

# ***Interactive comment on “Technical Note: A measure of watershed nonlinearity II: re-introducing an IFP inverse fractional power transform for streamflow recession analysis” by J. Y. Ding***

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## **Second response to Referee 2: Specific comments**

Comments by the Referee are shown in the Roman fonts, *but in Italic are the original texts quoted by the Referee as well as my responses*. I apologize for any editing errors of the comments.

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## Comment by Referee 2

The author has recently been involved in some interactive discussions on HESS-D papers, which is to be applauded. The current manuscript is basically, in his own terms, “*a consolidation and extension of these two sets of comment, plus two additional ones, all on streamflow recession*”. Again, this is to be stimulated as well, especially because the comments involved do not really ‘count’ as a scientific publication (for one thing, they’re not peer reviewed). This ‘consolidated’ note therefore seems essential to convey the author’s results.

The paper focuses on the use of a series of discharge transformations to assist characterization of streamflow recessions. Although I did find the subject as such interesting, I do have a number of concerns:

First, the proposed methodology requires that Brutsaert-Nieber parameter  $b$  is known in advance (i.e. the transform is a function of  $b$ ). Although the author has solved this problem by testing multiple estimates of  $b$ , I found the results not convincing. A linear regression is used to quantify the goodness of fit for multiple values of  $b$ . The resulting  $R^2$  is close to 1 for all cases, suggesting that it has no discriminative power. The methodology as applied thus is not able to select the ‘true’  $b$ . This in contrast to ‘traditional’ Brutsaert-Nieber analysis which does find an  $b$  (although different approaches result in different estimates of  $b$ , as recently shown by Stoelzle et al, but that’s another matter). This issue is not discussed in the paper.

Second, the proposed methodology is applied to individual recession events, resulting in event-specific estimates of  $a$  and  $b$ . Again, this is in contrast to ‘traditional’ Brutsaert-Nieber analysis which collapses all recession events to a single data cloud,

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resulting in unique  $a$  and  $b$  values. Especially when reasoning from the Boussinesq or the Manning equations, one should not expect inter-event variability in  $a$ , yet, this issue is not discussed at all.

Third, the proposed methodology is applied to a very limited duration dataset, for a large catchment. The author does acknowledge this, but the reasoning is rather weak (*“it has been because the recession flow data available for analysis were published in an open access journal such as HESS”*) and unconvincing because many streamflow data for smaller catchments, is freely available, e.g. the MOPEX data, or the LTER sites.

Above criticisms prevent me from recommending publication of the manuscript in its present form. In order to warrant publication, the proposed transformation method should be applied to a longer dataset from a more suitable catchment, using better indicators to distinguish 'good' from 'bad'  $b$ -values. Also the issue of temporal variability of parameters, and a more thorough comparison with 'traditional' Brutsaert-Nieber analysis should be included.

Also, the paper should be self-contained. References to HESDD-comments by the author should be eliminated from the paper. Arguments made in those comments should be consolidated into the present paper.

**My final recommendation is therefore “major revisions”.**

*Response: These major issues have been addressed in my initial response on major comments.*

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**Addendum by Author:** *Reviewing and responding to major objections, raised by Referee 2 as well as Referees 1 and 3, to the IFP transform model has helped collapse most of the technically as well as historically interesting comments into a simple mathematical problem.*

*That is: Brutsaert-Nieber (1977) model ( $-dQ/dt = aQ^b > 0, b \geq 0, a > 0$ ) is a simplest, nonlinear, differential equation, and it has an exact solution. It is first-order in the derivative term  $dQ/dt$  and nonlinear in the  $Q$  term. The solution can be obtained by separating the variables  $Q$  and  $t$  in the equation and integrating it from time zero to  $t$  (e.g. Sokolnikoff and Redheffer, 1958, pp. 12-13).*

*The solution happens to be the linearized IFP transform model (Ding, 1967, 1974). The Brutsaert-Nieber differential equation and its IFP transform solution constitute a kernel of truth in streamflow recession.*

*Through the IFP transform lens, I would like to respond to their specific comments as follows.*

## Specific comments by Referee 2:

[sec. 1] —The significance of “variation and persistence of the low flow” is illustrated with a reference to the Bible. I’m not sure if such a reference is appropriate in a scientific article target at a religiously diverse audience.

*Response: I appreciate the caution by the Referee. But the 18-word paraphrasing of a long Hebrew tale in Genesis is neutral in tone. This is in contrast to the evocative "Holy*

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*Grail" which headlined one recent HESS paper (Beven, 2006), and was referenced to by Troch et al. (2008) as in "... The search for a unifying theory is the Holy Grail in every scientific field ..."*

— *"During the public comment period of . . . I brought to their attention . . . refined and elaborated in a later comment . . . This note is a consolidation and extension. . ."*  
— Although I applaud the author's enthusiastic involvement in recent discussions in HESS, I don't think references to this activities should be part of a research paper.

*Response: The technical note is the end product of a long and tortuous search on my part for a glimpse of the elusive Holy Grail in mathematical hydrology. It had its genesis from my earliest comment (Ding, 1966) on a paper by Brutsaert (Ibrahim and Brutsaert, 1965).*

*Recently I was amazed and overwhelmed by the rich and diverse literature on stream-flow recession often involving sophisticated techniques. My initial HESS-D comment was inspired by Stoelzle et al (2013) in their comparative study of three standard implementaion schemes within the Brutsaert-Nieber model. Shortly thereafter I unearthed two earlier strands than mine of the RoSR solution technique developed analytically by Chapman (1964) and graphically by Takagi (Ishihara and Takagi, 1965). These were duly noted in a follow-up comment. (Mine eventually led to the development, in flood hydrology, of the vIUH variable instantaneous unit hydrograph model (Ding, 1974)). I feel the milestones for the long journey of re-discovery need be marked so as to both acknowledge those who had blazed the trail, and help guide hydrologists now and in the future.*

[2.1] — *"The heading of my most recent comment was titled on purpose . . . this*

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*hopefully has conveyed my view” — Idem*

*Response: Ditto.*

[2.1.1] — *The ‘classic’ Brutsaert-Nieber b-values 1, 1.5, 3 are mentioned, without mentioning too that they were derived from hydraulic groundwater theory (the nonlinear or linearized Boussinesq equation), applied to a flat aquifer, characterized by uniform conductivity.*

*Response: Please see Sect. 2.1.1, first paragraph, on the assumption of an idealized aquifer.*

[2.2] — *“This was discovered, . . . the latter then a graduate student . . . by me, then a graduate student at Guelph, Canada”* — The then status of the referenced authors is not relevant.

*Response: The independent discovery of the RoSR transform, by two graduate students as well as two other senior authors, took place a half-century ago. The boldness in our youth to have put forward an innovative solution technique, then and now, need be noted.*

— *“a parameter b value of 1.5 characterizes late-time recession in Brutsaert–Nieber model”* — No, it characterizes late-time recession from a Boussinesq aquifer under specific assumptions.

*Response: Please see Sect. 2.1.1, first paragraph, on the assumption of an idealized*

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aquifer; and Page 15665, Eq. (4) and Lines 15-16 on an alternate expression of the "Boussinesq" model.

[2.3]— "Their  $b$  values range from a lower limit of one" — What about the kinematic wave process that in theory, following steady-state initial conditions, would lead to a  $b = 0$ ?

*Response: I appreciate and accept the criticism. Also the outflow from an orifice is characterized by an  $N$  value of  $1/2$ , thus  $b = 2 - 1/N = 0$  (Ding, 1967, p.15.4). The lower limit for shape parameter  $b$  will be lowered to zero, as now specified in the **Addendum** above.*

— ". . . to an upper limit of 2" — What about the early-time  $b = 3$  as derived from the Boussinesq equation?

*Response: The Brutsaert-Nieber differential equation and its IFP transform solution both place no limits on the shape parameter  $b$  value, other than it being a non-negative number, the 'short' or 'early'-time  $b$  value of 3 included, as shown in the **Addendum** above.*

*But under the assumption of a monotonically increasing storage-discharge relation, there is no room for a parameter  $b$  value of 3. The Boussinesq equation, having  $b = 3$ , represents, for instance, a sudden drawdown of a groundwater table, either near a water well caused by a pumping test, or near the bank slope caused by a rapid drawdown of a reservoir. These rare cases occur prior to the beginning of typical drought flow analysis.*

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— “As an aside...” — I see no reason why this aside should be in the paper.

*Response: Please see response to Referee 3: Overview, under Equation 7.*

[3] — “*IFP . . . most suitable for low flow analysis*” — What about measurement noise and discrete values? These are most prominent at the lowest flows.

*Response: On a river, the rating curves are well defined by current metering at lower ends. Thus measurements of low flows are more accurate than high flows, especially when the rating curve has to extrapolate upward.*

*On a small watercourse during dry weather, it can be temporarily diverted to a measuring chamber or tank for an accurate determination of the flow volume. For outflows from small spings, they can be collected and measured using containers and a stopwatch (e.g. Malvicini et al., 2005).*

— “*R’s among all transforms for four events are similar, all close to or at 1.0*” — So there is almost no discriminative power in using *R*?

*Response: Agree. For a goodness-of-fit test, the correlation coefficient R is a necessary condition or criterion, but not a sufficient one.*

— “*ranges from (1.33, 0.07) to (1.5, 0.23)*” — Given the intra-event variability in *a* (which is not discussed at all!) why would one favour the lowest *a* for *b* = 1.33 and the highest *a* for *b* = 1.5?

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*Response: This shows, based on four recession events, the range of parameters  $b$  (1.33 to 1.5) and  $a$  (0.07 to 0.23), implying some uncertainty about their 'true' values.*

*As with typical recession flow analyses, the evapotranspiration in a water balance equation is assumed negligible. The impact of evapotranspiration on the model parameters is not considered in this introductory note, but need be in a detailed study. The attached Table 3 (Revised 2) now includes the start date of each of four recession events to complete the test dataset.*

*Ando et al. (1986, p.155), whose work is cited on Page 15666, Line 6, analyzed the seasonal variation of the scale parameter  $a$  for 21 mountainous basins in Japan, given a default shape parameter  $b$  value of 1.5. They showed on a daily depth basis, mm/d, the winter  $a$  values vary from slightly above zero to approximately 0.06, and that the seasonal  $a$  values vary from 1.29, 1.49, and 1.20 times the winter value for spring, summer, and autumn, respectively.*

*The parameters' uncertainty can be re-assessed or reduced by additional flow measurements in the field as discussed on Page 15663, Lines 8-11.*

*— “For the Spoon ...linear storage ...un-transformed” — which is  $b = 0$  to 1. How is this consistent with the earlier  $b = 1.33$  to 1.5 as derived from Manning's equation?*

*Response: These statistical inferences hold if the sole criterion, in addition to the correlation coefficient, for a goodness-of-fit test is the minimum variance of the scale parameter  $a$ , without recourse to a physical law.*

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[4] — “The range of  $b$  values varies from one to an upper limit of 2, rather than that of 3 adopted by Brutsaert and Nieber . . . for the early-time lower-envelopes” — The upper limit of 2 has been derived from a complete different approach (and assumptions) than the  $b = 3$

*Response: Agree. Parameters have meaning only within their model including its implementation scheme (e.g. Page 15669, Lines 26-27; Response to Referee 1, Part 2, Page C7643, Item 3 on the classical lower envelopes approach.)*

— “based solely on the highest correlation coefficients for each event” — Again, differences in  $R$  are very small

*Response: Agree.*

— “[This case study] has demonstrated only marginally the superiority of IFP transform over the conventional log transform method” — If the differences are “only marginal”, why is it then “superior”?

*Response: The log transform is a special, linear type of the IFP transform ( $b = 1$ ). Other nonlinear types, but not consistently of the same one, have higher correlation coefficients than, or same as, the log one, thus the qualified or equivocal statement.*

[Figure 1] — Is not mentioned in the manuscript

*Response: Please see Page 15666, Line 1.*

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## Additional references

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Sokolnikoff I.S. and Redheffer R.M., 1958, *Mathematics of physics and modern engineering*, McGraw-Hill, New York.

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Table 3 (Revised 2). Summary of Brutsaert-Nieber. model parameters for Spoon River, Illinois.

Event <sup>a</sup>	Start date	Length (day)	$Q(0)$ (mm/d)	Stats <sup>b</sup>	Type of IFP transform, $1/Q^{b-1}$					
					None	log	RoCR	RoSR	Recip	RoQ
				$b$	0	1.0	<b>1.33</b>	<b>1.5</b>	2.0	3.0
0	1994-05-15	9	0.84	$a$	0.05	0.07	0.08	<b>0.08</b>	0.10	0.16
				$R$	-0.98	-0.99	0.99	0.99	1.00	1.00
3	1986-07-19	4	0.56	$a$	0.06	0.13	<b>0.17</b>	0.19	0.29	0.63
				$R$	-1.00	-1.00	1.00	1.00	0.99	0.98
1	1988-05-03	4	0.29	$a$	0.01	0.05	<b>0.07</b>	0.09	0.18	0.66
				$R$	-1.00	-1.00	0.99	1.00	1.00	1.00
2	1988-06-13	8	0.10	$a$	0.01	0.07	0.15	<b>0.23</b>	0.84	10.88
				$R$	-0.97	-0.98	0.99	0.99	0.99	0.99
Mean				$\bar{a}$	0.03	0.08	<b>0.12</b>	<b>0.15</b>	0.35	3.08
Variance				$\sigma^2(a)$	0.0007	0.0012	0.0025	0.0055	0.1117	27.0751
Std.Div.				$\sigma(a)$	0.03	0.03	0.05	0.07	0.33	5.20

<sup>a</sup> Events arranged in the descending order of the initial flow value,  $Q(0)$ .

<sup>b</sup>  $b$  is the shape parameter (-),  $a$  the scale parameter  $[1/(d \text{ mm}^{b-1})]$ , and  $R$  the correlation coefficient.

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