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Technical Note: A simple generalization of the Brutsaert and Nieber analysis

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Abstract

The Brutsaert and Nieber (1977) analysis is a well known method that can estimate soil parameters given discharge data for some aquifers. It has been used for several cases where the observed late-time behavior of the recession suggests that the water stream that is adjacent to the aquifer has non-zero depth. However, its mathematical formulation is, strictly speaking, not capable of reproducing these real-case scenarios since the early time behavior is based on a solution for which the aquifer stream has zero depth (Polubarinova-Kochina, 1962). We propose a simple generalization for the Brutsaert and Nieber (1977) method that can estimate soil parameters for aquifers discharging into a water stream of finite non-zero depth. The generalization is based on already available solutions by Polubarinova-Kochina (1962), Chor et al. (2013) and Dias et al. (2014) and can be readily implemented with little effort. A sensitivity analysis shows that the modification can have significant impact on the predicted values of the drainable porosity.

1 Introduction

The Brutsaert and Nieber (1977) analysis (from now on referred to as BN77) has been widely used in hydrologic research to estimate aquifer parameters given some discharge data. This technique is based on "state-space"-like plots of $Q \times \mathrm{d}Q/\mathrm{d}t$, where $Q(t)$ is the aquifer discharge as a function of time. It is based on solutions for the Boussinesq equation for groundwater flow applied to a system as the one presented in Fig. 1, which shows a water channel of length L with one aquifer of length B on each side. Generally three solutions of the Boussinesq equation are considered for this method, which are the three solutions proposed by BN77: (i) the solution by Polubarinova-Kochina (1962) for a semi-infinite aquifer that deals with early-time behavior, (ii) the exact solution provided by Boussinesq (1904) adequate for later times and (iii) the linearized solution provided by Boussinesq (1903) that is also used for late-time behavior.

HESSD
11, 12519–12530, 2014

From the aforementioned solutions, only (iii) is able to deal with non-zero water-stream depths (H_0) adjacent to the aquifer (of initial water table height H). Recently, solution (i) – from now on we call "solution (i)" any solution for a semi-infinite aquifer where discharge is occurring – has been generalized by Chor et al. (2013) and Dias et al. (2014). The work by Dias et al. (2014) is of particular importance for the present work because it extends the early-time behavior to cases where the stream depth is different from zero.

Since BN77, some changes and improvements have been suggested (for a detailed review, see Rupp and Selker, 2006) but its main insight remains the same: that one should look at the rate of discharge as a function of discharge, or, mathematically (for the case of a power law),

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\alpha Q^{\beta}, \tag{1}$$

where Q is the water discharge, t is time and α and β are calibrated coefficients which can be compared to the predictions from the analytical solutions.

If one wishes to estimate only the soil hydraulic conductivity k_0 and the drainable porosity n_e, two of the three aforementioned solutions can be used. To apply BN77's theory in a case where the stream depth is not zero, one would have to ignore the fact that the solution by Polubarinova-Kochina (1962) does not account for that case. This is not stricly correct, since it is obtained with the consideration that $H_0 = 0$.

Based on these facts, we will focus on the BN77 method applied with (i) and (iii). We will generalize the implementation of (i) with existing solutions in order to provide a couple of equations completely compatible with more realistic scenarios.

HESSD
11, 12519–12530, 2014

2 Generalization of the early-time equation

Let ξ denote the Boltzmann variable for the one-dimensional Boussinesq equation (Chor et al., 2013; Dias et al., 2014),

$$\xi(x,t) = \frac{x}{\sqrt{4Dt}}, \tag{2}$$

where $D = Hk_0/n_e$, and ϕ denote a normalized water table height,

$$\phi = \frac{h}{H}, \tag{3}$$

where $h(x,t)$ is the water table height, x is the horizontal distance from the water stream and t is the time.

Let us also define

$$\psi \equiv \phi \frac{\mathrm{d}\phi}{\mathrm{d}\xi}, \tag{4}$$

which we apply to Darcy's law, along with Eqs. (2) and (3) to obtain

$$q(x,t) = \frac{H^{3/2}(n_e k_0)^{1/2}}{2} \frac{\psi(\xi(x,t), H_0/H)}{t^{1/2}} \tag{5}$$

where $q(x,t)$ is the flow rate per unit width at any point x of the aquifer. Since we are interested in the aquifer-stream interaction, we set $x = 0$, which produces

$$\begin{aligned} q(t) &= \frac{H^{3/2}(n_e k_0)^{1/2}}{2} \frac{\psi(\xi = 0, H_0/H)}{t^{1/2}} \\ &= \frac{H^{3/2}(n_e k_0)^{1/2}}{2} \frac{\psi_0(\phi_0)}{t^{1/2}}, \end{aligned} \tag{6}$$

HESSD
11, 12519–12530, 2014

where $\psi_0 \equiv \psi(\xi = 0)$ and $\phi_0 \equiv H_0/H$.

The value of ψ_0, as far as we know, cannot be obtained analytically and is generally obtained numerically or by means of approximations: its calculation will be dealt with later. For now, it suffices to note that ψ_0 is a function of ϕ_0 as given above.

Writing $\mathrm{d}Q/\mathrm{d}t = -\alpha_1 Q^{\beta_1}$, where the subscript 1 indicates the early-time solution, and $Q = 2Lq$ is the flow per unit length taken over the total length (L) of the tributary and main channel sections upstream from the gaging station, with q as in Eq. (6), yields $\beta_1 = 3$ and

$$\alpha_1 = \frac{1}{2H^3 k_0 n_e (\psi_0(\phi_0))^2 L^2} = \left[2H^3 k_0 n_e (\psi_0(\phi_0))^2 L^2\right]^{-1}. \tag{7}$$

Equation (7) is generally used with the assumption of $H_0 = 0$, which yields $\psi_0(0) = \Psi_0 \approx 0.6642$, which (substituting back into Eq. 7) gives the well known Eq. (18b) of BN77.

However, often the value of H_0 is not small enough in comparison with H in order for this approximation to be valid (Munster et al., 1996; Serrano and Workman, 1998; Barlow et al., 2000; Peterson and Connelly, 2001; Langhoff et al., 2006; Ha et al., 2008; Sena and de Melo, 2012). In these cases the misplaced assumption could lead to biased estimates of k_0 and n_e. These latter errors depend not only on the determination of α_1, but also on the late-time equations chosen and on the determination of the constants for that solution.

Evidence that the water depth of the adjoining stream is not negligible can be found (for example) in the work by Brutsaert and Lopez (1998), where the late-time data showed a decay with $\beta_2 \approx 1$, which in fact indicates that the watershed analyzed has a ratio H_0/H close to one (we use the subscript 2 to indicate the late-time solution). Indeed, for $\phi_0 = 0$, the exact analytical solution provided by (Boussinesq, 1904), which is valid for late times, gives $\beta_2 = 3/2$ (Brutsaert and Nieber, 1977), whereas numerical solutions of the Boussinesq equation (Kan, 2005) show that β_2 varies from 3/2 down to 1 as H_0/H varies from 0 to 1.

HESSD
11, 12519–12530, 2014

3 Comparison between both approaches

We dedicate this section to the estimation of the errors that arise by assuming that the stream depth H_0 is zero. For that purpose we take as a late-time equation the solution of the linearized Boussinesq equation presented by Boussinesq (1903),

$$h(x,t) = H_0 + \frac{4}{\pi}(H - H_0) \sum_{n=1,3,5...}^{\infty} \frac{1}{n} \sin\left(\frac{\pi n x}{2B}\right) \exp\left(-\frac{\pi^2 n^2 k_0 p H}{4 n_e B^2} t\right), \tag{8}$$

in which the water table height h is approximated as pH (for linearization purposes) and B is the length of the aquifer.

Equation (8) predicts $\beta_2 = 1$ and

$$\alpha_2 = \frac{\pi^2 k_0 p H L^2}{n_e A^2}, \tag{9}$$

where A is the area of the watershed, approximated by $2BL$.

Solution of Eqs. (9) and (7) gives, for n_e and k_0,

$$n_e = \left(\frac{p}{2}\right)^{1/2} \frac{\pi}{H \psi_0 A} (\alpha_2 \alpha_1)^{-1/2} \tag{10}$$

and

$$k_0 = \frac{A}{\sqrt{2p} H^2 L^2 \pi \psi_0} \left(\frac{\alpha_2}{\alpha_1}\right)^{1/2}. \tag{11}$$

In this formulation we assume both ψ_0 and p to be functions of $\phi_0 = H_0/H$, so we have $\psi_0(\phi_0)$ and $p(\phi_0)$, as was previously emphasized. We also assume that $p(\phi_0) = (1 - p_0)\phi_0 + p_0$, where $p_0 = 0.3465$, based on the fact that $p = 0.3465$ for $H_0 = 0$

HESSD
11, 12519–12530, 2014

T. L. Chor and N. L. Dias

(Brutsaert and Lopez, 1998). Setting $H_0 = 0$ (and therefore $\phi_0 = 0$) in this model will yield exactly the same equations as presented by Brutsaert and Lopez (1998).

To obtain $\psi_0(\phi_0)$ we use the approximation provided by Eq. (14) of Dias et al. (2014), since it is sufficiently accurate and simple to program, viz.

$$\psi_0(\phi_0) \approx \left(\Psi_0^d + a\phi_0^b\right)^{\frac{1}{d}} \left(1 - \phi_0^c\right) \left(1 + f\phi_0^g\right)^e, \tag{12}$$

with $a = 0.733841$, $b = 0.999223$, $c = 0.98359$, $d = 2.94568$, $e = 0.186587$, $f = 0.966673$, and $g = 0.93347$.

Figure 2 shows the ratio between k_0 and n_e when estimated with the assumption of $H_0 = 0$ and with the method presented here for different values of ϕ_0. Note that for ϕ_0 in the range $0 \leq \phi_0 \leq 0.6$ the errors introduced in the estimation of the hydraulic conductivity are small (the lowest point in the graph is 0.9 while the value at $\phi_0 = 0.6$ is around 1.16). However for n_e the deviation from the actual value is much larger. As an example, when $\phi_0 = 0.6$ the ratio $n_e(0)/n_e(0.6)$ is approximately 2.5. For cases where $\phi_0 > 0.6$ the errors in the estimation of both parameters are clearly quite large.

4 Conclusions

We have given an expression for early time aquifer discharge that generalizes the broadly used Eq. (18) of Brutsaert and Nieber (1977) for cases where H_0 is not small enough compared with H to make $\phi_0 = 0$ a valid approximation. This improvement, given mainly by Eq. (7), is easily applicable and requires no change in the original theory presented by BN77. Tests presented in Fig. 2 show that the estimation of the hydraulic conductivity is not greatly affected by this generalization, but the estimation of the drainable porosity does differ significantly from the true value.

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HESSD
11, 12519–12530, 2014

A simple generalization of the Brutsaert and Nieber analysis

T. L. Chor and N. L. Dias

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
◀◀ | ▶▶
◀ | ▶
Back | Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

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HESSD

11, 12519–12530, 2014

A simple generalization of the Brutsaert and Nieber analysis

T. L. Chor and N. L. Dias

Table A1. List of Symbols.

Symbol	Meaning	SI unit
A	Area of the watershed ($2BL$)	m^2
B	Total length of aquifer	m
D	Hk_0/n_e	$m^2\,s^{-1}$
h	Water table height	m
H	Water table height in aquifer at time zero	m
H_0	Depth of adjacent water stream	m
k_0	Hydraulic conductivity	$m\,s^{-1}$
L	Length of tributary channel	m
p	Linearization coefficient	1
p_0	Linearization coefficient for the homogeneous case	1
Q	Aquifer discharge	$m^3\,s^{-1}$
q	Aquifer discharge per unit length of the channel	$m^2\,s^{-1}$
n_e	Drainable porosity	1
t	Time	s
x	Horizontal distance from the aquifer–stream interface	m
α, α_1,α_2	Coefficient for the Brutsaert and Nieber analysis	$m^{3-3\beta}\,t^{\beta-2}$
β, β_1, β_2	Coefficient for the Brutsaert and Nieber analysis	1
ϕ	Normalized water table height (h/H)	1
ϕ_0	Normalized water table height at origin (H_0/H)	1
ψ	Normalized variable related to the discharge per unit length	1
ψ_0	The value of ψ at the origin $x = 0$	1
Ψ_0	The value of ψ_0 for the homogeneous case ($H_0 = 0$)	1
ξ	Boltzmann similarity variable	1

HESSD
11, 12519–12530, 2014

T. L. Chor and N. L. Dias

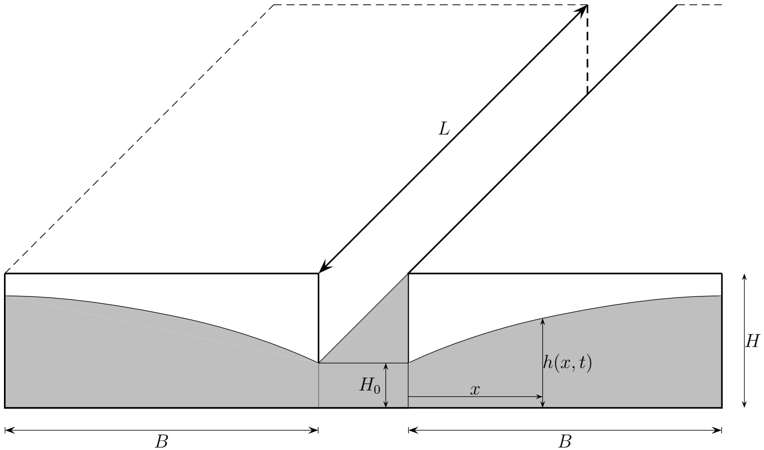

Figure 1. Schematic of a watershed of simple geometry during a hydrologic recession.

HESSD
11, 12519–12530, 2014

A simple generalization of the Brutsaert and Nieber analysis

T. L. Chor and N. L. Dias

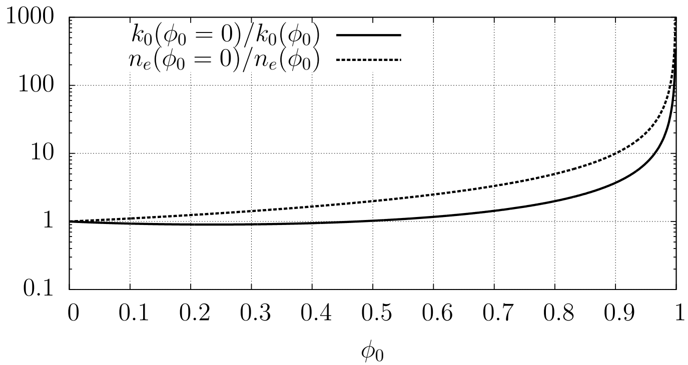

Figure 2. Differences when assuming $\phi_0 = 0$ in the estimation of the parameters n_e and k_0.

HESSD
11, 12519–12530, 2014

A simple generalization of the Brutsaert and Nieber analysis

T. L. Chor and N. L. Dias