Hydrol. Earth Syst. Sci. Discuss., 10, C3039–C3041, 2013 www.hydrol-earth-syst-sci-discuss.net/10/C3039/2013/

© Author(s) 2013. This work is distributed under the Creative Commons Attribute 3.0 License.



# **HESSD**

10, C3039-C3041, 2013

Interactive Comment

# Interactive comment on "Just two moments! A cautionary note against use of high-order moments in multifractal models in hydrology" by F. Lombardo et al.

### P. Furcolo (Referee)

p.furcolo@unisa.it

Received and published: 5 July 2013

The paper deals with a severe problem that any scientist dealing with multifractal analysis should be aware of: the reliability of (relatively) high order moments in the estimation of the scaling properties of a process from a record of data. Despite "traditional" statistics and several more specific papers advice against the use of high order sample moments, there is a wide amount of literature in the field of multifractal analysis where moments up to very high order are actually used in order to estimate the parameters of multifractal models.

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper



The Authors are aware of the problem and of the scientific literature on the topic (including Ossiander and Waymire 2000: last minute update) and show further interesting results that are worth being shared with the scientific community. The paper is well written, with a clear and well organized exposition, good and relevant figures. It shows the problem of statistical reliability of sample moments of increasing order under a different point of view and with reference to the model by Lombardo et al. (2012). The results converge to the results from other studies, such reinforcing the validity of the analysis made by the Authors.

This reviewer has just a couple of considerations that may be useful for a minor revision of the paper:

- 1) the paper correctly shows the evaluation of uncertainty of sample moments at different orders and at different scales. It is evident how uncertainty and bias can significantly alter the numerical results. Nevertheless, in the multifractal analysis, what actually is investigated is the scaling of the moments, which is related to the scaling of the partition coefficients or, equivalently, to ratios of moments at different scales. My feeling is that correlation among sample moments at different scales can play a "positive" role, in this case, allowing for an estimation of the scaling exponent which is less uncertain than the values of the moments themselves. More generally, an assessment of how uncertainty of the moments affects uncertainty of the scaling exponents may be included in the paper (even just by numerical investigation on the generated samples);
- 2) it is not clear from the text if the generated data correspond to a "bare" realization of the multifractal process or to a "dressed" one. The algebraic behaviour of the tail of the marginal distribution of the process at a given scale (and the divergence of the moments) generally depends on the dressing and can stem also from thin tailed generators. Furthermore, when analyzing synthetic data at different scales, starting from a bare realization at the smallest scale produces some bias in the scaling of the moments because of the different amount of dressing that arises at the different scales. This point is not really crucial to the focus of the paper, yet it is quite relevant in the

## **HESSD**

10, C3039-C3041, 2013

Interactive Comment

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper



multifractal modelling. This reviewer therefore suggests to add a better explanation of how the data are generated and (eventually) dressed and the possible implications on the numerical (as well as theoretical, if available) results.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., 10, 4627, 2013.

# **HESSD**

10, C3039-C3041, 2013

Interactive Comment

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

Discussion Paper

