Hydrology and
Earth System
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Interactive comment on "Technical Note: A measure of watershed nonlinearity II: re-introducing an IFP inverse fractional power transform for streamflow recession analysis" *by* J. Y. Ding

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Received and published: 1 April 2014

Response to Referee 3: Overview

I would like to thank Referee 3 for their thorough review and respond to their critical assessment of the IFP transform model.

Overview by Referee 3

This note describes and illustrates the use of the IFP (Inverse Fractional Power) transformation of flow to estimate the parameters of the Brutsaert-Nieber recession model, $-dQ/dt = aQ^b$. If the value of b can be selected appropriately, then the plot of transformed flow against time is linear, and parameter a can be estimated by linear regression. This IFP method is an alternative to estimating a and b by fitting a straight line to a cloud of points on log(-dQ/dt)-vs-log(Q) axes.

In my opinion, the IFP transformation may be of academic interest, but I do not think the paper has demonstrated any significant advantages over alternatives. The IFP method requires a two-stage estimation process, where b is chosen, and then the best value of a is found. This must be repeated until an optimal combination of a and b is obtained. I did not find that the examples revealing or compelling. In its current form it does not seem suitable for publication in HESS and I recommend Major Revisions.

Response by Author

To address coherently their numerous objections to the IFP transform model, I would like to go back to the origin of the Brutsaert-Nieber (1977) model and down to the basic of mathematics.

Equations 1 and 2

BN model is represented by a simple *nonlinear* differential equation (Eq. 1). It has a mathematical solution. Taking the logarithm of both sides of the equation:

$$log(-dQ/dt) = a + b\,logQ,\tag{1a}$$

but leaving intact the derivative term (dQ/dt), this does not solve the differential equation.

By contrast, separating the variables Q and t in the equation and integrating it from time 0 to t, one obtains a solution, which is the IFP transform model (Eq. 2a) (e.g. Sokolnikoff and Redheffer, 1958, pp. 12-13).

Thus the solution of BN model is the IFP transform, *not* the linear log-transform (Eq. 1a). In principle, the IFP transform is a *better* model than the linear log-transform one.

In the eye of the $Queen\ of\ Science$ (after mathematician E. T. Bell), the original BN equation (Eq. 1) and its IFP transform solution (Eq. 2a) constitute a kernel of truth and a law in streamflow recession. Others, such as Eq. 3 below for a nonlinear storage-discharge relation, are derivable from this law, which let me call, for discussion purposes, BND model, named after its discoverers.

This claim of mine that $BND\ model\ is\ a\ nature's\ law$ is not in dispute, but remains open to challenge by the Referees, c/o the Editor.

Viewed in this light, the note acknowledges the pioneering contribution of Brutsaert and Nieber, and my re-introduction of a half-century-old IFP transform as an exact solution helps solidify the $original$ BN equation into a hydrologic law.

Most, if not all, of my comments that Referee 3 finds objectionable are external to the

BND kernel and its derivatives. They can be removed from the final text, if the Editor so directs.

Equation 3

The nonlinear storage-discharge $(S - Q)$ relation is obtained by integrating Eq. 2a from time t to ∞. $S(t)$ represents the amount of storage available at time t for later release, i.e. an $available\ storage$ or $stock$, which will be eventually depleted, i.e. $S(\infty) = 0$ and $Q(\infty) = 0$.

By contrast, integrating from time 0 to t yields the amount of storage, $S(0) - S(t)$, that has been released at time t, i.e. a $spent\ storage$. The initial storage $S(0)$ is generally not known, and would have to be estimated from , say, a storage-discharge relation that remains to be determined, thus becoming a circular augument.

The dfference is critical between the two sets of the time limits for integration, the $future$ vs. the $past$. Following the second set causes some to have to *redefine* S as a *negative* storage (or storage *deficit*) where no such thing exists or needs to (e.g. Rupp and Woods, 2008)

Regarding the limits for shape parameter b, the IFP transform (Eq. 2a) places no limit on it, others than it being a real number. For application to streamflow recession, I appreciate and accept the criticisms from Referee 3 as well as Referees 1 and 2, and will revise its limits to: $0 \le b < 2$, the latter to avoid being trapped in a blackhole of the infinity.

Equation 7

"As an aside ... " meant to be a footnote to the text. This serves both as a reminder and word of encouragement that we hydrologists, members of one of the oldest professions, need not play a second fiddle to statisticians and alike. Hurst vs. Mandelbrot on analysis of the Nile River flood time series data is a classic example.

Table 3 (Revised)

Referee 3 also observes the lack of discriminating power of the correlation coefficient R among parameter pairs (b, a). Clearly this is an inadequacy of R as a statistical test: it is $necessary$, but $not\ sufficient$.

But I should add, this shows, on the other hand, the IFP transform model is robust, when b value is 1.5 and less. This is a theoretical value for hillslopes arrived at much earlier by Chapman (1964), Ishihara and Takagi (1965) and me (Ding, 1966). The robustness of the IFP transform lends support to my recommendation that the RoSR transform in which $b = 1.5$ be falsified for headwater catchments (see the final paragraph of the note).

References

Sokolnikoff I.S. and Redheffer R.M., 1958. Mathematics of physics and modern engineering, McGraw-Hill, New York.

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Table 3 (Revised). Summary of Brutsaert-Nieber model parameters for Spoon River, Illinois.

Event[a]	$Q(0)$ (mm/d)	Length (day)	Stats[b]	Type of IFP transform, $1/Q^{b-1}$					
				None	log	RoCR	RoSR	Recip	RoQ
			b	0	1.0	**1.33**	**1.5**	2.0	3.0
0	0.84	9	a	0.05	0.07	0.08	**0.08**	0.10	0.16
			R	-0.98	-0.99	0.99	0.99	1.00	1.00
3	0.56	4	a	0.06	0.13	**0.17**	0.19	0.29	0.63
			R	-1.00	-1.00	1.00	1.00	0.99	0.98
1	0.29	4	a	0.01	0.05	**0.07**	0.09	0.18	0.66
			R	-1.00	-1.00	0.99	1.00	1.00	1.00
2	0.10	8	a	0.01	0.07	0.15	**0.23**	0.84	10.88
			R	-0.97	-0.98	0.99	0.99	0.99	0.99
Mean			$\bar{a}$	0.03	0.08	**0.12**	**0.15**	0.35	3.08
Variance			$\sigma^2(a)$	0.0007	0.0012	0.0025	0.0055	0.1117	27.07
Std.Div.			$\sigma(a)$	0.03	0.03	0.05	0.07	0.33	5.20

[a] Events arranged in the descending order of the initial flow value.

[b] b is the shape parameter (-), a the scale parameter $[1/(\text{d mm}^{b-1})]$, and R the correlation coefficient.