Hydrol. Earth Syst. Sci. Discuss., 11, 12271–12290, 2014
www.hydrol-earth-syst-sci-discuss.net/11/12271/2014/
doi:10.5194/hessd-11-12271-2014

Hydrology and Earth System Sciences Discussions
Open Access

Is sinuosity a function of slope and bankfull discharge? – A case study of the meandering rivers in the Pannonian Basin

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Received: 15 September 2014 – Accepted: 12 October 2014 – Published: 4 November 2014

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Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

Pre-regulation channel sinuosities of the meandering rivers of the Pannonian Basin are analysed in order to define a mathematical model to estimate the influence of the bankfull discharge and the channel slope on them. As a primary database, data triplets of slope, discharge and sinuosity values were extracted from historical and modern datasets and pre-regulation historical topographic maps. Channel slope values were systematically modified to estimate figures valid before the river regulation works. The bankfull discharges were estimated from the average discharges using a robust yet complex method. The "classical" graphs of Leopold and Wolman (1957), Ackers and Charlton (1970b) and Schumm and Khan (1972) were compiled to a set up a theoretical surface, whose parameters are estimated by the real values of the above database, containing characteristics of the Pannonian Basin rivers.

As a result it occurred that there is a two-dimensional function of the bankfull discharges, which provides a good estimation of the most probable sinuosity values of the rivers with the given slope and discharge characteristics. The average RMS error of this estimation is around 15 % on this dataset and believed to be the effect of the non-analysed changes in the sediment discharge and size distribution.

1 Introduction

The manners of the rivers are affected by many parameters (e.g. water and sediment discharge, sediment grain size, channel slope, vegetation, tectonics, climate, tributaries . . .). The natural, uncontrolled rivers by changing their shape and pattern can answer to the impacts. The sinuosity is a very useful parameter to detect the changes. The connection between the slope and sinuosity was studied in flume experiments by Schumm and Khan (1972). Leopold and Wolmann (1957) and Ackers and Chalrton (1970b) defined the thresholds between the river patterns, according to the channel slope and water discharge values. Timár (2003) merged their results into a quasi-3-D

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diagram, which suggests that the results of the flume experiments (with very low water discharge) also exist along the natural rivers, at any discharge values. In this paper, this connection was studied, along the rivers of the Pannonian Basin (Fig. 1a).

The rivers are bedrock controlled in the mountain regions, and after getting to the plain, they became alluvial rivers. Depending on the main river parameters (especially the valley slope, the bankfull-discharge and the sediment load), they belong to the straight, meandering, braided or anastomosed river patterns. In this work the meandering alluvial rivers were studied. The rivers, unless their river beds are controlled, chose an optimal channel, depending on the slope, the water discharge and the transported sediment (Leopold and Wolman, 1960). The formation of the meanders increases the length of the river, accordingly decreases the channel slope and the velocity (Whitten and Brooks, 1972).

1.1 Changing slope – changing sinuosity

Schumm and Khan (1972) made flume experiments to detect the effects of the changing slope and changing sediment discharge to the river patterns. They find out, that the slope related to the sediment discharge; then determined their thresholds at different patterns. The river will be straight if the slope and sediment discharge is low. Increasing the slope, the water- and sediment discharge, the river starts to meandering then became braided. First, a meandering thalweg channel developed. If suspended load also existed, real meandering channel evolved, which has the sinuosity at least 1.3. The suspended load (kaolinite) stabilized the alternate bars, which emerged to form point bars. Schumm and Khan (1972) studied the relation between the slope and sediment discharge but not the grain size. At a given discharge, if the sediment is fine-grained, very likely less slope and sediment values needed to change the river pattern.

It is not so easy to extend the results of the flume experiments to real channels. During the modelling not all parameters can be analysed. But the results of Lane (1957) and Ackers and Charlton (1970b) fit to the model of Schumm and Khan (1972).

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At a given valley slope, the sinuosity is in inverse proportion to the sediment load (Schumm, 1963). At given water discharge and valley slope, if the sediment load is constant, a "calculated" sinuosity can be determined, which is a probable value. The system tries to keep this. If the valley became deeper, the sinuosity is increasing, until a maximum value, but after this threshold, if the dip is more increasing, the sinuosity will decrease. In this section, the river is not a self-organizing system but belong to the range of unorganised meandering (Timár, 2003).

1.2 Channel slope – bankfull discharge – river patterns

Leopold and Wolmann (1957) analyzed the connection between the channel slope and bankfull-discharge, at different river patterns. In their work, if the river sinuosity was lower than 1.5, the channel belonged to the straight pattern. They tried to detect the border-lines (depending on the slope and discharge) between the straight and meandering and the meandering and braided patterns. At a given discharge, the meandering rivers appeared at lower slopes than the braided. The straight channels appear at a wide range of channel slope.

Finally, Leopold and Wolmann (1957) separated the meandering and braided channels with the $s = 0.06 \cdot Q^{-0.44}$ equation (Q is in cubic foot / second). Leopold and Wolmann (1957) noted, the points of the graph derived from natural channels, so the different parameters can related to each other. There are lots of other factors, which can affect the behaviour of the rivers. However, they studied natural rivers, so the result displays a condition, which can occur along natural rivers.

Ackers and Charlton (1970b) defined the border lines between the meandering and straight patterns. Actually, they defined two lines: the upper limit of the straight pattern ($S < 0.001Q^{-0.12}$) and the lower limit of the meandering pattern ($S_v > 0.0014Q^{-0.12}$). Between the two lines, a mixed type appeared: a straight channel with bars. Later, Ackers (1982) gives the border line between the straight and meandering patterns, for sandy channels ($S_v = 0.0008Q^{-0.21}$). In these formulas, Q is in $m^3 s^{-1}$, but S_v means the valley slope.

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Knighton and Nanson (1993) completed the diagram with the anastomosing pattern, at low slope values. According to Henderson (1966), Carson (1984) and Fergusson (1987) the braiding depends on the grain size of the bed-load sediments: the larger the grain size, the greater will be the slope limit. According to van den Berg (1995), the channel slope is partly depends on the sinuosity, so he transformed it into valley slope (which is pattern-independent). In this way, the meandering and braided points overlapped. He found the meandering-braided transition is less dependent on the bankfull-discharge as in the original graph. Because of this he introduced the average of the annual floods, which is of the order of the bankfull discharge, but less dependent on the channel pattern.

The above mentioned works concern only of the sediment discharge, which is also an important factor of the river planform (Leopold and Wolman, 1957; Schumm, 1960) It was more involved in the work of Simpson and Smith (2001), studying 78 km section of the Milk River: 28 km meandering, 47 km braided, the last 3 km straight. They studied the slope and discharge values of the braided section. They found that the river does not belong to the range of braiding defined by Leopold and Wolmann (1957). The rivers studied by Leopold and Wolmann (1957) had gravel bed; the Milk River had sandy bed. Simpson and Smith (2001) suggested studying the silt-clay content, too, in these cases. The 65 % silt-clay content of the meandering section decreased to 18 % along the braided section. The channel become wider, the stream power and sediment transport decreased, and bars formed.

1.3 The quasi 3-D-graph

Timár (2003) merged the above mentioned graphs: the slope vs. sinuosity (based on flume experiments; Schumm and Khan, 1972) and the slope vs. discharge (based on natural rivers; Leopold and Wolman, 1957; Ackers and Charlton, 1970b). This quasi 3-D graph (Fig. 2a) suggests that the slope-sinuosity graph works for every discharge. The increasing slope increases the sinuosity until the sinuosity gets the maximum value

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of the given discharge. This value is the border of the self-organised and the unorganised meandering pattern (Timár, 2003).

Using the sinuosity values along the rivers of the Pannonian Basin (Petrovszki and Timár, 2010; Petrovszki et al., 2012), this graph was filled in with points, to create a real 3-D abstract surface.

2 Data and method

2.1 Sinuosity

The sinuosity values were calculated along the natural, uncontrolled meandering river beds of the Pannonian Basin (Petrovszki and Timár, 2010; Petrovszki et al., 2012). Nowadays, the rivers are controlled; the main river control works were made during the 19th century. The last natural channels can detect on the map sheets of the 2nd Military Survey of the Habsburg Empire. The sinuosity values were calculated with the formula of Schumm (1963), using a moving-window method and different window sizes (van Balen et al., 2008; Petrovszki and Timár, 2010; Petrovszki et al., 2012). To create the 3-D surface, the average sinuosity values calculated with 10 different window-sizes were used. Because of the calculation method, sinuosity values were available for every 50 m, but the slope and discharge values were less frequent.

2.2 Slope and discharge values

The slope and water discharge data were imported from the dataset of Viczián (1905). The survey was made after the river control works, so the slope values are higher like the original slopes were in their natural state. Viczián (1905) wanted to know the stream power of the rivers, so mainly the hilly and mountainous parts of the Pannonian Basin were surveyed. The slope and discharge values along the rivers of the Great Hungarian Plain were imported from Lászlóffy et al. (1965), so the lower slope and higher discharge points also could be analysed.

2.3 Bedrock control

The imported slope and discharge values needed some corrections. The first correction was: deleting the points along the antecedent valleys. These valleys are not alluvial but bedrock controlled, so these river sections are not belong to the self-organised meandering pattern.

Along the river Szamos (Someş) the antecedent river section is located near Jibou. In this area high sinuosity values appeared, mainly at higher window-sizes, because of the size of the bend. The average of the sinuosity values, calculated with 5–50 km window-size were used, so the points which were surveyed near Jibou, nearer than 25 km were deleted.

Not only one bedrock-controlled river section is located along the river Olt, so finally, no points were used from this river.

2.4 Calculating the bankfull discharge

On the original graphs (Leopold and Wolman, 1957; Ackers and Charlton, 1970b) the bankfull discharge was displayed. However, the used dataset measured only the mean water discharge. Williams (1978) collected 16 ways of determining the bankfull discharge, but none of them used the mean discharge. Williams (1978) compared the calculated bankfull discharges to the flow frequency, and defined their recurrence interval. Depending on the methods, it was between 1 and 5 years.

Viczián (1905) did not measure long time series of water discharges, either the maps did not display the cross sectional areas. So, these methods were not useable. Van den Berg (1995) shows the mean and bankfull discharges of many rivers. Using the meandering ones, a line was fitted on them (on a log-log scale). Along the small rivers the rate of the mean and bankfull discharge can change on a wide range. For the 1 $m^3 s^{-1}$ mean discharge, 10 and 1000 $m^3 s^{-1}$ bankfull discharge also appeared. Finally, the linear was fitted for the rivers with minimum 10 $m^3 s^{-1}$ mean discharge. For the lower discharges, no other line was fitted, because of the high deviation. The regression was

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made for 24 points, and its equation is: $\ln(Y) = 0.692\ln(X)+3.282$, where Y is the bankfull discharge, X is the mean discharge. Figure 1b shows the original points (van den Berg, 1995) and the fitted line (on a log-log scale; $R^2 = 0.727$). However, the line was fitted onto the points of natural rivers, which were affected by other parameters, too; the fitted line follows the trend of the points.

This fitting equation was used to generate bankfull discharges from the mean discharges. At the end of this method, the discharges become higher at least one order of magnitude (Fig. 1c).

2.5 Calculating the natural slope

During the main river control works in the 19th century the meandering rivers became straighter, their lengths shorted so their slope increased. Viczián (1905) measured the new slope values but for this present work, the original values needed. To correct these slopes, the notes of the river regulations were analyzed. These notes tell how the rivers were shortened. The natural slope of the river Tisza was published by Lászlóffy (1982). Along the rivers, surveyed by Viczián (1905) the slope corrections were made for every measured section of the rivers. The names of the villages next to the survey stations and their distance along the (controlled) river from the mouth were also described. The coordinates of these points were collected from the 2nd Military Survey maps, and their distance along the natural river could be defined. From these data, the difference between the distances along the natural and controlled rivers can calculated. In the area of the Great Hungarian Plain, the shortening could be about 40–50 %, while along the upper parts of the rivers, it often stayed under 10 %. The effect of the slope correction is not so significant like the mean-bankfull discharge conversion (Fig. 1c).

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2.6 Interpolation methods

The results can be displayed for every point, using a colour scale. However, the aim of the analysis was to verify the slope-sinuosity connection for a wide range of water-discharge, so a surface was fitted onto them.

First, different interpolation methods were tried, because these interpolations highlight the different features of the data. The best result was given by the local polynomial regression: a smoothed surface. This interpolation was used, because other parameters also affect the behaviour of the rivers. While all of these parameters could not be analyzed quantitatively (this was not the aim of this study), the slope-discharge-sinuosity connection was approached with this smoothed surface.

Lots of parameter cannot be modelled in flume experiments, e.g. the vegetation. The size of the flumes and the time of the experiments also should increase significantly, if the growing of the trees and brushes be waited. Nevertheless, the changing of the alluvium and the transported sediment is also a bit complicated.

In nature, several sort of sediment can find, and the sediment discharges also differ. Most of the flume experiments were made in sandy channels. The water can erode and move the sand quite fast, even if the water-discharge is low, so it is ideal for modelling. Ackers (1982) completed the results of Ackers and Charlton (1970a) with a new line, for sandy channels. Analysing the changing sediment discharge, the slope – bankfull discharge – sinuosity abstract surface could be improve.

2.7 Least squares regression

The "method of least squares" was invented by Gauss for fitting curves to measured points (coordinate pairs) on a planar Cartesian coordinate system. The simplest example is fitting a line to the points on a graph. Even if the points do not fit exactly on a single line, we want to draw a line that passes between the points.

In this example above the y coordinates of the measured points are regarded as "measured values" and the x coordinates are "measurement locations". The chosen

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linear function to be fitted is called "model". The coefficients of the model to be determined are called "unknowns". The error defined by Gauss, also known as "residual", can be expressed as the difference between the measured value and the value returned by the model function at a measurement location.

In our approach the sinuosity is approximated with a bivariate function of channel slope and discharge. So the sinuosity is z coordinate and the channel slope and discharge are x and y coordinates respectively. The model was chose as a quadratic (second order polynomial) function of x and y coordinates (a surface), that fits to the z coordinates. This problem is formulated in an array-matrix form (see Appendix for more details).

3 Results and discussion

Figure 1d shows the place of the points which were used for create the surface, on the bankfull-discharge – slope graph, after the above mentioned corrections. Figure 2b was created from these points, using local polynomial regression. At a given discharge, if there are enough points, the cross section is similar like the graph of Schumm and Khan (1972).

According to Stølum (1996) the expected sinuosity value is the π (3.14). Along the rivers of the Pannonian Basin, such high sinuosity value appears, if the bankfull discharge is enough high, however it is far from a general value, or even from a most probable figure. Also, as early as in the flume experiments of Schumm and Khan (1972) the maximum sinuosity occurred at the value of 1.3. In this study, the sinuosity values of the smallest rivers were around this value. At higher bankfull discharge higher sinuosity values appear. Figure 2d displays a cross section along the maximum sinuosity values of the fitted surface. At higher discharge values, the maximum sinuosity values are also higher. However, not only the higher discharge causes this, because it just affects the size of the bends. But more water can transport more sediment, which can affect the river pattern.

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The bankfull discharge–sinuosity relation, shown in Fig. 2d, probably works only in the Pannonian Basin, but in other areas, the graph should be somewhat similar. In nature, high sinuosity values also can appear at low discharges.

In the flume experiments of Schumm and Khan (1972) the discharge was $10^{-4}\,m^3\,s^{-1}$. At given bankfull discharges, the expected cross section should be similar like the graph of Schumm and Khan (1972; see the vertical graph of the Fig. 2a). There are not enough points at every bankfull discharge values to make a unique cross section along the original points. But between the bankfull discharge range of 20 and $30\,m^3\,s^{-1}$ the points have quite wide slope range. The resulted graph verifies the theory: after a given slope the sinuosity started to decrease (Fig. 2c).

The area where the increasing slope decreases the sinuosity called unorganized meandering (Timár, 2003) or wandering pattern (Miall, 1977) which is a transition pattern between the meandering and braided rivers. To make some checking, the places of the original points were analyzed on the maps. However, they are natural rivers, so other parameters also can affect them, the original river sections behave like the wandering rivers.

Another noticeable phenomenon appears in Fig. 2c. A curve was fitted along the points of the Pannonian Rivers (with bankfull discharge between 20 and $30\,m^3\,s^{-1}$), in the channel slope vs. sinuosity graph. Using the logarithmic slope scale, the curve is very similar to one of Schumm and Khan (1972). However, if we use a linear slope-scale, the fitted curve is rather a parabola (Fig. 2e and f). This similarity gives the idea to try to describe the slope-discharge-sinuosity connection with fitting parabolas, at every discharge value (Fig. 3a).

In the zone of the straight pattern (Fig. 3a), the sinuosity was set to unity by definition. At given discharges, the maximum sinuosity was chosen from the original data. Along the unorganised meandering section, other parabolas were fitted, and at the braided pattern, the sinuosity was set to 1.3. This value is selected as the average of the sinuosity values of this area. If the bankfull discharge was lower than $20\,m^3\,s^{-1}$, the sinuosity of the braided pattern was set to 1, because here the maximum sinuosity

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was lower than 1.3 (Fig. 3a). The resulted graph is a basic model: however in case of real rivers more or less difference occurs in any case. According to our opinion (which was not yet tested or analysed) the main cause of these differences is the effect by the sediment discharge and size distribution.

The other method to get a mathematical model to estimate the sinuosity values at given slope and discharge values was a surface fitting, using the least squares method (Fig. 3b; see Appendix). The channel slope–bankfull discharge plain was divided into 4 regions, and for every region a simple surface was fitted on the sinuosity values. In the straight pattern region, a horizontal plain was chosen with the sinuosity set to 1. In the other three pattern regions (self-organised meandering, unorganised meandering and braided patterns) three quadratic surfaces were fit. The parameters of these three surfaces were computed with constrains: the fitted surfaces at the margins of the regions have to be continuous.

The difference between the model-surfaces and the original dataset was calculated and displayed. (Fig. 3c). Comparing the original data to the parabolic model-surface, at lower discharges, the fitting is quite good, but high discharges the model is overestimated. Using the quadratic surface fitting method, no overestimation appeared at high discharges.

4 Conclusions

- Using the slope, discharge and sinuosity values of the rivers of the Pannonian Basin a model surface can be declared. This surface defines the probable sinuosity at given slope and discharge values. In this work 3 methods were analysed:
  - local polynomial fitting (using Surfer)
  - fitting parabolas at every water-discharge
  - quadratic surface fitting with least squares regression.

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– The curve of Schumm and Khan (1972) was displayed on a logarithmic channel slope scale. This slope-sinuosity connection is a parabola-like function. (This similarity gave the idea to make the parabolic fitting.)

– The slope, bankfull-discharge and sinuosity connection was displayed and model-surfaces were fitted (Fig. 3). Along the natural channels, the changing slope changes the sinuosity like in the flume experiments. The maximum RMS error remains under 15 %. The differences between the original data and the model surface are believed to be caused by the sediment load.

Appendix A

The model was chose as a quadratic function of x and y coordinates. Its general form:

$$z = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2.$$

For all the n measurements, the residuals form an array:

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & x_1^2 & x_1 y_1 & y_1^2 \\ 1 & x_2 & y_2 & x_2^2 & x_2 y_2 & y_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & y_n & x_n^2 & x_n y_n & y_n^2 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} - \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}.$$

According to the theory of least squares, the sum of the square of the residuals should be minimized.

In conventional form: $\sum_{i=1}^{n} v_i^2 := \min$

In array-matrix form: $\boldsymbol{v}^T \cdot \boldsymbol{v} := \min.$

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If we choose two neighboring regions, and we want to calculate the unknowns for both regions simultaneously, we can merge the arrays and the matrices. If we want the two surfaces coincide at the border of the two regions, we have to apply constrain. The constrains should be formulated in the following linearized way: $\mathbf{C} \cdot \boldsymbol{x} - \boldsymbol{w} = \mathbf{0}$ where $\boldsymbol{x}$ is the array of the unknowns, $\mathbf{C}$ is a matrix and $\boldsymbol{w}$ is an array.

Along the $y = mx + b$ line both model equations should return the same value. Substituting the line equation into both model equations, we get an equation that should be fulfilled at any x value:

$$a_0^{(I)} + a_1^{(I)}x + a_2^{(I)}(mx+b) + a_3^{(I)}x^2 + a_4^{(I)}x(mx+b) + a_5^{(I)}(mx+b)^2$$
$$= a_0^{(II)} + a_1^{(II)}x + a_2^{(II)}(mx+b) + a_3^{(II)}x^2 + a_4^{(II)}x(mx+b) + a_5^{(II)}(mx+b)^2.$$

To provide, that this equation always fulfilled, the constant term, the term multiplied by x and the terms multiplied by x^2 should be zero.

$$a_0^{(I)} + a_2^{(I)}b + a_5^{(I)}b^2 = a_0^{(II)} + a_2^{(II)}b + a_5^{(II)}b^2$$
$$a_1^{(I)} + a_2^{(I)}m + a_4^{(I)}b + 2a_5^{(I)}mb = a_1^{(II)} + a_2^{(II)}m + a_4^{(II)}b + 2a_5^{(II)}mb$$
$$a_3^{(I)} + a_4^{(I)}m + a_4^{(I)}b + a_5^{(I)}m^2 = a_3^{(II)} + a_4^{(II)}m + a_4^{(II)}b + a_5^{(II)}m^2.$$

These three equations after ordering also can be rewritten in the array-matrix form.

The condition equations, fulfilling the minima of the residuals, and fulfilling the constrain equations are called conditional least squares. In this case the sum to be minimized is:

$$\boldsymbol{v}^T\boldsymbol{v} - 2\boldsymbol{k}(\mathbf{C}\boldsymbol{x} - \boldsymbol{w}) := \min.$$

To find the extremum of the expression above, we derive it with respect to the unknowns ($\boldsymbol{x}$) and the Lagrange multipliers ($\boldsymbol{k}$). At the minimum these derivatives are zero. To

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rearrange these equations in a matrix form, expressing the unknowns and the Lagrange multipliers:

$$\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{k} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^T\mathbf{A} & -\mathbf{C}^T \\ -\mathbf{C} & \mathbf{0} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{A}^T \cdot \boldsymbol{l} \\ -\boldsymbol{w} \end{bmatrix}.$$

If we use the $\boldsymbol{x}$ array of unknowns calculated in this way in the model equations, the two surfaces already surely coincide at the border of the two regions.

We might add another constrains, like the two surfaces should form a smooth surface at the border. In this case we have to formulate the constrain and rearrange the constrain equations, in an array-matrix form and merge them to the **C** matrix and the $\boldsymbol{w}$ array.

Acknowledgements. The research is made in the frame of project OTKA-NK83400 (SourceSink Hungary); Water Management Research Group of the Hungarian Academy of Sciences. The European Union and the European Social Fund also have provided financial support to the project under the grant agreement no. TÁMOP 4.2.1./B-09/1/KMR-2010-0003.

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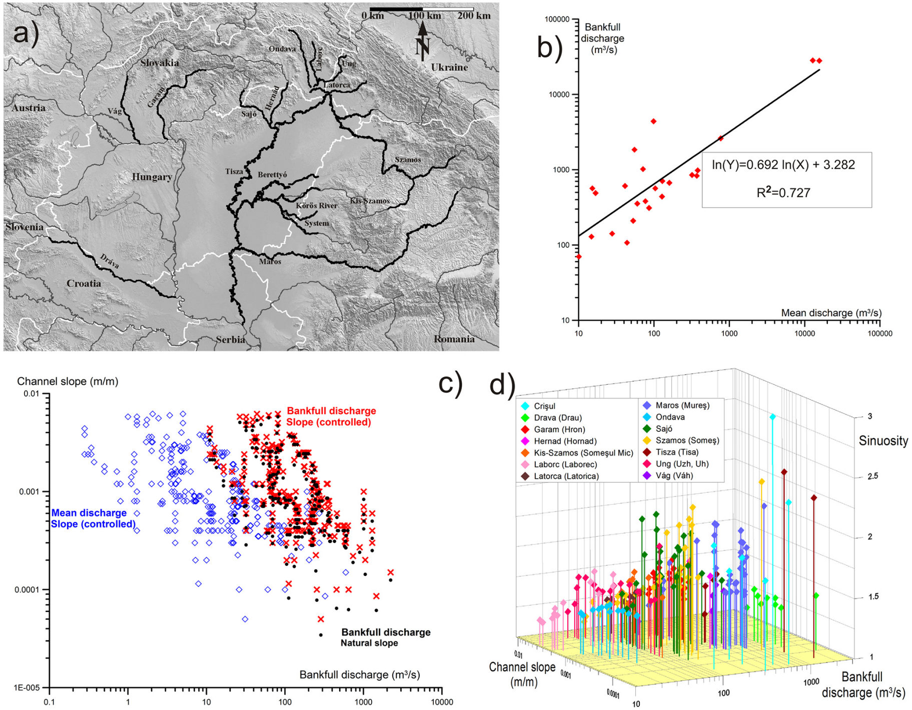

Figure 1. Corrections and points. **(a)** The water discharge, slope and sinuosity values of the displayed rivers (black) of the Pannonian Basin were used for this study; **(b)** the mean and bankfull discharges of van den Berg (1995), and the fitted line; **(c)** the result of the corrections; **(d)** the abstract surface was fitted onto these points (see black points on **c**). The different colours mean different rivers.

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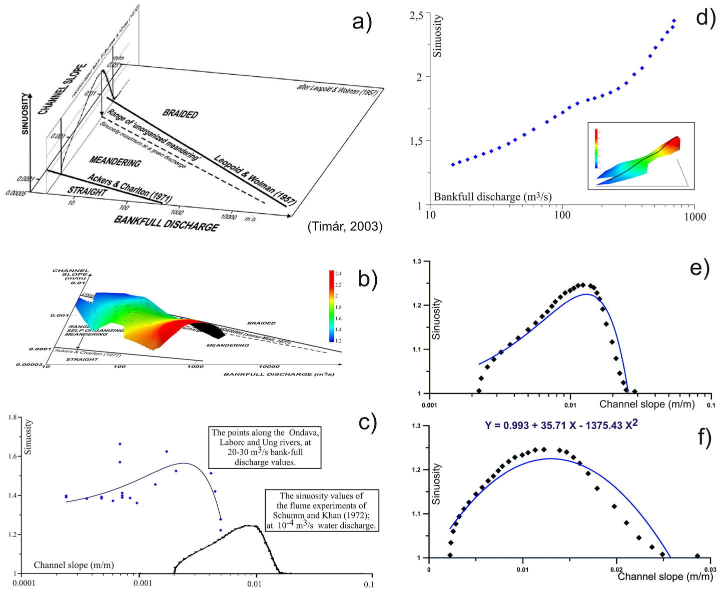

Figure 2. Theoretical and fitted cross sections. **(a)** The quasi 3 dimensional graph of Timár (2003) suggests, that the slope-sinuosity connection (Schumm and Khan, 1972) works for every bankfull discharge value. **(b)** The abstract surface shows the most probably sinuosity along the rivers of the Pannonian Basin (Fig. 1a and d), depending on the slope and bankfull discharge, using local polynomial regression. **(c)** The slope and sinuosity values at 20–30 $m^3 s^{-1}$ bankfull discharge values, and the fitted curve (polinom regression). The black line shows the result of Schumm and Khan (1972; 10^{-4} $m^3 s^{-1}$). Their original graph displayed this by depending on the valley slope, but it was converted to channel slope by Timár (2003). **(d)** Cross section along the maximum sinuosity values (at given discharges); at higher discharges, the sinuosity is also higher, but it is not a linear relation. **(e)** The channel slope–sinuosity graph of Schumm and Khan (1972; black dots); and the fitted parabola (blue line) using logarithmic channel slope scale and linear channel slope scale **(f)**.

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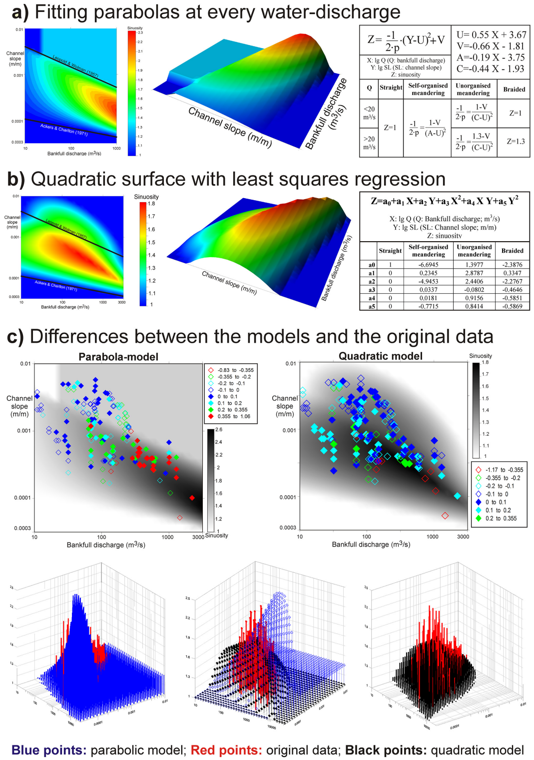

$$Z=\frac{-1}{2\cdot p}\cdot(Y-U)^2+V$$

X: lg Q (Q: bankfull discharge)
Y: lg SL (SL: channel slope)
Z: sinuosity

$U=0.55\,X+3.67$
$V=-0.66\,X-1.81$
$A=-0.19\,X-3.75$
$C=-0.44\,X-1.93$

Q	Straight	Self-organised meandering	Unorganised meandering	Braided
<20 m³/s	Z=1	$\frac{-1}{2\cdot p}=\frac{1-V}{(A-U)^2}$	$\frac{-1}{2\cdot p}=\frac{1-V}{(C-U)^2}$	Z=1
>20 m³/s			$\frac{-1}{2\cdot p}=\frac{1.3-V}{(C-U)^2}$	Z=1.3

$$Z=a_0+a_1X+a_2Y+a_3X^2+a_4XY+a_5Y^2$$

X: lg Q (Q: Bankfull discharge; m³/s)
Y: lg SL (SL: Channel slope; m/m)
Z: sinuosity

	Straight	Self-organised meandering	Unorganised meandering	Braided
a0	1	-6.6945	1.3977	-2.3876
a1	0	0.2345	2.8787	0.3347
a2	0	-4.9453	2.4406	-2.2767
a3	0	0.0337	-0.0802	-0.4646
a4	0	0.0181	0.9156	-0.5851
a5	0	-0.7715	0.8414	-0.5869

Figure 3. The parameters of the most probable sinuosity abstract surfaces, using different fitting methods. **(a)** At every discharge values, parabolas were fitted along the meandering river sections, for every slope and sinuosity values. Along the straight and braided patterns, constant values were used. **(b)** A quadratic surface was also fitted to estimate the most probable sinuosity values. Along the straight patterns, a constant value was used, along the other patterns, different parameters were calculated. **(c)** The differences between the model surfaces and the original data.

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