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Technical Note: A measure of watershed nonlinearity II: re-introducing an IFP inverse fractional power transform for streamflow recession analysis

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Abstract

This note illustrates, in the context of Brutsaert–Nieber (1977) model: $-dQ/dt = aQ^b$, the utility of a newly rediscovered inverse fractional power (IFP) transform of the flow rates. This method of streamflow recession analysis dates back a half-century. The IFP transform Δ_b on an operand Q is defined as $\Delta_b Q = 1/Q^{b-1}$. Brutsaert–Nieber model by IFP transform thus becomes: $\Delta_b Q(t) = \Delta_b Q(0) + (b-1)t$, if $b \neq 1$. The IFP transformed recession curve appears as a straight line on a semi-IFP plot. The method has both the advantage of being independent of the size of computational time step, and the disadvantage of being depending on the parameter b value. This is used to calibrate the Brutsaert–Nieber recession flow model in which b is a slope (or shape) parameter, and a is an intercept (or a scale parameter). It is applied to four observed events on the Spoon River in Illinois (4237 km²). The results show that the IFP transform method gives a narrower range of parameter b values than the regression method in a recession plot. Theoretically, an IFP transformed recession curve for large watersheds falls between those performed by the reciprocal of the cubic root (RoCR) transform and the reciprocal of the square root (RoSR) one. In general, the forgotten IFP transform method merits a fresh look, especially for hillslopes and zero-order catchments, the building blocks of a watershed system. In particular, because of its origin in hillslope hydrology, the 1-parameter RoSR transform need be falsified or verified for application to headwater catchments.

1 Introduction

Sustainable streamflow in a watershed is a prerequisite to the survival and prosperity of a civilization. Its uses include consumption by human and livestock, dilution or treatment of their wastes, irrigation of arable land, inland navigation, hydropower generation, fish habitat maintenance, to name a few. Its sustainability is characterized by the variation and persistence of the low flow, the latter effect aptly captured by what Mandelbrot

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and Wallis (1968) called the Joseph effect from their reading of the Genesis that in the land of ancient Egypt, seven years of harvest will be followed by seven years of famine. (The Noah flood represents for them the other extreme of the hydrologic regime.)

Prediction of the low flow during a prolonged rainless period, called the drought, dry weather, groundwater or base flow analysis, has long been based on a linear storage-discharge assumption, and the related semi-log plot for displaying the recession limb of a hydrograph (e.g. Gray, 1970).

In 1970's, Brutsaert and Nieber (1977) proposed a drought flow model which relates nonlinearly the rate of change of the flow to the flow itself: $dQ/dt = aQ^b$. This is best known for its dQ/dt vs. Q recession plot. The model eliminates the ambiguity of the start time of a recession hydrograph. However, the approximation of the infinitesimally time-step size dt by a finite difference Δt manifests itself in a scattering of the recession data points, called the cloud (e.g. Troch et al., 1993a; McMillan et al., 2010; Stoelzle et al., 2013; Chen and Wang, 2013). The originators recommend the use of early- and late-time lower envelopes for the data cloud, along both of which the effect of evaporation on the flow is reasoned on physical grounds to be minimal (e.g. Troch et al., 1993b; Brutsaert, 2005; Chen and Wang, 2013).

There have been numerous attempts to make use, or to reduce the size, of the data cloud in a recession plot. A first and obvious alternative is to fit the data cloud by a linear regression line (e.g. Troch et al., 1993a, b; Malvicini et al., 2005; Lyon and Troch, 2007).

In connection with the first one, Rupp and Selker (2006) devised a scheme of using a variable time-step size which scales to the magnitude of the drop in flow. The rescaled recession plot reduces the scatter at very low flows (e.g. Clark et al., 2009; Ajami et al., 2011; Staudinger et al., 2011).

A third and latest alternative is that of Kirchner (2009). He divides the Q scale into many increments, replaces the dQ/dt values within each of which by an average value, i.e. "binning" the data, and fits the data cloud by a polynomial, usually of a second degree (e.g. Teuling et al., 2010).

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Recently, Stoelzle et al. (2013) have surveyed the state of the art of streamflow recession analysis, with a special emphasis on Brutsaert–Nieber model. In their evaluation, they include the original lower envelopes (Brutsaert and Nieber, 1977), the data binning and polynomial fitting (Kirchner, 2009) as well as the linear regression methods, but excluding the variable time-step sizing of Rupp and Selker (2006). Their results suggest serious limitations to the comparability of streamflow recession characteristics derived with different methods.

During the public comment period of Stoelzle et al. (2013) paper, I brought to their attention a forgotten solution technique now called the inverse fractional power (IFP) transform. This was refined and elaborated in a later comment on Chen and Wang (2013) paper.

This note is a consolidation and extension of these two sets of comment, plus two additional ones, all on streamflow recession analysis, but some simply correcting errors, mainly mine (c.f. Farlin and Maloszewski, 2013; Troch et al., 2013). It will focus on the groundwater outflow model originated by Brutsaert and Nieber (1977), and make use of the recession flow data reported by Chen and Wang (2013). The latter provides four data segments for the Spoon River, a large watershed in Illinois, USA, and the centre piece of their nine-watershed study. The purpose is to illustrate, in the context of Brutsaert–Nieber model, the utility of a newly rediscovered IFP transform method whose origin traces back a half-century. This is essentially a retrospective piece of mine forming part two of the headlined HESS paper (Ding, 2011). It shifts the spotlight of the vIUH (variable instantaneous unit hydrograph) concept, a nonlinear system one, from floods to droughts. Comparison of the simple IFP transform method with other more sophisticated or complicated methods as recently reviewed by Stoelzle et al. (2013) will not be part of this note.

2 A brief mathematics of streamflow recession

2.1 Overview

The heading of my most recent comment was titled on purpose the “hydrograph recession as a maturing field of mathematical hydrology” (Ding, 2013b). This hopefully has conveyed my view that in the absence of the rainfall forcing and its effects, the falling or receding half of a hydrograph can be mathematically modelled starting from the principles of hydrology. Conceptually, the receding limb is simpler to model than its rising counterpart. Such a model can be falsified by a few spot measurements of the volume of water passing through a river section during the rainless period, i.e. by the dry weather flow. Compared to other field data collection programs, this seems not too onerously a task on small catchments.

Models, equations or methods representing the flow recession phenomenon for hillslopes and small watersheds described in the latest comment (Ding, 2013b) are re-capped and elaborated below. All of these are different faces or expressions of the same phenomenon. They have a shape (or, in Brutsaert–Nieber model, slope) parameter b , and a scale parameter (or intercept) a .

2.1.1 Brutsaert and Nieber model

Brutsaert and Nieber (1977) model is a hydraulic one for outflow from a cross section of an unconfined riparian aquifer having a horizontal impenetrable bottom layer. Being a one-dimensional model, it is most applicable to headwater catchments, not affected or dominated by the presence of streams of significant size. Of special interest are their derivations of the outflow behavior in term of the recession plot slope b : 3 for early recession, and 1.5 and 1 for late ones (Brutsaert and Nieber, 1977; Brutsaert, 2005).

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This standard model is now commonly written as follows:

$$-\frac{dQ}{dt} = aQ^b, \quad (1)$$

in which Q is the flow rate (mmd^{-1}), t is time (d), b is a shape parameter (dimensionless), and a is a scale parameter having a combination of the flow rate and the time units, $\text{mm}^{-(b-1)}\text{d}^{-(2-b)}$. In a $\log(-dQ/dt)$ vs. $\log(Q)$ plot, parameter b represents the slope of the recession curve, thus the name of slope parameter; and because when $Q = 1$, $a = -dQ/dt$, parameter a is thus the *intercept* of the curve at the $Q = 1$ line.

Conceptually, Eq. (1) is an expression of the growth or, in the present context, decay phenomena or functions found in nature and explored in science. For example, in a chemical reaction, the time rate of change in a concentration is proportional to the concentration raised to some power. The difference between the two is that flow Q is a time-rate variable, and $-dQ/dt$, the flow de-acceleration, is a second time-differential of the flow volume or concentration. Calculation of the de-acceleration requires a high resolution in time. For small watersheds, this is generally finer than the time resolution of daily or hourly flow measurements. Approximation of (dQ/dt) by the backward time-differencing one, $(Q(t) - Q(t - \Delta t))/\Delta t$, is thus the main source of the numerical instability in parameters characterizing the model. Approximation of $Q(t)$ by the backward time-averaging one, $(Q(t) + Q(t - \Delta t))/2$, is a secondary one.

2.1.2 IFP (inverse fractional power) transform

Rearranging Eq. (1) as: $Q^{-b}dQ = -adt$, and integrating from time 0 to t , I show the solution to be (Ding, 1974, 2013a, b):

$$1/Q^{b-1}(t) = 1/Q^{b-1}(0) + (b-1)at, \text{ if } b \neq 1 \quad (2a)$$

$$\log Q(t) = \log Q(0) - at, \text{ if } b = 1. \quad (2b)$$

Equation (2a) expresses a linear relation between an IFP transformed variable $1/Q^{b-1}(t)$ and the elapsed time t , and Eq. (2b) the same relation for a log transformed

early links between two approaches. For an illustration of a semi-RoSr plot, see Fig. 1 reprinted from Ishihara and Takagi (1965). This happened to be the first known one recorded in hydrology literature. As noted in Sect. 2.1.1 above, a parameter b value of 1.5 characterizes late-time recession in Brutsaert–Nieber model.

The application of the RoSR transform has rarely been reported in open literature, and if so, only opaquely, i.e. one had to read between the lines (e.g. Ando et al., 1986). But Takagi (1977) explicitly applied this transform for his recession analysis of rivers in Germany and Japan.

2.3 IFP (inverse fractional power) transform and the Tukey ladder of powers

Formally the IFP transform operator Δ_b on an operand Q is defined as:

$$\Delta_b Q = \begin{cases} 1/Q^{b-1} = Q/Q^b & \text{if } b \neq 1; \\ \log Q & \text{if } b = 1. \end{cases} \quad (5)$$

Brutsaert–Nieber model by IFP transform (Eqs. 2a and b) thus becomes:

$$\Delta_b Q(t) = \begin{cases} \Delta_b Q(0) + (b-1)at & \text{if } b \neq 1; \\ \Delta_b Q(0) - at & \text{if } b = 1. \end{cases} \quad (6)$$

$\Delta_b Q(t)$ may be abbreviated as $\Delta(t)$ with a given b value.

In top half of Eq. (5), its latter expression hints at an intriguing, counter-intuitive transformation of a time series. It weighs the data Q by a self-dependent, thus varying, and counter weight of $(1/Q^b)$. But this is compatible to the input-dependent variable IUH and a logical consequence of this dependency (Ding, 1974).

There are many types, some common or well-known, of IFP transform as tabulated in Table 1. These depend on values of the shape parameter b . For streamflow recession analysis, only some transforms are considered physically realistic. Their b values range from a lower limit of one to an upper limit of 2, the latter as indicated following

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Figure 2 shows, for Events 0 and 2, the original data and five or four transformed recession curves. In contrast to the log transform ($b = 1$), the IFP transform for $b > 1$ reverses the trend of the recession curve from downward to upward: the lower the observed flow value, the higher its transformed value; and that the higher the b value, the greater the transformation. The IFP transform, including the log one, is like a microscope with a varying magnifying power, gradually increasing its magnification towards the lowest flow value, thus most suitable for low flow analysis.

Table 3 summarizes results of the calculations for four events. Among them, Event 2 has both the lowest observed initial recession flow value and the second longest recession period, and as turns out, also the highest calibrated a value among four events, except for the RoCR transform. It shows that the correlation coefficients, R' s, among all transforms for four events are similar, all close to or at 1.0 in absolute number. As illustrated in Fig. 3, the parameter scatter diagram shows that for an event, as parameter b value increases, so does parameter a value reflecting the steepness of a transformed recession curve in Fig. 2; and that starting from $b = 1$, parameter a values diverge from the trend lines, especially for Event 2, but not Event 0, the base event.

On a large river such as the Spoon, the Manning (or Chezy) friction law governs the nonlinear storage–discharge relation expressed by Eq. (3). I have shown elsewhere in HESSD (Ding, 2011) that a typical shape parameter N value of 1.67 by Manning (or 1.5 by Chezy) gives a parameter b value of 1.4 (or 1.33) from Eq. (3): $b = 2 - 1/N$. A b value of 1.4 by Manning lies between b of 1.33 for the RoCR transform, and that of 1.5 for the RoSR characterizing an unconfined aquifer.

Based on results in Table 3 for four events for the Spoon River, the Brutsaert–Nieber parameter pair (b, a), calibrated by the IFP transform method, ranges from (1.33, 0.07) to (1.5, 0.23). These are a compromise between the hydrodynamic (i.e. based on Manning or Chezy) and the statistical (i.e. based on variance of the scale parameter) approaches.

For the Spoon, on the basis of a minimum variance of the scale parameter values, case can be made for a log transform model, i.e. a linear storage system, and even an un-transformed one, i.e. an autoregressive model of order one.

4 Discussion and conclusions

To summarize, the analytical results above show that the recession hydrograph for a large river can be represented by an IFP (inverse fractional power) transform model. Its transformed slope is independent of the size of time step, on which, however, depends the accuracy of the Brutsaert–Nieber model parameters when regressed from data on a recession plot. But it does depend on the shape parameter b value. The range of b values varies from one to an upper limit of 2, rather than that of 3 adopted by Brutsaert and Nieber (1977) and others (e.g. Chen and Wang, 2013) for the early-time lower-envelopes.

Table 4 shows that three of four events have much higher b values than 2, calculated from the $-dQ/dt$ vs. Q plot. It also shows the range of b values determined by IFP transform and based solely on the highest correlation coefficients for each event. Among three of four events, the linear regression method yields higher b values than the IFP transform one, and slightly lower for the fourth event.

The Spoon River, having a very large drainage area, is not a catchment I would choose to illustrate the utility of the newly rediscovered IFP transform. But it has been because the recession flow data available for analysis were published in an open access journal such as HESS by Chen and Wang (2013). As shown in Fig. 3, this case study has demonstrated glaring differences, in terms of the scatter of calibrated parameters, among several types of IFP transform. But as shown in Table 3, it has demonstrated only marginally the superiority of IFP transform over the conventional log transform method. But it does serve as a cautionary tale for nonlinear modelling exemplified by Brutsaert and Nieber (1977) that parameters have meaning only within their model which has its characteristic time-step size, Δt , or time scale (e.g. Ding, 2011), which

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is one day for the Spoon River. Results of model calibration to observed data need be so interpreted and applied accordingly. However, linearization by IFP transform of Brutsaert–Nieber model makes it time-scale invariant, thus rendering redundant any time-scale based optimization schemes, such as Rupp and Selker (2006).

To conclude, the forgotten IFP transform method re-introduced in this note merits a fresh look by the hydrology community for streamflow recession analysis, especially for headwater catchments including hillslopes, karsts, zero- or first-order streams of all shapes and sizes. Of it, the RoSR transform is a special case having a shape parameter b value of 1.5 derived from groundwater flow theory. Because of its root in hillslopes, the classical RoSR transform of 1960's, simple and elegant at once, is a model to be falsified for headwaters, i.e. used until proven otherwise.

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Table 1. Types of IFP (inverse fractional power) transform, $\Delta_b = 1/Q^{b-1}$.

Shape parameter b	IFP transform Δ_b	Symbol	Type
-2	Q^3	Cube	Cubic
-1	Q^2	Quad	Quadratic
0	Q	–	The original or un-transformed
1	$\log Q$	log	Logarithmic
1.33	$1/\sqrt[3]{Q}$	RoCR	Reciprocal of the cubic root
1.5	$1/\sqrt{Q}$	RoSR	Reciprocal of the square root
2	$1/Q$	Recip	Reciprocal
3	$1/Q^2$	RoQ	Reciprocal of the quadratic
4	$1/Q^3$	RoC	Reciprocal of the cubic

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Table 2. Transformed hydrographs for Spoon River, Illinois, May 1994 (Base event, 0). USGS Gage 05570000.

Date	Time	P	Q	Type of IFP Transform, $1/Q^{b-1}$				
				log	RoCR	RoSR	Recip	RoQ
May 1994	(d)	(mmd^{-1})	(mmd^{-1})					
15	0	0.40	0.84	-0.17	1.06	1.09	1.19	1.42
16	1	0	0.78	-0.25	1.09	1.13	1.28	1.64
17	2	0	0.71	-0.34	1.12	1.19	1.41	1.98
18	3	0	0.65	-0.43	1.15	1.24	1.54	2.37
19	4	0	0.61	-0.49	1.18	1.28	1.64	2.69
20	5	0	0.57	-0.56	1.20	1.32	1.75	3.08
21	6	0	0.56	-0.58	1.21	1.34	1.79	3.19
22	7	0	0.52	-0.65	1.24	1.39	1.92	3.70
23	8	0.81	0.50	-0.69	1.26	1.41	2.00	4.00
b (-)			0	1.00	1.33	1.50	2.00	3.00
Correlation R			-0.98	-0.99	0.99	0.99	1.00	1.00
Slope			-0.05	-0.07	0.02	0.04	0.10	0.33
Intercept			0.81	-0.20	1.07	1.10	1.20	1.37
a [$1/(\text{dmm}^{b-1})$]*			0.05	0.07	0.08	0.08	0.10	0.16

Source: Chen and Wang (2013).

* $a = \text{Slope}/(b - 1)$, if $b \neq 1$.



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Table 3. Summary of Brutsaert–Nieber model parameters for Spoon River, Illinois.

Event ^a	Q(0) (mmd^{-1})	Length (d)	Statis ^b	Type of IFP transform, $1/Q^{b-1}$					
				None	log	RoCR	RoSR	Recip	RoQ
			<i>b</i>	0	1.0	1.33	1.5	2.0	3.0
0	0.84	9	<i>a</i>	0.05	0.07	0.08	0.08	0.10	0.16
			<i>R</i>	-0.98	-0.99	0.99	0.99	1.00	1.00
3	0.56	4	<i>a</i>	0.06	0.13	0.17	0.19	0.29	0.63
			<i>R</i>	-1.00	-1.00	1.00	1.00	0.99	0.98
1	0.29	4	<i>a</i>	0.01	0.05	0.07	0.09	0.18	0.66
			<i>R</i>	-1.00	-1.00	0.99	1.00	1.00	1.00
2	0.10	8	<i>a</i>	0.01	0.07	0.15	0.23	0.84	10.88
			<i>R</i>	-0.97	-0.98	0.99	0.99	0.99	0.99

^a Events arranged in the descending order of the initial flow value.^b *b* is the shape parameter (-), *a* the scale parameter [$1/(\text{dmm}^{b-1})$], and *R* the correlation coefficient.

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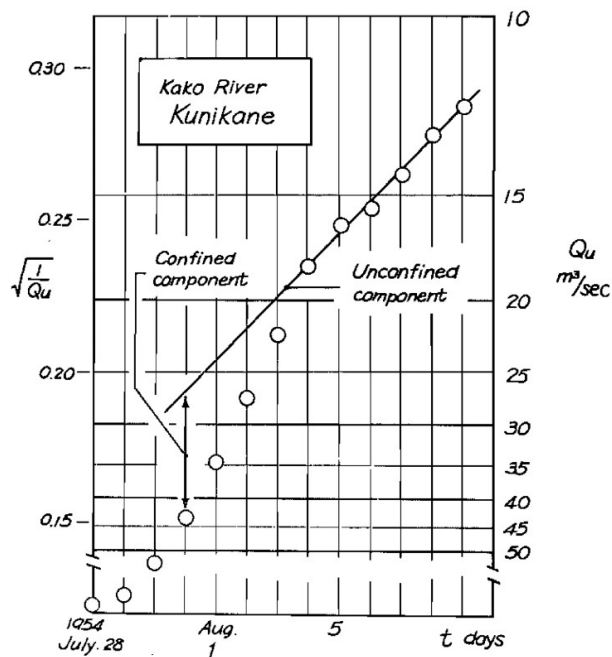


Fig. 1. A streamflow recession hydrograph on a semi-RoS plot for the Kako River at Kunikane, Japan (reprinted with permission from F. Takagi).

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IFP inverse fractional power transform

J. Y. Ding

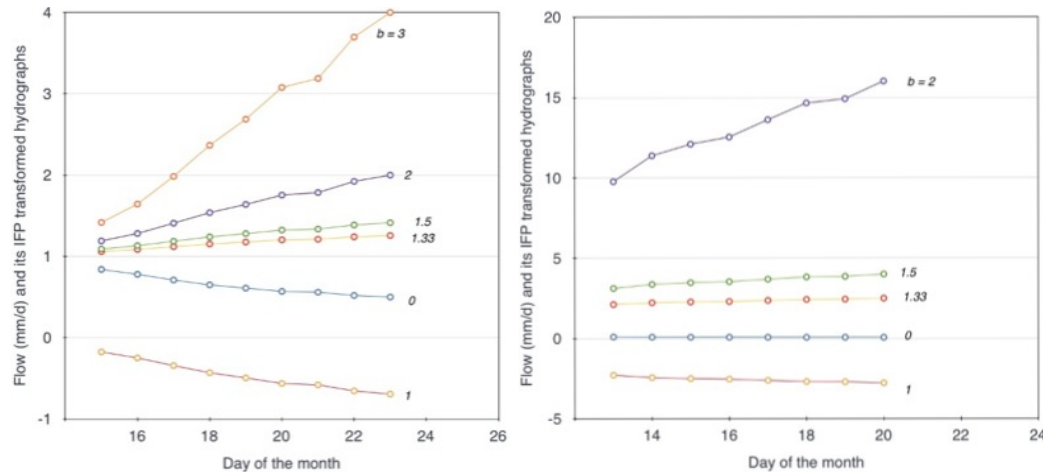


Fig. 2. The IFP (inverse fractional power) transformed hydrographs for the Spoon River, Illinois: on the left panel, May 1994 (Event 0), and right panel, June 1988 (Event 2). $b = 0$ denotes the non-transformed hydrograph; and $b = 1$, the log transformed one. Note the different vertical or flow scales between two panels. On the right panel and outside the plot area is the RoQ transformed hydrograph ($b = 3$) having transformed values from a low 95.55 to high 256.82.

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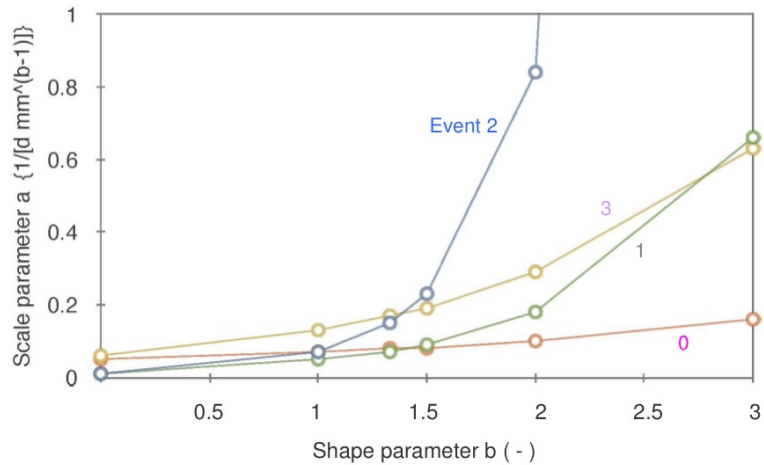



Fig. 3. A scatter diagram of the Brutsaert–Nieber model parameters, calibrated by the IFP (inverse fractional power) transform method, for the Spoon River, Illinois.