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Discussions

Interactive comment on "Technical Note: A measure of watershed nonlinearity II: re-introducing an IFP inverse fractional power transform for streamflow recession analysis" *by* J. Y. Ding

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Response to Referee 3: Main and Minor Points

Comments are shown in the Roman fonts, *and my responses in Italic*. I apologize for any editing errors of the comments.

Response: Reviewing and responding to major objections, raised by Referee 3 as well as Referees 1 and 2, to the IFP transform model has crystalized, in my mind, that the Brutsaert-Nieber differential equation and its IFP transform solution constitute a kernel of truth in streamflow recession.

Through the IFP transform lens, I would like to respond to their comments as follows. For clarity, the log transform of the Brutsaert-Nieber differential equation will be referred to as the log-derivative transform model.

Main Points by Referee 3

P15661 "However, the approximation of the infinitesimally time-step size dt by a finite difference Δt manifests itself in a scattering of the recession data points, called the cloud." In my opinion, this is not the dominant reason for the scattering which is observed. There are multiple causes, including non-uniqueness of the storage-discharge relationship, and heterogeneity within the catchment, as well as timestep effects.

Response: The log-derivative transform model is dependent on the timestep size. The scattering of the recession data points on a recession plot is the result of applying the "unsolved" Brutsaert-Nieber differential equation.

The amount of water stored on a catchment cannot be directly measured except for an idealized one. The storage-discharge relation will have to be inferred or derived from the observed hydrograph such as by Eq. 3, assuming that the effect of evapotranspiration on the measured flows is negligible. Whether or not the derived storage-discharge relation is unique, the catchment or aquifer is homogeneous for its size and (storage) type, the evapotranspiration rate is neglogible, etc., these can be as-

sessed against the physical assumptions after a correct mathematical model is chosen.

P15664/5 “Approximation of (dQ/dt) by the backward time-differencing one, (Q(t) – Q(t – Δ t))/Δ t, is thus the main source of the numerical instability in parameters characterizing the model.” And “Linearization of Brutsaert–Nieber model thus removes sources of numerical instability in its parameters.” It is unclear what is meant by “numerical instability in parameters”. No examples or references are provided. If the author is referring to the possibility of obtaining different a and b values at different timesteps, this needs to be made explicit, and the work of Rupp and Selker (2006) should be referenced here.

Response: The numerical instability in Brutsaert-Nieber parameters characterizing the nonlinear system is again the result of applying an "unsolved" differential equation. The nonlinear system’s instability manifests itself in its response as well as its parameters’ values, i.e. in a model, the response function and the parameters are one and the same.

P15665 “The physical constraint that S $\geq$ 0”. I do not agree that this constraint is necessary, and as a result I do not agree with the constraint that b$\leq$2. See Rupp and Woods (2008) for a demonstration that a negative value of S can be a physically meaningful measure of storage, if it represents a deficit.

Response: Please see Response on Overview, under Equation 3.

P15665 “thus b$\leq$2” If the author allows b=2, then the author has c=0 and N infinite. By the author’s own argument this does not seem reasonable, and yet the author adopts b=2 as physically reasonable. I do not see the author’s reasoning here (but see Rupp

and Woods (2008)).

Response: Please see Response on Overview, under Equation 3.

P15665 “its latter expression hints at an intriguing, counter-intuitive transformation of a time series.” I did not find this material helpful. Nor did I find the connection to the Tukey ladder of powers helpful. One could equally find other power transformations from other fields which had the same mathematical form, but this does not mean that there is a useful connection.

Response: The IFP transform model is a mathematical solution of Brutsaert-Nieber nonlinear differential equation for drought flow recession. It is thus grounded in hydrology, rather than statistics, which has a differnt role of falsifying a model that has to be first developed by us hydrologists.

P15667 The author never explains of justifies his use of best-fit correlation as his method of selecting a and b. One could hypothesise a number of different measures of the goodness of fit of the model to the data.

Response: Table 3 (Revised 2) and Figure 3 together summarize the results of standard statistical tests. These represent a sensitivity analysis of the IFP transform model response to changes in the parameter values.

Given the large drainage area of the Spoon River, and given a very small sample size, the case study need be looked at as an illustrated example of the IFP transform model application. This is explained both in the abstract and introduction.

The recession flow data of the large Spoon River constitute a very severe test of the assumption of a headwater catchment. This underpins the IFP transform model representing discharge from an idealized aquifer in a hillslope.

Given these limitations and keeping in mind the physical law, I've drawn the conclusion as to the range of Brutsaert-Nieber parameters in Page 15668, Line 24, Page 15669, Line 3. This has been clarified following the revised Table 3 as part of Response to Referee 2.

P15668 "the highest calibrated a value among four events" It is not meaningful to compare make numerical comparisons of the a values, because they are measured in different units (the units of a depend on the value of b, which takes on many values in these examples)

Response: For a common b value, the highest a value represents the upper limit of calibrated scale parameter. For $b \geq 2$, the very high a values are indicative of the nonlinear system's instability.

P15668 "as parameter b value increases, so does parameter a value reflecting the steepness of a transformed recession curve in Fig. 2" Again, it is not meaningful to compare values of a when b is not constant.

Response: The slope of an IFP transformed curve is a product of $(b-1)$ and a, not b or a alone.

P15668 "the Manning (or Chezy) friction law governs the nonlinear storage–discharge relation expressed by Eq. (3)." I do not see why a friction law of this type must govern the storage, because it is not clear that the storage is held in the river; it could equally be in a near-stream aquifer.

Response: I appreciate the caution by the Referee to differentiate types of storage: in-stream flow or near-stream aquifer. Other types include storages in an artesian aquifer ($b = 2 - 1/N = 1$, from $N = 1$), or behind an orifice ($b = 0$, from $N = 1/2$) (Ding, 1967, p.15.4). What counts in a derived storage-discharge relation is the type of storage in the downstream-most stretch of the watercourse. Given the large size of the Spoon River measuring 4237 km^2, it would be a surprise if the storage was not of in-stream one, thus not governed by a friction law.

Table 2, Table 3 and Figure 2 The author fits the IFP model to individual recessions which do not contain a wide range of flow values. As a result, flow varies almost linearly with time, and many of the power transformations also produce relationships which are close to linear. Correlations are never poorer than 0.97, and it does not seem meaningful to select an optimal b when almost every b value gives a strongly linear relationship.

Response: Please see Response on Overview, under Table 3 (Revised).

P15669 "Among three of four events, the linear regression method yields higher b values than the IFP transform one," The author rejects b values greater than 2 from the IFP method in compiling Table 4, so it is not surprising that the IFP values are smaller than the standard B-N values.

Response: I note the Referee does not find surprising the comparative results, event by event, between the traditional log-derivative transform and the newly re-discovered IFP transform, shown in Table 4.

Minor Points by Referee 3

P15667 “Events 2-4” There is no event 4 in Table 3.

Response: My thanks to the Referee for pointing out the error: "Events 2-4" will change to "Events 1-3".

Additional references

Ding, J. Y.: Flow routing by direct integration method, Proc., Int. Hydrol. Symp., Fort Collins, Colo., 1, 113–120, 1967.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 10, 15659, 2013.

Table 3 (Revised 2). Summary of Brutsaert-Nieber model parameters for Spoon River, Illinois.

Event[a]	Start date	Length (day)	$Q(0)$ (mm/d)	Stats[b]	Type of IFP transform, $1/Q^{b-1}$ None	log	RoCR	RoSR	Recip	RoQ
				b	0	1.0	**1.33**	**1.5**	2.0	3.0
0	1994-05-15	9	0.84	a	0.05	0.07	0.08	**0.08**	0.10	0.16
				R	-0.98	-0.99	0.99	0.99	1.00	1.00
3	1986-07-19	4	0.56	a	0.06	0.13	**0.17**	0.19	0.29	0.63
				R	-1.00	-1.00	1.00	1.00	0.99	0.98
1	1988-05-03	4	0.29	a	0.01	0.05	**0.07**	0.09	0.18	0.66
				R	-1.00	-1.00	0.99	1.00	1.00	1.00
2	1988-06-13	8	0.10	a	0.01	0.07	0.15	**0.23**	0.84	10.88
				R	-0.97	-0.98	0.99	0.99	0.99	0.99
Mean				$\bar{a}$	0.03	0.08	**0.12**	**0.15**	0.35	3.08
Variance				$\sigma^2(a)$	0.0007	0.0012	0.0025	0.0055	0.1117	27.0751
Std.Div.				$\sigma(a)$	0.03	0.03	0.05	0.07	0.33	5.20

[a] Events arranged in the descending order of the initial flow value, $Q(0)$.

[b] b is the shape parameter (-), a the scale parameter $[1/(\text{d mm}^{b-1})]$, and R the correlation coefficient.