017	0.016		0.225	0.986	0.415	0.394	
	Exp5	0.007	0.034	0.014	0.013		0.169	0.817	0.331	0.310	
Krems	Exp1	0.029	0.006	0.012	-0.008		0.686	0.148	0.283	-0.191	
	Exp2	0.013	0.020	0.017	-0.003		0.324	0.485	0.404	-0.069	
	Exp3	0.015	0.022	0.017	-0.003	0.020	0.362	0.531	0.404	-0.069	0.473
	Exp4	0.019	0.033	0.022	0.003		0.450	0.785	0.534	0.061	
	Exp5	0.020	0.022	0.021	0.001		0.478	0.536	0.493	0.019	

1193

1194 Table 8: Mean Bias for 2006 to 2009 between different rainfall realisations.

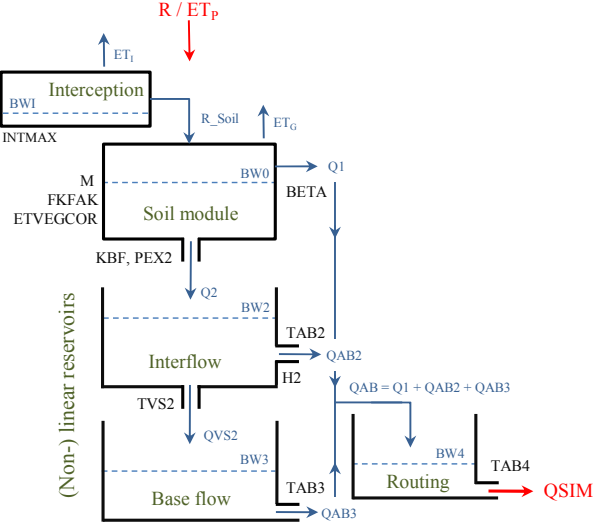
		Mean Bias [mm/d]	
		PInv - PObs	PInca - PObs
Schliefau	Exp1	0.14	
	Exp2	0.07	
	Exp3	0.22	0.02
	Exp4	0.42	
	Exp5	0.33	
Krems	Exp1	0.28	
	Exp2	0.40	
	Exp3	0.40	0.47
	Exp4	0.53	
	Exp5	0.49	

1195

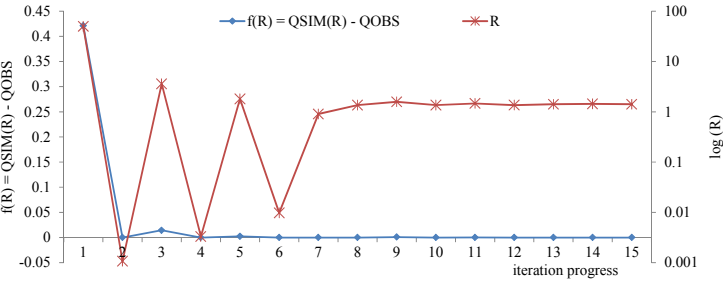
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1197 **Figures**



1198
1199 Figure 1: Structure, parameters and states of the forward model.



1201
1202 Figure 2: Illustration of the iteration progress for one model time step. Note that the right y-
1203 axis showing the inverse rainfall values (R) is in a logarithmic scale- [\(units in mm/h\)](#).

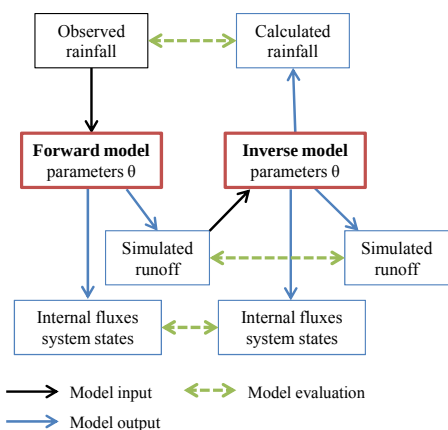


Figure 3: Setup of the virtual experiments and evaluation of the inverse model. All variables are calculated for every Monte Carlo run, in which parameters θ are varied.

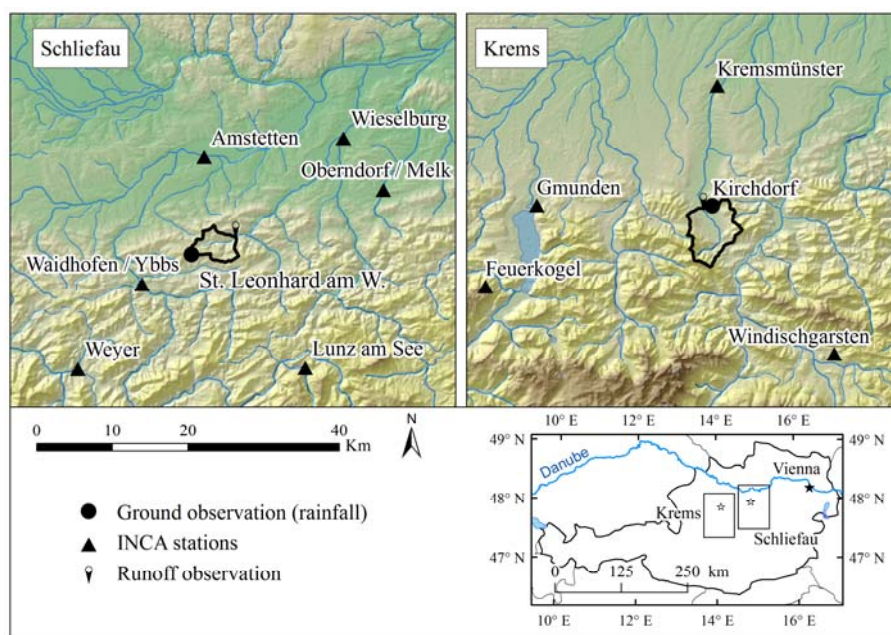


Figure 4: Schliefauf and Krems catchment and location of meteorological stations. Note that ground observation of rainfall is not part of the INCA stations network.

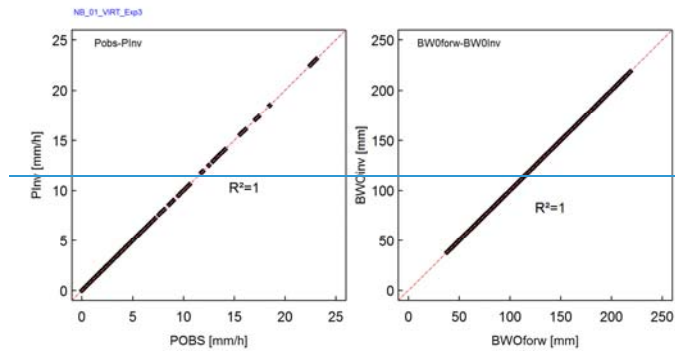


Figure 5: Virtual experiment with simulated runoff as input into the inverse model (Schliefau catchment): Identical observed and inverse rainfall (POBS-Plnv, left) and soil water content of forward and inverse model (BW0forw-BW0Inv, right).

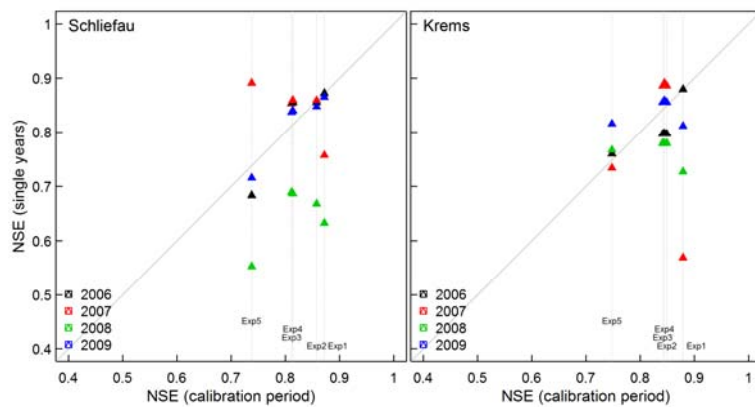
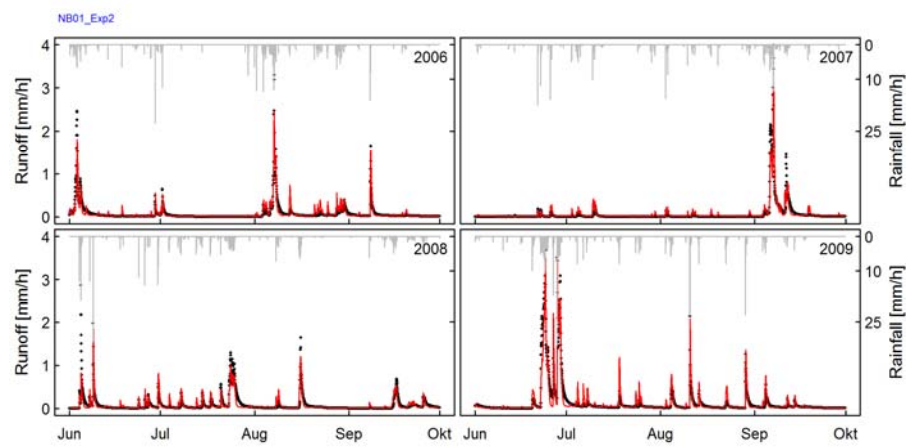


Figure 6: Nash-Sutcliffe-Efficiency (NSE) of the forward model for the calibration periods versus single years for the two study areas.

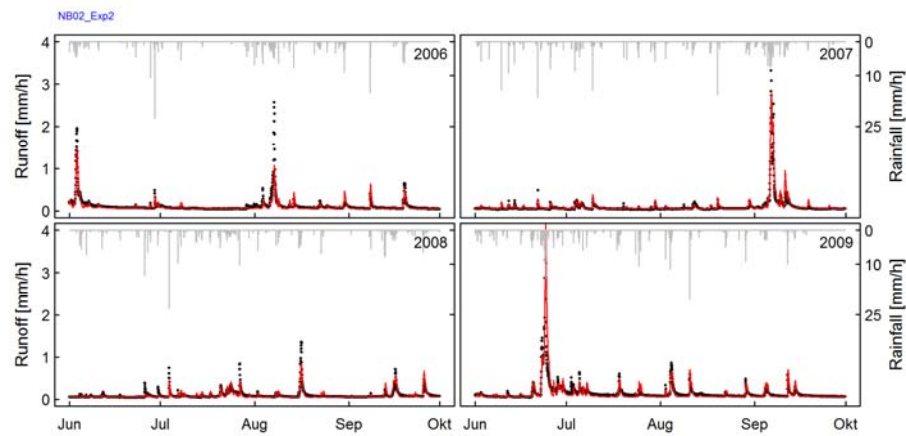
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1222 Figure 76: Schlieffau catchment: Observed (black points) and simulated (red) runoff of Exp2.

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1225 Figure 87: Krems catchment: Observed (black points) and simulated (red) runoff of Exp2.

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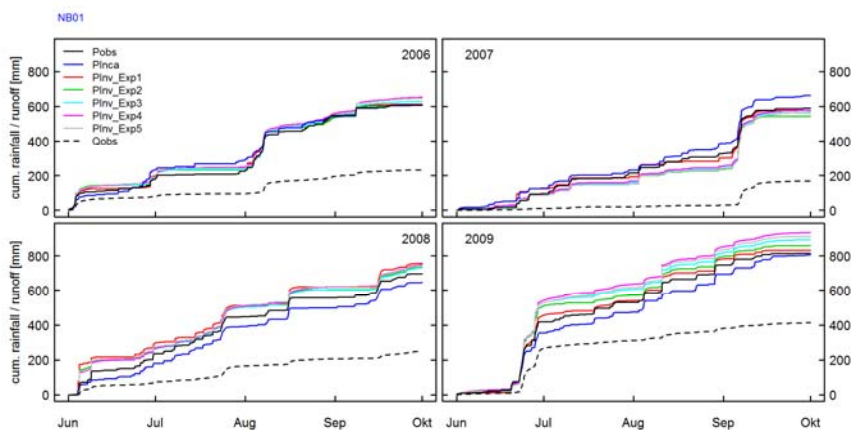


Figure 98: Schlieffau catchment: Cumulative rainfall curves for observed rainfall (PObs), INCA rainfall (Pinca) and the inverse rainfall of Exp1 to Exp5 (PInv). Cumulative sums of observed runoff are shown as dashed black lines.

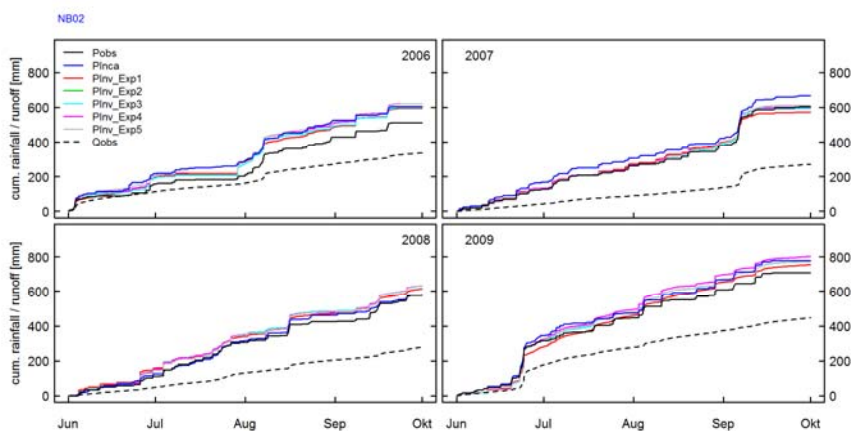


Figure 109: Krems catchment: Cumulative rainfall curves for observed rainfall (PObs), INCA rainfall (Pinca) and the inverse rainfall of Exp1 to Exp5. Cumulative sums of observed runoff are shown as dotted black lines.

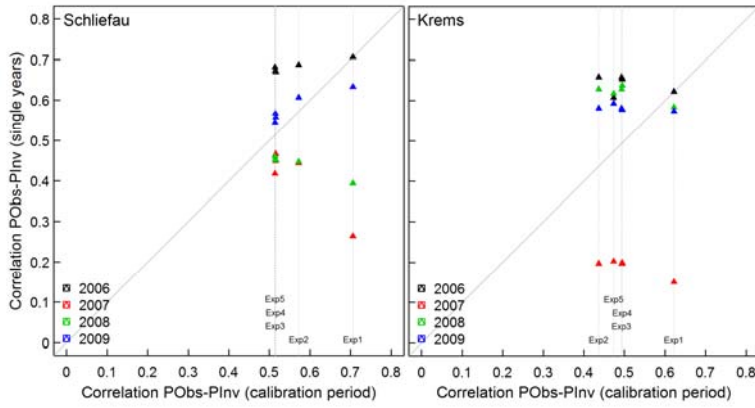


Figure 4410: Correlation between PObs-Plnv for the calibration periods of the simulation experiments Exp1 to Exp5 versus single years for the [two2](#) study areas.

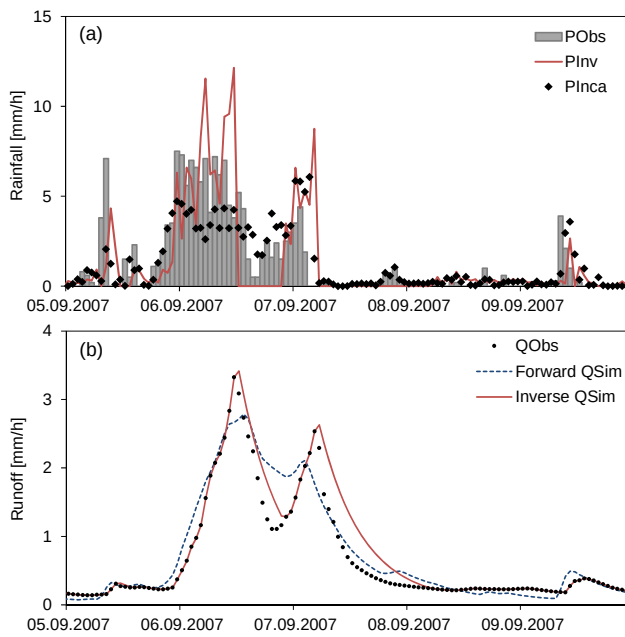
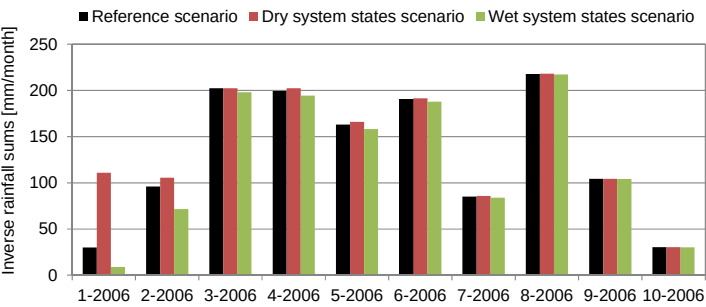


Figure 4211: Krems catchment: Temporal development of the different rainfall realisations (a) and runoff (b) for a flood event. Simulations originate from Exp3.

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1243

1244 Figure 4-12: Krems catchment: Monthly sums of inverse rainfall simulated in the scenarios
1245 "reference", "dry" and "wet" from Exp6.