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Discussions

Interactive comment on "Technical Note: A measure of watershed nonlinearity II: re-introducing an IFP inverse fractional power transform for streamflow recession analysis" *by* J. Y. Ding

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Response to Anonymous Referee #1

Part 2 - Brutsaert-Nieber model

The first response has addressed questions about the IFP transform. This second response will address those about Brutsaert-Nieber (BN) model as commonly implemented.

1. Sensitivity of the $-dQ/dt \sim Q$ recession plot to change in the time-step size Δt (Page C7376, Paragraph 2)

Referee 1 asks for a demonstration of the effect of Δt on the accuracy of the recession data points or cloud $(Q, -dQ/dt)$. To address this question, it will require exploring for me the newer and different paths trodden by Brutsaert and many others (e.g. Troch et al. , 2013; Mutzner et al., 2013). It is not the intent of this technical note to re-trace these footsteps, but below are a few hesitant ones of mine.

First, let me de-compose the BN recession plot ($-dQ/dt \sim Q$) back to two source plots having a common time axis t: $(Q \sim t)$, and $(dQ/dt \sim t)$. The first one is a familiar recession hydrograph (t, Q), and the second one a computed de-acceleration graph $(t, dQ/dt)$. Directions of computed errors for dQ/dt can be determined from its second derivative with respect to t .

BN model (Eq. 1) defines: $dQ/dt = -aQ^b$. The first derivative of it is: $d(dQ/dt)/dt = dQ^2/dt^2 = -abQ^{b-1}(dQ/dt) = a^2bQ^{2b-1}$.

The second derivative of dQ/dt is: $dQ^3/dt^3 = a^2b(2b-1)Q^{2b-2}(dQ/dt) = -a^3b(2b-1)Q^{3b-2}$.

Since the second derivative is negative, the curvature at point $(t, dQ/dt)$ is convex upward. Being the peak of a convex de-acceleration curve, $dQ/dt > \Delta Q/\Delta t$, if the latter is replaced by, say, a central difference approximation: $(Q(t+\Delta t) - Q(t-\Delta t))/(2\Delta t)$. Changing the sign, $-\Delta Q/\Delta t > -dQ/dt$.

In a recession plot, the $-\Delta Q/\Delta t \sim Q$ curve thus lies above $-dQ/dt \sim Q$ one. To maintain equality between both sides of BN model, parameter a, or b , or both on the right-hand side will have to increase in value to compensate for the increase in $-dQ/dt$ on the left-hand side. For individual recession events, this helps explain why values of parameter b determined by linear regression in a $-dQ/dt \sim Q$ plot are generally higher than its true values (Table 4).

2. Interpretations of the lower envelopes (Page C7377, Paragraph 1)

I appreciate the clarification by Referee 1 on the physical meaning of the lower envelopes for the recession data cloud. Indeed as BN (1977) originally intended, the lower envelope represents the slow release from the groundwater storage only. That it also represents a minimum rate of evapotranspiration is a later interpretation (e.g. Chen and Wang, 2013).

3. Advantages of the lower envelopes method (Page C7377, last paragraph)

I appreciate the clarification by Referee 1 on the advantages of the classical lower envelopes method. This also lends support to a statement of mine that parameters have meaning only within their model such as BN, and, I should add, its implementation scheme such as the lower envelopes method (Page 15669, Line 25, to Page 15670, Line 2).

Additional references

Mutzner, R., E. Bertuzzo, P. Tarolli, S. V. Weijs, L.Nicotina, S. Ceola, N. Tomasic, I. Rodriguez-Iturbe, M. B. Parlange, and A. Rinaldo (2013), Geomorphic signatures on Brutsaert baseflow recession analysis, Water Resour. Res., 49, 5462–5472, doi:10.1002/wrcr.20417.

Troch,P.A.,et al (2013), The importance of hydraulic groundwater theory in catchment hydrology: The legacy of Wilfried Brutsaert and Jean-Yves Parlange, Water Resour. Res., 49, doi:10.1002/wrcr.20407.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 10, 15659, 2013.