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Modeling root reinforcement using root-failure Weibull survival function

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Abstract

Root networks contribute to slope stability through complicated interactions that include mechanical compression and tension. Due to the spatial heterogeneity of root distribution and the dynamic of root turnover, the quantification of root reinforcement on steep slope is challenging and consequently the calculation of slope stability as well. Although the considerable advances in root reinforcement modeling, some important aspect remain neglected. In this study we address in particular to the role of root strength variability on the mechanical behaviors of a root bundle. Many factors may contribute to the variability of root mechanical properties even considering a single class of diameter. This work presents a new approach for quantifying root reinforcement that considers the variability of mechanical properties of each root diameter class. Using the data of laboratory tensile tests and field pullout tests, we calibrate the parameters of the Weibull survival function to implement the variability of root strength in a numerical model for the calculation of root reinforcement (RBMw). The results show that, for both laboratory and field datasets, the parameters of the Weibull distribution may be considered constant with the exponent equal to 2 and the normalized failure displacement equal to 1. Moreover, the results show that the variability of root strength in each root diameter class has a major influence on the behavior of a root bundle with important implications when considering different approaches in slope stability calculation. Sensitivity analysis shows that the calibration of the tensile force and the elasticity of the roots are the most important equations, as well as the root distribution. The new model allows the characterization of root reinforcement in terms of maximum pullout force, stiffness, and energy. Moreover, it simplifies the implementation of root reinforcement in slope stability models. The realistic quantification of root reinforcement for tensile, shear and compression behavior allows the consideration of the stabilization effects of root networks on steep slopes and the influence that this has on the triggering of shallow landslides.

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1 Introduction

Root reinforcement, the strength roots impart to soil, is recognized to be one of the most important contribution of vegetation to slope stability (Phillips and Watson , 1994; Sidle, 1992; Rickli and Graf, 2009). In the last 30 yr, three distinct methods have been employed to quantify root reinforcement. The approach of Wu et al. (1979) has been and is still used because of its simplicity. However, recent studies (Pollen, 2005; Schwarz et al., 2011; Cohen et al., 2011) have demonstrated that Wu et al. (1979) hypothesis that all roots break simultaneously can lead to order-of-magnitude error in the estimation of root reinforcemen and is thus untenable. More recently, Pollen et al. (2005) used the fiber bundle model with a stress-step loading to estimate root reinforcement. The advantage of this model is that roots of different dimensions do not all break at the same load. This approach, however, does not easily permit calculation of root elongation for realistic root bundles (e.g. roots with different apparent elasticities). To overcome this problem Schwarz et al. (2010c) implemented the strain-step loading approach in the Root Bundle Model (RBM). The main advantages of the RBM are: (1) calculation of the complete force-displacement curve of a bundle of roots, and (2) redistribution of forces on each single root based on their geometrical and mechanical properties (and not statistically imposed). In a further simplification of the RBM, Cohen et al. (2011) proposed an analytical solution implementing only the most relevant parameters (root-size distribution, root tensile force, Young's modulus, length, and tortuosity).

Since Wu et al. (1979) model, the main improvements in modeling root reinforcement are: (1) root strength is a function of diameter (Wu et al., 1979); (2) roots do not all break at the same time (Waldron and Dekessian, 1981; Pollen et al., 2005); (3) roots have different failure mechanisms, break or slip out (Waldron and Dekessian, 1981); (4) root geometry (length) and Young's modulus are functions of root diameter (Waldron and Dekessian, 1981; Schwarz et al., 2010b); (5) root tortuosity affects the apparent elasticity of roots and the failure mechanism (Schwarz et al., 2011).

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Based on empirical observations (Schwarz et al., 2011) and on micro-mechanic studies of roots (Loades et al., 2010), root mechanical properties are highly variable. Despite this evidence, all numerical and analytical models thus far implement the mechanical variability only as a function of root diameter, usually given as a distribution, assuming that roots within a diameter class are homogeneous. A more realistic assumption is that, for a given diameter or small diameter range, there is a variability due to the presence of "weak spots" related to the anatomy and geometry of roots (Loades et al., 2010). Root age, root constituents, environmental conditions in which roots grow are important factors that influence root biomechanics (Loades, 2007). All of these factors contribute to the variability of root mechanical properties. Thus, it is important to implement this variability in root reinforcement models and analyze how this variability affects the mechanical behavior of root bundles.

The objective of this work is to present this new approach for quantifying root reinforcement that considers the intrinsic variability of mechanical properties of roots of similar diameters. The new model is presented in Sect. 2. In Sect. 3 we present new field and laboratory root strength data used for the calibration and validation of the model (Sect. 4). A discussion of the model and comparisons with others is given in Sect. 5.

2 Model description

2.1 Root geometry and mechanics

We assume that each root is a linear-elastic fiber that breaks at a threshold displacement x_c. Estimating the tensile force in a root using the fundamental equation of linear elasticity requires knowledge of its geometry (diameter, length, tortuosity) and mechanical properties (maximum tensile force, Youngs's modulus).

Data on roots (Operstein and Frydman, 2000; Schmidt et al., 2001; Ammann et al., 2009; Schwarz et al., 2011; Giadrossich et al., 2012) provide support for modeling root length, L, maximum tensile force, F_{max}, and Young's modulus, E, as power-law functions of root diameter, i.e.

$$L(\phi) = L_0 \left(\frac{\phi}{\phi_0}\right)^{\gamma}, \tag{1}$$

$$F_{max}(\phi) = F_0 \left(\frac{\phi}{\phi_0}\right)^{\xi}, \tag{2}$$

$$E(\phi) = E_0 \left(\frac{\phi}{\phi_0}\right)^{\beta}, \tag{3}$$

where ϕ_0, L_0, F_0, and E_0 are scaling factors and γ, ξ, and β power-law exponents, where ϕ_0 is assumed to always equal 1 and will not be explicitely written in the following equations, e.g. $\frac{\phi}{\phi_0} = \phi$.

In natural soils, roots are not straight but tortuous and the force necessary to pull a root is small until the root is fully stretched out. A way to include this effect when using laboratory tensile tests to estimate root reinforcement in natural soils on slopes is to include a tortuosity coefficient that reduces the Young's modulus (e.g. Schwarz et al., 2010b), i.e.

$$E(\phi) = r E_0 \, \phi^{\beta}. \tag{4}$$

where the coefficient, r, ranges between 0.3 and 0.5. This coefficient does not affect the estimation of the maximum tensile force of a root, only the stretching (displacement) at which the maximum force is observed.

In this study we back-calculate the apparent value of the Young's modulus from field pullout tests using only measured displacement and tensile force (see Schwarz et al., 2010b).

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Using Eqs. (1), (2), and (4) together with the equation of elasticity, the tensile force, F, in a single root as a function of displacement, Δx, is

$$F(\phi, \Delta x) = \frac{r\pi E_0}{4L_0}\phi^{2+\beta-\gamma}\Delta x, \quad F(\phi, \Delta x) < F_{\max}(\phi), \tag{5}$$

and the displacement, x_c, at which that root fails is

$$\Delta x^{\mathrm{fit}}_{\max}(\phi) = \frac{4F_0 L_0}{r\pi E_0}\phi^{\gamma+\xi-\beta+2}. \tag{6}$$

2.2 Weibull survival function for roots

Given the range of the power-law exponents of Eqs. (1) to (3), Eq. (6) indicates that larger-diameter roots fail at greater displacements. In general, roots of increasing diameter will fail with increasing displacement. We hypothesize that the probability of a root to survive as a function of displacement is given by the two-parameter Weibull survival function

$$S(\Delta x) = \exp\left[-\left(\frac{\Delta x}{\lambda}\right)^{\varpi}\right], \tag{7}$$

where w is the Weibull exponent (shape factor) and λ the scaling factor. In addition to the effect of root diameter on its survival, roots of similar diameters will fail at different displacements owing to root-strength variability. Assuming that the scaling factor is the fitted maximum displacement at failure (Eq. 6), i.e. $\lambda = \Delta x^{\mathrm{fit}}_{\max}$, the normalized survival function is independent of root diameter and is given by

$$S(\Delta x^*) = \exp\left[-(\Delta x^*)^{\varpi}\right], \tag{8}$$

where

$$\Delta x^* = \frac{\Delta x}{\Delta x^{\mathrm{fit}}_{\max}(\phi)}. \tag{9}$$

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This function has only one parameter, namely the shape factor ω.

To construct this function from the data, we rank roots according to their normalized failure displacement (Eq. 9) where

$$\Delta x = \frac{4LF_{\max}^{\text{meas}}}{\pi\phi^2 rE} \tag{10}$$

and $F_{\max}^{\text{meas}}$ is the measured maximum force.

2.3 Root bundle reinforcement

The tensile force (root reinforcement) of a bundle of roots is obtained by summing the force contributions from each root multiplied by the survival function S

$$F_{\text{tot}}(\Delta x) = \sum_{i=1}^{N} F(\phi, \Delta x) S(\Delta x^*), \quad F(\phi, \Delta x) < F_{\max}(\phi) \tag{11}$$

where N is the number of roots. The RBMw implemented in a R code can be downloaded at the following link: www.ecorisq.org/openFTP/Schwarz.zip.

3 Results

3.1 Data and calibration of root parameters

To estimate parameters of the tensile force power-law function (Eq. 2) we use three sources of data obtained from roots of Spruce tree (*Picea abies* L.). For laboratory tensile tests, we excavated 43 roots from Üetliberg (47.349° N 8.491° E, altitude 860 m a.s.l.), a forested site dominated by Spruce tree near Zürich (Switzerland). Roots were cleaned and cut by hand, and tensile tests carried out with a universal testing machine (*LF-Plus Chatillon*) at the Department of Agriculture at the University of Sassari, Italy, within one week of sampling. Root diameter ranged from 0.6 to 2.8 mm. To complement the upper range of root diameter, we include in our analysis 53 measurements of root tensile strength from laboratory tests by Ammann et al. (2009) for the same specie but from a different study site in Switzerland (Gandberg, Schwanden GL). Their root diameter ranged from 3.5 to 10 mm. Finally, we include data from field pullout tests by Schwarz et al. (2011a) at Üetliberg. In these tests, 5 or more roots with diameters ranging from 0.9 to 3.7 mm were pulled in parallel with a testing machine.

Figure 1 shows measured tensile forces in roots from laboratory and field pullout tests. Regression of the data yields $\xi = 2.4$ and $F_0 = 8.9 \times 10^7$ N for laboratory tensile tests, and $\xi = 1.9$ and $F_0 = 5.5 \times 10^6$ N for the field pullout experiments. Tensile force is highly variable in both types of experiments (nonlinear regressions coefficients of 0.72 and 0.88 for laboratory and field tests, respectively).

Results indicate that the power-law functions of both types of measurements yield larger values than those reported in previous studies in Europe (Bischetti et al., 2005) but similar to second-order polynomial fits of Schmidt et al. (2001) for plants in the Oregon Coast Range as reported in Schwarz et al. (2012) (see Appendix A).

Data on Young's modulus for roots are scarce. Combining laboratory data from Operstein and Frydman (2000) on four Mediterranean shrub species and Ammann et al. (2009) on Spruce trees we obtain $\beta = -0.533$ and $E_0 = 3$ Pa.

Because there are scant data on root length versus root diameter, we combine existing measurements made on several roots (Schwarz et al., 2011; Giadrossich et al., 2012) which, when combine together, yield $\gamma = 0.575$ and $L_0 = 18.5$ m.

With the root parameters now calibrated, it is possible to explore the force-displacement behavior of a single root diameter class and the effects of root-strength variability using the survival function. Figure 2 shows Eq. (11) for one root ($N = 1$) for three values of the Weibull exponent ($\omega = 2, 10, 100$, respectively dotted, dashed, and solid lines) for three root diameter class (1, 2, and 3 mm, respectively red, black and blue). Also shown in Fig. 2 are the field-measured mean and standard deviation of the maximum pullout force and the displacement at maximum pullout force for these three root diameter classes (points and error bars; see Schwarz et al., 2011, for original data). The three curves show the sensitivity of the model to the values of the exponent ϖ. For $\varpi = 100$, which indicates little variability in root mechanical behavior within a diameter class, the model reproduces the average value of the root diameter class together with linear increase in force with displacement until failure expected from a single root. For decreasing values of ϖ, the curves become smoother, revealing the increased variability of root strength within a diameter class and resulting in a maximum force much lower than the average with a spread of the reinforcement force over a much larger range of displacement. Although the calculation use $N = 1$, these smoother curves should not be interpreted as the behavior of a single root, but as the mean behavior of many roots belonging to one diameter class.

3.2 Calibration of the survival function

Figures 3 and 4 show the survival function obtained from root pullout tests for laboratory and field data, respectively. Also shown are modeled survival functions using Eq. (7) for different values of ϖ. The best fits are found using $\varpi = 2.3$ and 2.4 for laboratory and field data, respectively.

3.3 Model validation: root-bundle reinforcement

Considering the root diameter distribution of a bundle of roots it is possible to calculate the displacement-force behavior for the whole bundle, that is meant to characterize the root reinforcement under tensile solicitations. Figure 5 shows how the mechanical behavior of a bundle of roots may change considering values of the Weibull exponent $w = 2$ and $w = 100$, respectively, compared to the results of field pullout experiments of a bundle of roots (Schwarz et al., 2011). The measured behavior of the bundle shows a peak of pullout force of 600 N by a displacement of 0.02 m. The data show an almost linear increase up to the peak and a non–linear decay afterwords. The curve calculated with $w = 100$ shows three sharp peaks corresponding to the failure of the three classes of root diameter that make up the bundle (0.001, 0.002, and 0.003 m respectively). In this case the maximal peak of pullout force reaches almost 933 N at a displacement of 0.047 m, whereas for the curve obtained with $w = 2$ the maximal pullout force peaks at about 490 N at a displacement of 0.037 m. For $w = 2.4$ we obtain a maximal pullout force of 517 N at a displacement of 0.036 m. The same calculation using the model of Wu et al. (1979) would lead to constant value of 1209 N (indicated in the figure as horizontal red line).

For the same conditions but changing the diameter-maximal force equation (calibrated on the laboratory tensile tests, and not on the pullout experiments) would lead to a maximal pullout force of 349 N by a displacement of 0.027 m.

4 Discussion

As argued in the previous section, the normalized failure displacement (Δx^*) applied to the Weibull function should be equal to one. Nevertheless our experimental observations have shown that the variability of the normalized failure displacement is from 0.8 to 1.4. We ascribe the uncertainly to two kind of errors. The first one is the variability of maximal root tensile force. As the model sensitivity ultimately depend by the relation by tensile force and root diameter, variability of normalized failure displacement depends on the wide of the cloud points. Variability of maximal root tensile force must be checked and residuals analysis became fundamental. So that, we should exclude outliers and diameter class that are not represented by a minimum number of data. The second type of error is the variability of calculation using different algorithms for fitting the regression of the diameter-force relationship, such as the ones implemented in *R software* or in *Microsoft Excel*. Fitting with different methods result in a big difference of results. The data analysis routines in Microsoft Excel are sometimes defined primitive (McCullough and Heiser, 2008) and we decided to use the "nls" function implemented in the R software to make a comparison, since this function use a different fitting approach. Basically, the fit that results in a normalized failure displacement equal

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to 1 should be considered the best one. The result of the fitting is also influenced by the calibration of the Eqs. (1) and (3). In Appendix A we reported the fitting data by different combination of software and diameter classes showing this differences.

The Weibull exponent w results constant independently from the root diameter class considered ($w = 2$). Even if it is intuitive to assume that the value of w should decrease with increasing root diameter classes due to the higher probability for the bigger classes to have "weak spots" related to the anatomy and geometry of the root (Ammann et al., 2009), it seems that this effect is masked from the scaling factor (λ), indicating that the probability to have "week spots" is more related to the root length and not to the root diameter.

The RBMw presents two major advantages in comparison to the RBM (Schwarz et al., 2011): (1) it accounts strength variability of each root diameter class, (2) the mathematical formulation is simple and can be easily implemented in numerical models (just 4 eqs. – Eqs. 1, 2, 4 and 7). The fact that pullout mechanisms is not considered in the RBMw is in part indirectly considered when calibration is done with field pullout experiments. In this case, the fit of the force-diameter function consider both pullout and breakage, whereas the residual pullout force after the peak value is neglected (which allow a conservative estimation of root reinforcement).

Differences in calculated root reinforcement using calibration with tensile tests and pullout test may be due to different factors: different period of sampling, different mechanisms of breakage, different water content, gauging length (Zhang et al., 2012). It is not possible in this study to identify quantitatively the main reasons. Further field pull-out experiments are considered the best data for a realistic calibration of the RBMw for specific conditions, and a diversified combination of factors would lead to a important dataset for the characterization of the possible contribution of root to reinforcement.

The comparison of the results obtained with the RBMw and the Wu method confirm that the Wu method could lead to errors up to 150 %, as showed in previous studies (Schwarz et al., 2011). However, it is interesting to observe that considering different root distribution for a constant root area ration (RAR) the quotient between the maximal

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pullout force of the bundle calculated with the RBMw and the force calculated with the Wu model is almost constant to a value of 0.4. This consideration is valid only if root distribution are heterogeneous and the *w* exponent is equal to 2 (a sensitivity analysis of this parameter is shown in the Fig. A3). Field data have shown that the combination of the root distribution 1 in Fig. 7 and a value of 2 for the *w* exponent is the most realistic one. Thus, the application of a reduction coefficient equal to 0.4 to make a first estimation of the maximum root reinforcement can be applied. However, for a complete characterization of the root reinforcement the calculation of a displacement-force curve is more appropriated because it gives indication about the initial stiffness of the root bundle and the total dissipated energy. The quantification of the root bundle stiffness is important because force redistribution on a slope during the triggering of a shallow landslide depend on the stiffness of the material, as shown by Schwarz and Cohen (2011). Moreover, the meaning of the total dissipated energy through the breakage and root-soil friction of a root bundle during slope failure was discussed by Ekanayake and Phillips (1999). Especially for not over-consolidated soil material, for which it is difficult to properly define a failure threshold for flat and broad stress-displacement curves, the energy approach method solves this problem by considering the total mobilized energy.

It was mentioned that the use of field experiment for calibration of the model are considered the most appropriated, moreover, this approach could allow the implementation of the elasticity considering the Hooke's law where pullout force is proportional to the displacement and doesn't need the explicit calibration of root geometric parameters such as the root length and Young's modulus of root material. This approach would lead to the following equation:

$$F(\phi, \Delta x) = H(\phi)\,\Delta x \tag{12}$$

where $H(\phi)$ corresponds to a spring constant as function of root diameter ϕ, which resume the mechanical properties of the root-soil system under specific conditions (root diameter, tree species, stand, soil type, and moisture conditions). The application of the Hooke's law would simplify the calculation and reduce the number of parameter considered in the calculation with the consequence to reduce the source of errors. This approach is possible only using time consuming and complicated field pullout experiments.

5 Conclusions

A new approach for a realistic quantification of root reinforcement that consider the strength variability of each root diameter class is presented using Weibull survival functions. The RBMw allows the characterization of root reinforcement in term of maximum pullout force, stiffness and energy. Moreover, the new mathematical formulation simplify the implementation of root reinforcement in slope stability models such as SO Slope (Schwarz and Cohen, 2011). More datasets of field pullout experiment are needed for the further validation of the RBMw under different combination of factors (soil type, soil moisture, and tree species). The same approach of the RBMw can be applied to characterize the reinforcement of roots under compression (Schwarz and Cohen, 2011). The realistic quantification of root reinforcement for tensile, shear and compression behavior will allow the consideration of the stabilization effects of root networks on steep slopes (Schwarz et al., 2012) and the influence that this has on the triggering of shallow landslides (Schwarz and Cohen, 2011). The application of such a slope stability model contributes to improving the quantification root reinforcement for use of protection forests and it supports the rational formulation of guidelines for the management of these forests.

A In order to verify which could be the source of errors that produce a normalized failure displacement different than 1 in the calibration of the Weibull distribution, we fitted the diameter-force curves for different number of class diameter, excluding classes with few samples, and using two softwares (Excel and R). The results shown in Fig. A1 indicate that the different methods lead to quite different values of the equation parameters. The meaning of these differences in term of root strength variability is shown in Fig. A2. Summarizing, small changes in the fitting of the root diameter- force curve lead

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to considerable changes in the scaling factor (λ), and thus in the value of normalized failure displacement (Δx^*) of the cumulative Weibull probability distribution.

As mentioned in the discussion, assuming a constant reduction coefficient to estimate the maximum pullout force of a root bundle using the Wu model seems to be acceptable under certain conditions. Figure A3 shows that the application of a w exponent of 2 strongly reduce the differences of maximum pullout force in comparison to a value of the w exponent equal to 100. Moreover, the results show that the coefficient change considering root distribution of few diameter classes.

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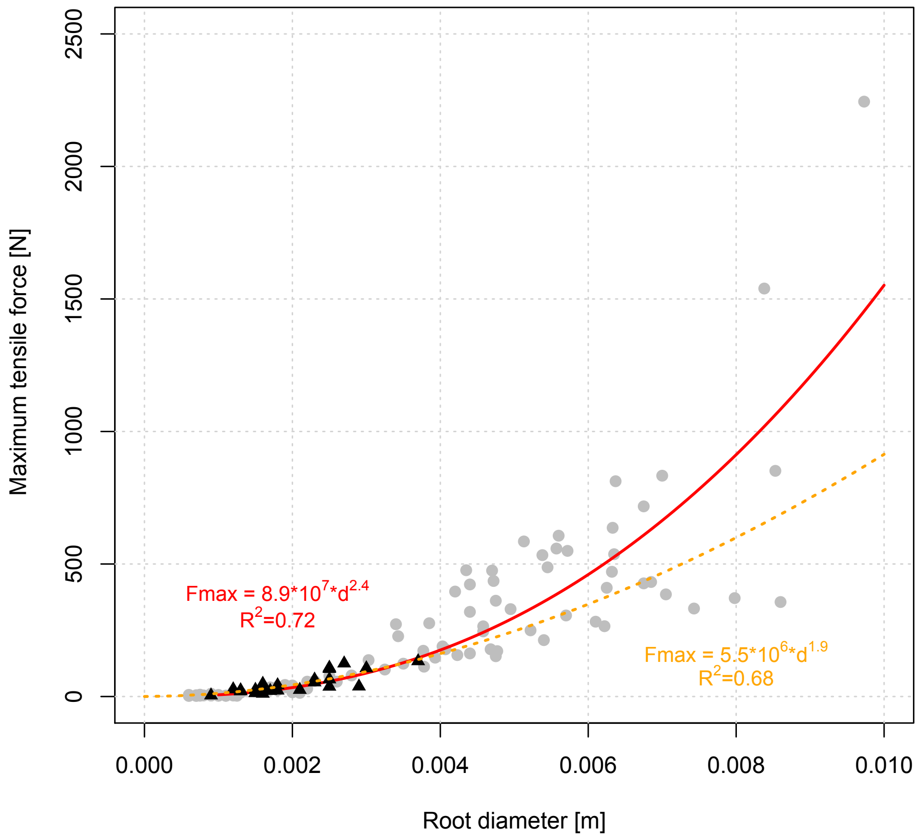

Fig. 1. Maximum measured tensile forces of spruce roots (*Picea abies* L.) as a function of root diameter. Black triangles are data from field pullout experiments (Schwarz et al., 2011) and grey circles are data of laboratory tensile tests. The red line is the fit for the laboratory tests; the orange dashed line is the fit of the field tests.

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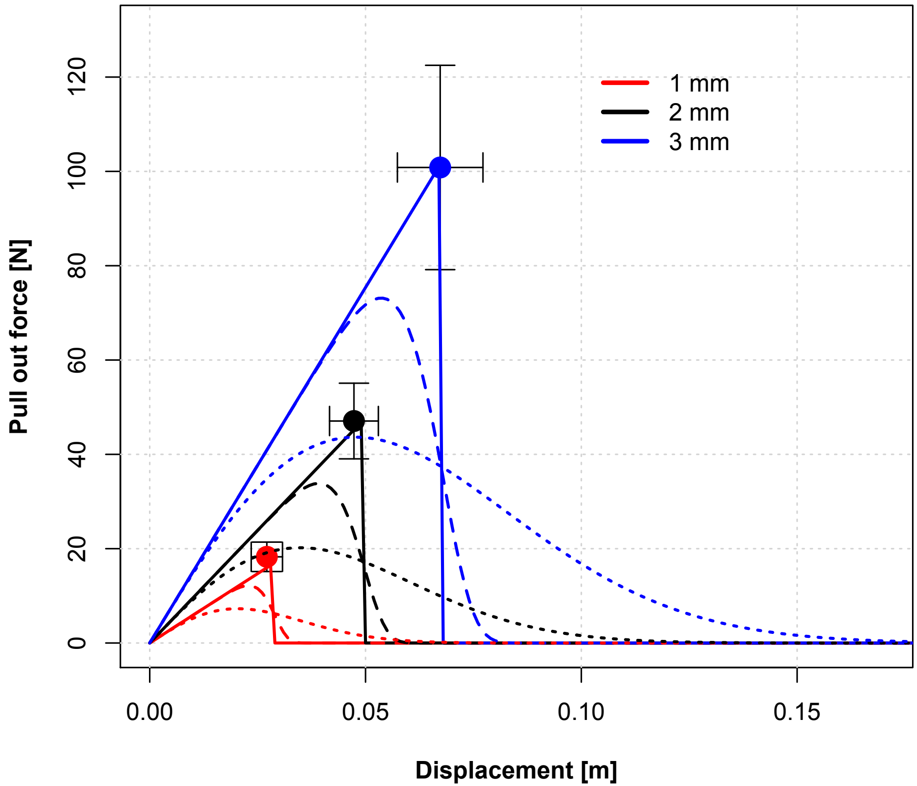

Fig. 2. Force-displacement behavior of 1 (red), 2 (black) and 3 (blue) mm diameter root classes. Points and error bars indicate mean and standard deviation measured in field pullout experiments (Schwarz et al., 2011). Curves show the modeled force-displacement for three values of the Weibull exponent, $\omega = 2$ (dotted line), $\omega = 10$ (dashed line), and $\omega = 100$ (solid line).

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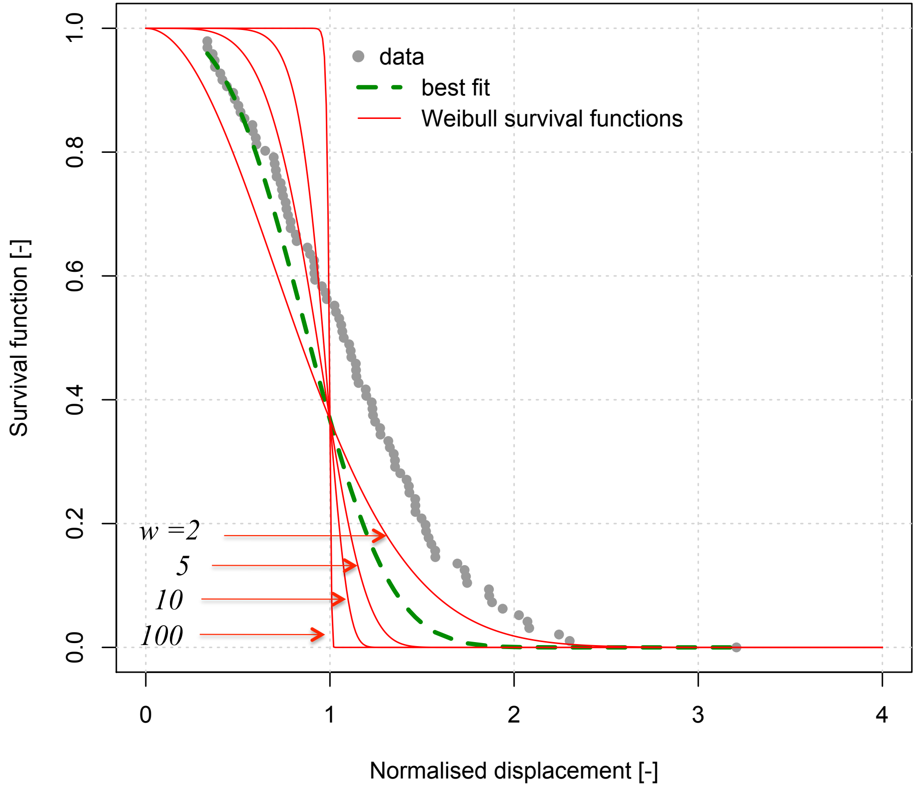

Fig. 3. Survival function for laboratory tensile test data. Grey dots are measurements, the green dashed line shows the best fit using a Weibull exponent $\omega = 2.3$. Red lines show the survival functions for $\omega = 2, 5, 10, 100$.

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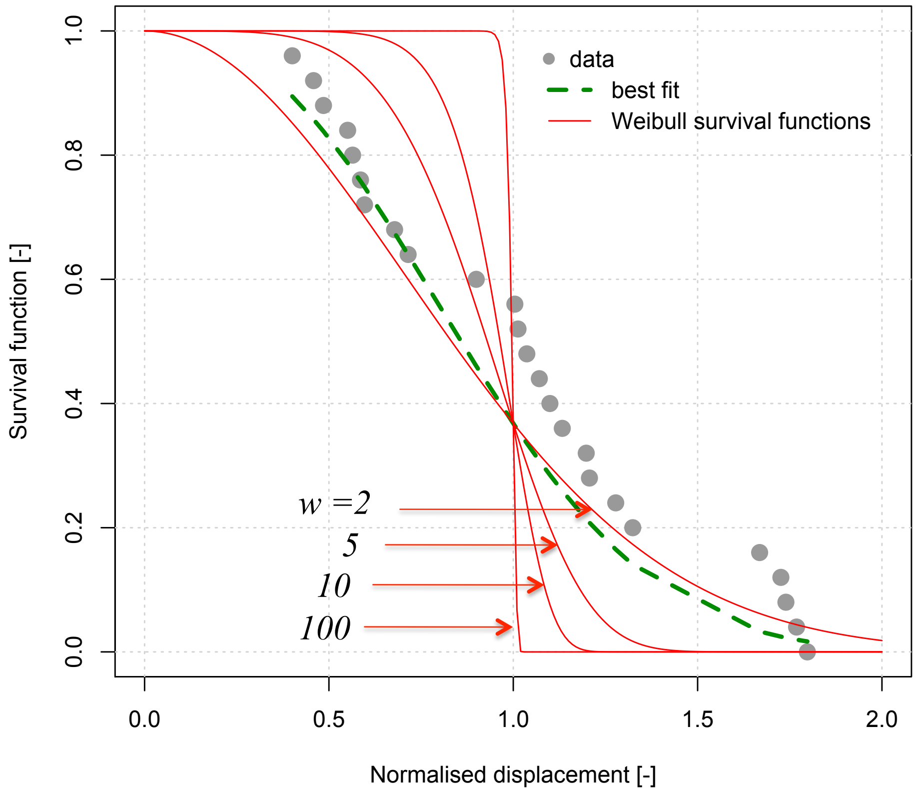

Fig. 4. Cumulative probability distribution of the normalized displacement at the point of maximal pullout force for the field pullout tests data. The grey points are the measured data, the green dashed line show the best fit of the Weibull distribution (obtained for a *normalized failure displacement* = 1 and *exponent* = 2.4), and the red lines show the sensibility of the Weibull function to the *exponent* values (2, 5, 10, 100).

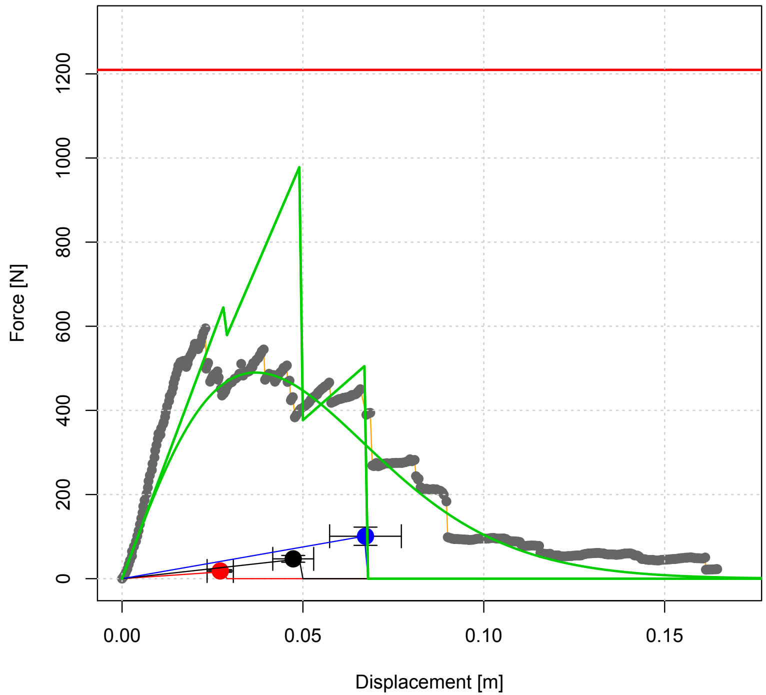

Fig. 5. Measured and simulated force-displacement behavior of a bundle of roots. The grey points (linked by the orange line) are the measured data of field pullout experiments (Schwarz et al., 2011a) for a root bundle composed by 7 roots of 0.001 m diameter, 13 roots with 0.002 m diameter, and 5 roots with 0.003 m diameter. The green lines show the prediction of the RBMw considering two values of the Weibull *exponent*, 100 and 2 respectively. The red-black-blue lines and points show the behaviors of the single roots, as link to Fig. 4. The continuous red line on the top of the plot indicates the estimated value of root reinforcement using the Wu model (Wu et al., 1979).

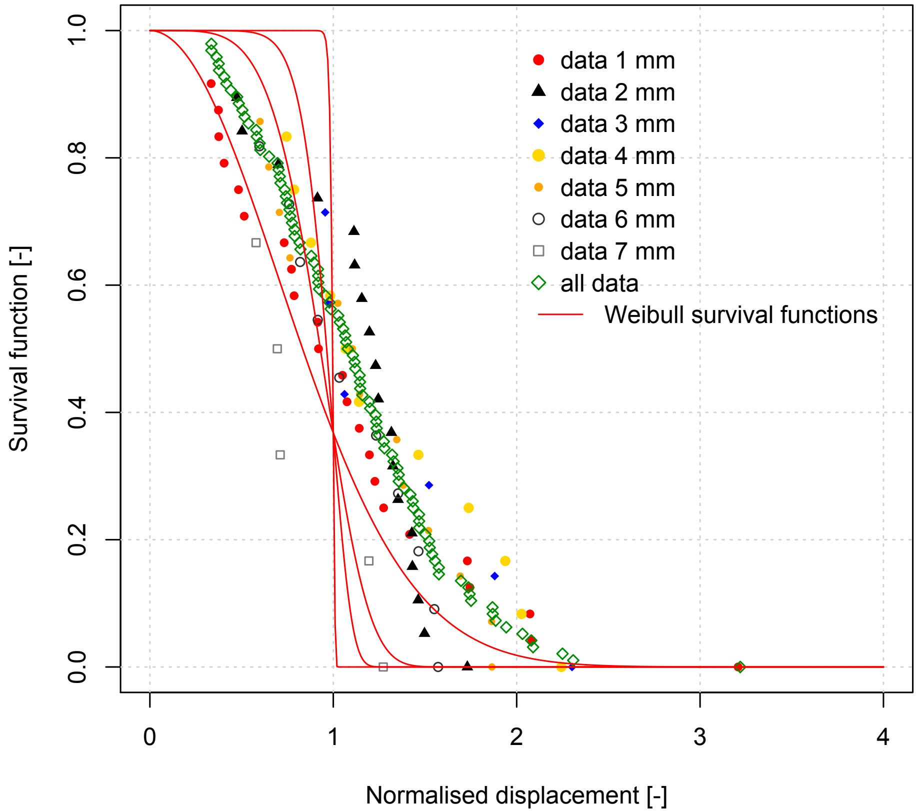

Fig. 6. Survival function calculated for each root diameter classes (from 1 to 7 mm) and for all the data of the tensile tests.

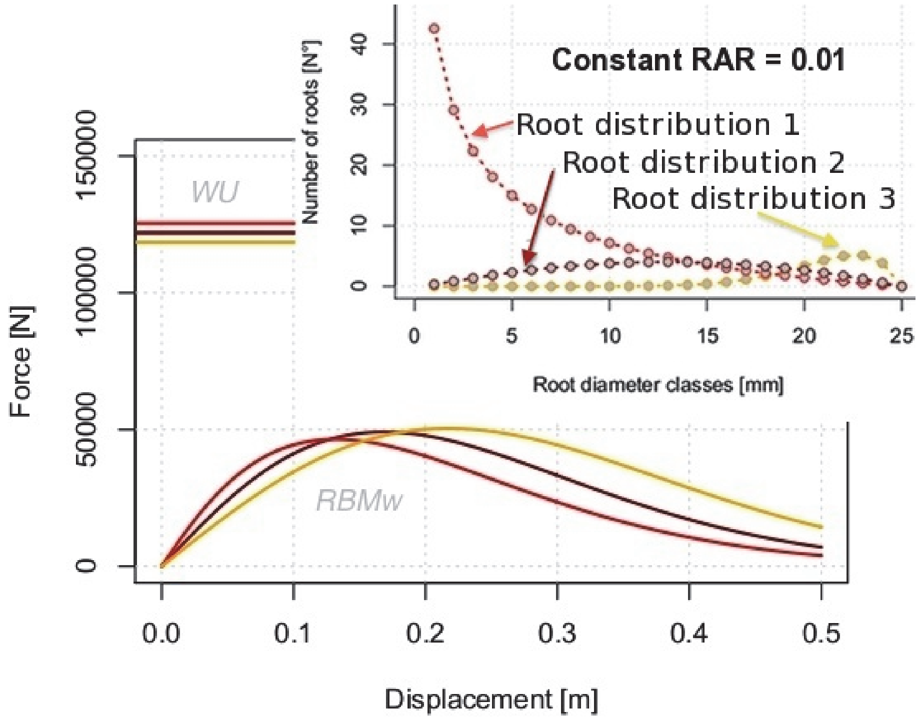

Fig. 7. Sensitivity analysis of the displacement-force curve of a bundle considering three different root distribution (see insert) with a constant root area ration (RAR) equal to 0.01, calculated with the RBMw and the Wu model. All the geometric and mechanical parameter were kept constant.

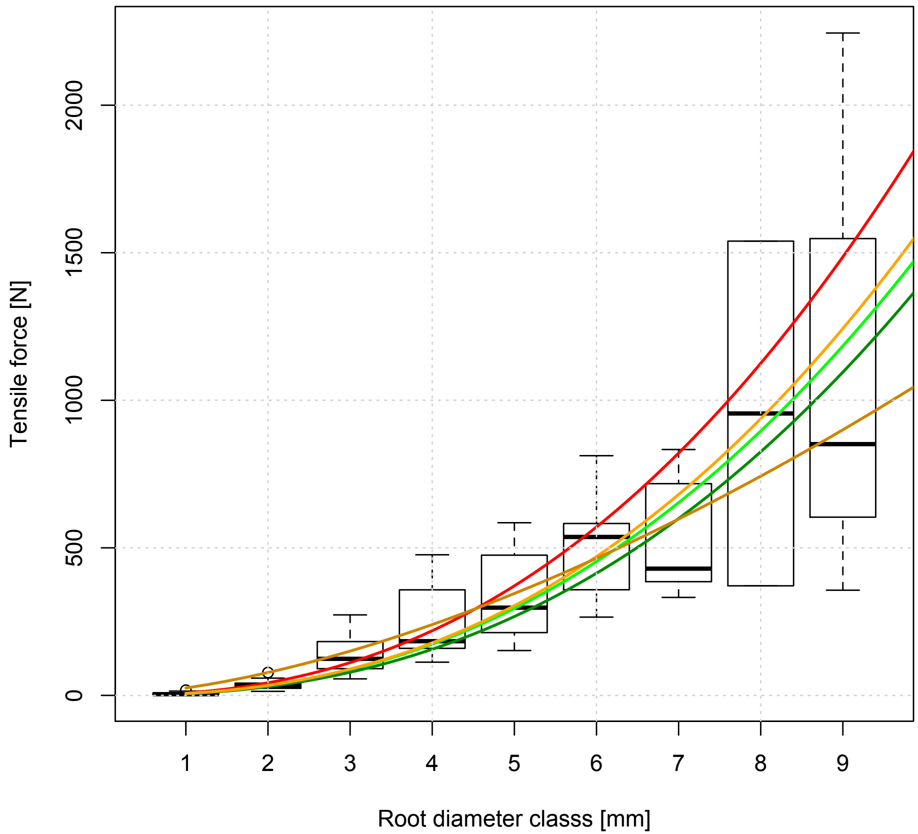

Fig. A1. Boxplot of the maximal tensile force measurement in function of root diameter classes. The three different fitting curves correspond to the fit of all data (green1 = xls, green4 = *R*), the fit of data up to 0.007 m diameter (orange1 = xls, orange3 = *R*), and finally the iteratively fitted curve to obtain a normalized failure displacement equal to 1 (red). ($n1 = 23$, $n2 = 17$, $n3 = 7$, $n4 = 12$, $n5 = 14$, $n6 = 11$, $n7 = 6$, $n8 = 2$, $n9 = 3$).

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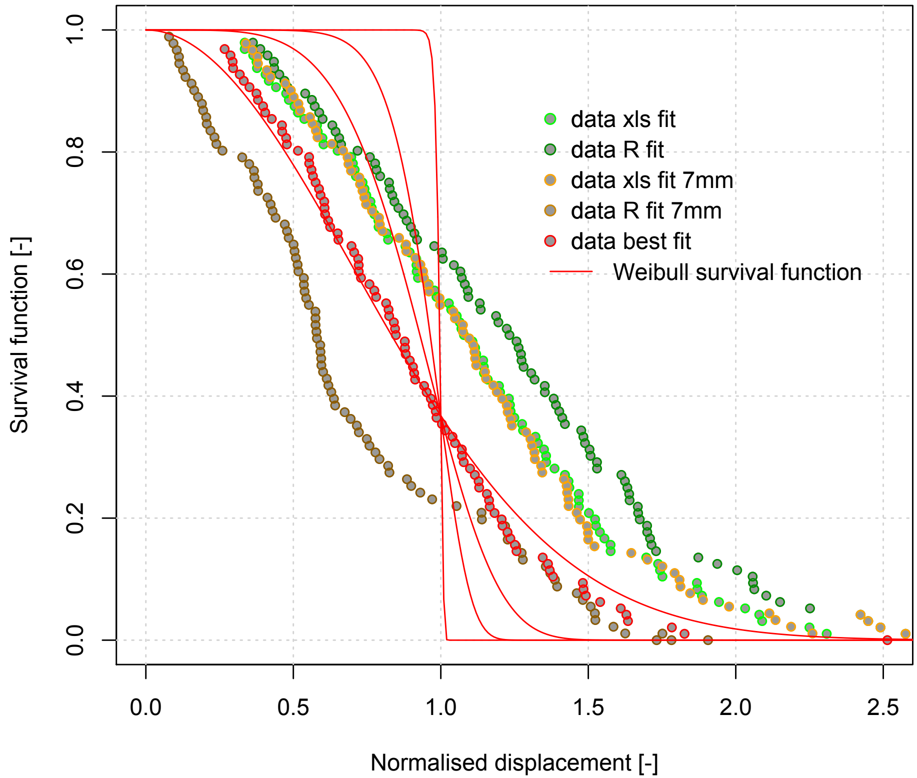

Fig. A2. Survival function of the normalized displacement calculated with different displacement-tensile force curves shown in Fig. A2.

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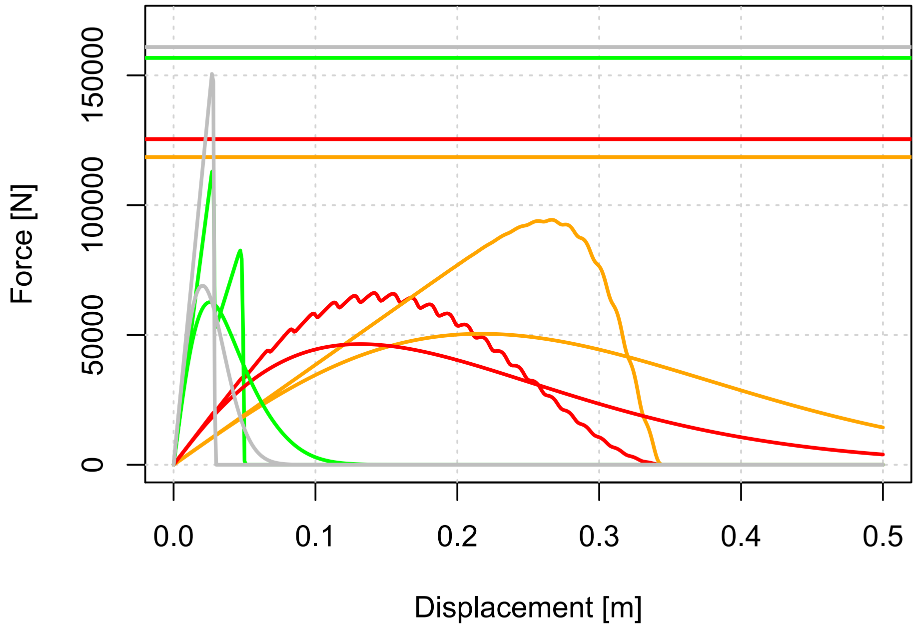

Fig. A3. Sensitivity analysis of the displacement-force curve of root bundles considering different root distribution (red = RD1 in Fig. 6, orange = RD3 in Fig. 6, green = two root diameter class distribution, gray = one root diameter class distribution) with a constant root area ration (RAR) equal to 0.01, calculated with the RBMw and the Wu model. For the calculation with the RBMw a value of 2 (lower curve) and 100 (upper curve) were used for the exponent *w*.

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