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Imperfect scaling in distributions of radar-derived rainfall fields

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Comments to Authors

Paper summary

The reviewed manuscript studies how the distribution of 5-min rainfall intensity varies with the scale of spatial averaging. To do so, the authors fit a maximally skewed α -stable distribution model to the log-transformed rainfall intensities from 26 storms in Belgium, and study how the multiplicative weights of a discrete cascade depend on the scale of spatial averaging. Similar to other studies in the past (see General Comment #2 below), the authors find that the multiplicative weights are not iid, indicating dependence of their distribution on the scale of spatial averaging, the large scale rainfall intensity, as well as a power deficit at high frequencies.

General Comment #1

In the Introduction, the authors provide a literature review on multifractal rainfall scaling. Except for the technical part associated with the construction of discrete multiplicative processes, it would be useful if the authors could point to the large number of studies showing that rainfall fields follow a more general scale invariance condition (than simple scaling) known as stochastic self similarity or multifractality; see e.g. Tessier *et al.* (1993), Perica and Foufoula-Georgiou (1996a,b), Harris *et al.* (1996), Olsson (1998), Deidda *et al.* (1999, 2004, 2006), Gütner *et al.* (2001), Ahrens (2003), Nykanen and Harris (2003), Gebremichael *et al.* (2004, 2006), Veneziano and Langousis (2005a), Veneziano *et al.* (2006a, 2007, 2009), Langousis and Veneziano (2007), Langousis *et al.* (2009, 2013), Veneziano and Lepore (2012), Langousis and Kaleris (2013), Veneziano and Yoon (2013), and the reviews in Veneziano *et al.* (2006b) and Veneziano and Langousis (2010).

General Comment #2

In the Concluding section, the authors should discuss more their findings and relate them to observed deviations of rainfall from exact multifractal scaling, such as breaks in the power-law behavior of the spectral density (Fraedrich and Larnder, 1993; Olsson, 1995; Menabde *et al.*, 1997),

lack of scaling of the non-rainy intervals (Schmitt *et al.*, 1998; Olsson, 1998; Güntner *et al.*, 2001; Veneziano *et al.*, 2006a; Langousis and Veneziano, 2007), differences in scaling during the intense and moderate phases of rainstorms (Venugopal *et al.* 2006), the power deficit at high-frequencies relative to multifractal models (Perica and Foufoula-Georgiou, 1996a,b; Menabde *et al.*, 1997; Menabde and Sivapalan, 2000), and dependencies of the multiplicative weights on the scale of spatial averaging and the large scale rainfall intensity (e.g. Veneziano *et al.*, 2006a; Rupp *et al.*, 2009, and Serinaldi, 2010).

General Comment #3 (technical soundness)

One concern is the transformation in equation (5). Note that the resulting field (referred to as "conservative") is not scaling. The case is similar to the results obtained when using the gradient amplitude method, where the transformed field scales in a multifractal way independently from the scaling of the original field (see e.g. Veneziano and Iacobellis, 1999; Veneziano and Langousis, 2010; Neuman 2010a,b; 2012; Guadagnini and Neuman, 2011). It is my understanding that the authors use the transformation in equation (5) solely for illustration purposes, and perform their scaling analysis using the original rainfall fields. If this is the case, I suggest that the authors remove this part of the analysis and associated Figures. In case the scaling analysis has been conducted using the transformed (i.e. "conservative") fields, the authors are advised to change their approach and apply the scaling analysis to the original rainfall fields.

Another concern is the issue of zero rainfall. It is not clear to me how equation (11) applies in the case when $R_{k+1} = 0$. I suggest that the authors provide a brief explanation.

General Comment #4 (page 11394, lines 1-20)

While necessary, the log-log linear dependence of different moment orders on the scale of spatial averaging is not a sufficient condition for multifractal scale invariance; i.e. simple scaling is also associated with log-log linear plots. To study the type of rainfall scaling (i.e. simple or multifractal), the authors should specify the particular form of the moment scaling function $K(q)$; see e.g. Veneziano and Langousis (2005a), Veneziano *et al.* (2006b), Langousis and Veneziano (2007), and the review in Veneziano and Langousis (2010). The authors are encouraged to further investigate the multifractal character of the rainfall signal, by extending their analysis to include calculation of the moment scaling function. Please note that for logstable cascade generators, the moment scaling function receives a specific form; see e.g. Veneziano and Furcolo (1999).

Specific Comment #1 (page 11388, line 25)

Detailed analyses on the statistical properties of the dressing factor for discrete multifractal cascades, have been conducted by Veneziano and Furcolo (2003), Veneziano and Langousis

(2005b), Langousis and Veneziano (2007) and Langousis *et al.* (2009). The first study developed a numerical procedure to obtain the exact distribution of the dressing factor for discrete multifractal measures, while the other studies devised approximations for hydrological applications; see the reviews in Veneziano *et al.* (2006b) and Veneziano and Langousis (2010).

Specific Comment #2 (page 11392, line 09)

Several studies have tried to link the maximum scale up to which multifractal scale invariance holds, to the characteristics (i.e. spatial extent and lifetime) of rainstorms; see e.g. Veneziano and Langousis (2005a), Langousis and Veneziano (2007), and Langousis and Kaleris (2013).

Specific Comment #3 (equation 11)

Please note the typo in the subscripts of equation (11).

Specific Comment #3 (Figures 10-17)

Figures 10-17 are not easy to understand. The authors should provide a more comprehensive discussion of their findings; see also General Comment #2.

Contribution and audience

Apart from the issues raised in General Comments #3 and #4, which the authors should carefully address, the presented work appears to be solid and of interest to a wide audience of hydrologists.

Prior publication

To my knowledge, neither the same nor very similar work has been published elsewhere.

Recommendation

For the reasons mentioned above, it is recommended that the paper is published in *HESS* after moderate revisions.

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