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Uncertainty analysis of a spatially-explicit annual water-balance model: case study of the Cape Fear catchment, NC

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Abstract

There is an increasing demand for assessment of water provisioning ecosystem services. While simple models with low data and expertise requirements are attractive, their use as decision-aid tools should be supported by uncertainty characterization.

We assessed the performance of the InVEST annual water yield model, a popular tool for ecosystem service assessment based on the Budyko framework. Our study involved the comparison of ten subcatchments in the Cape Fear watershed, NC, ranging in size and land use configuration. We analyzed the model sensitivity to the eco-hydrological parameters and the effect of extrapolating a lumped theory to a fully distributed model. Comparison of the model predictions with observations and with a lumped water balance model confirmed that the model is able to represent differences in land uses. Our results also emphasize the effect of climate input errors, especially annual precipitation, and errors in the eco-hydrological parameter Z , which are both comparable to the model structure uncertainties. In practice, our case study supports the use of the model for predicting land use change effect on water provisioning, although its use for identifying areas of high water yield will be influenced by precipitation errors. While the results are inherently local, analysis of the model structure suggests that many insights from this study will hold globally. Further work toward characterization of uncertainties in such simple models will help identify the regions and decision contexts where the model predictions may be used with confidence.

1 Introduction

The interactions between hydrology and land-use and land-management decisions have received increased attention in recent years. The International Association of Hydrological Sciences (IAHS) recently declared this decade *Panta Rhei* – everything flows – to focus on the changing dynamics of the water cycle in connection with

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Carolina. This study quantifies the effect of parameter uncertainty on model outputs through sensitivity analyses; compares the distributed application of the water balance to the catchment-scale application; and quantifies the accuracy of calibrated and uncalibrated versions of the model by comparing model predictions to observations.

- 5 From a practical standpoint, this work helps InVEST model users identify modeling uncertainties and proposes simple and replicable methods that can be used to quantify their effect on water services.

2 Methods

Errors in hydrologic model predictions can be separated into three sources: the structural error associated with model formulation and scale, error in parameter selection, and error in the model inputs. To assess these three sources, we applied the InVEST annual model to ten subcatchments in the Cape Fear basin, NC. Their co-location implies a similarity in climate and seasonality and facilitates a focus on variations in land-use, size and topography (Hrachowitz et al., 2013). The following sections provide the description of the model and case study, the methods for the sensitivity analyses, the assessment of input data errors, and the evaluation of model performance.

2.1 InVEST annual water yield model

2.1.1 Background theory

20 The Budyko curve is a unique empirical function that relates the ratio of actual to potential evapotranspiration (averaged over a catchment and over many years) to the ratio of precipitation to potential evapotranspiration (Budyko, 1979). The function is bounded by two limits – an energy limit in which actual evapotranspiration is equal to potential, and a water limit for which actual evapotranspiration is equal to precipitation.

25 Due to spatial and temporal variability in climate forcing, the asynchronicity of water

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supply (P) and demand (PET), the imperfect capacity of the root zone to buffer that asynchronicity, and lateral redistribution of water within the catchment, the Budyko curve lies below those two limits (Fig. 1).

To describe the degree to which long-term catchment water-balances deviate from the theoretical limits, a number of scholars have proposed one-parameter functions that can replicate the Budyko curve (e.g., Fu, 1981; Choudhury, 1999; Zhang et al., 2004; Wang and Tang, 2014). The InVEST water yield model employs the formulation by Zhang et al. (2004), which incorporates a parameter, ω :

$$\frac{AET}{P} = 1 + \frac{PET}{P} - \left[1 + \left(\frac{PET}{P} \right)^\omega \right]^{1/\omega} \quad (1)$$

AET is the actual evapotranspiration (mm), P is precipitation (mm), PET is the potential evapotranspiration (mm). Larger values of ω indicate those basins that are more “efficient” in converting precipitation to transpiration, e.g., those with precipitation synchronous with PET and/or with deeper root zones. Gentine et al. (2011) and Troch et al. (2013) have shown that the natural co-evolution of vegetation, climate, and topography may lead to basins for which the effects implicitly captured by ω counter-balance each other, offering an explanation for the observed convergence of data along the Budyko curve. The intent of the InVEST model, however, is to predict the effects of human-induced changes, i.e., to examine catchments for which natural co-evolution is disrupted.

2.1.2 Spatially-explicit application to land-use change

Model overview

To represent parcel-level changes to the landscape, InVEST represents explicitly the spatial variability in precipitation and PET, soil depth, and vegetation. The model is GIS-based, using rasters of climate and soil properties as inputs (see Tallis et al., 2013 for full details).

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For vegetated land uses, InVEST applies the Zhang formulation in a spatially explicit way at the pixel scale (10 to 100 m on a side):

$$\frac{AET_i}{P_i} = 1 + \frac{PET_i}{P_i} - \left[1 + \left(\frac{PET_i}{P_i} \right)^{\omega_i} \right]^{1/\omega_i} \quad (2)$$

In contrast to Eq. (1), P , PET , ω , and AET are all functions of the local position, indicated by the subscript i .

The parameter ω is further deconstructed to separate the effects of soil depth, rainfall frequency, and other factors, following an approach proposed by Donohue et al. (2012):

$$\omega_i = Z \frac{AWC_i}{P_i} + 1.25, \quad (3)$$

where AWC_i is the plant-available water content (depth), and Z is an empirical parameter. The constant, 1.25, in Eq. (2) reflects the minimum value of ω corresponding to bare soil, following Donohue et al. (2012). In this representation, differences in land-use and land-cover affect both PET , through changes to the crop coefficient, K_c , and Z , through changes to the root depth and plant-available water content.

For open water, wetlands, and urban land-uses, InVEST computes AET_i directly as a user-defined proportion of PET_i , with classical approaches such as the FAO 56 guidelines (Allen et al., 1998) or local knowledge used to determine the appropriate proportion (Tallis et al., 2014). The simple representation of these LULCs, compared to the vegetated land uses modeled with Eqs. (2) and (3), reflects the focus of the model on vegetation-dominated landscapes.

Total evapotranspiration from a catchment is computed as the sum of AET_i attributed to each cell, and water yield is obtained by subtracting this value from the total precipitation.

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Selection of the Z parameter

The empirical constant Z captures catchment-wide characteristics of climate seasonality, rainfall intensity, and topography that are not described by the plant-available water content (AWC) and annual precipitation P . Given the empirical nature of the model, the value of the Z parameter remains uncertain. In this work, we examine the three methods for the determination of Z that are proposed in the InVEST user's guide (Tallis et al., 2014). The first draws upon recent work that suggests that Z is positively correlated with the average annual number of rain events per year, N , and that Z may be approximated by $N/5$ (Donohue et al., 2012). This implies that Z captures rainfall patterns, distinguishing between catchments with similar annual precipitation but different intensity. The second method relies on globally available estimates of ω (e.g. Liang and Liu, 2014; Xu et al., 2013). Z is inferred from these published values of ω by inverting Eq. (2) with values of AWC and P averaged over the catchment. In the third method, Z is determined via calibration to streamflow data (see Sect. 2.5).

2.2 Cape Fear study area

The Cape Fear catchment is a 23 600 km² area in North Carolina. Its major land uses are forest (40 %), wetland (15 %), grassland (14 %), and agriculture (12 %), mainly in the lower parts of the watershed and including intensive swine and poultry farms. Urban and agricultural development has generated significant groundwater extraction throughout the catchment.

The climate is humid subtropical, with a precipitation average of ~ 1200 mm over the 2002–2012 study period (Table A1). This period was used for the analyses based on the longest period available for climate data, observed streamflow, and matching LULC map. The available precipitation data comprise the PRISM dataset (Gilliland, 2003) and a network of eight rain gauges maintained by the USGS (USGS, 2014). For our analyses, we use the PRISM data and two additional rasters interpolated from

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variance among the county withdrawals. The average withdrawal rate, 39 mm year⁻¹, was subtracted from the predicted water yields for comparison with observations.

2.3 Sensitivity to Z and K_c

Step one in our assessment of the InVEST model was a local sensitivity analysis of water yield to the Z parameter and the crop coefficient, K_c , for forest – the dominant LU class. The sensitivity of the model to Z can also be interpreted as the sensitivity to AWC, when the raster values are varied homogeneously over the catchment, since these parameters play a similar role in the model structure (Eq. 3).

As noted above, large uncertainties surround the selection of the Z parameter (Tallis et al., 2014). For what we term the “baseline” case, we set Z equal to one-fifth the number of rain days per year ($Z = N/5$). Based on historic precipitation data (SERCC, 2014), the average number of rain days per year is approximately 110, giving a value of Z of 22. We used this value as a baseline for all subcatchments, and allowed the parameter to vary between 1 and 30 for the sensitivity analyses. This range was estimated from Eq. (3) used with extreme values of P and AWC found in our catchments, and extreme values of ω (2.1 and 3.75) found in the study by Zhang et al. (2004).

Forest was the dominant LULC in all basins, with its cover ranging from 43 to 72 % of subcatchments. We therefore decided to use the crop factor K_c -forest for the sensitivity analyses, and a baseline value of 1 for K_c forest was obtained from the FAO 56 guidelines (Allen et al., 1998). Uncertainties on this value are large since it remains difficult to provide accurate estimates of the actual evapotranspiration of forest (McMahon et al., 2013). We set the upper bound to 1.1, because values greater than this are unlikely (McMahon et al., 2013), and set the lower bound to 0.7.

For the two parameters, we performed sensitivity analyses with the ranges defined above. The results are presented as a change in predicted water yield compared to the baseline run, thus assessing absolute sensitivity. Precipitation and reference

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evapotranspiration used for these runs were from the PRISM and the FAO datasets, respectively (see Sects. 2.5 and 4 for insights into the error introduced by climate data).

2.4 Comparison of distributed and lumped application of the water-balance model

Although the InVEST annual water yield model is based on the well-studied Budyko framework, it departs from its classical application by applying the partitioning model at the pixel scale. To our knowledge, the effect of the pixel-by-pixel calculation performed by InVEST has not been previously studied. Therefore, we compared the model predictions to those obtained by applying the Zhang model at the catchment scale, therefore applying the Budyko framework in a more classical way. Application of such a lumped model requires a value of ω , which we derived from Eq. (3) with average values of P , PET , and AWC , and with Z set to the baseline value of 22, as would be done in a typical ungauged application. We thus obtained, for each subcatchment, an estimate of areal AET and water yields for the vegetated areas. AET for urban areas and wetlands was calculated separately, following the same method as InVEST, and total water yield was calculated as the area-weighted average of yield from the vegetated and urban areas.

2.5 Performance of the InVEST model

To quantify the accuracy and precision associated with the InVEST water-yield model, we assessed model performance by comparison with observed data for each of the ten subwatersheds in the Cape Fear area. We measured performance with the model bias, i.e. the relative difference between predicted and observed yields, and also with the subcatchment ranking by water yields. The ability of the model to predict ranking is important for applications where prioritization of areas of low and high yields is needed (Guswa et al., 2014).

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2.5.1 Uncalibrated model

We first examined the performance of the model when Z was determined without calibration. We considered calculating Z both from the number of rain days and from a global value of ω , to evaluate the appropriateness of these recommended methods.

5 In addition to assessing overall model performance, we also assessed the correlation between model performance and the proportion of forest in the catchment. These analyses aimed to identify a potential bias that may be corrected by modifying the LULC-specific crop factor K_c .

2.5.2 Calibrated model

10 To separate the effects of error associated with model structure from error attributed to parameter estimation, we also determined the value of Z via calibration. We calibrated to individual watersheds, identifying for each subcatchment the Z value that resulted in a zero error in the water yield. We examined the similarity of Z values across the ten basins, allowing us to assess the robustness of the model structure since we expect

15 Z to depend on larger-scale climate and geology and not on local-scale land-use. We also considered the performance of the model with a single value of Z applied to all subcatchments, determined by minimizing the average bias for all basins. This allowed us to assess the uncertainty in prediction of water yield due to model structure, i.e., the inherent uncertainty to applying Eqs. (2) and (3) to different basins even when the

20 parameter, Z , is chosen by best fit.

2.5.3 Comparison with errors in climate inputs

To provide context for the uncertainty in the predictions of water yield from the InVEST model, we compared the prediction error to the uncertainty in water yield that arises from uncertainty in climate (i.e., variability in the rasters of P and ET_0). Uncertainties

25 in climatic data and their impact on rainfall–runoff models are commonly cited in the

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literature (McGlynn et al., 2012; McMahon et al., 2013). To be an effective decision-support tool, errors attributed to model structure and parameter selection should be on par with or less than the irreducible error associated with uncertainty in the inputs.

As illustrated in Table 1, the mean precipitation differed significantly across subcatchments: the differences between the PRISM and USGS datasets, with the spline or IDW interpolation methods, respectively, were -14 and -13 %. The difference was more spatially heterogeneous with the spline method, with some subcatchments receiving less precipitation relative to the baseline (PRISM dataset) and others receiving more. The reference evapotranspiration data also showed significant differences across sources, although the FAO and Climate Office sources showed good agreement. The MODIS values were 22 % higher on average than those from the other two sources. Differences between the Climate Office and FAO data were spatially variable, being positive for some subcatchments and negative for others.

To assess the uncertainty in water yield due to variability in climate inputs (precipitation and reference evapotranspiration), we examined the sensitivity of the baseline model results to spatially homogeneous increases and decreases in climate forcing. We considered climate inputs that are 10 % greater and 10 % less than the baseline.

3 Results

3.1 Sensitivity of water yield to climate, Z , and K_c

Water yield predictions are very sensitive to climate inputs. The sensitivity is higher for precipitation than ET_0 . A 10 % increase in precipitation resulted in a 30 % increase in yield, while the same increase in ET_0 resulted in a 15 % decrease in yield.

In contrast to the climate variables, water yield is less sensitive to values of Z : for example, a change in Z from the baseline value of 22 to a value of 10 results in an increase in yield of approximately 27 % (Fig. 3). However, given the large uncertainties

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in the Z parameter, potential errors in water yield can be large: for example, the water yield is 155 % higher when Z is at its minimum value, relative to the baseline case with $Z = 22$. The sensitivity to Z is catchment-specific, as expected, since its effect on yield is modulated by AWC and P , both of which are spatially variable. In addition, the relative sensitivity of yield to Z decreased with increasing values of Z and increased with increasing values of the aridity index (PET/P , results not shown).

The model was found to be more sensitive to K_c (Fig. 3) with a 30 % change in K_c resulting in a 41 % change in the water yield. However, given the expected range of K_c values, the effect of parameter uncertainty on the yield prediction is lower than for Z .

3.2 Comparison of spatially explicit and lumped models

Across the ten subcatchments, the water yields predicted by the spatially explicit InVEST model were on average 10 % lower than the outputs from the lumped Zhang model (Table 2). For eight of the ten catchments, the spatially explicit model predicted lower yields than the lumped model, and differences ranged from from –24 to 14 %. The two catchments for which the lumped model predicted lower yield than the InVEST model were the Morgan Creek and Cane Creek catchments, which have the highest proportions of forest and the lowest proportions of urbanized area across the ten catchments (Table 2).

3.3 Performance of the InVEST model

3.3.1 Uncalibrated model

Figure 4a presents predictions of water yield from the invest model when the Z -parameter is determined from the number of rain days ($Z = 22$). The performance of the model for the baseline run was fair, with the bias between predicted and observed values averaging –16 % for all subcatchments. This bias ranged from –53 to –1 %, implying that this choice of Z leads to a systematic underestimation of water yield. With

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the exception of one catchment, the biases ranged from -25 to -1 %. The outlier with an error of -53 %, Rockfish catchment, is relatively small (237 km^2), and the observed water yield is also an outlier, being the highest in the dataset (367 mm). This area is also characterized by sandy soils; the plant available water content averages 0.11 , compared to values between 0.17 and 0.20 for the other subcatchments. This suggests that the catchment may exhibit a unique behavior, which we will highlight in the following analyses. Across all basins, predicted yields range from 163 – 322 mm year^{-1} vs. an observed range of 177 – 368 mm year^{-1} .

Figure 4b presents the ranking of catchments in terms of their observed and predicted yields. Discarding the outlier catchment, the figure indicates that the model accurately predicts the high and low ranking catchments, while there is some dispersion in ranks for the five mid-range yields, which vary from 236 to 289 mm year^{-1} .

For the second case, when Z is determined from published values of ω , the model performance was not satisfying. The Z value found for all subcatchments averaged 6 , which results in a large model bias (averaging 68 %).

3.3.2 Calibrated model

When Z is determined through calibration for each subcatchment, values of the parameter range from 6 to 20 . The calibrated value of 6 was obtained for the Rockfish catchment; discarding that outlier catchment, values range from 10 to 20 , averaging 14.5 . This variability translates into relatively small changes in water yield – the average difference among the basins is 27 %. The single Z value obtained by minimizing the average subcatchment bias ($Z = 14$) is similar to these individual Z values. With this calibrated value, the error in yield for all subcatchments ranges from -38 to 14 % with a median value of -3 %. Predicted yields range from 183 to 336 mm year^{-1} vs. an observed range from 177 to 368 mm year^{-1} . Figure 4a presents model predictions of water yield vs. the observed values across the ten catchments. Open circles represent results from the calibrated INVEST model, while black bars represent the uncertainty

in yield due to a 50 % uncertainty in water withdrawals. Gray bars represent the uncertainty in predicted yield due to a 10 % uncertainty in precipitation.

Model bias is not correlated with forest cover ($R^2 = 0.01$), nor with any other LULC (Table 1). The absence of systematic bias suggests that K_c values are in a realistic range, with no significant error due to LULC parameter selection. No significant bias was detected with regard to catchment size, suggesting that this characteristic did not systematically influence the model predictions either.

4 Discussion

4.1 Sensitivity to Z and K_c

Variability in the Z parameter, which is linearly related to ω , results in a shift of the Zhang curve, which affects water yield predictions (Fig. 1). Our results suggest that the sensitivity of water yield to Z is low compared to the climate inputs, and decreases for larger values of Z (Fig. 3). This is consistent with the Zhang model for which the sensitivity to ω , decreases with increasing values of ω (Fig. 1). Due to this low sensitivity, small errors in estimating Z are likely to have limited impact on the reliability of water yield predictions.

The sensitivity to Z also provides a sense of the sensitivity to AWC, which is a function of the local ecohydrological properties: plant available water content, root depth and soil depth (cf. Tallis et al., 2014 for details). Examination of Eq. (3) suggests that a relative change in Z has the same effect as a relative change in these ecohydrological parameters: a 50 % error in these parameters, if assumed homogeneous over the catchment, will have the same response as a 50 % error in Z . Given the typical confidence interval for these measurable parameters, the uncertainty on these parameters will have a smaller effect on model outputs than the uncertainty in Z .

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When analyzing the model sensitivity to K_c , two things are to be considered. First, the K_c value affects only the portion of the landscape covered with forest, and this reduces its effect. Because total water yield is the sum of the yields from the different parts of the landscape, parameters affecting only a portion of the landscape will have a smaller effect. Second, it is worth noting that the K_c coefficient directly affects PET for a given LULC, since the latter is the product of K_c by ET_0 . Examining the sensitivity of the model to K_c is therefore equivalent to a displacement along the Zhang curve, rather than a shift of this curve (Fig. 1).

The results of the sensitivity analyses indicate that embedded in the Zhang model is the concept that the dominant effects of land-use and land-cover change on water yield will be via the effects on K_c and PET rather than through changes to root depth and plant-available water content.

4.2 Comparison of spatially explicit and lumped models

Comparison of the model predictions with the classical lumped model application suggests three insights. First, it provides a sense of the effect of the pixel-by-pixel application of the Budyko theory, which has not received much attention in the literature. Because of its non-linear nature, the average response of Eq. (2) applied across the landscape in a spatially explicit way is not equivalent to the response of the function applied to the entire watershed, characterized by average parameters. Our results suggest that this discretization effect is not large for the Cape Fear watersheds, with the difference between the lumped and explicit models ranging from -24 to $+14$ %. This range is consistent with the typical errors expected from the application of simple empirical models. This point can be illustrated by the performance of the lumped model when compared with the observations: bias ranges from -36 to 29 %. It is worth noting that the larger, positive biases (> 22 %) were obtained for the two subcatchments that had > 25 % urban cover, and the three basins with the least urban cover (Cane Creek, Rockfish, and Morgan Creek) had the largest underestimates of yield. These results suggest that the contribution from urban areas was overestimated by the simple model.

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Second, the good agreement between the InVEST model and the lumped model allows to draw on the large body of work investigating the performance of the latter model. For example, Zhou et al. (2012) report a bias of less than 20 % in a long-term study of 150 large basins worldwide; similarly, Zhang et al. (2004) report a mean absolute error of < 60 mm in their study of over 470 catchments worldwide, corresponding to a bias < 10 % for the majority of the catchments. Other local examples may be drawn by users to understand how the Budyko theory may apply locally (e.g. Yang et al., 2007 in China). Overall, there is a large ongoing effort to improve the parameterization and predictive use of the Budyko framework (Donohue et al., 2012; Liang and Liu, 2014; Xu et al., 2013). Future progress may therefore be used to refine the InVEST model interpretation in different geographic contexts. We note, however, that the agreement between the lumped model and the catchment model is context specific. As illustrated in Table 2, the differences between the lumped model and the InVEST model will vary between catchments, such that extrapolation of the results from such studies will need to be done cautiously.

The final point is based on the observation that yields predicted by the spatially explicit model were consistently less than those predicted by the lumped model. This difference could be due to differences in mean parameter values or due to the non-linearity in Eq. (2). Looking at Fig. 1, the concave nature of the Zhang curve indicates that the average response over a range of climates will lead to lower evapotranspiration and higher yields than if the equation were applied to the mean climate. Similarly, application over a range of values of ω would also lead to higher yield than what is predicted using the mean yield (Fig. 1). In this case, the lower yields predicted by the explicit model are due to differences in the mean values of ω between the lumped and explicit models. This indicates that the empirical expression for Z , developed for a lumped application (e.g., Donohue et al., 2012), may give values of Z (and, therefore, ω) that are too large when used in a spatially explicit model. Use of a smaller value of Z in the spatially explicit model would increase yield, although further studies would be

study, the length of the precipitation and streamflow data did not allow conducting such temporal analyses. Regional analyses where space is substituted for time thus represent a powerful way to assess the ability of the model to capture differences in LULC configuration.

5 **4.3.2 Ungauged catchments**

Another important lesson from the analyses is that the calibrated Z value is relatively close to the baseline value, which was derived independently from the average annual number of rain events. Based on Fig. 3, using one value or the other would result in a difference in yield of approximately 10 %. This error is small compared to other
10 model uncertainties, suggesting that this method for determining Z is robust. The underprediction of yield for ungauged catchments could be explained by errors in the precipitation raster, the Z parameter, and the treatment of water withdrawals. Based on Eq. (2), the negative bias implies the underestimation of the precipitation data or overestimation of the Z coefficient. As already noted, errors in precipitation data are
15 difficult to characterize. However, precipitation was more likely underestimated in this study since it did not include snowfall.

Conversely, the method relying on a constant ω value was not found satisfying for this case study, since it resulted in large overestimation of the water yields. Using $\omega = 4$, the Z value found for individual subcatchments ranged from 4 to 8, averaging 6, a value
20 that results in a large model bias (averaging 68 %).

With regard to relative yield values, the model was able to predict subcatchment ranks fairly accurately (Fig. 4b), which means that priority areas would be correctly identified. The uncertainties in ranking for medium yield catchments (ranking from 3 to 6) could be partly explained by their similarity (observed yields range from 236 to
25 278 mm) and the uncertainty in the water abstraction, as suggested by the overlapping error bars in Fig. 4a. Interestingly, although these results were obtained with the calibrated value of Z , they are only slightly sensitive to the value of Z , since the ranking of subcatchments is largely maintained when the value of Z changes. The ranking of

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subcatchments based on the baseline run, for example, was identical to the one with $Z = 14$.

4.4 Practical implications

In this final section, we discuss the results with a focus on practical implications for model users.

Our analyses suggest that the uncertainty introduced by *variability in the precipitation inputs* is high, comparable or higher than the uncertainty introduced by the parameter Z and the use of the lumped model theory on a pixel-by-pixel basis. This suggests that confidence intervals for climate data deserve particular attention (especially if interpolating local data from weather stations). The comparison of three climate input data sources suggested that large errors may occur when using point data or datasets obtained with different modeling assumptions. These results confirm a wide body of research that highlight the importance of precipitation inputs for rainfall runoff models (McGlynn et al., 2012; Zhou et al., 2012) and in particular for the InVEST model (Boithias et al., 2014; Sánchez-Canales et al., 2012). Although it was outside the scope of this study to investigate which climate datasets are less prone to errors, our results also draw attention to spatially heterogeneous errors. If model users are interested in the relative ranking of subcatchments, the spatial distribution of errors should be specifically investigated (e.g. probability of a systematic bias in mountainous areas).

The model is not very sensitive to *uncertainty in Z* over a modest range (e.g., 14–22). This is consistent with the conclusions from Sánchez-Canales et al. (2012), who report a low sensitivity to Z in a Mediterranean watershed, for which Z varied between 7 and 9. Since the viable range of Z is quite wide, however, it is possible that large uncertainties in that parameter will translate to significant uncertainty in yield (Fig. 3). Our analyses provided further insights into the methods for Z selection and highlighted that the sensitivity of the model to Z decreased with increasing values of Z . Based on the examination of Eq. (2), this property will apply generally. Therefore, in temperate

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climates where values of Z are high (based on the interpretation of Z as the number of annual rain events), the model outputs are likely to be less sensitive to this parameter.

Our study also presented a method to detect a *bias related to the LULC parameters*, when multiple observations are available in a catchment. Because K_c values are LULC-specific, the correlation between model performance and K_c values can be used to identify a possible error in the parameter and rectify the values accordingly. No bias was found in this study, bringing confidence in the ability of the model to capture the differences in LULC. We note that these correlation analyses rely on nested subcatchments that are not independent from each other, which decreases the significance of the relationship: five subcatchments are independent, while the other five partially overlap. However, proportions of forest cover varied widely between all subcatchments (from 43 to 72 %), which justifies our interpretation of the analyses.

We conclude this section with a perspective on the model performance assessment, highlighting key *limitations in the calibration/testing* exercise. First, we note that some water transfers are missing in the model, including irrigation and water abstraction. The model represents agriculture in the same way that it does natural vegetation, and irrigation is not included explicitly. Second, in the Cape Fear catchment, the magnitudes of the water withdrawals are small but this aspect of the modeling may be improved in future applications. In particular, distinction between uses of groundwater (crop irrigation or drinking water) are necessary to account for the fate of water extraction: evapotranspiration in the case of irrigation water, or return flow to the river in the case of drinking water (e.g. Terrado et al., 2014). Additionally, performance was evaluated at the catchment scale. A potential benefit of a spatially explicit model, however, is the ability to predict patterns of water yield within a basin. To properly evaluate that capability, further work should focus on comparing the InVEST model to more sophisticated fully distributed models.

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5 Conclusion

Our study aimed to assess the performance of the InVEST annual water yield, a tool that is gaining interest in the ecosystem services community. While such simple models with low requirements for data and level of expertise are needed for practical applications, greater attention should be paid to characterizing the modeling uncertainties. Our assessment of the potential input errors, sensitivity analyses and comparison with observations in the Cape Fear catchment add to this body of work. Key results of the analyses are as follow:

- In the Cape Fear catchment, the InVEST model was most sensitive to uncertainty in the precipitation forcing.
- Errors in climate input data may be significant and non-spatially homogeneous, resulting in uncertainties in the assessment of zones of high and low water yields.
- The study supports the recommendations for setting the Z parameter based on the number of rain events, or via calibration with available observed data.
- Based on the average bias and the explained variance in yield among the subcatchments, the model performance was fair to high, suggesting that the effects of land-use and land-cover are adequately captured by the model.
- The errors potentially introduced by a pixel-level application of the Budyko theory will depend on catchment configuration; in Cape Fear, they remained small, comparable to the climate and structural errors of the empirical model.
- Water abstractions and irrigation processes that are not represented in simple models may confuse the calibration exercise, especially in data scarce environments where the ecosystem services approach is gaining momentum.

While the sensitivity analyses results are inherently local, the methods outlined in this study provide a template that can be used in most InVEST model applications. The

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analyses do not require hydrologic expertise and are facilitated by the model batch-processing capabilities. Since rigorous uncertainty analyses are currently not the norm in the ecosystem services community, such simple guidance is essential to help users interpret models correctly and conduct more robust assessment of the water-related ecosystem services.

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Table 1. Precipitation and evapotranspiration in Cape Fear according to different data sources. Mean and standard deviation values are obtained from the 10 subcatchments. The relative difference between baseline data (i.e. PRISM and FAO sources, respectively, for P and ET_0), and the alternative data sources, is given as the mean and the range for the ten subcatchments.

	Annual P (mm)			Annual ET_0 (mm)		
	PRISM	Spline	IDW	FAO	ClimOffice	MODIS
Mean (\pm st. deviation)	1118 \pm 11	966 \pm 81	975 \pm 38	1200 \pm 18	1189 \pm 56	1459 \pm 19
Relative difference from baseline data (mean difference and range)		-14 % [-23; 2] %	-13 % [-17; -4] %		-1 % [-8; 5] %	+22 % [14; 24] %

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Table 2. Summary of mean flow, precipitation, reference evapotranspiration, and land use characteristics of the ten study subcatchments. LULC classes shrubland, swine farm, open water and barren represented $\leq 2\%$ and are not reported here. Predicted mean flow values are results from the InVEST model with Z set to 14. P and ET_0 are precipitation and reference evapotranspiration, respectively.

ID	Name	Area (km ²)	Observed flow (mm)	Predicted flow (mm)	<i>P</i> (mm)	ET ₀ (mm)	% Forest	% Grassland	% Agriculture	% Pasture	% Wetland	% Urban
2105769	Cape Fear at Kelly	13 567	278	239	1112	1212	49	13	9	6	6	13
2105500	Cape Fear at Tarheel	12 535	265	249	1109	1207	51	13	9	6	3	14
2102500	Cape Fear at Lillington	8973	236	254	1110	1196	55	10	9	8	1	14
2104220	Rockfish at Raeford	237	368	226	1118	1240	62	18	1	0	7	8
2102000	DeepRiver at Moncure	3727	250	248	1113	1203	58	9	7	11	0	11
2097314	NewHope at Blands	197	357	336	1143	1199	49	5	2	2	3	39
2100500	DeepRiver at Ramseur	913	289	314	1112	1177	43	9	9	10	0	27
2096960	HawRiver at Bynum	3294	278	287	1110	1181	48	10	14	9	0	17
2097464	Morgan at WhiteCross	22	177	201	1133	1198	72	7	10	5	0	5
2096846	Cane at Orange	20	202	183	1123	1192	71	6	11	6	0	4

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Table 3. Bias between the water yields obtained from the InVEST model (baseline value $Z = 22$), the lumped Zhang model, and observed data. The average, minimum, and maximum bias values for all the subcatchments are reported. Note that comparison with observations discards the Rockfish subcatchment which was identified as an outlier (see text for details).

	Average	Min	Max
InVEST/Lumped model	−0.10	−0.24	0.14
InVEST/Observations	−0.16	−0.53	−0.01
Lumped model/Observations	0.04	−0.36	0.29

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Table A1. Data sources and statistics for model inputs. Raster statistics are for the entire Cape Fear catchment delineated in Fig. 2.

Data	Type	Value (Mean and range)	Source	Range for sensitivity analyses
Precipitation	Raster	1180 mm [1030; 1450] mm	PRISM* (Gilliland, 2003) (USGS, 2014)	±20 %
Reference evapotranspira- tion	Raster	1240 mm [1160; 1310] mm	FAO* MODIS (Mu et al., 2012) Climate Office (NCSU, 2014)	±10 %
DEM	Raster	90 m [0; 250] m	(USGS, 2013a)	n.a.
LULC	Raster	Cf. Appendix	(NASS, 2013)	n.a.
Soil depth	Raster	1710 mm [0; 2110] mm	(USGS, 2013b)	n.a.
PAWC	Raster	0.18 [0.07; 0.52]	(USGS, 2013b)	n.a.
Root depth	Per LULC class	See Table A1	(Allen et al., 1998)	n.a.
K_c	Per LULC class	See Table A1	(Allen et al., 1998)	[−30 %; +10 %]
Z	Constant	22*	(Tallis et al., 2014)	[1; 30]

* Indicates the data source used for the baseline run (see Sect. 3.2).

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Table A2. Biophysical table used for the baseline InVEST model run, giving the root depth and crop coefficient K_c for each Land use/Land cover (LULC) class (values from Allen et al., 1998).

LULC	Root depth (mm)	K_c
Ag-Corn	1500	0.75
Ag-other	1100	0.7
Grass	1100	0.9
Forest	5000	1
Wetland	na	1.1
Urban	na	0.4

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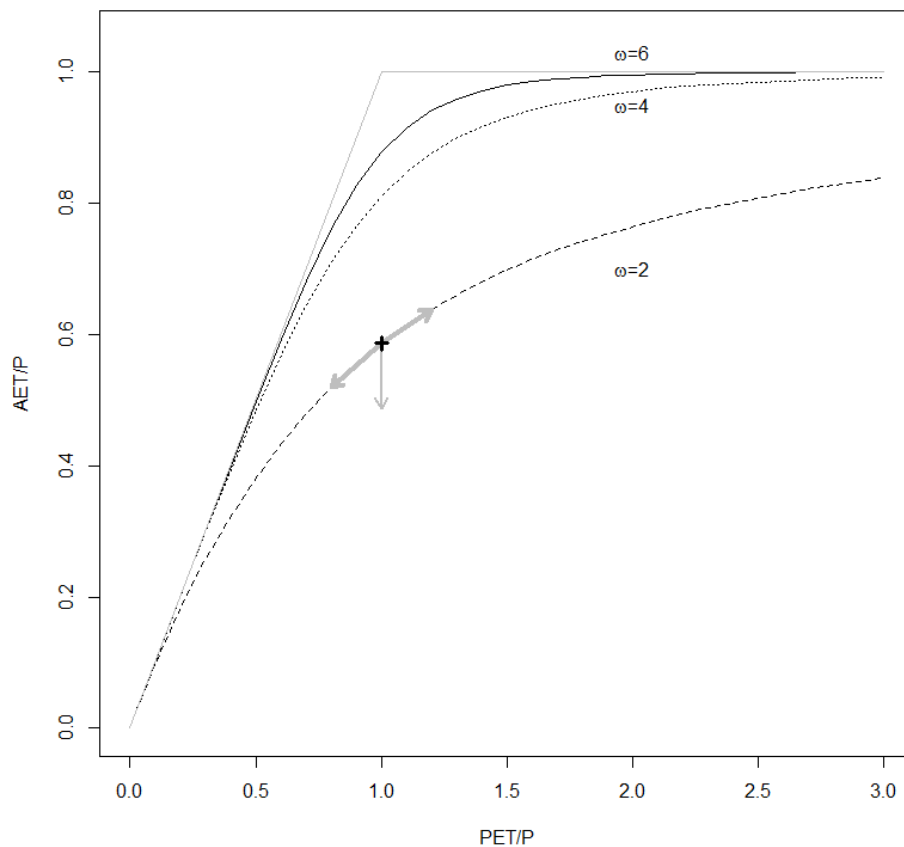


Figure 1. Zhang model (Eq. 1), shown for ω values of 2, 4, and 6. Grey lines represent the energy and water limits. Arrows illustrate the effect of a change in the climate forcing (thick arrows) and a change in the ω parameter, a function of Z , precipitation, and soil properties (thin arrow, see Eq. 3 for details).

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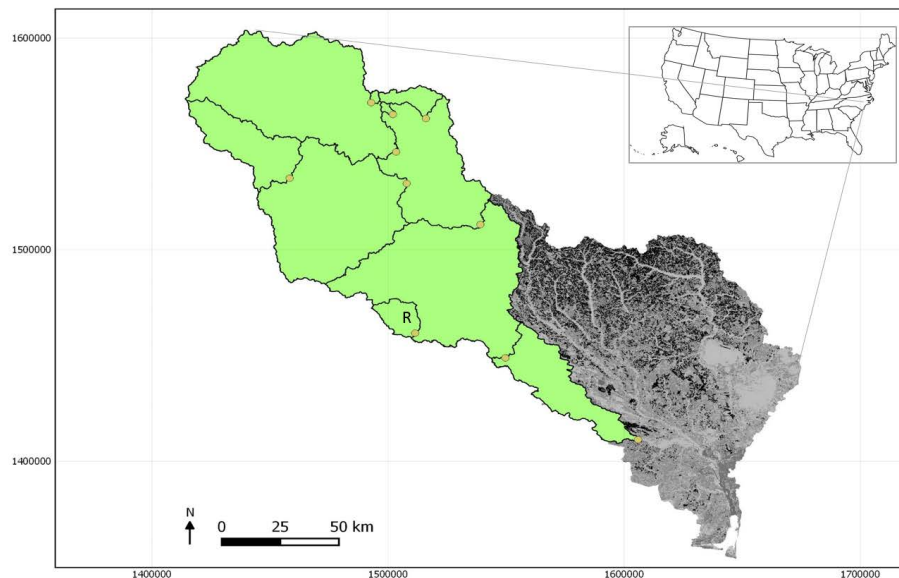
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Figure 2. Cape Fear catchment showing locations of the stream gauges and subwatersheds used in the study. The Rockfish catchment, discussed in the text, is indicated by a R.

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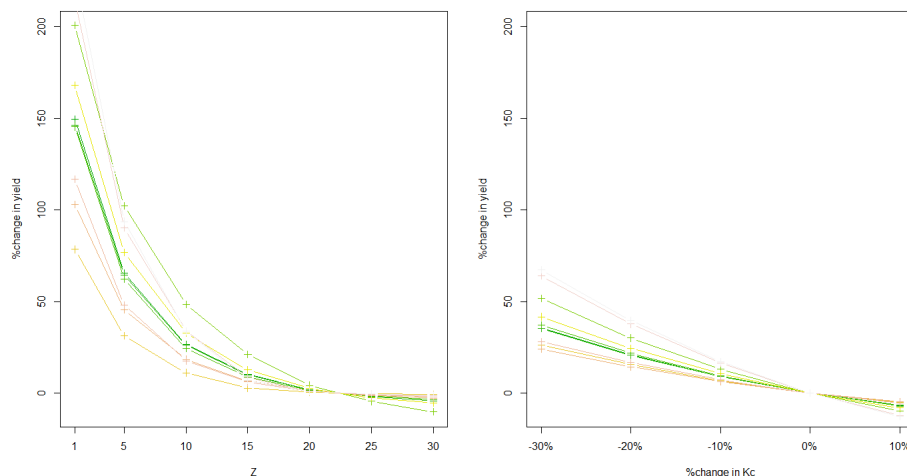


Figure 3. Sensitivity of the water yield output to the Z coefficient and crop coefficient for forest LULC (K_c). Changes are relative to the baseline run (where $Z = 22$ and $K_c = 1$). On the left hand side plot, absolute Z values are plotted on the x axis to facilitate the discussion on the Z coefficient. Each curve represents a subcatchment.

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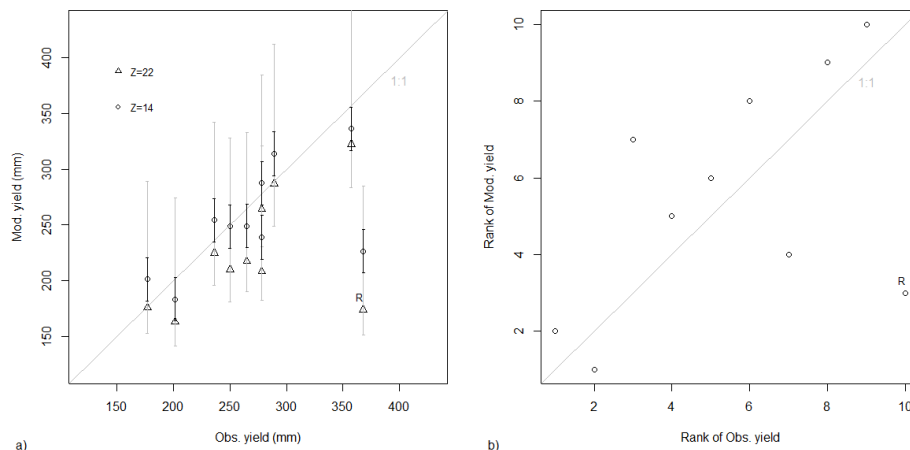


Figure 4. (a) Comparison between modeled yields (corrected for water withdrawal) and observed yields, both for the baseline run ($Z = 22$), and the calibrated run ($Z = 14$). Black error bars represent the uncertainty on the value for water withdrawal, while grey bars represent a 10% error in the precipitation input. (b) Comparison of subcatchment ranks. The outlier (Rockfish) subcatchment is noted with a R on each figure (see text for details).

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