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**MODELING THE SNOW SURFACE TEMPERATURE WITH A ONE-LAYER
ENERGY BALANCE SNOWMELT MODEL**

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Short title: Utah Energy Balance snowmelt model

22

23

Abstract

24 Snow surface temperature is a key control on and result of dynamically coupled energy
25 exchanges at the snow surface. The snow surface temperature is the result of the balance
26 between external forcing (incoming radiation) and energy exchanges above the surface that
27 depend on surface temperature (outgoing longwave radiation and turbulent fluxes) and the
28 transport of energy into the snow by conduction and meltwater influx. Because of the
29 strong insulating properties of snow, thermal gradients in snow packs are large and
30 nonlinear, a fact that has led many to advocate multiple layer snowmelt models over single
31 layer models. In an effort to keep snowmelt modeling simple and parsimonious, the Utah
32 Energy Balance (UEB) snowmelt model used only one layer but allowed the snow surface
33 temperature to be different from the snow average temperature by using an equilibrium
34 gradient parameterization based on the surface energy balance. Although this procedure
35 was considered an improvement over the ordinary single layer snowmelt models, it still
36 resulted in discrepancies between modeled and measured snowpack energy contents. In
37 this paper we evaluate the equilibrium gradient approach, the force-restore approach, and a
38 modified force-restore approach when they are integrated as part of a complete energy and
39 mass balance snowmelt model. The force-restore and modified force-restore approaches
40 have not been incorporated into the UEB in early versions, even though Luce and Tartoton
41 have done work in calculating the energy components using these approaches. In addition,
42 we evaluate a scheme for representing the penetration of a refreezing front in cold periods

43 following melt. We introduce a method to adjust effective conductivity to account for the
44 presence of ground near to a shallow snow surface. These parameterizations were tested
45 against data from the Central Sierra Snow Laboratory, CA, Utah State University
46 experimental farm, UT, and Subnivean snow laboratory at Niwot Ridge, CO. These tests
47 compare modeled and measured snow surface temperature, snow energy content, snow
48 water equivalent, and snowmelt outflow. We found that with these refinements the model
49 is able to better represent the snowpack energy balance and internal energy content while
50 still retaining a parsimonious one layer format.

51

52 Keyword: Energy Balance snowmelt model, refreezing, snow, snow water equivalent,
53 surface temperature of snow.

54 **1. Introduction**

55 Snowmelt is an important source of water in the western United States and much of
56 the world. Modeling snowmelt is important for water resources management and the
57 assessment of spring snowmelt flood risk. The processes involved in snowmelt have been
58 widely described (U.S. Army Corps of Engineers, 1956; Gray and Male, 1981; Bras, 1990;
59 Dingman, 1994; Linsley *et al.*, 1975; Viessman *et al.*, 2002). In snowmelt modeling, the
60 heat flux between the snowpack and the atmosphere is partially governed by the snow
61 surface temperature (Gray and Male, 1981; Dingman, 1994; Dozier, 1989) which depends
62 on the conductive heat flux into the snow. Modeling conductive heat flux through the
63 snowpack is a complex problem due to the changing nature of the snowpack through the
64 influences of heating and cooling history. One of the primary reasons for the poor

65 performance of single layer models in comparative validations is the poor representation of
66 internal snowpack heat transfer processes (Blöschl and Kirnbauer, 1991; Koivasulo and
67 Heikenkeimo, 1999). Some snowmelt models use finite difference solutions of the heat
68 equation (Anderson, 1976; Dickinson *et al.*, 1993; Flerchinger and Saxton, 1989; Jordan,
69 1991; Yen, 1967). Possible inaccuracies in modeling the internal snowpack properties
70 could lead to errors in estimating the snowpack and snow surface temperature (Colbeck and
71 Anderson, 1982). Models such as CROCUS (Vionnet *et al.*, 2012) have made considerable
72 progress in representing the detail of within snow processes. There has also been recent
73 progress towards using Richards equation to model meltwater flow in snow using multiple
74 layers (Wever *et al.*, 2014). However Wever *et al.*, did note that there are challenging
75 numerical issues associated with inhomogeneities in grain size and density, and precise
76 quantification of the parameters that impact the model is a challenge. Furthermore, there is
77 an increasing realization that lateral inhomogeneities in snowpacks are important (e.g.
78 Wankiewicz, 1979; Higuchi and Tanaka, 1982; Kattelmann and Dozier, 1999; Williams et
79 al., 2010; and Eiriksson et al., 2013). These inhomogeneities result in lateral variability
80 across a range of scales and fingering in the way that meltwater enters and flows through
81 snow that is different from the matrix flow represented in one-dimensional finite difference
82 solutions. This suggests that even our most complex snowpack models must seek a way to
83 parameterize unmeasurable sub-element scale variability. In the single layer approach we
84 model the surface temperature that provides the connection between the snow and the
85 atmosphere above in a relatively straightforward way that avoids modeling the complexity
86 of uncertain within snow processes.

87 Modeling needs a balance between representing details that are important to the
88 purpose, or question being addressed and avoiding complexity and inaccuracy for details
89 that are less important. There is no one right solution and in this paper we examine and
90 evaluate single layer solutions that avoid some of the complexity of multilayer models for
91 our purposes, which are the quantification of overall surface energy exchanges and
92 meltwater produced by a snowmelt model for hydrological studies.

93 The UEB snowmelt model (Tarboton *et al.*, 1995; Tarboton and Luce, 1996, You
94 2004) is a physically-based point energy and mass balance model for snow accumulation
95 and melt. The snowpack is characterized using two primary state variables, namely, snow
96 water equivalent, W , (m) and the internal energy of the snowpack and top layer of soil, U ,
97 (kJ m^{-2}). The physical basis of the model is the conservation of mass and energy. Snow
98 surface temperature, a key variable in calculating latent and sensible heat fluxes and
99 outgoing longwave radiation, is modeled using a thin surface skin or equilibrium gradient
100 approach. The surface skin is assumed to have zero heat capacity. Snow surface
101 temperature is calculated from the energy balance at the surface of the snowpack by
102 equating incoming and outgoing fluxes between the snow mass and the air above; this
103 allows the snow surface skin temperature to be different from the average temperature of
104 the snowpack as reflected by the energy content. This thus reflects the key insulating effect
105 of snow on the surface energy balance without the introduction of additional layers and
106 their resultant complexity and the potential for error where there is insufficient information
107 to properly model this complexity.

108 The UEB model was initially tested against snow accumulation and melt
109 measurements and was found to perform well. Later tests included comparisons against

110 internal energy through measurement of the temperature profile in a snowpack (Tarboton,
111 1994). These tests indicated a discrepancy between the modeled and the measured internal
112 energy (Tarboton, 1994; Tarboton and Luce, 1996). Luce (2000) and Luce and Tarboton
113 (2010) analyzed the snowpack energy fluxes from a season of measurements collected at
114 the USU drainage farm in Cache Valley, Utah to evaluate the reasons for the discrepancies
115 in the internal energy. One cause was the estimation of longwave radiation inputs based on
116 air temperatures in an environment subject to frequent temperature inversions and resultant
117 fog. Another cause of the discrepancies was the parameterization of snow surface
118 temperature. These problems had been offsetting each other in a way that when the
119 longwave radiation inputs were corrected, the modeled surface temperatures no longer
120 matched measurements. To address this problem, Luce (2000) and Luce and Tarboton
121 (2001, 2010) evaluated various alternative parameterizations against the currently used
122 equilibrium gradient approach. These included the force-restore approach (e.g. Deardorff,
123 1978; Dickinson *et al.*, 1993; Hu and Islam, 1995) and a modified force-restore approach
124 that was suggested (Luce 2000; Luce and Tarboton, 2001, 2010) to improve the
125 representation of snow surface temperature and help improve the representation of energy
126 content in the snowpack. However these evaluations were driven by measured surface
127 temperature and did not include coupled modeling of the snow energy balance driven by
128 atmospheric forcing. In this paper these suggestions are implemented and tested within the
129 UEB snowmelt model.

130 Snowmelt generated at the snow surface is initially held in the snowpack as liquid
131 water up to the liquid holding capacity. When the surface forcing changes to cooling, this
132 water refreezes and a refreezing front penetrates into the snow. The rate of penetration of

133 the refreezing front is governed by the rate of heat loss, the latent heat of fusion, and the
134 temperature gradient in the layer above the refreezing front. The original UEB model
135 (Tarboton, 1994; Tarboton and Luce, 1996) used the equilibrium gradient approach to
136 estimate snow surface temperature and did not account for the presence of liquid water
137 during refreezing periods with the result that the snow surface temperature is modeled as
138 too low with too little heat loss during these periods. Multiple-layer snow models (e.g.
139 Flerchinger and Saxton, 1989; Jordan, 1991) account for this effect because the liquid
140 content and temperature of each layer is explicitly represented. Here we present and test a
141 formulation for representing this refreezing effect in the single layer UEB model. In
142 addition to the two changes mentioned above we also introduce a method to adjust the
143 effective thermal conductivity of shallow snowpacks to account for the combined effect of
144 snow and the ground below the snow.

145 **2. Model Description**

146 *2.1 Mass and energy balance equations*

147
148 The original UEB model is described by Tarboton *et al.*, (1995) and Tarboton and
149 Luce (1996). Here we evaluate modifications introduced to refine the representation of
150 surface temperature, including the modified force-restore approach, refreezing of liquid
151 water and conductivity adjustments for shallow snow (You, 2004). In separate work, we
152 have evaluated the addition of a vegetation layer to UEB (Mahat and Tarboton, 2012;
153 Mahat *et al.*, 2013). We refer to the Tarboton *et al.*, (1995) model as the *original* UEB

154 model. The model examined here we refer to as *surface* UEB. This is a single layer model
 155 used to model snow accumulation in the open and is also the beneath canopy part of
 156 *vegetation* UEB that models snow accumulation and melt in forested environments.
 157 Vegetation UEB comprises two layers, a surface layer that is surface UEB and a vegetation
 158 layer that was evaluated by Mahat and Tarboton (2012) and Mahat et al., (2013). A
 159 comprehensive review of the surface layer model is given here so that the reader can
 160 understand the context for the modifications that were made. Where we do not use a
 161 qualifier the methods are the same in surface UEB and the original UEB.

162 In the UEB model (Tarboton *et al.*, 1995; Tarboton and Luce, 1996), the time
 163 evolution of the snowpack is driven by the energy exchange between the snowpack, the air
 164 above and the soil below according to mass and energy balance equations through snow
 165 water equivalent, W , and energy content, U ,

166

$$167 \quad \frac{dU}{dt} = Q_{sn} + Q_{li} - Q_{le} + Q_p + Q_g + Q_h + Q_e - Q_m, \quad (\text{kJ m}^{-2} \text{h}^{-1}) \quad (1)$$

168

$$169 \quad \frac{dW}{dt} = P_r + P_s - M_r - E, \quad (\text{m h}^{-1}) \quad (2)$$

170 where Q_{sn} is the net shortwave energy received by the snowpack, Q_{li} is the incoming
 171 longwave radiation, Q_{le} is outgoing longwave radiation, Q_p is the energy advected by
 172 precipitation into the snow, Q_g is the ground heat flux to the combination of snow and the
 173 upper layer of soil, Q_h is the sensible heat flux to/from the snow with sign convention that
 174 flux to the snow is positive, Q_e is the latent heat flux to/from the snow with sign convention

175 that flux to the snow is positive, and Q_m is the advected heat removed by meltwater. P_r is
176 the rate of precipitation as rain; P_s is the rate of precipitation as snow; M_r is the meltwater
177 outflow rate; and E is the sublimation rate; t is time (h). Internal energy U is not defined
178 relative to absolute zero, but rather relative to the melting point. U is thus taken as 0 kJ m^{-2}
179 when the snowpack is frozen at 0°C and contains no liquid water. With this definition
180 negative internal energies correspond to the cold content (e.g., Dingman, 1994 p182) and
181 positive internal energies reflect change in phase of some fraction of snow from frozen to
182 liquid. The model requires inputs of air temperature, wind speed, humidity, and incident
183 radiation that are used to drive the energy balance, and precipitation that is used to drive the
184 mass balance. Precipitation is partitioned into snowfall or rainfall based upon air
185 temperature (U.S. Army Corps of Engineers, 1956). In locations where snow is subject to
186 redistribution due to wind blown drifting or sliding, an accumulation factor (Tarboton *et al.*,
187 1995; Tarboton and Luce, 1996; Luce *et al.*, 1998) is used to adjust the snowfall inputs.

188 The use of energy content as a state variable means that the model does not
189 explicitly prognose snowpack temperature. Since snowpack temperature is important for
190 energy fluxes into the snow, it needs to be obtained diagnostically from internal energy and
191 snow water equivalent as follows:

192
193 If $U < 0$ $T_{ave} = U / (\rho_w W C_i + \rho_g D_e C_g)$ All solid phase (3 a)

194 If $0 < U < \rho_w W h_f$ $T_{ave} = 0^\circ\text{C}$ with $L_f = U / (\rho_w h_f W)$ Solid and liquid mixture (3 b)

195 If $U > \rho_w W h_f$ $T_{ave} = \frac{U - \rho_w W h_f}{\rho_g D_e C_g + \rho_w W C_w}$ All liquid (3 c)

196

197 In the equations above, T_{ave} denotes snowpack average temperature ($^{\circ}\text{C}$), h_f denotes
198 the latent heat of fusion (333.5 kJ kg^{-1}), ρ_w the density of water (1000 kg m^{-3}), C_i the
199 specific heat of ice ($2.09 \text{ kJ kg}^{-1} \text{ }^{\circ}\text{C}^{-1}$), ρ_g the soil density, C_g the specific heat of soil, C_w the
200 specific heat of water ($4.18 \text{ kJ kg}^{-1} \text{ }^{\circ}\text{C}^{-1}$), D_e the depth of soil that interacts thermally with
201 the snowpack and L_f the liquid fraction by mass. The basis for equations (3 a) to (3 c) is
202 that the heat required to melt the entire snow water equivalent at $0 \text{ }^{\circ}\text{C}$ is $\rho_w W h_f$ (kJ m^{-2}).
203 Where U is between 0 and this quantity, the liquid fraction is determined by proportioning,
204 i.e. $L_f = U / (\rho_w h_f W)$. The heat capacity of the snow combined with thermally interacting soil
205 layer is $\rho_w W C_i + \rho_g D_e C_g$ ($\text{kJ }^{\circ}\text{C}^{-1} \text{ m}^{-2}$), so in the case that $U < 0$, dividing U by this combined
206 heat capacity gives T_{ave} . Where $U > \rho_w W h_f$ the snow contains sufficient energy to melt
207 completely and the temperature of the remaining liquid phase is given by (3 c). Practically,
208 the condition in Equation (3 c) only occurs when W is zero since a completely liquid
209 snowpack cannot exist; it becomes melt runoff. Nevertheless, this equation is included for
210 completeness to keep track of the energy content during periods of intermittent snow cover.
211 With T_{ave} representing the temperature of the ground, Eq. (3c) handles the possibility of
212 snowfall melting immediately due to coming in contact with warm ground.

213 The net shortwave radiation is calculated from incident shortwave radiation and
214 albedo calculated as a function of snow age and solar illumination angle following
215 Dickinson *et al.* (1993). The incident shortwave radiation is either measured or estimated
216 from the diurnal temperature range (Bristow and Campbell, 1984). On sloping surfaces,
217 incident radiation is adjusted for slope and aspect (e.g. Dingman, 1994).

218 In the albedo model, which follows Dickinson *et al.* (1993) and is described in
219 detail in Tarboton and Luce (1996), the dimensionless age of the snow surface, τ , is
220 retained as a state variable, and is updated with each time step, dependent on snow surface
221 temperature and snowfall. Reflectance is computed for two bands; visible ($< 0.7 \mu\text{m}$) and
222 near infrared ($> 0.7 \mu\text{m}$) with adjustments for illumination angle and snow age. Then
223 albedo is taken as the average of the two reflectances. A parameter d_{NewS} (m) represents the
224 depth of snowfall that is assumed to restore the snow surface to new conditions ($\tau = 0$).
225 With snowfall, P_s , less than d_{NewS} in a time step the dimensionless age is reduced by a factor
226 $(1 - P_s/d_{NewS})$

227 When the snowpack is shallow (depth $z < h = 0.1$ m) the effective surface albedo, A ,
228 is taken as $r_\alpha \alpha_{bg} + (1 - r_\alpha) \alpha_s$ where $r_\alpha = (1 - z/h)e^{-z/2h}$. This interpolates between the snow albedo,
229 α_s , and bare ground albedo, α_{bg} , with the exponential term approximating the exponential
230 extinction of radiation penetration of snow scaled to $1/e^2$ at depth h .

231 The incident longwave radiation is estimated based on air temperature, T_a (K) using
232 the Stefan-Boltzmann equation. The emissivity of air is estimated using Satterlund's (1979)
233 equation for clear conditions. The presence of clouds increases downward longwave
234 radiation. This is modeled by estimating the cloud cover fraction based on the Bristow and
235 Campbell (1984) atmospheric transmission factor (see details in Tarboton and Luce, 1996) .
236 The outgoing longwave radiation is calculated from the snow surface temperature using the
237 Stefan-Boltzmann equation, with emissivity of snow, ϵ_s , taken as 0.99.

238 The latent heat flux, Q_e and sensible heat flux, Q_h are modeled using bulk
239 aerodynamic formulae (Anderson, 1976):

240

$$241 \quad Q_h = \rho_a C_p (T_a - T_s) K_h \quad (4)$$

242 and

$$243 \quad Q_e = \rho_a h_v (q_s - q_a) K_e, \quad (5)$$

244

245 where ρ_a is the density of air, C_p is the specific heat of air at constant pressure
246 ($1.005 \text{ kJ kg}^{-1} \text{ }^\circ\text{C}^{-1}$), h_v is the latent heat of vaporization (sublimation) of ice (2834 kJ kg^{-1}),
247 q_a is the air specific humidity, q_s is the specific humidity at the snow surface which is
248 assumed to be saturated relative to the vapor pressure over ice (e.g., Lowe, 1977), and K_h
249 and K_e are turbulent transfer conductances for sensible and latent heat respectively. Under
250 neutral atmospheric conditions K_e and K_h are given by

251

$$252 \quad K_n = \frac{k_v^2 u}{[\ln(z_m / z_0)]^2} \quad (6)$$

253

254 where z_m is the measurement height for wind speed, air temperature, and humidity, u is the
255 wind speed, k_v is von Kármán's constant (0.4), and z_0 is the aerodynamic roughness. When
256 there is a temperature gradient near the surface, buoyancy effects may enhance or dampen
257 the turbulent transfers, necessitating adjustments to K_n . We use

258

$$259 \quad K_h = K_n \frac{1}{\Phi_M \Phi_H} \quad (7)$$

260

261 and

262

263
$$K_e = K_n \frac{1}{\Phi_M \Phi_E} \quad (8)$$

264

265 where Φ_M , Φ_H , Φ_E are the stability functions for momentum, sensible heat, and water vapor,

266 respectively. The stability functions are estimated using the bulk Richardson number:

267

268
$$R_i = \frac{gz_m(T_a - T_s)}{\frac{1}{2}(T_a + T_s)u^2}, \quad (9)$$

269

270 where g is gravity acceleration (9.8 m s^{-2}). For stable conditions ($R_i > 0$), we use the

271 approximation of Price and Dunne (1976),

272

273
$$\frac{1}{\Phi_M \Phi_H} = \frac{1}{\Phi_M \Phi_E} = \frac{1}{1 + 10R_i}. \quad (10)$$

274

275 For unstable conditions ($R_i < 0$) we use (Dyer and Hicks, 1970; Anderson, 1976;

276 Jordan, 1991),

277

278
$$\frac{1}{\Phi_M \Phi_H} = \frac{1}{\Phi_M \Phi_E} = (1 - 16R_i)^{0.75}. \quad (11)$$

279

280 Because information for estimating turbulence under extremely unstable conditions
281 is poor, we capped the value of $1/\Phi_M\Phi_H$ at 3, which occurs near $R_i = -0.2$. Anderson (1976)
282 shows that iterative solutions of Deardorff's (1968) empirical equations begin to level off
283 for more strongly unstable situations as the value of 3 is approached. Strongly unstable
284 conditions are rare over snow, but this is in the model code for completeness. These
285 stability corrections assume that sensible and latent heat transfer coefficients are equal,
286 $K_h=K_e$.

287 *2.2 Original quantification of surface energy flux*

288 An important characteristic of the UEB model is its separate representation of
289 surface temperature and average snowpack temperature. This facilitates reasonable
290 modeling of surface energy exchanges that depend on snow surface temperature, while
291 retaining a parsimonious single layer model. In this paper we apply new parameterizations
292 for the snow surface temperature introduced by Luce and Tarboton (2010) and test them in
293 the context of a full surface energy balance. The sum of energy fluxes in Equation (1) from
294 above the snowpack are referred to as the surface energy forcing.

295

$$296 \quad Q_{forcing}(T_s) = Q_{sn} + Q_{li} + Q_h(T_s) + Q_e(T_s) + Q_p - Q_{le}(T_s) \quad (12)$$

297

298 The sensible heat, latent heat, and outgoing longwave radiation are functionally dependent
299 on the surface temperature, T_s . In the original model, the heat conducted into the snow, Q_{cs} ,
300 is calculated as a function of the snow surface temperature, T_s , and average snowpack
301 temperature, T_{ave} .

302

303
$$Q_{cs}(T_s, T_{ave}) = k\rho_s C_i \frac{(T_s - T_{ave})}{Z_e} = K_s \rho_s C_i (T_s - T_{ave}) \quad (13)$$

304

305 where ρ_s is the snow density (kg m^{-3}), k the snow thermal diffusivity ($\text{m}^2 \text{h}^{-1}$), Z_e the
306 effective depth over which the temperature gradient acts (m), and $K_s = k/Z_e$ is termed snow
307 surface conductance. In the original model, because there is uncertainty in values for Z_e
308 and k , K_s was used as a calibration parameter.

309 The energy balance at the surface is given by:

310

311
$$Q_{cs}(T_s, T_{ave}) = Q_{forcing}(T_s). \quad (14)$$

312

313 Equation (14) is solved numerically for T_s using the Newton-Raphson method
314 backed up by a more robust bisection approach. The Newton-Raphson scheme is used first
315 because it is more efficient. It tests for convergence and in time steps (a small percentage
316 depending on the data) when it does not converge, the model resorts to a more robust
317 bisection approach that is guaranteed to converge because the equation giving temperature
318 flux into the snow based on surface temperature is monotonic. This is the case for all the
319 surface temperature parameterizations evaluated. Thus the new approach for surface
320 temperature does not alter the numerical stability. Physically, T_s is constrained to be no
321 greater than 0°C when there is snow present. When the equilibrium solution produces a
322 solution of $T_s > 0^\circ\text{C}$, this means that conduction into the snow cannot accommodate all the
323 energy input through surface forcing, and the extra energy will produce meltwater at the

324 surface, which then infiltrates into the lower parts of the snowpack and, if $U < 0$, refreezes,
325 representing the meltwater advection process for transport of energy into the snow. In
326 these cases the surface energy flux terms in Equation (1) are calculated using $T_s = 0$ °C to
327 model the snow energy content change.

328

329 3. Alternative Models of Surface Heat Conduction

330

331 Heat flow in a snowpack can be described using the diffusive heat transfer equation
332 and assuming homogeneity of snow properties (Yen, 1967)

333

$$334 \quad \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2}, \quad (15)$$

335

336 where T is the temperature (°C), z is depth relative to snow surface (m), and k is the thermal
337 diffusivity of snow ($\text{m}^2 \text{h}^{-1}$). Thermal diffusivity is related to thermal conductivity and
338 specific heat by:

339

$$340 \quad k = \frac{\lambda}{C_i \rho_s}, \quad (16)$$

341

342 where λ is the thermal conductivity of snow ($\text{kJ m}^{-1} \text{K}^{-1} \text{h}^{-1}$). For semi-infinite boundary
343 conditions ($0 < z < \infty$) with sinusoidal temperature fluctuation at the upper boundary ($z=0$):

344

345
$$T(0,t) = \langle T \rangle + A \sin(\omega t), \quad (17)$$

346

347 the differential equation (15) has solution (Berg and McGregor, 1966):

348

349
$$T(z,t) = \langle T \rangle + A e^{-\frac{z}{d}} \sin\left(\omega_1 t - \frac{z}{d}\right) \quad (18)$$

350

351 In this solution, A is the amplitude of the imposed temperature fluctuation at the surface, ω

352 is the frequency, $\langle T \rangle$, the average about which surface temperature fluctuations are

353 centered, and d is the damping depth for a given frequency. At the snow surface, the

354 primary forcing is diurnal, suggesting $\omega = \omega_1 = 2\pi/24 \text{ h}^{-1}$, with the damping depth,

355
$$d = d_1 = \sqrt{\frac{2k}{\omega_1}}, \text{ corresponding to frequency } \omega_1.$$

356 Equation (18) indicates that temperature oscillations are damped by a factor $1/e$ for

357 each increment of depth d_1 , and the time-averaged temperature at each depth is $\langle T \rangle$.

358 Equation (18) can be differentiated on the depth (z) to evaluate the temperature gradient,

359 and the surface energy flux (at $z=0$) can be written as:

360

361
$$Q_{cs} = -\lambda \frac{\partial T}{\partial z}(0,t) = \frac{\lambda A}{d_1} [\sin(\omega_1 t) + \cos(\omega_1 t)]. \quad (19)$$

362

363 Recognizing that $\omega_1 \cos(\omega_1 t)$ is the derivative of $\sin(\omega_1 t)$ with respect to t , and
364 substituting Equation (17) and its time derivative into Equation (19) yields:

365

$$366 \quad Q_{cs} = \frac{\lambda}{d_1 \omega_1} \frac{\partial T}{\partial t}(0, t) + \frac{\lambda}{d_1} (T(0, t) - \langle T \rangle). \quad (20)$$

367

368 This expresses the surface heat flux as a function of both the time derivative of
369 surface temperature and the difference between the current surface temperature and the
370 time averaged surface temperature (Luce and Tarboton, 2010). This analytic solution for
371 the simplified setting of a semi-infinite domain with sinusoidal surface temperature forcing
372 serves as the basis for the numerical approximations of surface temperature, T_s , that are
373 evaluated.

374 **3.1 Equilibrium gradient approach**

375 The original equilibrium gradient method of surface temperature parameterization
376 used in Equation (13) can be seen to be an approximation to Equation (20) that ignores the
377 time derivative of the surface temperature term and approximates the average temperature
378 at the surface over time, $\langle T \rangle$, by the snowpack average temperature, T_{ave} , while using
379 actual surface temperature, T_s , in place of the sinusoidal forcing $T(0, t)$. This method
380 approximates the energy flux as a gradient between the surface temperature and average
381 temperature of snow over an effective distance Z_e , equivalent to d_l . In the original UEB
382 model Z_e was absorbed into the parameter K_s that was calibrated, however here d_l is related
383 to the diurnal frequency, so to retain this calibration capability we use $Z_e = r d_l$ (i.e., the

384 damping depth d_l scaled by a dimensionless adjustable parameter r) and write Equation (13)
385 in the form showing the similarity to Equation (20):

386

$$387 \quad Q_{cs} = \frac{\lambda}{rd_1} (T_s - T_{ave}). \quad (21)$$

388

389 **3.2 Force-restore approach**

390 The force-restore parameterization (e.g. Deardorff, 1978; Dickinson *et al.*, 1993; Hu
391 and Islam, 1995) is:

392

$$393 \quad Q_{cs} = \frac{\lambda}{d_1} \frac{1}{\omega_1 \Delta t} (T_s - T_{slag1}) + \frac{\lambda}{rd_1} (T_s - T_{ave}), \quad (22)$$

394

395 (Luce and Tarbton, 2010). Here Δt is the time step and T_{slag1} is the surface temperature of
396 snow in the previous time step. A finite difference approximation has been used for the
397 time derivative and $\langle T \rangle$ has been replaced by the depth average snowpack temperature T_{ave} .

398 Again, we have scaled the damping depth by a parameter r .

399 **3.3 Modified force-restore approach**

400 Luce (2000) and Luce and Tarboton (2001; 2010) found that the diurnal cycle may
401 be superimposed on a temperature gradient that varied at longer weekly to seasonal time
402 scales, causing variations in the temperature gradient and heat fluxes with depth. Luce
403 (2000) and Luce and Tarboton (2001; 2010) suggested that the heat flux and the surface

404 temperature could be estimated using the following modification to the force-restore
405 equation:

406

$$407 \quad Q_{cs} = \frac{\lambda}{d_1} \frac{1}{\omega_1 \Delta t} (T_s - T_{s_{lag1}}) + \frac{\lambda}{rd_1} (T_s - \bar{T}_s) + \frac{\lambda}{d_{lf}} (\bar{T}_s - \bar{T}_{ave}), \quad (23)$$

408

409 where \bar{T}_s is the average surface temperature estimated for the previous 24 hours, and \bar{T}_{ave} is
410 the 24 hour time average of the depth average snowpack temperature. The 3rd term
411 represents the superimposed gradient, a lower frequency effect, approximated using an
412 equilibrium gradient approach similar to Equation (21). In this parameterization d_{lf} is the
413 damping depth associated with the longer time scale forcing having lower frequency ω_{lf} , i.e.

414 $d_{lf} = \sqrt{\frac{2k}{\omega_{lf}}}$. In Equation (23) since the appropriate low frequency parameter (ω_{lf}) is not

415 known *a priori*, Luce (2000) and Luce and Tarboton (2001; 2010) suggested that d_{lf} be
416 calibrated.

417 **3.4 Theory of meltwater refreezing**

418 The approaches described above solve for surface temperature based upon a balance
419 between surface forcing and the capacity of the snow near the surface to conduct heat into
420 or out of the snowpack. However, during a cooling period following melting where there is
421 liquid water present in the snow, the depression of snow surface temperature is inhibited by
422 the energy required to refreeze liquid water near the surface before a temperature gradient
423 can be established and conduction can occur. The net effect of this is that when there is

424 liquid water present the snow surface stays warmer longer and heat loss at night and in
425 cooling periods is more rapid. To accommodate this effect we have developed a
426 parameterization for the penetration of a refreezing front and conduction of heat between
427 the surface and refreezing front while there is liquid water present in the snow.

428 When snow energy content U is greater than 0, liquid water exists in the snowpack.
429 The snowpack is assumed to be isothermal at 0 °C. Using the relationship between energy
430 content and liquid fraction (Equation 3 b), the equivalent depth of liquid water in the
431 snowpack w_m (m) is calculated as:

432

$$433 \quad w_m = L_f W = \frac{U}{\rho_w h_f} \quad (24)$$

434

435 The capillary holding capacity of the snow is defined as mass fraction liquid
436 holding capacity, L_c , times snow water equivalent $L_c W$, which implies that the maximum
437 density of capillary water, ρ_m , is $\rho_m = \frac{L_c W \rho_w}{D} = L_c \rho_s$, where D is the depth of snowpack.

438 We assume that prior to melt outflow, when the liquid water content is less than the
439 capillary holding capacity, the meltwater is held at the maximum density of capillary water
440 in the upper portion of the snowpack. The justification for this assumption is that energy
441 generating melt primarily originates at the surface. With this assumption the depth to
442 which meltwater has penetrated is:

443

444
$$d_w = \frac{w_m \rho_w}{\rho_m} = \frac{U}{\rho_w h_f} \frac{\rho_w}{\rho_m} = \frac{U}{\rho_m h_f}. \quad (25)$$

445

446 This describes the state of the snowpack prior to the onset of a refreezing episode
 447 during which $Q_{forcing}$ is negative. The negative forcing will result in refreezing that
 448 penetrates down from the surface as illustrated in Figure 1. The rate of increase of the
 449 depth to the refreezing front, d_r , is given by:

450

451
$$\frac{dd_r}{dt} = -\frac{Q(T_s)}{\rho_m h_f}, \quad (26)$$

452

453 where $Q(T_s)$ is the heat flux just above the refreezing front, here indicated to be a function
 454 of surface temperature T_s . The sign convention is that heat flux is positive into the snow
 455 which is why there is a negative sign in Equation (26).

456 We assume a linear temperature gradient above the refreezing front with $Q(T_s)$
 457 given by

458

459
$$Q(T_s) = \lambda \frac{T_s}{d_r}. \quad (27)$$

460

461 We use an equilibrium approach for surface temperature that balances the surface
 462 forcing with the conduction into the snow above the refreezing front, neglecting any heat

463 stored in the snow between the refreezing front and the surface (as this will be small
464 because the heat capacity of snow is less than the latent heat of fusion). This is written
465

$$466 \quad Q(T_s) = Q_{forcing}(T_s). \quad (28)$$

467
468 To solve for $d_r(t)$ the dependence of $Q_{forcing}(T_s)$ on T_s is linearized,
469

$$470 \quad Q_{forcing}(T_s) = a - bT_s. \quad (29)$$

471
472 Here a is the forcing surface energy flux when the surface temperature of snow is 0°C , and
473 b is the slope of surface forcing flux to surface temperature function. This is a positive
474 value since $Q(T_s)$ decreases with T_s . a is obtained by putting $T_s=0$ into $Q_{forcing}(T_s)$. b is
475 obtained by putting a small negative (below freezing) T_s into $Q_{forcing}(T_s)$ and solving (29).
476 If a is greater than 0, then the surface forcing is positive and meltwater is being generated at
477 the surface so d_r is set to 0. When a becomes less than 0, the snowpack starts refreezing.
478 Combining Equations (27) and (29) gives:

$$479 \quad \frac{\lambda}{d_r} T_s = a - bT_s, \quad (30)$$

481
482 T_s can then be expressed as:
483

484
$$T_s = \frac{a}{\frac{\lambda}{d_r} + b}. \quad (31)$$

485

486 Substituting this T_s into (27) then the result into (26) gives:

487

488
$$\frac{dd_r}{dt} = -\frac{\lambda a}{\rho_m h_f (\lambda + b d_r)}, \quad (32)$$

489

490 Integrating Equation (32) starting from the initial refreezing depth d_{r1} during a time step,

491 we get:

492

493
$$\lambda d_r + \frac{b}{2} d_r^2 - \left(\lambda d_{r1} + \frac{b}{2} d_{r1}^2 \right) = -\frac{a\lambda}{\rho_m h_f} \Delta t \quad (33)$$

494

495 This has solution

496

497
$$d_r = \frac{-\lambda + \sqrt{\lambda^2 + 2b\left(\lambda d_{r1} + \frac{b}{2} d_{r1}^2 - \frac{a\lambda\Delta t}{\rho_m h_f}\right)}}{b}. \quad (34)$$

498

499 Only the positive root has been retained since only positive values of d_r are physically
 500 interpretable and b is a value greater than 0. When d_r is greater than rd_l , the effective depth
 501 associated with diurnal temperature fluctuations, or all meltwater is refrozen, the model

502 reverts back to the surface temperature parameterization without refreezing of meltwater as
503 described above.

504 ***3.5 Adjustment of thermal conductivity, λ , for shallow snowpack***

505 In equations (13), (21), (22) and (23) the temperature gradient is calculated over an
506 effective depth ($Z_e = rd_1$) estimated from the depth of penetration of surface temperature
507 forcing at a diurnal frequency. When the snow is shallow this depth may extend into the
508 ground below the snow cover. In such cases the thermal conductivity used in the surface
509 temperature parameterizations above needs to reflect the combined conductivity of snow
510 and soil below. We therefore take the effective thermal conductivity of the snowpack, λ_e ,
511 as the harmonic mean to the effective depth, Z_e , where the amplitude is damped by the
512 same factor as it would be for deep snow (see Figure 2). In deep snow the amplitude of
513 diurnal temperature fluctuations at depth Z_e is damped by (Equation 18) $e^{-Z_e/d_1} = e^{-r}$. In
514 the combined snow/soil system, given r , we first solve for the depth into the soil z_2 at which
515 the amplitude of diurnal temperature fluctuations is damped by this same factor e^{-r} . Then
516 λ_e is obtained by taking the harmonic mean to this depth. The thermal diffusivity of the
517 ground below the snow, k_g , is related to the thermal conductivity, λ_g , heat capacity, C_g , and
518 density, ρ_g , of the ground through:

519

$$520 \quad k_g = \frac{\lambda_g}{C_g \rho_g}. \quad (35)$$

521

522 The diurnal damping depth, d_g , associated with this ground thermal diffusivity is:

523

524
$$d_g = \sqrt{\frac{2k_g}{\omega_1}}. \quad (36)$$

525

526 The amplitude of diurnal temperature fluctuation at depth z_2 into the ground, relative to the

527 surface temperature fluctuation is therefore damped by $e^{-z_s/d_1} e^{-z_2/d_g}$. Equating this to e^{-r}

528 we obtain:

529

530
$$\frac{z_s}{d_1} + \frac{z_2}{d_g} = r. \quad (37)$$

531

532 Thus z_2 is:

533

534
$$z_2 = d_g \left(r - \frac{z_s}{d_1} \right). \quad (38)$$

535

536

537 The effective thermal conductivity, λ_e , and the effective depth, Z_e , for the shallow

538 snowpack are then estimated through:

539

540
$$Z_e = z_s + z_2 = z_s + d_g \left(r - \frac{z_s}{d_1} \right), \quad (39)$$

541

$$\frac{1}{\lambda_e} = \frac{\frac{z_s}{\lambda} + \frac{z_2}{\lambda_g}}{Z_e}, \quad (40)$$

543

544 Equation (40) is used to obtain the effective thermal conductivity near the surface when the
545 snow is shallow. This is used in the parameterizations for surface temperature that
546 calculate the surface heat flux between the snowpack and the atmosphere as well as
547 conduction into the snow.

548 Summarizing our model improvements, the force restore and modified force restore
549 approach have been included in the new surface UEB snowmelt model to better
550 parameterize the surface temperature of snow. A new refreezing scheme was developed to
551 model heat loss following partial melt through modeling the penetration of a refreezing
552 front into the snowpack. The model was changed to adjust effective thermal conductivity
553 used in the surface temperature parameterization for a shallow snowpack where the
554 penetration depth for diurnal temperature fluctuations extends into the ground.

555 **4. Study Sites and Data**

556

557 The new surface UEB model was calibrated and tested using data from three
558 locations in the Western U.S.

559

560 *Utah State University Drainage and Evapotranspiration Experimental Farm.*
561 The USU drainage and evapotranspiration experimental farm is located in Cache
562 Valley near Logan, Utah, USA (41.6° N, 111.6° W, 1350 m elevation). The weather
563 station and instrumentation were in a small fenced enclosure at the center of an open field
564 with no obstructions to wind in any direction for at least 500 m. Cache Valley is a flat-
565 bottomed valley surrounded by mountains that reach elevations of 3000 m. During the
566 period of this experiment the ground was snow covered from November 20, 1992 to March
567 22, 1993. Air temperatures ranged from -23 °C to 16 °C and there was 190 mm of
568 precipitation (mostly snow, but some rain). The snow accumulated to a maximum depth of
569 0.5 m with maximum water equivalent of 0.14 m. Data collected included measurements
570 of snow water equivalent, snow surface temperature, temperatures within the snowpack and
571 the upper soil layer, and the meteorological variables necessary to drive UEB at 30 minute
572 time steps.

573 Shallow soil temperatures were measured using two thermocouples placed below
574 the ground surface at depths of 25 mm and 75 mm. Another thermocouple was placed at
575 the ground surface. The snowpack temperature was measured using thermocouples
576 suspended at 50, 125, 200, 275 and 350 mm above the ground surface on fishing line strung
577 between two upright posts. These temperature measurements were corrected for high
578 frequency fluctuations in the panel reference temperature (Luce and Tarboton 2010).
579 Snowpack surface temperature was measured with two Everest Interscience model 4000
580 infrared thermometers. Internal energy content of the snowpack was calculated from the
581 temperature profile of the snowpack and upper soil layer accounting for the near surface
582 nonlinearity through an analytic integral of Equation (18) as described by Luce (2000),

583 Luce and Tarboton (2010). Snow water equivalent was measured using a snow tube. Snow
584 pits provided measurements of density and depth. On each measurement occasion snow
585 water equivalent was measured at eight locations (fewer when snow had disappeared from
586 some) and averaged.

587 A complete dataset including the air temperature, wind speed, relative humidity,
588 incident shortwave radiation, outgoing shortwave radiation, temperature profile through the
589 snow and surface temperature of snowpack was available from January 26, 1993 to March
590 22, 1993 when the snow completely melted away. The data at USU DF was used in this
591 study to calibrate the new surface UEB model.

592

593 *Central Sierra Snow Laboratory*

594 The Central Sierra Snow Laboratory located 1 km east of Soda Springs, California,
595 measures and archives comprehensive data relevant to snow. It is located at 39°19' N,
596 120°22' W, at an elevation of 2100 m. The meteorological data are reported each hour and
597 consist of temperature, radiation, humidity, precipitation, and wind measurements at two
598 levels in a 40 m by 50 m clearing and in a mixed conifer canopy with 95% forest cover.
599 Snow depths and water equivalent are measured daily (except on weekends) and eight
600 lysimeters record melt outflow each hour. The data from the open site used in this study
601 were collected between November 14, 1985 and July 1, 1986 when the snowpack
602 disappeared at the open site at a 6 hour time step. A total of 124 snow water equivalent
603 measurements in addition to hourly lysimeter data were available for this time period. This
604 dataset was used to test the new surface UEB model.

605

606 *Niwot Ridge, Colorado*

607 Another dataset used to test the new model comes from Subnivean snow laboratory
608 at Niwot Ridge on the eastern slope of the Front Range of Colorado (3517 m MSL, 40°03'
609 N, 105°35' W) collected during the 1995~1996 winter seasons. The instrument site is
610 located in a relatively flat area above the treeline within a broad saddle of the ridge. The
611 high elevation and exposure of Niwot Ridge, and typically dry atmospheric conditions,
612 result in large clear-sky atmospheric transmissivity, high solar insolation, and low
613 magnitudes of incident longwave radiation, low air temperatures, and high wind velocities.
614 The dataset includes measurements of air temperature, wind speed, relative humidity, and
615 incident shortwave radiation from April 28, 1996 to September 30, 1996 with a time step of
616 2 hours. Measured lysimeter data are also available although there are concerns as to how
617 representative it is due to preferential flow paths (finger-flow) in the snow resulting in
618 under-catch of meltwater (Cline, 1997a). This dataset was used for further validation of the
619 new surface UEB model in a setting with spatial variability and wind exposure.

620 **5. Results**

621 The new surface UEB model with the modified force-restore surface temperature
622 parameterization was calibrated against the data from the USUDF to adjust some
623 parameters and reflect the model changes. The model was then tested at the CSSL site.
624 The model was validated using data from the Niwot ridge site, testing to some degree the
625 physical basis and transferability of the model parameters.

626 At USUDF, Luce (2000) and Luce and Tarboton (2010) found evidence that the
627 estimates of the incoming longwave radiation used in the original model testing (Tarboton

628 *et al.*, 1995; Tarboton and Luce, 1996) were too low due to frequent inversions during
629 winter. Luce (2000) estimated the downward longwave radiation flux from the total
630 snowpack energy balance during non-melt periods given all other energy components such
631 as ground heat flux, net shortwave radiation, turbulent fluxes and outgoing longwave
632 radiation. The corrected longwave estimates were validated against cloud and fog
633 observations at a nearby airport. In validating the new surface energy approximation, we
634 used the measured shortwave radiation, the downward longwave radiation estimated by
635 Luce (2000), and the measured ground heat flux to drive implementations of surface UEB
636 with each of the three alternative surface temperature parameterizations given above
637 (Equilibrium gradient, Force-restore and Modified Force-restore). The new surface model
638 includes parameters from the original UEB model as well as new parameters introduced
639 with the enhancements. Although there is some degree of circularity in using the total
640 energy balance as an estimator of one stream of incoming energy, none of the alternative
641 surface temperature parameterizations and none of the refreezing components were used in
642 making the estimates. Consequently, comparisons among alternative model choices are
643 nominally unaffected by the partially calibrated longwave radiation estimates at the
644 USUDF location, and the results should be viewed in the context of a comparison for
645 different approaches and incremental improvement rather than as a validation per se. Table
646 1 gives parameter values indicating which are new, and which were adjusted from their
647 original UEB values to fit the data at USUDF as discussed below.

648 *5.1 Modeled internal energy of snow*

649 Figure 3 shows the time series of measured snow, ground and snow surface
650 temperatures at the USU Drainage Farm that were used to calculate the internal energy

651 content of the snowpack. Because this measured internal energy is only based on
652 temperatures and does not account for any liquid water present, measured internal energy
653 content is only comparable to modeled internal energy during cold periods when liquid
654 water is not present. During warm periods, the modeled energy content is expected to go
655 above zero while measured energy content remains close to (just below) zero. The three
656 approaches for surface temperature approximation described above were included as
657 options in the new surface UEB. (The original UEB model only had the gradient approach).
658 The comparisons between the modeled and measured internal energy values (Figure 4)
659 focus on periods when the snow is cold and liquid water is not present. These comparisons
660 appear similar to the initial work of Luce (2000, Figure 2-5) and Luce and Tarboton (2001;
661 2010) that indicates that the modified force restore snow surface temperature
662 approximation compares best to the internal energy content of snowpack. Here we note
663 that these results differ from the earlier work of Luce (2000) and Luce and Tarboton (2001;
664 2010) in that the new results are complete model simulations driven by inputs of air
665 temperature, humidity, radiation and wind with surface temperature calculated by the
666 model. The earlier work used the measured surface temperature to drive calculations of
667 internal energy estimating only the conduction into the snow, which does not test
668 interactions of the new scheme with energy fluxes dependent on surface temperature. The
669 results here are from a free running model forced by weather inputs that do test the
670 modeling of dynamic interactions among the surface energy exchanges and surface
671 temperature. Some parameters and physical properties quantified earlier (Luce and
672 Tarboton, 2001; 2010) were used here. Following the success of the modified force-restore

673 surface temperature approach relative to the other approaches at the USUDF, the modified
674 force-restore was used in all subsequent evaluations at the other sites.

675 Comparisons between modeled and measured variables at USUDF are shown in
676 Figures 5, 6, 7, and 8. Figure 5 includes measured snow water equivalent and the results
677 from five model runs. Four model runs are from the new surface UEB model using the
678 parameters listed in table 1, each initialized on a different date indicated by the letters (a)
679 through (d) following periods of severe weather and likely erroneous inputs. The fifth
680 model run is from the original UEB model with its original parameters reported by
681 Tarboton (1994). Figure 6 shows the measured and modeled energy content from the new
682 surface UEB model run initialized on 1/26/1993 together with a model run using the code
683 prior to the addition of the refreezing parameterization. Note that with the addition of the
684 refreezing parameterization, lower energy content better in line with measurements is
685 obtained than without the refreezing parameterization.

686 Figure 7 shows measured and modeled energy content from the original UEB model,
687 indicating a large discrepancy in energy content. This problem was identified by this
688 comparison to internal energy computed from temperature profile measurements (Figure 3).
689 This discrepancy has been resolved (Figure 6) through the combination of modifications
690 reported in this paper (modified Force-Restore, surface refreezing and shallow snow
691 conductivity adjustment). These results point to the importance of comparing models to
692 measurements of their internal state as without the direct comparison to energy content the
693 discrepancy with the original UEB may not have been identified.

694 *5.2 Modeled snow water equivalent and meltwater*

695 Figure 8 shows surface temperature comparisons for two time intervals chosen to be
696 illustrative of periods prior to the onset of melt and during the period when snow is melting.
697 The model runs shown in Figure 8 (a) were initialized on Jan. 26, 1993. The original UEB
698 model run shown in Figure 8 (b) is the same as in Figure 8 (a) while the new surface UEB
699 model run shown was initialized on Mar. 9, 1993. Note that these surface temperature
700 comparisons, such as were used in the development of the original UEB do not indicate the
701 energy discrepancy that full profile temperature measurements reveal.

702 The new surface UEB model and the calibrated model parameters were then tested
703 using the 1985 -1986 data from the CSSL, CA. Comparisons of the modeled and the
704 measured variables are shown in Figures 9, 10, 11, and 12. The modeled results fit the
705 measurements well. The discussion section further examines these results.

706 The new surface UEB model was also tested using 1996 data from the Subnivean
707 Snow Laboratory at Niwot Ridge, CO. Modeled and observed snow water equivalent are
708 compared in Figure 13. The model was initialized with the beginning observed snow water
709 equivalent value of 1.4 m. Melt outflows that totaled to 0.23 m were recorded. These were
710 used to infer the snow water equivalent back through time. However, as shown in Figure
711 13, there is a discrepancy between the measured total melt (0.23 m) and observed initial
712 snow water equivalent (1.4 m). This is presumed to be due to preferential meltwater
713 drainage flow paths in the snow as reported previously at this location (Cline, 1997b). An
714 adjustment factor was calculated as $\frac{W_{ini} + \sum p}{\sum m}$, where W_{ini} is the initial measured snow
715 water equivalent, $\sum p$ is the total precipitation during the modeling time, and $\sum m$ is the
716 total measured meltwater outflow.

717 5.3 Modeled albedo

718 The USUDF instrumentation included a net radiometer and downward and upward
719 pointing pyranometers. These were used to obtain a measured estimate of Albedo that was
720 compared to albedo as simulated by the original model and new surface UEB model
721 (Figure 14). These results indicated that albedo was not being refreshed to new snow
722 values following snowfall. This was corrected by changing the threshold of new snow
723 water equivalent that restores albedo to the new snow cover, d_{NewS} , to 0.002 m; this was
724 previously 0.01 m.

725

726 6. Discussion

727 The most significant change introduced into the surface UEB model was the
728 change to the surface temperature parameterization. Figure 9 shows the snow water
729 equivalent data originally used to validate the UEB model, together with surface
730 temperature comparisons, such as Figure 8 and melt outflow comparisons such as Figure
731 10. These results looked satisfactory at the time, but once measurements of internal energy
732 (Figure 7) were obtained it was realized that the original UEB had problems representing
733 internal energy and this deficiency was traced in part to the surface temperature
734 parameterization (Luce and Tarboton 2010). Incorporating the modified force restore
735 approach they suggested into the UEB model resulted in improvements in snowpack
736 internal energy estimates (Figure 4).

737 Density and thermal conductivity are the primary parameters introduced in the
738 new parameterization of surface temperature (equations 21, 22 and 23). Variability in

739 thermal conductivity as a function of snow density is to be expected as both are determined
740 by the snow's microstructure but are not uniquely related to each other. Measurements of
741 the thermal conductivity of snow are thoroughly reviewed by Sturm et al. (1997). In the
742 literature there is variability in the values reported for thermal conductivity (Anderson,
743 1976; Gray and Male, 1981; Lee, 1980). Anderson (1976, p30 Figure 3.1) shows that the
744 thermal conductivity of the snowpack may change over a wide range from $0.15 \text{ kJ m}^{-1} \text{ h}^{-1}$
745 K^{-1} to $7.5 \text{ kJ m}^{-1} \text{ h}^{-1} \text{ K}^{-1}$ at a density of 200 kg m^{-3} . Lee (1980) also reported a range from
746 $0.25 \text{ kJ m}^{-1} \text{ h}^{-1} \text{ K}^{-1}$ at a density of 100 kg m^{-3} to $5.3 \text{ kJ m}^{-1} \text{ h}^{-1} \text{ K}^{-1}$ at a density of 700 kg m^{-3} .
747 Gray and Male (1981) indicated that thermal conductivity changes are nonlinear from
748 $0.18 \text{ kJ m}^{-1} \text{ h}^{-1} \text{ K}^{-1}$ at a density of about 175 kg m^{-3} to $5.76 \text{ kJ m}^{-1} \text{ h}^{-1} \text{ K}^{-1}$ at a density of
749 800 kg m^{-3} . The UEB model retains a degree of simplicity by not modeling surface density
750 and thermal conductivity as time varying quantities. The surface UEB uses a single
751 thermal conductivity value and snow density, and the values of $\lambda_s = 0.33 \text{ kJ m}^{-1} \text{ h}^{-1} \text{ K}^{-1}$ and
752 $\rho_s = 200 \text{ kg m}^{-3}$ were calibrated to fit the internal energy measurements of Figure 4
753 considering the snow thermal properties inferred from frequency analysis by Luce and
754 Tarboton (2010). Snow density is reflective of the density of the snow surface, involved in
755 surface energy exchanges, rather than the snowpack as a whole. Modeling the thermal
756 conductivity as a function of density may improve the performance of snowmelt models if
757 the density was able to be appropriately modeled. However, the errors in modeling the
758 density may introduce errors in modeling the surface heat conduction and the internal
759 energy content.

760 A value of $r=1$ was used for the dimensionless damping depth factor. This
761 nominal value corresponds to a gradient over the depth to which diurnal temperature

762 fluctuations are attenuated by a factor of $1/e$. The soil thermal conductivity parameter also
763 plays a role in the model when the snowpack is shallow (Equation 40) and was set to a
764 value of $6.5 \text{ kJ m}^{-1} \text{ h}^{-1} \text{ K}^{-1}$, within the range of soil heat conductivity reported for the Logan
765 Area (Hanks and Ashcroft, 1980; Luce, 2000). The low frequency forcing frequency value,
766 w_{lf} , was set to 0.0654 rad/h based on Luce and Tarboton (2010).

767 It is interesting to note that with a new surface temperature parameterization
768 calibrated to USUDF data, the model better represents the CSSL snow water equivalent
769 data (Figure 9) and cumulative melt data (Figure 10) early in the season. This model
770 successfully resolves the failure to capture early-season melt, a problem which is a fairly
771 common feature of single-layer models (Slater et al. 2001). The model now holds energy
772 content closer to zero and is able to represent early season melt, correcting the relatively
773 small early season discrepancy in comparisons to CSSL data that was present in the original
774 UEB model calibrations. Small discrepancies still exist in the modeled snow water
775 equivalent and the measurement snow water equivalent at the high accumulation period.
776 This may be due to remaining model errors and some uncertainty (undercatch) in the
777 snowfall measurements that are inputs. The disappearance date of the snow at CSSL was
778 still modeled about one week later than the observed, which may be due to errors in
779 modeling the decrease of albedo perhaps due to contamination of the snow or due to the
780 increase of longwave radiation from the nearby forest canopy.

781 Representation of observed snow water equivalent at USUDF in a single model
782 run proved to be difficult. We attributed this to uncertainty and likely erroneous input
783 quantities during windy and stormy severe weather periods. Snowfall was recorded in a
784 heated unshielded precipitation gauge so is uncertain and likely to suffer from undercatch.

785 There was also snow drifting resulting in accumulation and scour associated with strong
786 winds, and griming of the instruments recording radiation.

787 One of the problems discovered with the original UEB model was that it offsets
788 the bias due to the surface temperature parameterization by a bias in heat loss following
789 surface melting (Figure 6). Following a period of snowmelt, the observed energy content is
790 observed to fall below 0 but the modeled energy content remained above 0. Without the
791 refreezing parameterization surface temperature immediately drops in a cooling period,
792 limiting the heat loss by reducing the outgoing longwave radiation. The parameterization
793 of the refreezing front corrected this to some extent (Figure 6) keeping the surface
794 temperature warmer and sustaining greater outgoing longwave radiation energy losses, the
795 extra energy loss going to refreeze liquid water present and allowing the model energy
796 content to drop more in line with the observations.

797 Melt outflow rates were not measured at USUDF. The changes in surface
798 temperature and refreezing parameterization changed the modeled amount of liquid water,
799 which changed melt outflow. We used measured melt outflow at CSSL (Figure 11) to
800 adjust the snow hydraulic conductivity to 200 m h^{-1} , a value still within the range from 20
801 m h^{-1} to 300 m h^{-1} reported in the literature (Gray and Male, 1981). Liquid holding
802 capacity was adjusted to 0.02 to better fit melt outflow.

803 D_e and z_0 were adjusted based on the research of Luce (2000) and Luce and
804 Tartboton (2010) where a value of 0.1 m was suggested for the soil effective depth and a
805 value 0.01 m suggested for the surface aerodynamic roughness of snow z_0 in the calculation
806 of turbulent heat flux.

807 The Albedo measurements at USUDF enabled refinement of the parameter
808 quantifying the new snow water equivalent that restores albedo to the new snow cover,
809 resulting in a more responsive modeling of albedo, consistent with observations (Figure
810 14). However, there is an offset between modeled and observed albedo in this figure,
811 which, we believe, is due to downward pointing limited-band pyranometers not being
812 appropriate for measuring snow reflectance. However they do still provide us with relative
813 measurements useful in quantifying the timing and responsiveness of albedo changes.

814 As was observed at the USU drainage farm, the new surface model also gave a
815 good approximation of the surface temperature of snow (Figure 12) at the CSSL snow
816 laboratory. Both the new model and the original model perform well in approximating the
817 surface temperature of snow at CSSL site. However, the new model corrects the offsets
818 between the modeling of snow surface temperature and the modeling of the internal energy
819 of the snowpack in the original model. Here we note that uncertainties exist in the
820 measurements, e.g., the measurement of surface temperature of snow has positive value
821 during some daytime periods.

822 The comparison between modeled and measured snow water equivalent at Niwot
823 Ridge inferred from observed initial snow water equivalent and melt outflow is given in
824 Figure 13. This shows that after the adjustment to correct the discrepancy between initial
825 snow water equivalent and measured melt, the back-calculated snow water equivalent
826 compares well with modeled snow water equivalent. Due to the adjustment involved this is
827 really only a check on the timing of the ablation.

828 **7. Conclusions**

829 This paper has: (1) Evaluated the force restore and modified force restore
830 temperature parameterizations developed for a single layer snowmelt model in a complete
831 energy balance free-running model driven by only atmospheric forcing; (2) Introduced and
832 evaluated a new parameterization for the refreezing of liquid water near the surface in an
833 energy balance snowmelt model; and (3) Introduced a refinement to adjust thermal
834 conductivity parameters for shallow snowpacks. Collectively these contributions have
835 solved the issue of overestimating the energy loss of snowpack and underestimating the
836 average snow temperature in an earlier version of the UEB snowmelt model. With these
837 refinements, the model was better able to represent internal energy content, snow surface
838 temperature, early and late season snowmelt and albedo quite well. Through this modeling
839 work the understanding of snow surface energy exchanges and how they can be more
840 effectively modeled has improved.

841 This work has integrated information from a number of measurement sources to
842 validate and improve parameterization of processes in the model. Without the temperature
843 profile measurements that quantified internal energy, the energy content discrepancy would
844 have been hard to identify.

845 The new surface UEB snowmelt model has been calibrated and tested against
846 datasets from the USU Drainage Farm and CSSL snow laboratory and performed well at
847 these two sites. The paper also included tests against some data from Niwot ridge,
848 Colorado. However some discrepancies still exist between the modeled variables and the
849 observations. Also some variables cannot be strictly compared or compared against a

850 complete dataset. A more complete dataset of the liquid water content, together with
851 continuous observation of snow water equivalent, snow surface temperature, melt, and
852 depth, is necessary for a comprehensive test of the model improvements given here. This
853 speaks to the need for integrated measurements of multiple variables at each of multiple
854 sites to more fully constrain snow mass and energy processes to further improve snow
855 models. Such datasets are becoming available (Morin *et al.*, 2012) and it is important for
856 future studies to take advantage of such datasets, and for more of such datasets to be
857 collected.

858 Surface UEB is a single layer model designed to be parsimonious, yet use
859 physically based calculations for the energy and mass exchanges at the snow surface so as
860 to be transferable, with limited calibration, to other locations. This transferability was
861 evaluated to a limited extent in this paper by using multiple somewhat geographically
862 dispersed test sites in Utah, Colorado and California. The results thus provide some level of
863 confidence in the transferability of the model, though further testing at additional sites
864 would add to the confidence in the model transferability, or lead to further improvements.
865 Surface UEB uses a limited number of state variables so as to be easy to apply in a spatially
866 distributed fashion. It focuses on surface energy exchanges and surface temperature as the
867 variable at the interface between the surface and atmosphere governing energy exchanges.
868 It avoids attempting to represent the internal energy exchanges between snowpack layers
869 thereby avoiding the introduction of errors due to the challenges in representing these
870 complex internal snow processes. UEB compared favorably against more complex layered
871 models in a recent model intercomparison (Rutter *et al.*, 2009). Further evaluation of

872 surface UEB together with other models in different climate and topographic settings, as
873 suggested in Rutter et al. (2009), should be pursued.

874 **Acknowledgments**

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877 should not be interpreted as necessarily representing official policies, either expressed or
878 implied, of the U.S. Government.

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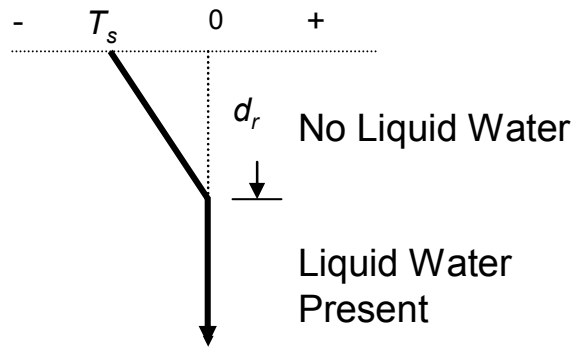
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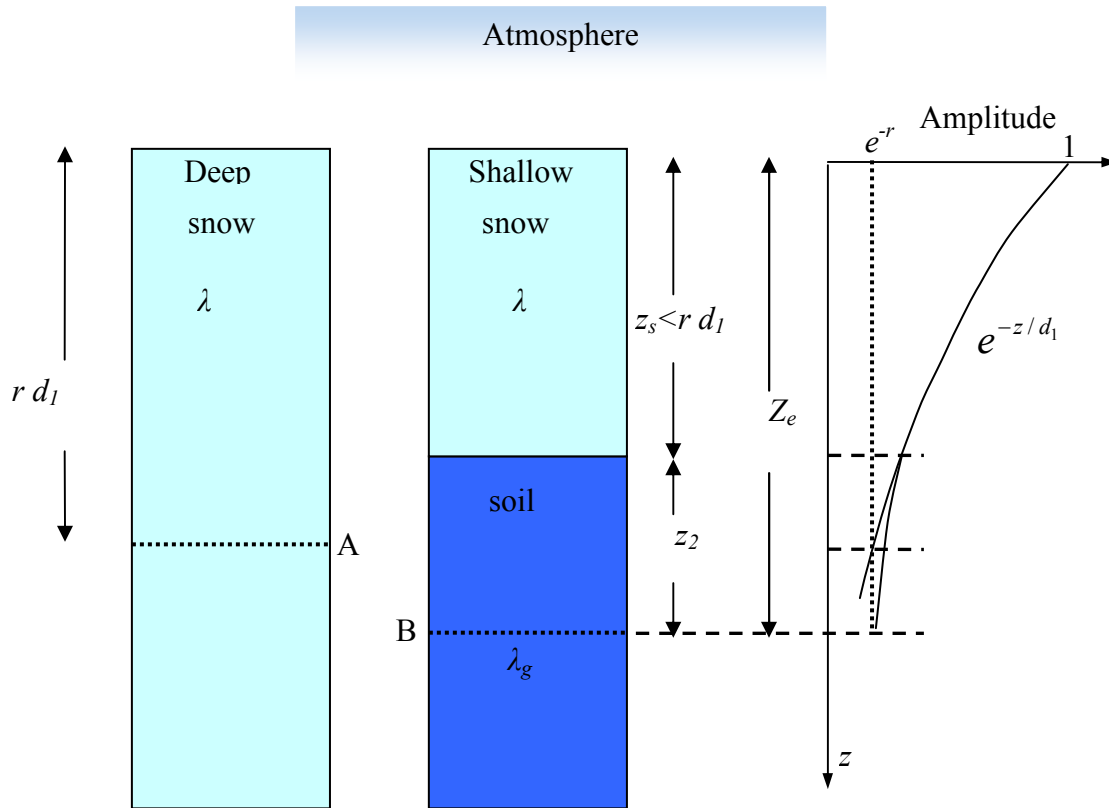
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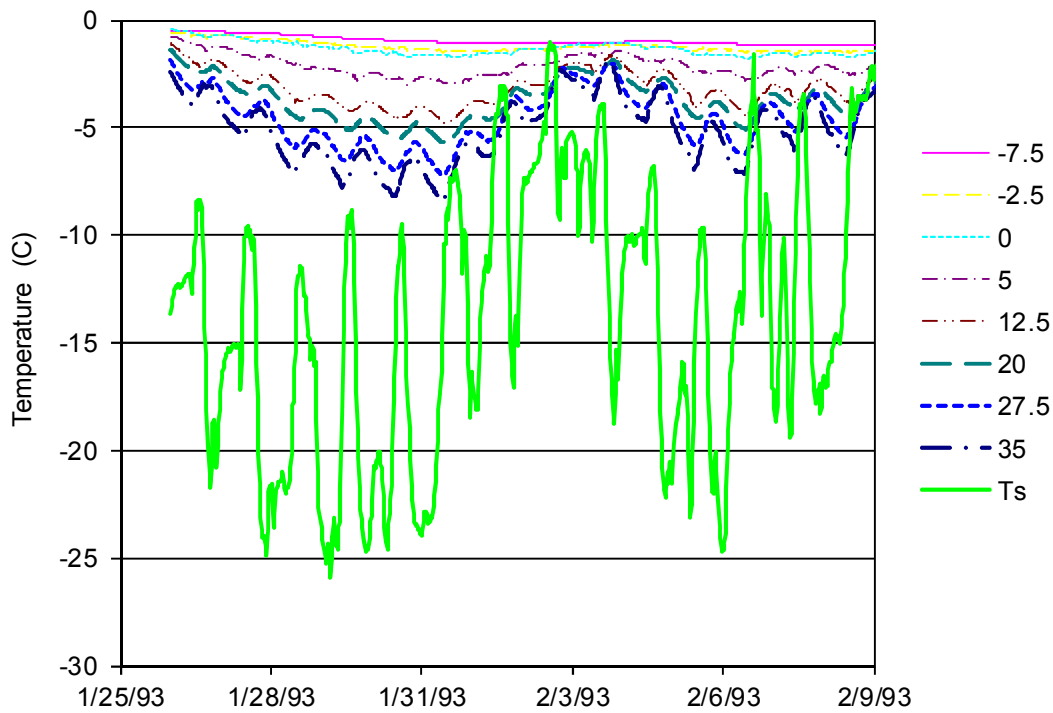
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1090 Figure 2 Heat conduction scheme for combined snow/soil system. The dashed lines at
1091 depths A and B indicate the depths at which temperature fluctuation amplitude is damped
1092 by e^{-r} in the deep snow and combined snow/soil system respectively.

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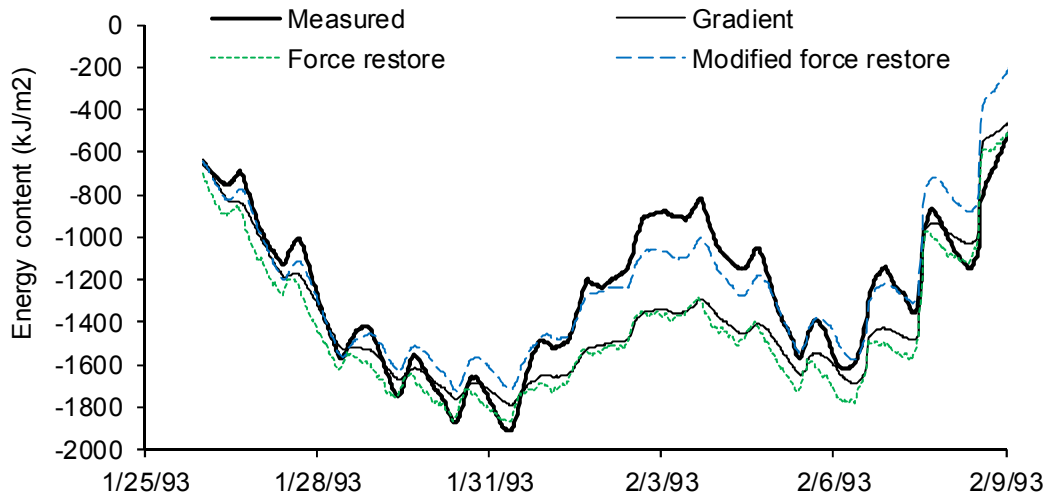
1096 Figure 3 Measured snow, ground, and snow surface temperatures at the USU Drainage
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 1098 temperatures are from thermocouples labeled according to their height relative to the
 1099 ground surface. Negative heights are below the ground surface and positive heights above
 1100 the ground surface. 0 refers to the measured temperature at the ground surface.

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1106 Figure 4 Comparisons of internal energy of snowpack during the first two freezing weeks at
1107 the USU Drainage Farm. Measured is the internal energy of snowpack calculated from the
1108 temperature profile (Figure 3). Gradient, Force restore, and Modified force restore
1109 represent the modeled internal energy of snowpack using the equilibrium approach, the
1110 force-restore approach, and the modified force restore approach respectively.

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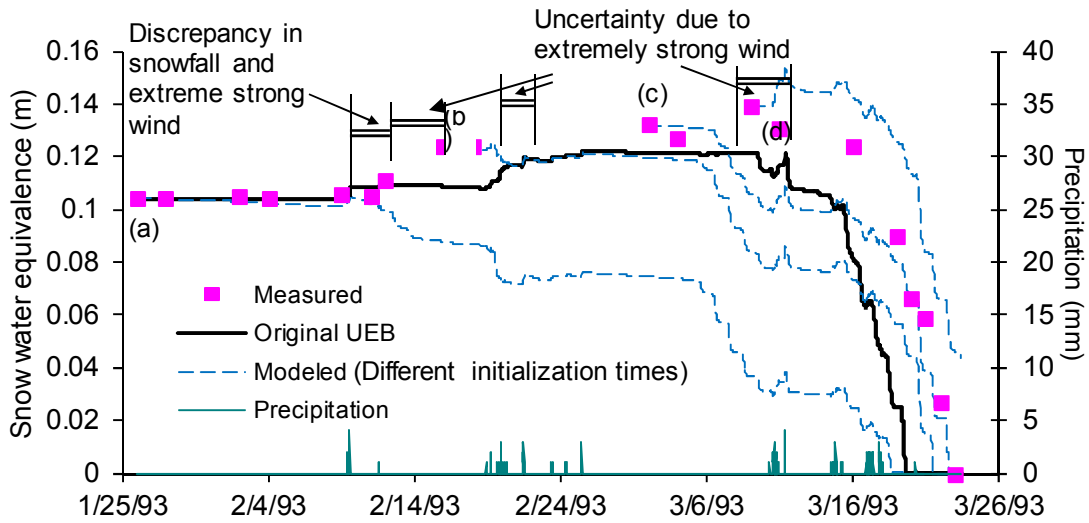


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Precipitation input is shown (spiky line at the bottom) relative to the axis at the right.

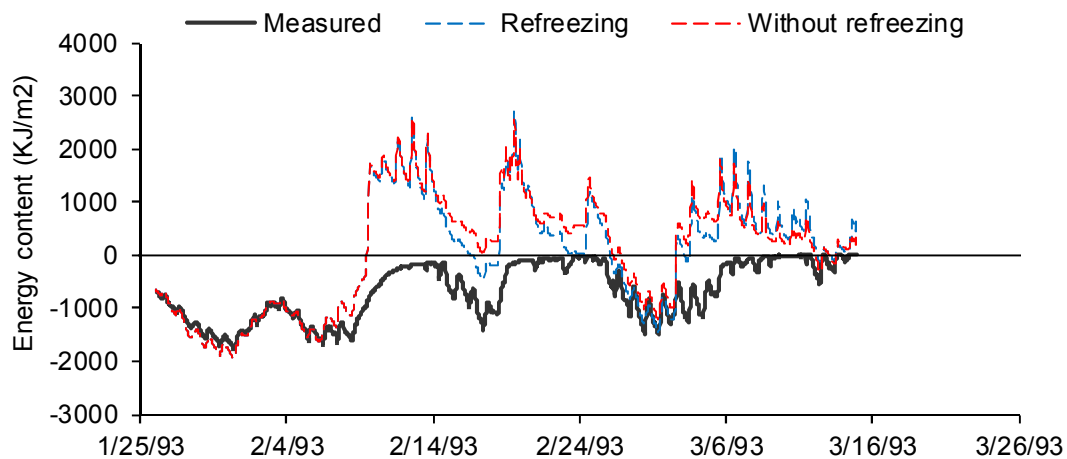
Letters (a) through (d) indicate points where the model was re-initialized following periods of likely erroneous inputs due to severe weather.

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1120 Figure 6 Comparisons of internal energy of snowpack in 1993 at the USU Drainage Farm.

1121 The wide solid line is the measured values. “Refreezing” represents the modeled internal

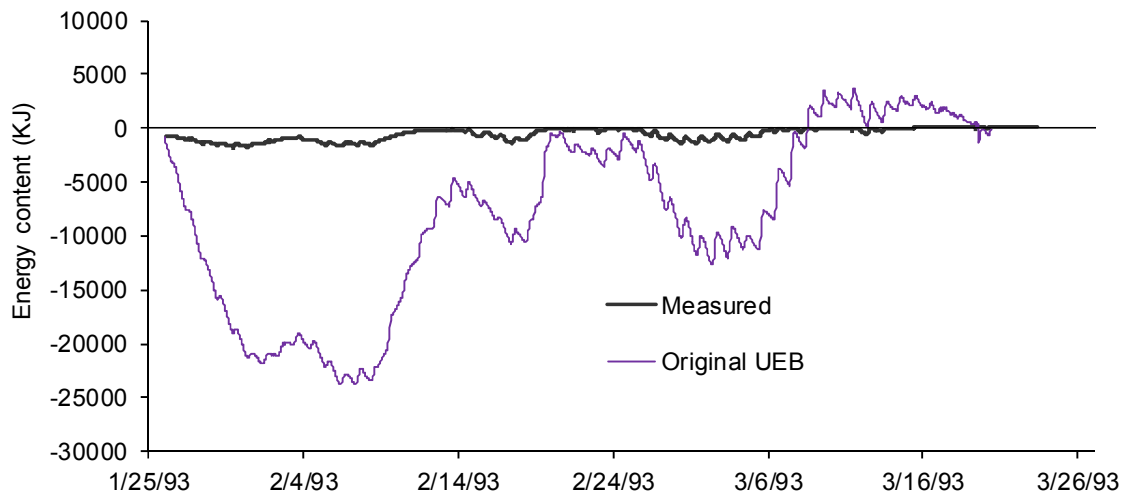
1122 energy of snowpack with new surface UEB model. “Without refreezing” represents the

1123 model without the refreezing scheme.

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1128 Figure 7 Comparisons between the measured and modeled internal energy of the snowpack

1129 at the USU Drainage Farm in the original model.

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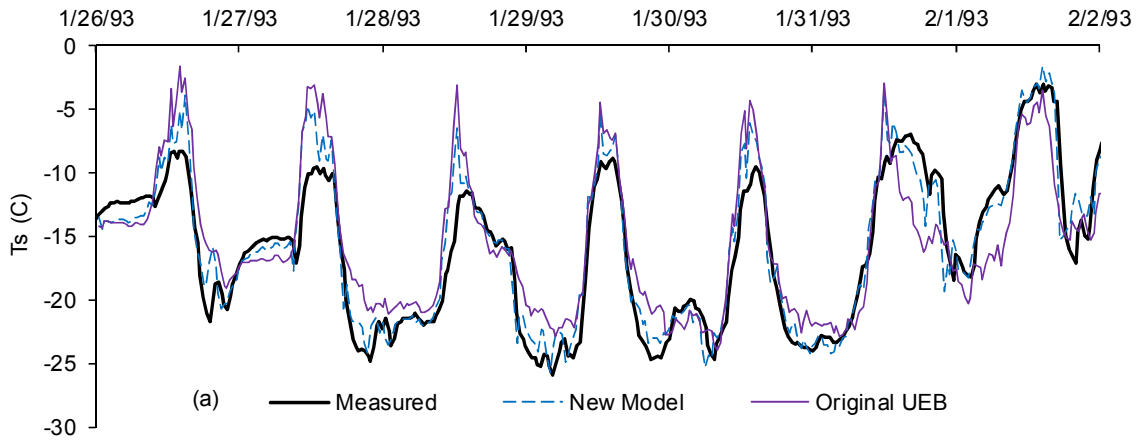
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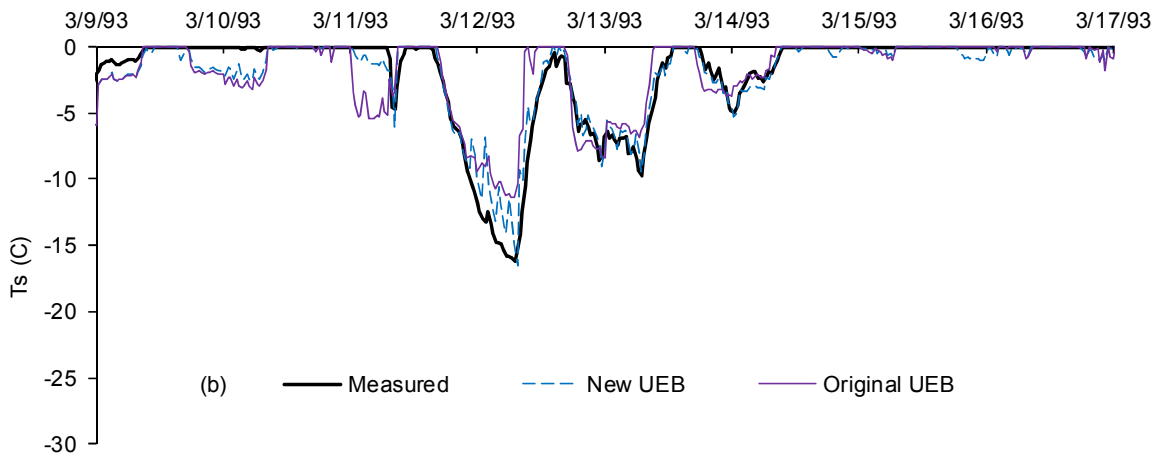
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1138 Figure 8 Comparisons of snow surface temperature in 1993 at the USU Drainage Farm. (a)

1139 the first two subfreezing weeks, and (b) end of the modeling period when the snowpack is

1140 occasionally in an isothermal state.

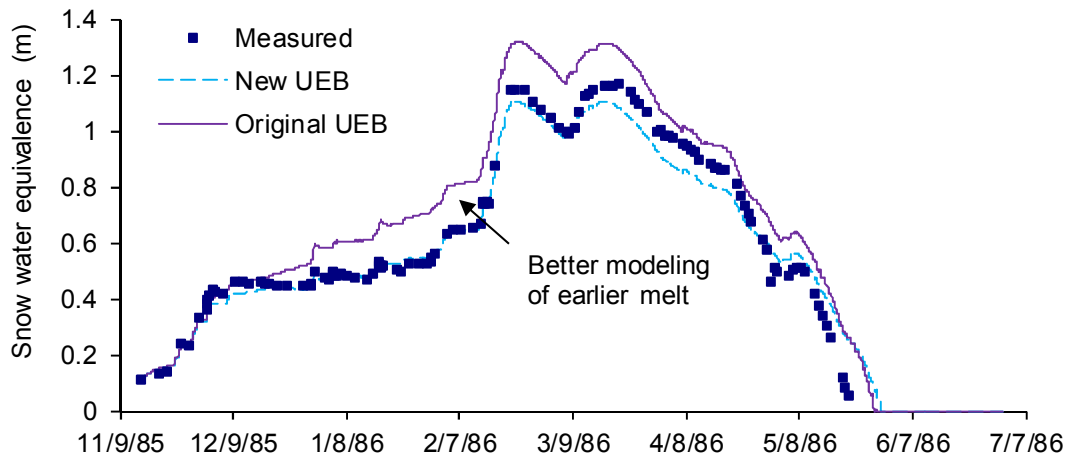
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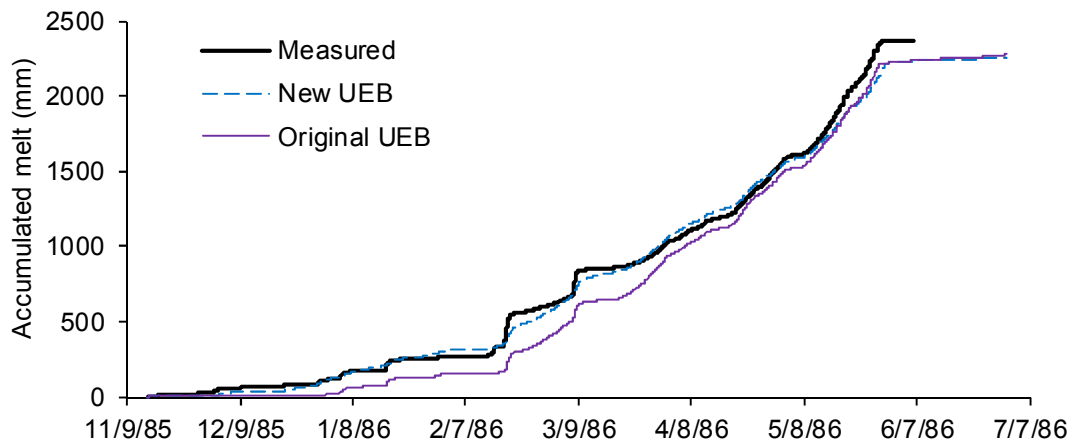
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1147 Figure 9 Comparisons of snow water equivalent in 1986 at CSSL.

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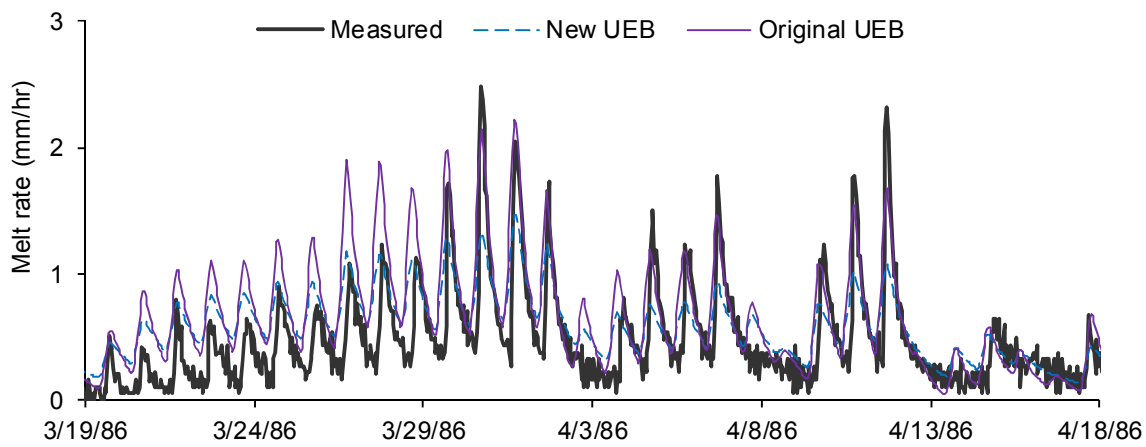
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1152 Figure 10 Comparisons of accumulative melt in 1986 at CSSL.

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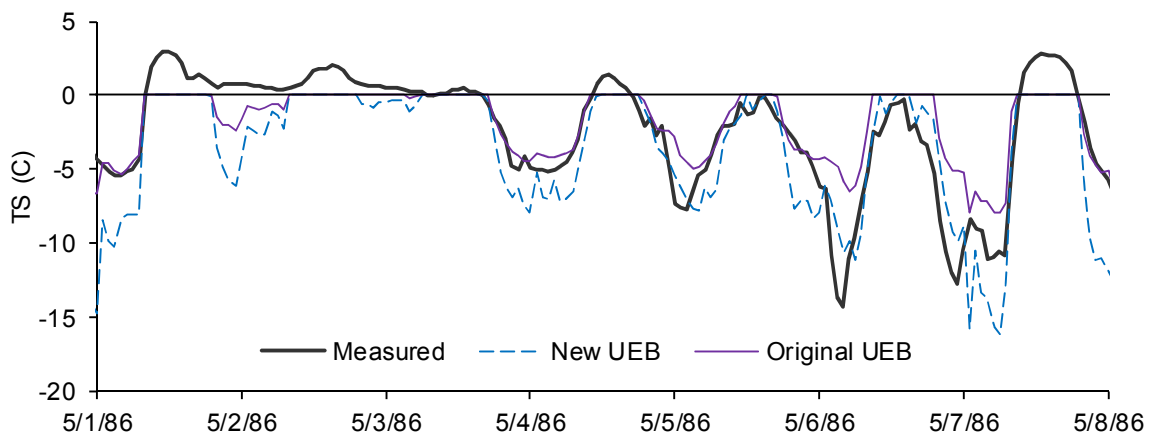
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1158 Figure 11 Comparisons of meltwater outflow rate in 1986 at CSSL

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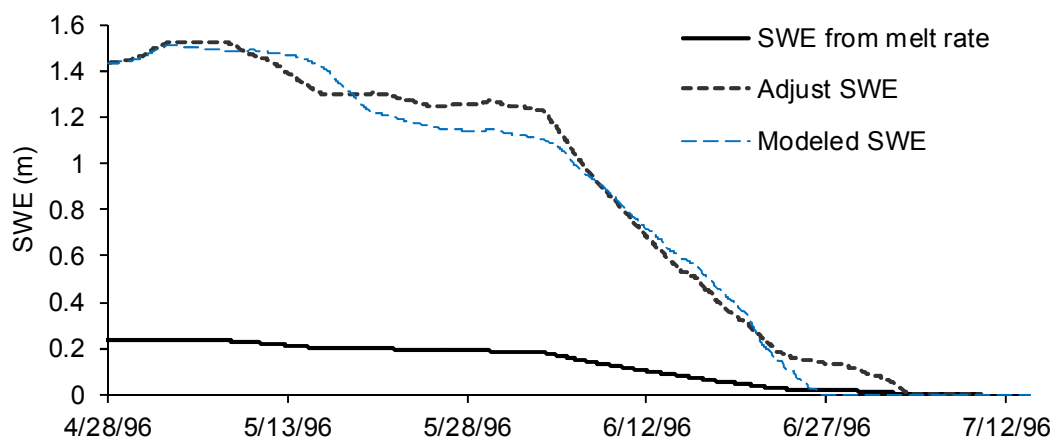
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1163 Figure 12 Comparisons of surface temperature of snow in 1986 at CSSL

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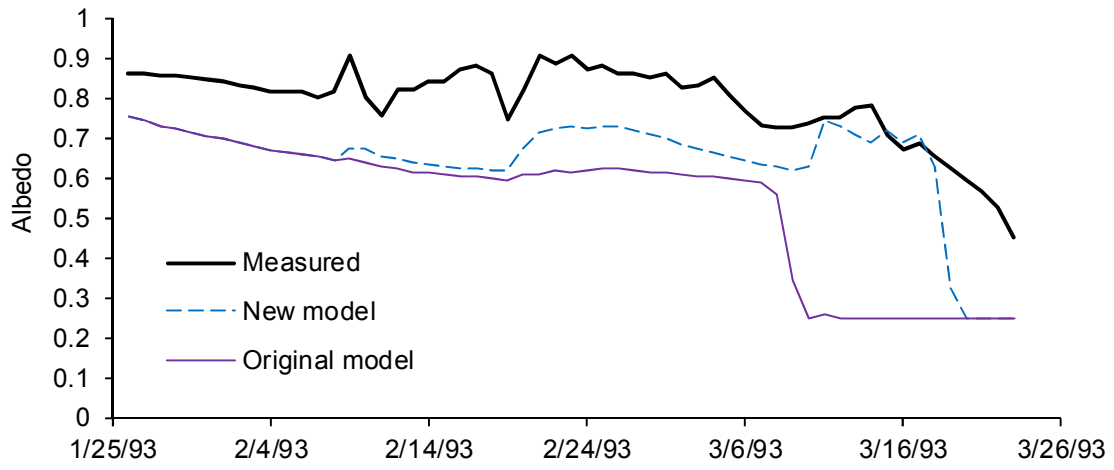


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1167 Figure 13 Comparisons of snow water equivalent in 1996 at Subnivean Snow Laboratory at
1168 Niwot Ridge watershed, CO.

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1172 Figure 14 Comparison of measured and modeled albedo at the USU drainage farm.

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1175 Table 1 Model parameter values

Parameters	Value
Thermal conductivity of snow λ_s	**0.33 $\text{kJ m}^{-1} \text{K}^{-1} \text{h}^{-1}$
Thermal conductivity of soil λ_g	**6.5 $\text{kJ m}^{-1} \text{K}^{-1} \text{h}^{-1}$
Low frequency forcing frequency ω_{lf}	**0.0654 radians h^{-1} ($\omega_1/4$)
Dimensionless damping depth factor r	**1
Threshold depth for fresh snow d_{NewS}	**0.002 m
Saturated hydraulic conductivity K_{sat}	*200 m h^{-1}
Surface aerodynamic roughness z_o	*0.01 m
Capillary retention fraction L_c	*0.02
Soil effective depth D_e	*0.1 m
Snow density ρ_s	*200 kg m^{-3}
Ground heat capacity C_g	2.09 $\text{kJ kg}^{-1} \text{K}^{-1}$
Density of soil layer ρ_g	1700 kg m^{-3}
Emissivity of snow ϵ_s	0.99
Temperature above which precipitation is rain T_r	3°C
Temperature below which precipitation is snow T_{sn}	-1 °C
Wind/air temperature measurement height z_m	2 m
Bare ground albedo α_{bg}	0.25
New snow near infrared band reflectance α_{iro}	65%
New snow visible band reflectance α_{vo}	85%

1176 ** These parameters are new, i.e., they were not present in the Original UEB

1177 * These parameters were calibrated to have new values.

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1195 T_s is the measured surface temperature of snow from an infrared sensor. Other
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1197 ground surface. Negative heights are below the ground surface and positive heights above
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1207 dashed lines are the modeled values with new model starts at different times. Precipitation
1208 input is shown (spiky line at the bottom) relative to the axis at the right. Letters (a) through

1209 (d) indicate points where the model was re-initialized following periods of likely erroneous
1210 inputs due to severe weather.

1211

1212 Figure 6 Comparisons of internal energy of snowpack in 1993 at the USU Drainage Farm.

1213 The wide solid line is the measured values. “Refreezing” represents the modeled internal

1214 energy of snowpack with new surface UEB model. “Without refreezing” represents the

1215 model without the refreezing scheme.

1216

1217 Figure 7 Comparisons between the measured and modeled internal energy of the snowpack

1218 at the USU Drainage Farm in the original model.

1219

1220 Figure 8 Comparisons of snow surface temperature in 1993 at the USU Drainage Farm. (a)

1221 the first two subfreezing weeks, and (b) end of the modeling period when the snowpack is

1222 occasionally in an isothermal state.

1223

1224 Figure 9 Comparisons of snow water equivalent in 1986 at CSSL.

1225

1226 Figure 10 Comparisons of accumulative melt in 1986 at CSSL.

1227

1228 Figure 11 Comparisons of meltwater outflow rate in 1986 at CSSL

1229

1230 Figure 12 Comparisons of surface temperature of snow in 1986 at CSSL

1231

1232 Figure 13 Comparisons of snow water equivalent in 1996 at Subnivean Snow Laboratory at

1233 Niwot Ridge watershed, CO.

1234

1235 Figure 14 Comparison of measured and modeled albedo at the USU drainage farm.

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