We would like to thank the reviewers for their constructive and encouraging comments. Please find our responses below.

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Response to Referee 1 (E. Ansink):

This paper is on the application of the bankruptcy literature to the problem of sharing scarce river water, focusing on spatial and temporal variation in water availability. A case study provides an illustration of the proposed method.

I like part of this paper (see final bits of this report), but I also have three major comments. My first comment is that the paper ignores the backbone of the bankruptcy literature, or claims literature, by ignoring the rules' properties. My two other comments are on the mathematical specification of the method and the novelty of the paper's contribution.

We would like to thank you for your carful review and helpful comments. It must be noted that the responses below have benefitted from our direct email correspondence after receiving your comments. We highly appreciate your patience, kind hints and constructive comments. The manuscript has been revised substantially to address your concerns.

1. Two immediate questions that arise when reading the paper are:

(a) Why focus on these 4 bankruptcy rules and ignore the many other rules?,

Good question. This has two main reasons:

1. The selected rules are among the most commonly used rules in the literature, as you also mention in your work (Ansink and Weikard, 2012).

2. One of the main objectives here was to develop optimization models that allow application of original bankruptcy rules to river sharing problems with respect to flow variability constraints. Formulation of each rule as an optimization model is not very easy and straightforward, so we have included what we were able to formulate so far. Future studies might be able develop other models based on different bankruptcy rules.

The benefit of developing an optimization model is that people can simply use the rules without a need to worry about understanding the details of calculations and going through a step-by-step analysis in complex river problems with different physical constraints.

The revised manuscript clarifies these points.

and (b) Why perform a stability analysis on the bankruptcy solutions?

Analyzing the stability of allocation solutions is common in the water resources literature (e.g., Dinar and Howitt (1997), Teasley and McKinney (2011), Madani and Dinar (2012), Read et al. (2014)). Once different allocation solutions are developed, stability of solutions is normally evaluated using quantitative measures to determine the solutions with higher potential acceptability. Also, the water resource literature normally prefers to suggest more than one allocation option to the decision makers to increase the likelihood of developing a resolution through expansion of the feasible solution set. This is perhaps one of the main reasons that the water resource literature normally develops alternative allocation solutions first and then evaluates their quality.

The revised manuscript discusses "stability analysis" in details (please see lines 204-244.)

In my view, the answer to both questions is that the authors ignored the backbone of the bankruptcy literature, or claims literature, by ignoring the rules' properties. In this literature, the attractiveness of rules is based on their properties, such as monotonicity or independence properties, with respect to various possible perturbations of the problem at hand (for an overview of these properties see Herrero and Villar, 2001; Thomson, 2003).

By ignoring this aspect of the bankruptcy literature, it appears that the authors have selected arbitrary rules. By applying these arbitrary rules to a case study, the outcomes are also arbitrary, which is why they need some stability analysis to evaluate the attractiveness of the outcomes. This stability analysis is redundant had the authors not ignored the properties of the rules.

We agree with you that an alternative to stability analysis (ex-post approach) is to adopt an axiomatic (ex-ante) approach (perhaps more popular in the economics literature.) Ideally, the results from both approaches should coincide and this can be approved by future research.

Nevertheless, providing more options to the negotiators in real negotiations will expand the feasible solution set and create more opportunities for cooperation. Also in practice, it might be harder to develop agreement over an ex-ante approach before clarifying to the beneficiaries what they would exactly obtain under this approach. Stakeholders are more likely to be able to evaluate an allocation solution which clearly indicates their gain (water volume or utility from water allocation) rather than an allocation method whose outcomes are not clear. An axiomatic approach might be more attractive where the social planner (central allocator) has superpower and can implement the social planner solution developed based on a range of reasonable principles.

Also, please note that in the main difference between the river bankruptcy problem and normal bankruptcy problems (or water bankruptcy problems with unequal access) is the asymmetric accessibility of resource at a given time and location. Therefore, even if the solution principles are reasonable, the actual solutions will be affected by the physical limitations of the system. Indeed, the same solution principles might not yield similar results in two river bankruptcy problems with similar claims and total water availability as the physics of the system make them different. This difference might make the axiomatic approach less practical as it does not provide clear information to the parties about their actual gains.

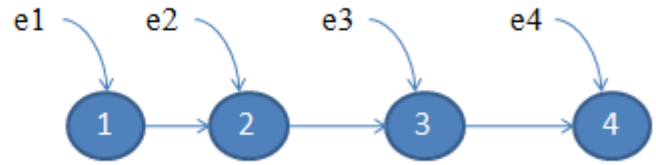
Please see lines 204-244 of the revised manuscript.

From the paper, it becomes clear that the authors are interested in variability. Given this interest they could have chosen or designed properties that are desirable with respect to this variability and subsequently derive the rules that satisfy these properties. In fact, for river runoff variability this is the procedure that has been followed by Ansink and Weikard (2013) with respect to so-called composition properties.

Please note that our objective was not to design new rules. Instead, we wanted to adopt the existing and commonly used rules for river systems. We proposed optimization models which develop allocation solutions with minimal deviation from the results of original bankruptcy solutions with respect to resource access constraints. Therefore, when there are no resource availability restrictions, our results match the results of original bankruptcy solutions.

Given that the basis for developing these optimization models have been the original allocation rules, we believe our results satisfy the composition properties used by Ansink and Weikard (2013). Nevertheless, our results are closer to the results of the original rules when compared with the new allocation rules developed by Ansink and Weikard (2012) using the composition properties approach. As an example, we solved the three numerical examples from Ansink and Weikard (2012).

These examples involve four riparian beneficiaries on a liner river system. The Inflow column indicates the amount of contribution of each beneficiary to the river system and the Claim column shows the beneficiaries' claims. The Ansink & Weikard column shows the results of the method developed by composition properties (as suggested by the reviewers) based on the P, CEA and CEL rules. The Zarezadeh et al. column shows the results obtained using the optimization models developed in this study based on the same bankruptcy rules. The No Restriction Case column shows the results based on the P, CEA and CEL rules in a situation where all parties have equal access to the total resource. This case represents the original bankruptcy case where parties have no restriction in receiving the amount of asset suggested by the bankruptcy rules. In the water resources context, this situation is common in groundwater bankruptcy problems (Madani and Dinar, 2013).



	Inflow	Claim	Ansink & Weikard's			Zarezadeh et al.			No Restriction Case		
			P	CEA	CEL	P	CEA	CEL	P	CEA	CEL
Case 1	80	50	29	40	20	32	40	33	32	40	33
	10	10	6	10	0	6	10	0	6	10	0
	10	20	13	20	10	13	20	3	13	20	3
	10	90	62	40	80	58	40	73	58	40	73
Case 2	80	50	25	40	10	29	30	30	29	30	30
	10	30	16	25	10	17	30	10	17	30	10
	10	20	12	18	10	12	20	0	12	20	0
	10	90	57	28	80	52	30	70	52	30	70
Case 3	80	50	33	40	30	38	50	40	38	50	40
	30	10	8	10	0	8	10	0	8	10	0
	10	20	16	20	15	15	20	10	15	20	10
	10	90	73	60	85	69	50	80	69	50	80

Comparison of the results clearly show the superior performance of the suggested optimization models in producing results which are closer to the original bankruptcy results in the no restriction case. Indeed, the results in all three cases show that in these examples, the physics of the system (shape of the network and water availability) creates no restriction for implementing the original bankruptcy results. However, the results based on Ansink & Weikard's method match the original bankruptcy results only in one of the nine examples solved (CEA rule in Case 1).

To compare the two methods in a case where physical characteristics of the system make the original bankruptcy solutions infeasible we tested an additional case. In this case, the original P solution is not feasible as the available water in the territory of agent 1 is not more than 10 units. Again, the results show that the proposed solution does a better job in matching the results of the original bankruptcy results in comparison with the Ansink and Weikard's method. Moreover, the suggested method treats all parties equally and does not discriminate parties based on their location as done by the Ansink and Weikard's method, resulting in favoring the downstream parties (Mianabadi et al., 2013).

	Inflow	Claim	Ansink & Weikard			Zarezadeh et al.			No Restriction Case		
			P	CEA	CEL	P	CEA	CEL	P	CEA	CEL
Case 4	10	50	3.13	5.00	0	10.00	10.00	0	10.53	10.00	0
	10	30	4.22	7.50	0	6.42	10.00	0	6.32	10.00	0
	10	20	4.53	8.75	0	4.28	10.00	0	4.21	10.00	0
	10	90	28.13	18.75	40	19.28	10.00	40	18.95	10.00	40

The revised manuscript discusses the advantages of the proposed approach over the method proposed by Ansink and Weikard (2012) (Lines 170-204).

2. The specification of the proportional rule is not clear, because:

(a) The interpretation of the objective function is not clear, i.e. why are you minimizing the difference between $\lambda_{P_{i,t}}$ and the product of the other λ's?; My gut feeling is that for this specification to lead to a proportional solution requires that all claims are equal, but there is so little motivation that I cannot prove this. Please motivate and elaborate on how we should interpret this rule and what is its relation to the standard proportional rule in the bankruptcy literature (cf. Thomson, 2003).

As mentioned, the optimization models have been developed to apply well-known bankruptcy rules to river problems in which the physical shape of the river network and flow variability might prevent implementation of original bankruptcy solutions. These optimization models have been formulated (with unique objective functions and constraints) such that they produce results which are identical to original bankruptcy solutions when there are no physical restrictions. In other cases, the models try to minimize the difference between the original bankruptcy solutions and feasible allocation solutions, with respect to the physical characteristics of the system.

In case of the P rule optimization model, the objective function helps minimizing the maximum satisfaction rate (allocation/claim). Therefore, in absence of physical restrictions the satisfaction rates are all equal (please see the results Cases 1-3 above). The second term in the objective function helps minimizing the difference between the satisfaction rates of the beneficiaries and developing a unique allocation solution. Therefore, as indicated in cases 1-3, there is no need for all claims to be equal "for this specification to lead to a proportional solution" (as suggested by the reviewer.)

The revised version provides additional explanations on formulation of the proportional optimization model to address your concern.

(b) Equation (1) does not specify the variable over which the objective function is minimized. Is $\lambda_{P_{i,t}}$ in (1) and (11) endogenous?

As discussed all λ's are decision variables and are determined by the optimization model.

The revised version explicitly identifies the decision variables from known variables of each optimization model.

Similar comments apply to the other 3 rules introduced in Section 3.

The revised version provides additional explanations on formulation of each optimization model to address your concern. Also, general characteristics of the optimization models have been illustrated in lines 273-331.

Thanks for your comment!

3. One of the paper's objectives is (final sentence Section 1): "developing a new water allocation mechanism that . . . provides allocations solutions with respect to the temporal and spatial variability of water flows in trans-boundary river systems."

In Section 2 it becomes clear that this variability is modeled as constraints in an optimization model. For temporal variability this implies that solutions are computed for each period separately, which is a weak interpretation of the paper's objective.

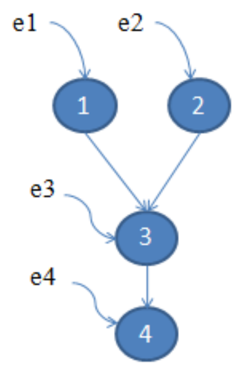
We would like to highlight that your observation is valid in the example case which represents an unregulated system for which the problem was solved in different time steps separately. But, the introduced framework can handle regulated systems in which water allocation can be done over the whole planning period by one run of the optimization model with respect to flow timing variability. Furthermore, additional constraints (e.g. minimum monthly (or daily) allocation for environmental flows or maintaining hydropower head) can be added to the model, if needed. This type of water allocation exercise with respect to temporal variability of flows is very common in the water resources management and reservoir operations literature.

This has been clarified in the revised version to address your concern. Please see lines 170-204 and 341-359.

For spatial variability, this implies that solutions are constrained by water flowing downstream (i.e. equation (5)), which is not really novel (cf. ˙Ilkılıç and Kayı, 2012, and other papers), and for which more elegant alternatives are available (Ansink and Weikard, 2012).

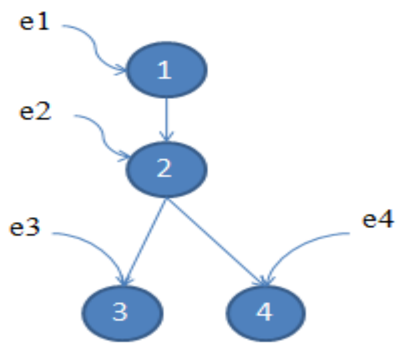
We are afraid that this is not an accurate interpretation of the capability of the proposed method in capturing the physical characteristics of river systems. The model proposed in this study does not have the major limitations of the models the reviewer refers to. The Ilkılıç and Kayı's model (2012) has "no restrictions on the possible networks between sources and agents" whereas in the river sharing problem the physical characteristics of the system impose restrictions on networks between sources (river sections) and agents (riparian parties). The Ansink and Weikard's model (2012) only works for linear systems in which all riparians fall on the same line. The proposed modeling framework is specific to river networks and is applicable to all river networks, regardless of their shape, number of tributaries, etc. This model can fully capture the physical characteristics and spatial variability of the system.

We think judgment about the 'elegance' of the solution methods is very subjective. As shown in cases 1-3 the results of our method match the original bankruptcy solutions and in case 4 in which the original bankruptcy solutions is not feasible under the P rule, our results are very close to the original P solutions. The following three cases show the results of the application of the proposed P, CEA, and CEL models to non-linear river networks, which to the best of our knowledge, cannot be solved using the Ansink and Weikard's method, which the reviewer finds 'more elegant'.



	Inflow	Claim	Zarezadeh et al.			No Restriction Case		
			P	CEA	CEL	P	CEA	CEL
Case 5	10	50	10.0	10.0	10.0	16.7	12.5	23.3
	20	30	12.0	13.3	10.0	10.0	12.5	3.3
	10	20	8.0	13.3	0.0	6.7	12.5	0.0
	10	50	20.0	13.3	30.0	16.7	12.5	23.3

Case 6	10	50	10.0	10.0	10.0	21.7	12.5	33.3
	20	30	16.7	13.3	20.0	13.0	12.5	13.3
	10	20	13.3	13.3	10.0	8.7	12.5	3.3
	10	15	10.0	13.3	10.0	6.5	12.5	0.0



	Inflow	Claim	Zarezadeh et al.			No Restriction Case		
			P	CEA	CEL	P	CEA	CEL
Case 7	10	50	10.0	10.0	10.0	16.67	12.5	23.3
	20	30	15.0	13.3	5.0	10.00	12.5	3.3
	10	20	10.0	13.3	10.0	6.67	12.5	0
	10	50	15.0	13.3	25.0	16.67	12.5	23.3

For clarification of the above points, we have highlighted the main advantages of the proposed method in the revised version of the paper. Please see lines 170-204.

Also, spatial variability, as modeled in this paper cannot deal with excess flow. Suppose that downstream inflow is high while downstream claims are low. How is excess water allocated? Equation (4) assures that no riparian receives more than his claim, but the system is closed since the most downstream riparian m cannot dispose more than (fixed) SD.

When the downstream user has more water than its claim, this user is not part of the bankruptcy game because it automatically gets more than what it needs. So, there is no need to include the beneficiaries with excess flows in the analysis. This means that the excess water allocation is not an issue to worry about. Also in practice, a downstream riparian party that receives more water than it claims (demands) is not expected to be part of water conflicts.

This has been clarified in the revised text. Please see lines 319-331.

To close on a more encouraging note, I appreciate several aspects of this paper, including:

1. I enjoyed the thorough explanation of the benefits and advantages of applying bankruptcy rules to the sharing of scarce river water. As the authors explain, sometimes benefit sharing is just not realistic or there may be insufficient information to apply benefit-sharing approaches grounded in cooperative or noncooperative game theory (such as Van den Brink et al., 2012; Ambec and Sprumont, 2002; Ambec et al., 2013).

2. I appreciate the careful consideration of the basis for (exogenous) claims in the case study section. The three explored alternatives are relevant for actual negotiations and should therefore be considered in a cooperative analysis as presented in this paper.

Thanks for your encouraging feedback. We appreciate your thoughtful comments and patience in the personal email discussion. Your constructive comments have significantly helped us in improving the clarity of this work.

References

Ambec, S., A. Dinar, and D. McKinney (2013). Water sharing agreements sustainable to reduced flows. Forthcoming in Journal of Environmental Economics and Management.
Ambec, S. and Y. Sprumont (2002). Sharing a river. Journal of Economic Theory 107(2), 453–462.
Ansink, E. and H.-P.Weikard (2012). Sequential sharing rules for river sharing problems. Social Choice and Welfare 38(2), 187–210.
Ansink, E. and H.-P.Weikard (2013). Composition properties in the river claims problem. MPRA Working Paper 51618.
Herrero, C. and A. Villar (2001). The three musketeers: Four classical solutions to bankruptcy problems. Mathematical Social Sciences 42(3), 307–328.
˙Ilkılıç, R. and C. Kayı (2012). Allocation rules on networks. Working Paper 118, Universidad del Rosario.
Thomson, W. (2003). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: A survey. Mathematical Social Sciences 45(3), 249–297.
Van den Brink, R., G. van der Laan, and N. Moes (2012). Fair agreements for sharing international rivers with multiple springs and externalities. Journal of Environmental Economics and Management 63(3), 388–403.

References

Ansink, E. and H.-P.Weikard (2012). Sequential sharing rules for river sharing problems. Social Choice and Welfare 38(2), 187–210.
Ansink, E. and H.-P.Weikard (2013). Composition properties in the river claims problem. MPRA Working Paper 51618.
Dinar, A. and R.E. Howitt (1997). Mechanisms for allocation of environmental control cost: empirical tests of acceptability and stability, J. Environmental Management, 49(2), 183–203.
Ilkılıç, R. and C. Kayı (2012). Allocation rules on networks. Working Paper 118, Universidad del Rosario.
Madani K. and A. Dinar (2012) Cooperative Institutions for Sustainable Common Pool Resource Management: Application to Groundwater, Water Resources Research, 48(9), W09553.

Madani K. and A. Dinar (2013) Exogenous Regulatory Institutions for Sustainable Common Pool Resource Management: Application to Groundwater, Water Resources and Economics, 2-3: 57–76.
Mianabadi, H., Mostert, E., Zarghami, M., and Giesen, N. van de. (2013) Transboundary water resources allocation using bankruptcy theory; Case study of Euphrates and Tigris Rivers, Proceedings of the Transboundary water management across borders and interfaces: present and future challenges International Conference & Workshops (TWAM,2013). 16- 20 March, Aveiro, Portugal.
Read, L., B. Inanloo, B., and K. Madani (2014) Optimality versus stability in water resource allocation, J of Environmental Management, 133: 343–354.
Teasley, R. L. and D. C. McKinney (2011) Calculating the benefits of trans-boundary river basin cooperation: the Syr Darya basin, Water Resources Planning and Management, 137(6), 481-490.

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Response to Referee 2:

The manuscript very nicely presents a bankruptcy approach for allocating water among stakeholders in trans-boundary basins. Water allocation schemes are a relevant topic even for developed areas with plenty of information. The paper is very well written, easy to understand and provides very clear theoretical development and numerical examples on how bankruptcy might work and traditional game theory may not. I liked very much the exercise of putting the theory in. There is some innovation in the manuscript in terms of evaluating stability of the solutions; however, he paper would benefit from some improvements including a better set of conclusions, some roadmaps and organization as well as other minor issues.

We would like to thank you for your thoughtful comments. Please find our responses below.

Major Issues

Contribution of the paper seems more or less clear. It talks about noncooperative and cooperative game theory and information needs for these approaches to be useful. It argues that quantitative results are limited in these and that the paper's approach based on rules is relatively new. A better job could be done in pages 13858, 13859 and 13860 to summarize how the paper approach fits in the larger literature context. Perhaps a simple chart that maps the families of water allocation methods over two or three dimensions would be very helpful and highlight the contribution of the manuscript.

Thanks for the suggestion. In the revised version, we have tried to discussion on how the bankruptcy approach complements the existing literature by an extensive discussion presented in lines 66-103.

Conclusions are somehow vague. I recommend sharpening the key findings and linking them with specific results or their discussion.

Agreed. The conclusions section has been revised as suggested.

Some additional subheadings would improve readability of the paper. For example over River bankruptcy problems have one referring to bankruptcy in the context of other approaches, then start in p. 13860 with a second subheading elements of bankruptcy problems, and lastly a subheading for innovations. Perhaps a better title for the third main heading could be 'Banrkuptcy methods' and consider changing heading 2. Think in the paper flow and the purpose of each main section. Do something similar for the remaining headings.

Great suggestion! Subheadings have been added to improve the readability.

The authors are knowledgeable about the subject and seem to be very productive publication-wise. Yet better recognizing previous efforts on similar approaches should be better reflected in the literature review and the discussion of their results. I encourage to compare or simply find commonalities and differences between these results and similar transboundary basin water allocation papers.

Thanks for the suggestion. In the revised version, we have elaborated on the main differences and similarities between this work and the previous transboundary water allocation solutions, especially with those which have tried using bankruptcy methods. Please see lines 66-103, 149-162 and 177-204.

Address some issues with the mathematical formulations. For example in equations 1-11. What is the objective function about? Please elaborate.

Because of the physical and temporal restrictions as well as unequal access to the river, allocations based on simple (original) bankruptcy rules might not be feasible in practice in many cases. In other words, enough water might not be available at a given location at a given time to match the outcome of bankruptcy methods that assume equal access and are appropriate for one-shot games. Therefore, the major contribution of this work is formulating river water allocation bankruptcy problems as optimization models which allow solving the river bankruptcy problem with consideration of temporal and spatial constraints in the system. In absence of temporal and spatial constraints, the optimization models' results match the results of original bankruptcy rules that do not consider any physical and temporal restrictions. When such constraints exist, the optimization models try to minimize the difference between the allocation results and what could have been allocated based on the original bankruptcy rules if such constraints did not exist.

Each optimization model has been developed based on one of the commonly used bankruptcy methods and it searches for the essential variables of these methods to be able to develop allocations. To help the search, some additional variables had to be included in the optimization models. Each model tries to minimize the difference between its results and the results of the original bankruptcy methods. This difference is zero when the physical structure of the system, and temporal and spatial variability of flow creates no restriction in allocation.

The objective function of each model and the constraints that are unique to each optimization model were necessary to be able to have a working model in each case. For example, Equations 1-11 try to apply the proportional rule to the river problem. In the original bankruptcy problem, a unique allocation rate λ_P is used. Given the flow variability in river systems having a unique allocation rate might not be feasible. Therefore, the proposed model tries to find the feasible allocation rate for each riparian while trying to minimize the maximum allocation rate and the difference between the allocation rates. In absence of flow variability restrictions, the allocation rates are equal (so the second term in the objective function will be minimized) and the maximum allocation rate is equal to the outcome of original proportional rule which considers no accessibility restriction.

Based on the received comments, we realized that the structure and purpose of the optimization models were not clear to the readers. So, the revised version provides additional explanations on formulation of each optimization model to address the reviewers' concern. Also, general characteristics of the optimization models have been illustrated in lines 273-331.

How is the program solved for all time steps in the year? Notation is very confusing. Please guide the reader better.

As discussed in the paper, the example case represents an unregulated system. Therefore, calculations are done in each time step (monthly in this case) separately. The final allocation is the sum of monthly allocations. In regulated systems, however, one model would be sufficient for the whole planning horizon (e.g. a year) as long as the model components are adjusted accordingly. In such cases, other constraints (e.g. minimum monthly environmental allocations or water allocations for maintaining the necessary energy head for hydroelectricity production) can be also added to the model.

We have clarified these points (lines 341-359) in the revised version.

I liked the stability section, but how was the voting figured out for estimating plurality indexes of the allocation methods? (In table 1).

The assumption is that each party selects the allocation scheme that gives it the highest share. A sentence was added in the revised version to clarify this further.

The bankruptcy stability index is then one of the innovations of the paper. Please have a better explanation of equations 27 and 28. How do these work and how are they an improvement over Loehman et al. 1978. What is the term v_i? Please clarify these equations.

Agreed. We will add more explanation to highlight our contribution. In short, Loehman et al.'s formula (1978) is not directly applicable to bankruptcy problems as it would always selects the same rule (CEL) and would not be sensitive to changes in claims and allocation values. Therefore, in this paper we suggest replacing claim (C_i) in the formula with adjusted claim (υ_i), as defined by Equations 27 and 28.

To address your concern, this section has been fully revised. Please see lines 573-607.

Minor issues

Figures and tables need some improvements.

Table 1. Caption, put in parentheses 'acceptability' or something with a qualifier of what does the plurality index indicates. It just adds a sentence to the caption and makes the paper easier to read.

Table 2. Likewise please provide a better roadmap. Higher the value then means . . . Just add a sentence.

Figure 1, enlarge nodes and improve color scheme for black and white printing. In the caption perhaps discuss more what is going on.

Figure 2. Enlarge scale on the lower right. Perhaps overlay a schematic like the one in figure 1 indicating inflows, sinks, demand and supply. Improve color scheme for B& W printing.

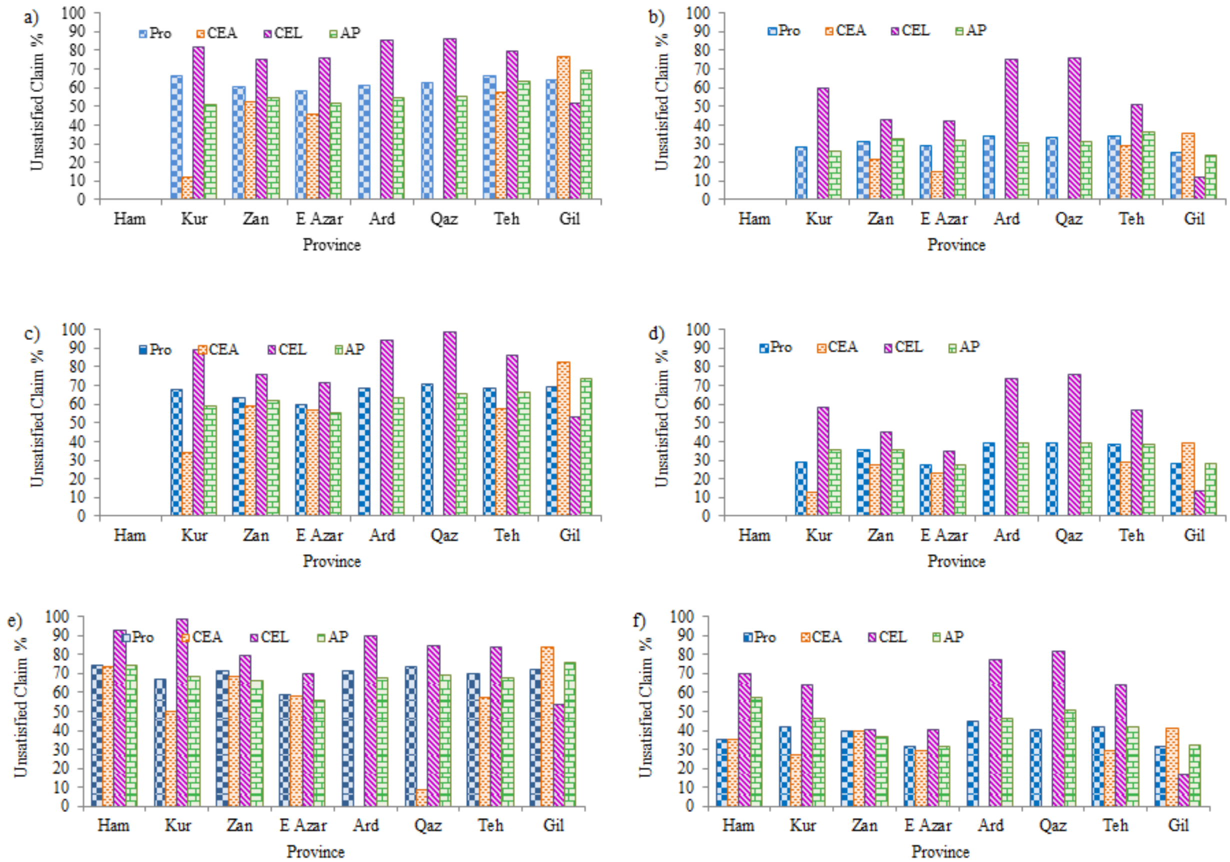
Figure 3. Enlarge fonts. Perhaps eliminate smoothing in lines since this isnot really a continuous but a discrete parameter.

Figure 4. Vertical axis, mention annual. Use different line types for the availability.

Figure 5. Eliminate smoothing in the monthly flows since the way the problem is solve is at a monthly time step, right? Enlarge fonts in legend. Vertical axis perhaps mention monthly (since these are flows and demands.

Figure 6. Hard to read, but is challenging to accommodate all this information. Perhaps of more value is to graph scarcity rather than allocations. That can tell better the story.

The table and figures will be revised as suggested. Thanks for your suggestions.
We tried your suggestion for Figure 6 (the figure below). We feel the original figure is more appropriate. So, we have kept it as it was. However, its caption was revised for clarification.



We appreciate your thoughtful comments!

References

Loehman, E., Orlando, J., Tschirhart, J., and Winstion, A. (1979): Cost allocation for a regional wastewater treatment system, Water Resources Research, 15(2), 193–202.

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Response to Referee 3:

This paper adapts bankruptcy methods to determine allocations of water in a transboundary river system when water supply is insufficient. I think that it makes a contribution, but it raises several issues that it does not address, and that what it does say it could say better. I recommend “Major Revision” in light of the comments below.

We would like to thank you for your valuable comments. Please find our responses below.

The Introduction points out that there are many international rivers in the world; nonetheless the main example, a river that lies entirely in Iran, is not one of them. In fact, there is no discussion of the conditions under which an interprovincial river (or, in the USA, an interstate river) is effectively equivalent to an international river.

A river system is considered to be transboundary as long as the river basin boundaries do not match the political boundaries. Therefore, as you correctly say, interprovincial or interstate rivers are effectively equivalent to an international river.

The revised version now clarifies this (please see lines 465-467). Thanks for your comment!

Another issue with the example is that it has two main branches (Fig. 2), but the theory (Fig. 1) assumes a linear river (with no tributaries, effectively), so it is not clear how the theory applies to the example.

Figure 1 is just a simple schematic of a transboundary river system. The formulation of the problem as a river network allows for solving bankruptcy problems for rivers irrespective of its shape, number of branches, number of beneficiaries, etc. The suggested optimization models are functional as long as mass balances are accurately defined, and nodes and links are correctly introduced in the model. So, their application is not limited to linear systems.

We have revised Figure 1. The new schematic river system has is non-linear. Also, explanation has been added to the text to clarify that the proposed model is applicable to non-linear systems. Please see lines 295-297, 334-340, and 636-637.

The authors claim that they are going to develop a new water allocation mechanism that (1) requires minimal utility information from the riparians, (2) is not limited to problems in which cooperation must result in quantifiable benefits, and (3) provides allocations that can vary temporally and spatially as flows vary. Do they achieve these objectives?

They do a good job on (1);

Thanks for the encouraging feedback.

(2) I don't understand;

The word "extra" has been dropped from the statement used by the reviewer. The exact statement in the paper is as follows:

"(2) its application is not limited to problems in which cooperation must result in **extra** quantifiable benefit."

The commonly used transboundary water allocation methods (e.g. cooperative game theory, social planner solution) rely on "incremental" benefit of cooperation. In other words, for these methods to be functional, cooperation among the parties must result in extra system's benefits and such benefits must be quantifiable. Although cooperation seems to be beneficial in most cases, quantitative valuation of incremental benefits is challenging and controversial, especially in transboundary systems where parties might not be willing to reveal their true utility functions. Therefore, one of the advantages of the suggested framework is that it does not require any information on the added value of cooperation.

and they can accomplish (3) by rerunning the analysis every time inputs or demands change, which is an easy solution but doesn't yield any insights.

We are afraid that this is not an accurate and fair observation.

The suggested modeling framework fully captures the spatial variability of flows and physical characteristics of the river system. This does not have anything to do with rerunning the analysis every time inputs and demands change. Direct allocation based on the original bankruptcy rules, which assume equal access to the resource and have been designed for one-shot games, would not have been possible without developing these models. So, we believe that new insights are produced and one can realize if the river network and flow variability allow for implementation of the original bankruptcy rules or not. In cases that implementation of original solutions is not possible (very likely in river systems) the methods help developing allocation solutions that are close to the original allocation solutions.

The presented example represents an unregulated system. In this case the problem was solved in different time steps separately. But, the introduced framework can also handle regulated systems in which water allocation can be done over the whole planning period by one run of the optimization model. Furthermore, additional constraints (e.g. minimum monthly allocation for environmental flows or maintaining hydropower head) can be added to the model, if needed.

These points have been clarified in the revised manuscript. Please see lines 170-204 and 341-359.

In fact, what they do is adapt several standard bankruptcy procedures to the river water allocation problem.

Yes. Indeed, this is the major contribution of the work and has not been a straightforward job. Developing optimization models which make classic bankruptcy methods applicable to river systems should be a major contribution. Please see lines 170-204 and 260-359.

A bankruptcy procedure produces an allocation of assets among claimants when the total of the claims exceeds the assets. Usually bankruptcy procedures are expressed in money, but here the assets and the claims are amounts of water. Because bankruptcy rules adjudicate claims only, they take no account of the importance (utility) any particular claim: All claims are of equal weight, and differ only in amount. Because of this property, objective (1) is automatically satisfied.

Agreed. Thanks for the encouraging conclusion.

I think that the authors understand bankruptcy methods, at least in a technical way, but there is an issue about applicability that they do not address. Bankruptcy methods have been applied to water resources previously (the Caspian Sea, for example), but the previous applications have all (quite rightly) involved parties in symmetric positions. For example, any riparian can draw water from a lake, or by withdrawing less water make a transfer to any other riparian, who can then withdraw more.

This is not the case for provinces (or states, or countries) along a river, as in Figure 1. For example, if $I_{3;t}$ happens to be very large, can province 3 transfer some of its excess water to province 1? Not easily, because water runs downhill, so the flow in province 2 must be non-negative. Perhaps this is not a practical issue, since in typical river basins the inflows are highest near the source. But it is an issue for the fairness of a bankruptcy method, which is not fair if all of the transfers must be downhill. This methodology cannot be expected to show the widely perceived strategic advantage that geography gives to upstream riparians over downstream.

We think this argument is general when it comes to transboundary water allocation and not unique to river bankruptcy problems. Indeed, the characteristics that the reviewer highlights (dissymmetric positions, unequal access, etc.) are what makes the tansnboundary water allocation problems interesting to study and very challenging to solve. The case the reviewer suggests is very far from reality. We are not sure if we know of any transboundary water conflict in the world in which the upstream party is pushing the downstream party to transfer water upstream! Indeed, the upstream party would have a very hard time to sell such an argument and it should not matter if such an unreasonable party does not find the solution fair as the base request seems to be very unfair itself! Nevertheless, we claim that even if such a case exists, our model can properly handle it.

The advantage of the proposed model is that it can capture the physical characteristics of the system. If an upstream nation requests water transfer from downstream, this should be done through a tunnel, channel, pipe, etc. In that case a new link can be added from the downstream riparian node to the upstream riparian node. As long as mass balance equations are correct in the model, the model is capable of solving the problem, regardless of the shape of the water networks and the direction of flows. It is true that water flows in only one direction through each link and that flows should be non-negative, but there is no restrictions on the number of links between two given nodes. Therefore, if the case that the reviewer suggests exists a backward link can be added that links node 3 to node 1 to ensure that party 1 has access to the water produced in the area under the control of party 3.

The revised manuscript explicitly states that the proposed models can be applied to any river network, irrespective of its shape. Please see lines 295-297, 334-340, and 636-637.

In a technical sense, the authors could address the problem by a condition like $O_{i;t}$ >=0. The fact that they don't do this suggests that they haven't thought of this possibility, which in practice is admittedly unlikely to be feasible.

As discussed, the suggested case can be easily solved by adding links between nodes. In reality, water transfer from downstream to upstream is not going to happen through the main river channel. So, there is a need for using a pipe, channel, aqueduct, etc. to transfer water. Similarly, a new link can be added to the model to make sure the water network is accurately represented.

While we agree with you that we have not thought about this possibility as such a situation is highly unlikely emerge in practice, we believe a network model like ours can handle the suggested case with no problem.

Please see lines 295-297, 334-340, and 636-637.

But the possibility of any transfer is essential to the fairness of bankruptcy allocations, which have little to recommend them if they cannot be seen as "fair." This point seems to make conclusions about fairness fundamentally flawed.

We do not agree with the reviewer on the essence of the water transfer possibility from downstream to upstream to develop 'fair' solutions to transboundary water allocation problems. In our opinion, it is not reasonable for an upstream riparian to ask for more than the amount of water that is produced in its boundaries. However, if such situation exists, the model is still capable of developing solutions.

The description of an allocation rule, such as **3.1**, would be easier to understand if it were clarified what are the data and what is specific to the method. I think that the data are the initial conditions (at $t = 0$) and the inflows, $In_{i,t}$.

Yes. The input data are the initial conditions, inflows and claims. The rest are decision variables, determined by the optimization models.

As suggested, the revised manuscript now identifies the optimization decision variables after mathematical presentation of each optimization model.

As the authors point out, (2)–(9) are common to all rules, so the presentations would be simpler if they appeared only once.

Thanks for the suggestion. We have followed your suggestion in the revised manuscript. Please see lines 298-334.

The calculation of $\lambda_{P_{i,t}}$ is clear, but it's not clear how to calculate λ_{P_t}, since (11) is only a constraint, and all λ's are positive.

We are not sure if we can follow the comment. All λ's are decision variables and will be determined by the optimization model. Constraint 11 sets λ_{P_t} equal to the maximum $\lambda_{P_{i,t}}$.

The revised manuscript explains the relations between different cutback proportions. Pleas see lines 371-378.

This paper adds some material on stability, but it does not make much of a contribution. The "plurality index" amounts to the result when provinces "vote" for their most-preferred method. As any Social Choice text will demonstrate, plurality voting is a poor way to carry out a group decision. (There is no agreement on which way is best, but there is general agreement that plurality is the worst of the methods in common use.)

We agree with the reviewer that plurality rule is not the best social choice rule. However, the literature has referred to this rule as the easiest method for finding the most popular allocation rule (e.g. Dinar and Howitt (1997)). Also, despite its limitations, the plurality rule is commonly used in most elections, reflecting its popularity in practice.

Here, we first used this method to find the most popular method. Considering the limitations of this method (as discussed by the reviewer), we introduced a more sophisticated stability assessment method to evaluate the acceptability of the popular solution. Our finding that the most popular method might not be the most stable method based on the introduced stability index is a good proof for our agreement with the reviewer's argument that the social choice might not be always feasible.

Please see lines 563-602.

The power index introduced in (25) is also rather simple (assuming that X_i and x_i are intended to be the same), as the denominator on the right side of (25) is the same for all values of i, so player i's power is simply the amount by which i's allocation falls short of i's claim. Minimizing the variability of the stability (actually, instability) measure S_α

can then be understood as favoring allocations that are, as nearly as possible, equally far below the corresponding claims. It is therefore hardly surprising that the CEL (Constrained Equal Losses) rule comes out best according to this index.

We are afraid that there is a misunderstanding here. As discussed in the paper, the original index (originally Equation 25 and now Equation 26) is not applicable to the problem. Therefore, we modified this index and introduced Equations 28-33 which define the power index for bankruptcy problems.

Based on the proposed equations the reviewer's argument is not valid. The simple proof is that CEL is the least stable allocation scheme under dry conditions! Our findings clearly show that the stability of the solution is highly sensitive to the levels of claim, water availability, and feasible allocations.

We have revised this section to further clarify the differences between our proposed stability index and the one which is commonly used in the cooperative game theory literature. Please see lines 591-602.

We thank you again for your constructive comments.

References

Dinar, A. and R.E. Howitt (1997) Mechanisms for allocation of environmental control cost: empirical tests of acceptability and stability, J. Environmental Management, 49(2), 183–203.