

Dear Dr. Ding,

Thank you for your insightful comments and suggestions. The followings are our responses to each of your comments.

Hydrograph recession as a maturing field of mathematical hydrology.

1. Introduction

I enjoy reading in an open forum the discussion paper by Chen and Wang (2013). From a personal perspective of what may be called the “mathematical hydrology” or mathematics of hydrology, I would like to offer comment on their recession analysis methodology described in their Sect. 2.1

Thank you for your constructive comments which are helpful to improve the manuscript.

2. Methodology

They start their equation (Eq. 1) from the Brutsaert-Nieber recession flow model, $-dQ/dt = aQ^b$ (e.g., Brutsaert, 2005). Integrating Eq. (1) yields:

$$1/Q^{b-1}(t) = 1/Q^{b-1}(0) + (b-1)at \quad \dots\dots\dots(D1)$$

and in a conventional form:

$$Q(t) = [Q^{-(b-1)}(0) + (b-1)at]^{-1/(b-1)} \quad \dots\dots\dots(D2)$$

Equation (D2) in fact is an “Eve” of most nonlinear baseflow models (e.g., Ding, 1966; Brutsaert, 2005). This is an analytical solution of the Boussinesq equation representing an outflow hydrograph from a cross section, perpendicular to a stream, of an unconfined aquifer. It is thus most applicable to hillslopes and zero-order catchments, and then to first-order streams, i.e. small watersheds. Equation (D1) represents a linear relation between the inverse fractional power (IFP) transformed discharge $1/Q^{b-1}(t)$ and the elapsed time t , so that the recession curve appears as a straight line on a semi-IFP plot (Ding, 1966, 2012). Being a linear form, the IFP transformed recession line is independent of the size of time step (Δt), compared to the Δt -dependent ($-dQ/dt$) term in the recession plot. The storage can be inferred from observed baseflow by integrating an elementary volume, Qdt , from time t to infinite:

$$S(t) = \int_t^\infty Qdt = S(0) + [1/(2-b)][1/a]Q^{2-b}(t) \quad \dots\dots\dots(D3)$$

This form is identical to their Eqs. (5a) and (5b). Their Eq. (3), based on reasoning on physical grounds, can be derived backward by differentiating Eq. (D3),(5a) or (5b).

Thank you for your interesting representation of IFP transformed representation of discharge. This provides a different view on recession analysis. The IFP transformed recession line is independent of the size of time step (Δt) but dependent on the value of parameter b which is. The $-dQ/dt \sim Q$ analysis eliminates the impact of initial recession time but dependent on the time step.

3. Effect of evaporation

In the derivations outlined above, evaporation has not been considered. Thus recession parameters a and b are independent of it, and need not be estimated at the lower envelop where the impact of evaporation is minimal (cf., page 5772, lines 18-19).

Evaporation is accounted for in their water balance equation (Eq. 2): $dS/dt = -Q - E$. This implies there are more flux pathways to the storage than the flow one alone. Among others, evaporation is thought to deplete or reduce the in-stream flow which in turn depletes the feeding or contributing aquifer storage. The flow is measured or measurable, but the storage is not, but inferable from flow measurements. The storage thus inferred is part of the contributing storage and need be adjusted upward for evapotranspiration loss.

For the purpose of this comment, the evaporation term (E) is considered in my view to represent the channel evaporation. In Tables 2a and 2b for the Spoon River (4,237 km²) and Nodaway River (1,972 km²), respectively, both not small in size by any measure, the estimated (channel) evaporation is all higher than the corresponding observed baseflow, thus not negligible (both rates mostly between 0.5 to 2 mm/d).

Thank you. One of the assumptions of the conceptual model is that the contributing storage-discharge function is single-valued relation. The values of recession parameters (a and b) are dependent on groundwater characteristics, and are not affected by evaporation. The lower envelope is used to estimate the parameter values due to 1) Given Q , groundwater discharge is corresponding to smaller recession rate ($-dQ/dt$); 2) The impact of E on recession rate is minimum. The storage in unsaturated and saturated zones is treated as one storage component, and the evaporation term (E) includes evaporation from both in-stream flow and land surface.

4. Supplement

Figures 1 and 1S together show the recession plots of $\log(-dQ/dt)$ vs. $\log Q$ for all nine study watersheds. I notice each of the data clouds can be fitted, objectively, by an additional linear regression line. This would simplify considerably the evaporation estimation procedure. It may be beyond the current scope of Chen and Wang (2013) paper, but I encourage them to explore this statistical, regression alternative of fitting recession parameters.

Thank you for your suggestion. In this study, the lower envelopes are determined by eyes. If the regression line is used, evaporation can only be estimated for the data points over the regression line.

Additional references

Brutsaert W.: Catchment-wide base flow parameterizations, Sec. 10.6 of "Hydrology: An Introduction," Cambridge Univ. Press, 2005. ISBN-10 0-521-82479-6. Ding, J. Y.: Discussion of "Inflow hydrographs from large unconfined aquifers," by H. A. Ibrahim and W. Brutsaert, J. Irrig. Drain. Div., Am. Soc. Civ. Eng., 92(IR1):104-107, 1966.

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