Lombardo et al. (submitted) caution against using estimates of moments h higher than 2 when parameterizing multifractal models. The authors should be aware that it has already been established that there exists a critical range of moments (bounded by H_c^-, H_c^+) for which the two most common estimators (both based on empirical moment scaling) of the underlying structure function converge to the structure function itself. Outside of this range, the behavior of the empirical estimators is unknown. Ossiander and Waymire (2000) provide a rigorous theoretical basis for this critical range.

As an example, Ossiander and Waymire (2000) also demonstrate that, for a multifractal model that generates a gamma-distributed quantity, the upper critical value H_c^+ is ~3.13. For this particular example, they also show that for $h > H_c^+/2$ (see their Fig. 2b), true values lie outside of the 95% confidence interval of the estimations. Therefore, these findings are generally consistent with the position of Lombardo et al. (submitted).

Ossiander and Waymire (2000), therefore, have already provided a warning against using higher moments. Their particular focus was on discrete multiplicative cascade models. Lombardo et al. (submitted) demonstrate that this warning also pertains to the disaggregation model of Lombardo et al. (2012) with a variety of distributions. Together, the results of Ossiander and Waymire (2000) and Lombardo et al. (submitted) suggest that a reasonable heuristic rule would be to use moments up to, but not exceeding, $h = 2$.

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