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Hydrology and Earth System Sciences
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Discussions

Interactive comment on "An evaluation of analytical streambank flux methods and connections to end-member mixing models: a comparison of a new method and traditional methods" *by* M. Exner-Kittridge et al.

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1 Introduction

We (the authors) would first like to thank the Referees for their thorough comments and the time they spent reviewing the paper. Both Referees provided valuable comments. We will address each comment by both Referees in sequence.

2 Anonymous Referee 1

Response to the comment *First, I would like to see a more careful theoretical explanation...*

A precise explanation of the assumptions for the derivations of Eqs. (21) to (24) are given at the beginning of Section (2.2). The assumptions that represent the physical attributes of gross gains and losses along a stream reach are *"simultaneous and uniform gains and losses throughout the stream reach and stationarity in time"*. Unlike the Gain-Loss and Loss-Gain equations where a quick glance at the equations can provide an intuitive understanding of the spatial assumptions of the models, the Simultaneous equations must be understood in the physical context by the underlying assumptions and the fundamental mass balance equations used in the derivations. We can add the above statement to the end of the derivations for Eqs. (21) to (24). Both Figs. (2) and (3) were illustrative attempts to allow the reader to visually and conceptually understand the assumptions of the three methods.

Response to the comment *Regarding eq 21-24: The derivation...*

We thankfully acknowledge the referee for pointing out this fact. It is true, that for $Q_{final} = Q_{init}$ the Equations 21-24 are ill defined, but this is a so called "removable discontinuity" (http://en.wikipedia.org/wiki/Classification_of_discontinuities), as for the limit $Q_{final} \rightarrow Q_{init}$, all equations have a finite and well defined value. Let's take the expression:

$$\frac{Q_{init} - Q_{final}}{\ln\left[\frac{Q_{final}}{Q_{init}}\right]} \tag{1}$$

which appears in Eqs. 21-24, and causes the apparent discontinuity. Now, let's call

$Q_{final} := x$ and $Q_{init} := x_0$ for simplicity, and calculate the limit $x \to x_0$:

$$\lim_{x \to x_0} \frac{x_0 - x}{\ln\left[\frac{x}{x_0}\right]} \tag{2}$$

In fact, this is a limit of the type $\frac{0}{0}$, but can be solved applying *L'Hôpital*'s rule (http://en.wikipedia.org/wiki/L'H%C3%B4pital's_rule):

$$\lim_{x \to x_0} \frac{x_0 - x}{\ln\left[\frac{x}{x_0}\right]} = \lim_{x \to x_0} \frac{-1}{(\frac{x_0}{x})(\frac{1}{x_0})} = -x_0 \tag{3}$$

This means that, for $Q_{final} \to Q_{init}$, expression (1) has the value $-Q_{init} = -Q_{final}$. Now we come back to Eqs. 21-24. For the case $Q_{final} \to Q_{init}$, Eq. (21) in the manuscript will now take the form:

$$Q_{in,Sim} = (-Q_{init}) \cdot \ln\left[\frac{C_{final} - C_{in}}{C_{init} - C_{in}}\right] = Q_{init} \cdot \ln\left[\frac{C_{init} - C_{in}}{C_{final} - C_{in}}\right] \tag{4}$$

Equations 22-24 will simplify in a similar fashion, for the case $Q_{final} \to Q_{init}$.

Independently, if one takes Eq. (17) in the manuscript, this is the point where one has considered $q_{in} - q_{out} \neq 0$ in the derivation, and the integration is performed on the RHS. If one considers $Q_{init} = Q_{final}$, equivalent to $q_{in} = q_{out}$ due to mass balance, Eq. (17) would simplify to:

$$\int_{C_{init}}^{C(x)} \frac{dC}{C(x) - C_{in}} = -q_{in} \int_0^x \frac{dx}{Q_{init}} \tag{5}$$

and integrating:

$$\ln \frac{C(x) - C_{in}}{C_{init} - C_{in}} = -q_{in} \cdot \frac{x}{Q_{init}} \tag{6}$$

Evaluating for $x = L$, and rearranging for $q_{in} \cdot L = Q_{in}$, gives the expression:

$$Q_{in} = (-Q_{init}) \cdot \ln \frac{C_{final} - C_{in}}{C_{init} - C_{in}} = Q_{init} \cdot \ln \frac{C_{init} - C_{in}}{C_{final} - C_{in}} \tag{7}$$

which is identical to Eq. (4) in this manuscript.

The analytical derivation presented here shows that the model proposed is robust, as Eqs. 21-24 in the manuscript represent the general expressions for all possible combinations of Q_{init}, Q_{final}, C_{init} and C_{final}. The particular case $Q_{init} = Q_{final}$, which can be derived independently (Eqs. 5-7 in this reply), gives the same result as the general model (Eqs. 21-24 in the manuscript) if one considers the limit $Q_{final} \to Q_{init}$. As stated by the Referee, there should not be any theoretical reason for Q_{in} and Q_{out} to be estimated for the case $Q_{init} = Q_{final}$, and we have shown here that this is not the case. Possibly more importantly are the practical implications as mentioned by the Referee. In natural stream conditions with tremendous heterogeneity, Q_{init} will almost never be truly equal to Q_{final} in the mathematical sense. Additionally, even if $Q_{init} = Q_{final}$ occurred naturally, the measurement error associated with the discharge measurement method would ensure Q_{init} would never equal Q_{final}.

A set of equations, similar to Eq. (4) in this reply, will be added for Eqs. 21-24 in the manuscript for the particular case $Q_{init} = Q_{final}$, acknowledging the fact that these are special cases of the already present ones considering the limit $Q_{final} \to Q_{init}$.

These facts are completely independent with the term $dCdQ$ in Eq. (14). The removal of this term is not arbitrary, and is acknowledged in the text in page 10427, line 4 *"Neglecting second order differentials..."*. The term $dCdQ$ appears after multiplying discharge Q and concentration C on the right side of the differential slice considered in

Figure 3. Here, both outgoing discharge and concentration have varied an infinitesimal amount with respect of the incoming quantities, and so the product reads:

$$(Q + dQ) \cdot (C + dC) = QC + CdQ + QdC + dQdC \tag{8}$$

If the incremental variations dQ and dC were finite, it would make physical sense to retain the product $dQdC \rightarrow \Delta Q \Delta C$, as we could assign some value to this quantity, but these are diffential (infinitesimal) increments, and second order differentials need to be neglected when compared with first order differentials ($dQdC << CdQ$ and $dQdC << QdC$) in order to obtain a solution for the differential equation presented in Eq. (14). From a purely mathematical point of view, if we define the quantity mass flow $M = Q \cdot C$, then its value after the differential slice in Figure 3 would be:

$$M + \frac{\partial M}{\partial x} dx = M + dM = (Q \cdot C) + d(Q \cdot C) \tag{9}$$

and by definition, applying the product rule of differentiation, one gets:

$$d(Q \cdot C) = C \cdot dQ + Q \cdot dC \tag{10}$$

where the term $dQdC$ does not even appear.

Response to the comment *Regarding section 4.2: I cannot follow...*

We are very sorry that section (4.2) was confusing for the Referee. Looking through the section again with the Referee's comments in mind, we can fully understand the confusion. We will remove Eq. (33) and perform the EMMA derivation with Q_{out} without trying to (confusingly) connect the EMMA with Streambank fluxes until the derivation is finished. There was also confusion related to the spatial sequence of flows associated with Eqs.(36) and (37), which we believe was mainly due to a combination of

the confusion due to a poor connection with the streambank fluxes and non-intuitive variable names for flows. All flows in the EMMA derivation including the final form do not imply a spatial sequence. For example, $Q_{out,s2}$ is only the portion of Q_{out} from the second source, and subsequently $Q_{out,s2} \cdot C_{s2}$ is the total load lost anywhere by the second source. One of the critical points that needs to come across in the section is that EMMA is insensitive to the spatial distribution of streambank fluxes and furthermore is insensitive to Q_{out} entirely. We will try to better describe and illustrate the conceptual differences between EMMA and streambank fluxes. We will add in some simple examples for this purpose.

For example, lets imagine a stream reach with a Q_{up} and Q_{down}, where Q_{up} is the upstream discharge and Q_{down} is the downstream discharge (we will use the variable names recommended by the Referees). We then continuously inject a bromide tracer somewhere upstream of Q_{up} to ensure that the tracer is completely mixed at Q_{up}. We then measure Q_{up} and Q_{down} in addition to the bromide concentrations at the locations Q_{up} and Q_{down} that we will call C_{up} and C_{down}. The measurements are $Q_{up} = 2$ l/s, $Q_{down} = 3$ l/s, $C_{up} = 100$ mg/l, and $C_{down} = 50$ mg/l. If we would like to estimate the streambank fluxes, we could plug the numbers into the relevant equations and get the following: $Q_{in,L-G} = 1.5$, $Q_{out,L-G} = 0.5$, $Q_{in,G-L} = 2$, $Q_{out,G-L} = 1$, $Q_{in,sim} = 1.71$, and $Q_{out,sim} = 0.71$. Separately, we may also want to determine the amount of Q_{in} left in Q_{down} (EMMA).

Instead of using the EMMA equation, we could simply look at the Loss-Gain and Gain-Loss equations and calculate from the gross gains and losses what the amount of Q_{in} should be left in Q_{down} for each method. The simultaneous method is a bit more complicated than a simple calculation, so we will leave it out for now. For the Gain-Loss, we add 2 l/s (Q_{in}) to the initial 2 l/s (Q_{up}) then after the mixture we remove 1 l/s (Q_{out}). The result at Q_{down} is that 1.5 l/s came from Q_{up} and 1.5 l/s came from Q_{in} for the Gain-Loss method. Performing the same procedure for Loss-Gain, we first remove 0.5 l/s (Q_{out}) from the initial 2 l/s (Q_{up}), then we add 1.5 l/s (Q_{in}). Again, both sources con-

tribute 1.5 l/s to Q_{down}. Indeed, if a similar procedure is performed on the Simultaneous method the result would also be the same for the proportions of Q_{down}. Although we will estimate gross gains and losses differently depending on the spatial distribution assumptions, we will always produce the same result for EMMA regardless of the spatial distributions. The fact that the EMMA equations and Loss-Gain equations look the same is due to the spatial assumption of the Loss-Gain that coincidentally has the Q_{in} entering the stream at the end, which ensures that the $Q_{in,L-G}$ will always be equal to amount of Q_{in} remaining at Q_{down}. Nevertheless, EMMA is conceptually different from streambank fluxes as EMMA does not imply a knowledge of the gross gains and losses and streambank fluxes do not imply a knowledge of the source proportions at a downstream discharge location.

The discussion revolving around the conceptual differences between EMMA and streambank fluxes would not be in the paper if there was no confusion between the two in the scientific community. Unfortunately, we did find some confusion in the literature, and that is why we wanted to clarify the issue.

Response to the comment *I suggest the authors reconsider their variable naming scheme.*

We completely agree with changing the variable names. All of the variables with $init$ will instead become up, all of the variables with $final$ will become $down$, all of the variables with in will become $gain$, and all of the variables with out will become $loss$. It also seems like a good idea to add "min" and "max" in parentheses to the Loss-Gain and Gain-Loss terms in the tables and figures to make them more immediately intuitive.

Response to the comment *I suggest the authors de-emphasize the idea...*

If we have stated directly or indirectly in the text that our evaluation method is to determine what method is more accurate to reality, then we will surely modify our text. We do state several times in the text that our goal is to evaluate what method is more accurate to our numerical stream simulations. More specifically in the introduction we state *The goal of our study is to quantitatively evaluate the accuracy and sensitivity of the new method against the existing steady-state streambank flux tracer methods. This evaluation is performed through a combination of analytical comparisons and numerical stream simulations as described in the following sections.* Although the Referee may not be interested in the accuracies of different methods, we were interested and we thought others may be interested too. We also must unfortunately disagree with the statement by the Referee *The fact that another method that is always going to be somewhere between these two is generally more accurate is almost a foregone conclusion*. If indeed the Simultaneous method produced results that were overall close to the Gain-Loss equation rather than the Loss-Gain equation, then the Loss-Gain equation likely would have been the superior performer.

Response to the comment *Title – Is "streambank flux" a general enough term...*

You are probably right, we actually just liked the name "streambank". Combining your other comment about the title, we thought of another title. *An evaluation of stream flux mixing models: Comparing estimates of gross exchanges based of different spatial flow distribution assumptions*.

Response to the comment referring to *Pg 10430 lines 7-13*

We again agree. We will put in the text a short description about the necessity to continuously monitor concentrations downstream and integrate the concentration over

time with references to papers like Payn et al. 2008, Payn et al. 2009, and Covino et al. 2011.

Response to the comment referring to *Pg 10432 lines 1-3*

We initially thought that we should use non-dimensional values for everything, but we finally decided that the paper might seem too abstract without values people could relate to. We decided on using 1000 m and 2000 m for a couple reasons. First as the Referee surmised, these reach lengths would scale very well with the switching lengths of 100 m and 200 m that we determined from the DTS papers and would therefore be easy to mentally convert to different lengths. If a reader wanted to extrapolate to a 500 m reach, then it could be possible. Second, we did think that the reach lengths were relatively appropriate for stream tracer tests. For example, Covino et al. 2011 performed tests for the same purpose and had reach lengths of 800, 1000, 1050, 2246, and 3744 to name a few. What will determine the appropriate stream reach length for a tracer test is the discharge of the stream and the available mass of tracer. The reach length also needs to be sufficiently long to ensure complete mixing. We will include the above comments in the paper.

Response to the comment referring to *Pg 10440 line 2*

Thank you. We will correct it.

Response to the comment referring to *Figure 5*

The distributions of Q_{out} and Q_{in} are purely a function of the input distribution of Q_{init}

and the ARIMA model. We specifically defined Q_{init} to have an equal distribution from 1 to 5 l/s to represent semi-realistic discharges in a small stream reach. The resulting normal distributions for Q_{out} and Q_{in} are not surprising. We could change the input Q_{init} to have smaller values or larger values, but shifting the Q_{init} range would also only shift the ranges of the Q_{out} and Q_{in} as well. This would be similar to changing the units from l/s to ml/s. The results of the method comparisons should not change, but these small flux values in the practical application of the methods may be problematic due to the errors involved with the measurements of discharges and concentrations.

3 Anonymous Referee 2

Response to the comment *As pointed out by referee 1...*

As we mentioned in the previous response to Referee 1, these variable names will be changed.

Response to the comment *At equation 11, I think "x" is not really "stream length"...*

Thank you for finding this error. We will change it.

Response to the comment *I followed the derivation to equation 21, but could not...*

Equation (22) from the manuscript comes simply from Eq. (21) plus the overall mass balance equation for the stream reach, Eq. (2) in the manuscript. Rearranging Eq. (2)

in the manuscript we get:

$$Q_{out} = Q_{init} - Q_{final} + Q_{in} \tag{11}$$

And now substituting Eq. (21) for Q_{in}:

$$Q_{out} = (Q_{init} - Q_{final}) + (Q_{init} - Q_{final}) \frac{\ln[\frac{C_{final}-C_{in}}{C_{init}-C_{in}}]}{\ln[\frac{Q_{final}}{Q_{init}}]} \tag{12}$$

$$Q_{out} = (Q_{init} - Q_{final}) \cdot (1 + \frac{\ln[\frac{C_{final}-C_{in}}{C_{init}-C_{in}}]}{\ln[\frac{Q_{final}}{Q_{init}}]}) \tag{13}$$

$$Q_{out} = (Q_{init} - Q_{final}) \cdot \frac{\ln[\frac{Q_{final}}{Q_{init}}] + \ln[\frac{C_{final}-C_{in}}{C_{init}-C_{in}}]}{\ln[\frac{Q_{final}}{Q_{init}}]} \tag{14}$$

$$Q_{out} = (Q_{init} - Q_{final}) \cdot \frac{\ln[\frac{Q_{final}\cdot(C_{final}-C_{in})}{Q_{init}\cdot(C_{init}-C_{in})}]}{\ln[\frac{Q_{final}}{Q_{init}}]} \tag{15}$$

We are especially grateful to the referee in this point, as a typo has been found in Eq. (22) of the manuscript. If one compares Eq. (22) in the manuscript to Eq. (15) in this reply, they are identical, except for the terms $Q_{in}C_{in}$ in the numerator inside the logarithm. They need to be corrected to $Q_{final}C_{in}$ and $Q_{init}C_{in}$ respectively, as can be seen in Eq. (14) derived in this reply. Fortunately, this typo does not affect Eq. (24) in the manuscript and the subsequent simulation evaluations. We will include both the additional derivation of Eq. (22) as listed above and the typo correction of Eq. (22).

Response to the comment *Referee 1 had some good comments about the unexpected behavior...*

Unfortunately, we do not follow the derivation of the Referee in this point. The case of Eq. (23) is considered, with $Q_{out} = 0$ and $C_{in} \approx 0$. As the Referee states, for this situation the mass flow for the tracer is constant and equal to $Q_0C_0 = Q_{init}C_{init} = Q_{final}C_{final}$. One can therefore define $Q_{init} = \frac{Q_0C_0}{C_{init}}$ and $Q_{final} = \frac{Q_0C_0}{C_{final}}$, even though there is no need, as these discharges are considered measured quantities. Nevertheless, substituting these expressions in Eq. (23), as the Referee suggests, one gets:

$$Q_{in,Sim} = (Q_{init} - Q_{final}) \cdot \frac{\ln[\frac{C_{final}}{C_{init}}]}{\ln[\frac{Q_{final}}{Q_{init}}]} \tag{16}$$

$$Q_{in,Sim} = (Q_{init} - Q_{final}) \cdot \frac{\ln[\frac{C_{final}}{C_{init}}]}{\ln[\frac{\frac{Q_0C_0}{C_{final}}}{\frac{Q_0C_0}{C_{init}}}]} \tag{17}$$

$$Q_{in,Sim} = (Q_{init} - Q_{final}) \cdot \frac{\ln[\frac{C_{final}}{C_{init}}]}{\ln[\frac{C_{init}}{C_{final}}]} \tag{18}$$

$$Q_{in,Sim} = (Q_{init} - Q_{final}) \cdot (-1) = Q_{final} - Q_{init} \tag{19}$$

which is the expression it should reduce to as emphasized by the Referee, due to simple mass balance.

Response to the comment *The derivation of equation 25 was also quite unclear...*

We will clarify the statement and add one additional equation as an example. For the derivation, we can use any one of the three streambank flux methods (6 possible equations). For simplicity, we will use the $Q_{in,L-G}$ equation from Eq. (5). As the value of $Q_{in,L-G}$ will be the same before and after the tracer injection (assuming steady-state), we can make two versions of the $Q_{in,L-G}$ before and after the tracer injection with a different C_{init} and C_{final} prior to the injection of the tracer and post injection of the tracer.

$$Q_{final}\left(\frac{C_{final,prior}-C_{init,prior}}{C_{in}-C_{init,prior}}\right)=Q_{final}\left(\frac{C_{final,post}-C_{init,post}}{C_{in}-C_{init,post}}\right) \tag{20}$$

With some rearrangement, we come to our final equation from Eq. (25) (with changes in the variable names to be more clear).

$$C_{in}=\frac{C_{init,prior}C_{final,post}-C_{final,prior}C_{init,post}}{C_{init,prior}-C_{init,post}-C_{final,prior}+C_{final,post}} \tag{21}$$

Response to the comment *This is an interesting paper but not clear...*

We will make several modifications to the paper based on the replies to Referee 1 that will hopefully improve the clarity and the understanding of the equations.