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prediction of  
flow-duration  
curves**

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# Geostatistical prediction of flow-duration curves

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## Abstract

We present in this study an adaptation of Topological kriging (or Top-kriging), which makes the geostatistical procedure capable of predicting flow-duration curves (FDCs) in ungauged catchments. Previous applications of Top-kriging mainly focused on the prediction of point streamflow indices (e.g. flood quantiles, low-flow indices, etc.). In this study Top-kriging is used to predict FDCs in ungauged sites as a weighted average of standardised empirical FDCs through the traditional linear-weighting scheme of kriging methods. Our study focuses on the prediction of period-of-record FDCs for 18 unregulated catchments located in Central Italy, for which daily streamflow series with length from 5 to 40 yr are available, together with information on climate referring to the same time-span of each daily streamflow sequence. Empirical FDCs are standardised by a reference streamflow value (i.e. mean annual flow, or mean annual precipitation times the catchment drainage area) and the overall deviation of the curves from this reference value is then used for expressing the hydrological similarity between catchments and for deriving the geostatistical weights. We performed an extensive leave-one-out cross-validation to quantify the accuracy of the proposed technique, and to compare it to traditional regionalisation models that were recently developed for the same study region. The cross-validation points out that Top-kriging is a reliable approach for predicting FDCs, which can significantly outperform traditional regional models in ungauged basins.

## 1 Introduction

An empirical Flow Duration Curve (FDC) graphically represents the percentage of time (or duration) in which the streamflow can be equalled or exceeded over a historical period of time (see e.g. Vogel and Fennessey, 1994). Empirical FDCs are often used to represent the streamflow regime of a given catchment when an adequate number of streamflow observations are available. A deterministic hydrologist would probably

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refer to an FDC as a key signature of the hydrological behaviour of a given basin, as it results from the interplay of climate, size, morphology, and permeability of the basin; a statistical hydrologist would refer to an FDC as the exceedance probability, or equivalently the complement to the probability distribution function (cdf) of streamflows (see e.g. Castellarin et al., 2013).

Because of their ability to provide a simple yet comprehensive graphical view of the overall historical variability of streamflows in a river basin, from floods to low-flows, and their peculiarity of being readily understandable by those who do not have a strong hydrological background, empirical FDCs are routinely used in several water-related studies and engineering applications such as hydropower generation, design of water supply systems, irrigation planning and management, wasteload allocation, sedimentation studies, habitat suitability, etc. (see e.g. Vogel and Fennessey, 1995).

The literature reports two different representations of empirical flow-duration curves, depending on the reference period of time (see Vogel and Fennessey, 1994): (i) period-of-record flow duration curves (POR-FDCs), constructed on the basis of the entire observation period and (ii) annual flow duration curves (AFDCs), constructed year-wise. The two representations are complementary to each other and should be selected by practitioners depending on the water problem at hand (Castellarin et al., 2004b). For instance, AFDCs are useful for quantifying the streamflow regime in a typical hydrological year, or in a particularly wet or dry year (see Vogel and Fennessey, 1994); POR-FDCs are a steady-state representation of the long-term streamflow regime and can be effectively used, for instance, for patching and extending streamflow data (Hughes and Smakhtin, 1996).

In many practical applications one has to predict FDCs at ungauged catchments or catchments for which the available hydrometric information is sparse (see e.g. Castellarin et al., 2013). This task is often addressed by developing regional models of FDCs. The scientific literature proposes several of such models that adopt different approaches to the problem: some model regard the curves as the exceedance probability function of streamflows and regionalise the parameters of theoretical frequency



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distributions (see Fennessey and Vogel, 1990; LeBoutillier and Waylen, 1993; Castellarin et al., 2007; Mendicino and Senatore, 2013); similarly, some other adopt a suitable mathematical expression for representing the curves and regionalise the expression parameters (Franchini and Suppo, 1996; Mendicino and Senatore, 2013); finally, some other do not make any attempt mathematically represent the curves, they rather standardise empirical curves constructed for gauged catchments that are hydrologically similar to the target site (i.e. characterised by a similar physiographic, pedologic and climatic conditions, also referred to as donor sites, see e.g. Kjeldsen et al., 2000) by an index streamflow (e.g. mean annual flow), and then average the dimensionless curves to predict the standardised FDC for the study catchment. The averaging procedure may (see e.g. Ganora et al., 2009), or may not (see e.g. Smakhtin et al., 1997), adopt a weighting scheme, which gives more importance to donor sites that are more hydrologically similar to the target site. The literature commonly groups these regionalisation procedures into parametric (i.e. procedures that parameterise FDCs and then regionalise parameters, like the first two examples) and non-parametric procedures (i.e. procedure that dispense with a parameterisation of the curves, like the third example, see e.g. Castellarin et al., 2004a, 2013).

It is a common argument that an accurate representation of FDCs for daily streamflows requires probabilistic models (or mathematical expressions) with four or more parameters (LeBoutillier and Waylen, 1993; Castellarin et al., 2007), which control the position, scale and shape of the distribution. This hampers the construction of reliable regional models, due to the large uncertainty that is commonly associated with regional relationships that express the shape parameters in terms of physiographic and climatic catchment descriptors (see Castellarin et al., 2007). As a result, classical approaches to FDCs regionalisation based on averaging standardised curves constructed for neighbouring gauged sites (Smakhtin et al., 1997) have been recently revisited through a mathematical model that enables the user to quantify the dissimilarity between empirical FDCs and associate this dissimilarity with a distance in the multidimensional space of catchment descriptors (Ganora et al., 2009). An innovative feature of this approach

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is the possibility to weight each empirical FDC according to the distance between each gauged basin and the target site in the space of catchment descriptors, therefore accounting for the hydrological similarity of the donor sites with the site of interest. Like many of the traditional approaches proposed in the literature, though, the approach proposed in Ganora et al. (2009) (1) requires a preliminary subdivision of the study area into homogeneous pooling-groups of sites (i.e. clustering), (2) predicts a standardised (i.e. dimensionless) FDC for the target site, which needs then to be multiplied by a dimensional scale index (e.g. an indirect estimate of mean annual streamflow) in order to be of practical use. Both steps are critical phases of a regionalisation process. In particular concerning step (1), geostatistical regionalisation approaches have been shown to be particularly effective in dispensing with the preliminary identification of homogeneous pooling-group of sites while using regional hydrological information for predicting streamflow indices in ungauged catchments (e.g. flood quantiles, low-flow-indices, etc.: see e.g. Chokmani and Ouarda, 2004; Skøien et al., 2006; Castiglioni et al., 2009, 2011; Archfield et al., 2013; Laaha et al., 2013); yet no geostatistical procedure has been developed that specifically addresses the problem of FDC regionalisation.

Our paper focuses on the derivation of a geostatistical technique that addresses both limitations mentioned above for the prediction of FDCs in ungauged sites. We adopt Topological kriging or Top-kriging, which is a block-kriging with variable support area that interpolates streamflow-indices along stream networks (see e.g. Skøien et al., 2006). Top-kriging has been proved to be particularly successful in predicting point streamflow values (e.g. low-flow and flood quantiles, mean annual flood, stream temperatures, etc.) in various geographical and climatic contexts without a delineation of homogeneous regions (see e.g. Merz et al., 2008; Castiglioni et al., 2011; Vormoor et al., 2011; Archfield et al., 2013; Laaha et al., 2013).

We adopt Top-kriging as the core tool for predicting standardised (i.e. divided by mean annual flow) and dimensional long-term daily FDCs on the basis of empirical period-of-record curves (POR-FDCs, hereafter referred to as FDCs for the sake of brevity) constructed for neighbouring streamgauges.

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The idea behind our study is (i) to identify a meaningful empirical point value (or index) that fully characterises the whole empirical FDC, (ii) to model the spatial correlation structure, or the spatial variability, of this point index over the study region through Top-kriging and (iii) to use this very spatial correlation model to predict FDCs in ungauged sites by weighting neighbouring empirical FDCs. We present two possible applications of the proposed procedure, the first one predicts standardised FDCs, that is FDCs divided by Mean Annual Flow (MAF), the second one predicts FDCs divided by the product between Mean Annual Precipitation and drainage area. MAP is generally easier to predict than MAF in ungauged sites, due to the higher density of raingauging networks relative to streamgauging ones. The second application can therefore be used to obtain a prediction of the dimensional FDCs for the target size.

The approach is developed and tested through a comprehensive leave-one-out cross-validation procedure for a rather wide geographical region located in Eastern-Central Italy including 18 unregulated river basins. Castellarin et al. (2007) propose regional models of long-term daily FDCs for this area, which we use in this study as benchmark models for comparing the accuracy and reliability of the proposed approach.

## 2 Geostatistical hydrological prediction in ungauged sites

### 2.1 Top-kriging

Top-kriging is a powerful geostatistical procedure proposed by Skøien et al. (2006) which performs hydrological predictions at ungauged sites along stream-networks on the basis of the empirical information collected at neighbouring gauging stations. As kriging techniques, the spatial interpolation is obtained in Top-kriging by a linear combination of the empirical values; therefore, the unknown value of the streamflow index of interest at prediction location  $x_0$ ,  $\hat{Z}(x_0)$ , can be estimated as a weighted average of

the variable measured in the neighborhood:

$$\hat{Z}(x_0) = \sum_{i=1}^n \lambda_i Z(x_i) \quad (1)$$

where  $\lambda_i$  is the kriging weight for the empirical value  $Z(x_i)$  at location  $x_i$ , and  $n$  is the number of neighbouring stations used for interpolation. Kriging weights  $\lambda_i$  can be found by solving the typical ordinary kriging linear system 2, with the constrain of unbiased estimation 2b:

$$\sum_{j=1}^n \gamma_{i,j} \lambda_j + \theta = \gamma_{0,i} \quad i = 1, \dots, n \quad (2a)$$

$$\sum_{j=1}^n \lambda_j = 1 \quad (2b)$$

where  $\theta$  is the Lagrange parameter and  $\gamma_{i,j}$  is the semi-variance between catchment  $i$  and  $j$ . The semi-variance is also referred to as variogram in geostatistics and represents the space variability of the regionalised variable  $Z$ . A peculiar feature of Top-kriging is to consider the variable defined over a non-zero support  $S$  (i.e. the catchment drainage area) (Cressie, 1993; Skøien et al., 2006); this implies that the kriging system 2 remains the same, but the gamma values between the measurements need to be obtained by regularization, that is the smoothing effect of support area  $S$  on the point variogram, which is computed by applying an integral average of the variable  $Z$  over  $S$ . After this, the point variogram can be back-calculated by fitting aggregated variogram values to the sample variogram (details can be found in Skøien et al., 2006).

## 2.2 Total negative deviation (TND)

Top-kriging could in principle be directly applied to interpolate single streamflow values associated with a given duration (i.e. streamflow quantiles). Therefore, similarly

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standardised (i.e. divided by mean annual flow) FDCs: two hydrologically similar catchments will show similar standardised curves, hence a small area between the curves, whereby two basins that are with completely different in terms of hydrological behaviour will be characterised by highly different FDCs, and therefore the area between the curves will be high. Following this background idea, we propose to summarise the entire FDC through a point index which we term Total Negative Deviation (TND) between a dimensionless (i.e. standardised by a reference streamflow value) FDC and one,

$$\text{TND} = \sum_{i=1}^m |q_i - 1| \Delta_i \quad (3)$$

where  $q_i$  represents the  $i$ th empirical dimensionless streamflow value,  $\Delta_i$  is half of the frequency interval between the  $i + 1$ th and  $i - 1$ th streamflow values, and the summation includes only  $i = 1, \dots, m$  dimensionless streamflow values that are lower than 1 (i.e. negative deviation).

Empirical TND values are proportional to the filled areas in Fig. 1, where black thick curves represent the empirical FDCs. More specifically, Fig. 1 represents the dimensionless empirical FDCs that were constructed for three streamgauges by using two standardisation methods: in one case the curve is standardised by the mean annual flow (standardisation by MAF,  $\text{TND}_1$ , top panels of Fig. 1); in the other case the curve is standardised by  $\text{MAP}^*$ , that is a reference streamflow equal to the catchment area  $A$  times the mean annual precipitation (standardisation by  $\text{MAP}^*$ ,  $\text{TND}_2$ , bottom panels in Fig. 1) (see details on standardisation procedure in Sect. 3.2).

TND defined by Eq. (3) and illustrated in Fig. 1 is very informative on the shape of the FDC, which, in turn, is controlled by climatic, physiographic and geo-pedological characteristics of the catchment. Distinctions between the dominant hydrological functions in different seasons within the same catchment can be highlighted by TND. Catchments that are dominated by rapidly responding near-surface runoff processes have steeper FDC slopes, and therefore larger TND, while FDCs are less steep where slower responding runoff generation processes prevail, and under these circumstances TND

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will be smaller. This is related to functional similarity: catchments that store and retain more water should have smaller TND. The magnitude of TND is related not only to the climate but also to how efficiently the catchment partitions water into runoff.

### 3 Top-kriging of flow-duration curves

#### 3.1 Construction of empirical FDCs

The construction of empirical FDCs for gauged sites is straightforward: (i) pooling all observed streamflows in one sample, (ii) ranking the observed streamflows in ascending order and (iii) plotting each ordered observation vs. its corresponding duration, which is usually dimensionless. The duration of the  $i$ th observation in the ordered sample is equal to an estimate of the exceedance probability of the observation,  $1 - F_i$ . If  $F_i$  is estimated using a Weibull plotting position, the duration  $d_i$  is,

$$d_i = \text{Prob}\{Q > q_i\} = 1 - \frac{i}{N + 1} \quad (4)$$

where  $N$  is the length of daily streamflows observed in a gauged site and  $i = 1, \dots, N$  is the  $i$ th position in the rearranged sample.

A common representation of FDCs reports log-flows on the y-axis and the duration on the x-axis (see Fig. 1). Another common representation adopts a log-normal space instead, in which log-transformation of streamflows are still reported on the y-axis, while the x-axis reports duration expressed as a normal standard variate  $z$ ,

$$z_i = \Phi^{-1}(1 - d_i) \quad (5)$$

where  $\Phi$  is the cdf of the standard normal distribution. The combination of the two transformations improves significantly the readability of the FDC (see Fig. 2), the log-transformation enhances the representation of observed streamflows, which usually

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spans over two or more orders of magnitude, while expressing the duration as a standard normal variate improves the visualization of small and large durations, that is flood- and low-flows, respectively.

### 3.2 Computation of empirical TND values

5 According to what anticipated in Sect. 2.2, two different standardisation procedures are considered for computing TND values:

#### TND<sub>1</sub>

TND values are computed after standardisation by Mean Annual Flow (MAF), that is the traditional way to standardise FDCs.

#### 10 TND<sub>2</sub>

TND values are computed for FDCs that are standardised by a rescaled Mean Annual Precipitation (MAP\*). The standardisation is performed by dividing each streamflow value by the empirical catchment-scale MAP value, rescaled to basin size as,

$$\text{MAP}^* = \text{MAP} \cdot A \cdot \text{CF} \quad (6)$$

15 where  $A$  is the catchment area and  $\text{CF}$  is a unit-conversion factor (e.g. if streamflows are in  $\text{m}^3 \text{s}^{-1}$ ,  $\text{MAP}$  in  $\text{mm}$  and  $A$  in  $\text{km}^2$ , then  $\text{CF} = 3.171 \times 10^{-5}$ ). Once the dimensionless FDC is predicted for an ungauged site, then a dimensional FDC can be obtained by multiplying the curve by a local catchment-scale estimate of  $\text{MAP}^*$ .

20 The idea behind the choice of two different standardisations of FDCs derives from two different purposes: ( $\text{TND}_1$ ) MAF standardisation is the traditional choice when an index-flow regionalisation approach, with MAF being the index-flow), is used to regionalise FDCs (see Castellarin et al., 2004b; Ganora et al., 2009). Such an approach, as already mentioned, needs then an appropriate regional model for predicting the

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index-flow in ungauged basins (e.g. a multiregression model) in fact, once a standardised FDC is predicted for an ungauged site, then a dimensional FDC can be obtained by multiplying the dimensionless curve by an estimate of MAF for the site of interest, which is a critical and delicate step in the regionalisation procedure (see e.g. Brath et al., 2001; Castellarin et al., 2004a); (TND<sub>2</sub>) MAP\* standardisation enables one to derive dimensionless FDCs to be used for regionalisation, and to predict a dimensional curve, which is ultimately what practitioners really need for addressing the water problem at hand, simply by multiplying the dimensionless FDC by MAP and catchment area. The discriminant between the two ways resides in the fact that the uncertainty associated with predictions of MAP is generally significantly smaller than the uncertainty associated with predictions of MAF for ungauged sites, in virtue of the large availability of raingauges and the accuracy of geostatistical procedure for interpolating point observations (see e.g. Brath et al., 2003; Castellarin et al., 2004a).

Concerning the practical computation of TND values, that is TND<sub>1</sub> or TND<sub>2</sub> empirical values, the record length generally varies among the available streamgauges. Therefore, before applying 3 one needs to set a maximum duration  $d_{\max}$  that can be used in order to compute the TND values consistently for all sites in the region.  $d_{\max}$  can be set according to the minimum record length in the region (e.g. if the minimum record length in the region is 5 yr, one should set  $d_{\max} = (5 \times 365)/(5 \times 365 + 1)$ ).

Once a suitable reference streamflow is selected for performing the standardisation of the curves (i.e. MAF or MAP\*), one can easily identify the number of durations  $m$  for which the empirical dimensionless streamflow values are lower than 1 (i.e. streamflow values lower than MAF or MAP\*) and compute TND according to 3. For instance, once computed the standard-normal duration  $z_i$  associated with each standardised streamflow quantile  $q_i$ ,  $\Delta_i$  in 3 can be computed as,

$$\Delta_i = 0.5(z_{i+1} - z_{i-1}) \quad \text{for } i < m \quad (7a)$$

$$\Delta_i = 0.5(z_i - z_{i-1}) \quad \text{for } i = m \quad (7b)$$

### 3.3 Geostatistical interpolation of TND and FDCs

Empirical TND (i.e.  $TND_1$  and  $TND_2$ ) values are site specific and can be interpolated with geostatistical techniques. Top-kriging can be applied as illustrated in the stepwise description by Skøien (2013) through the suite of R-functions included in the R-package `r_top`, which can be accessed from the Comprehensive R Archive Network (CRAN, <http://cran.r-project.org/>). The application of Top-kriging formally requires exactly the same steps in both cases (i.e. for empirical  $TND_1$  and  $TND_2$  values). For the sake of brevity, we will recall these steps by referring to the set of empirical  $TND_1$  values only.

The point sample variogram for each standardisation (see Sect. 3.2) can be computed using the binned variogram technique, for which sample points are aggregated in distance classes or bins, under the hypothesis of isotropy, i.e. the variogram does not vary with direction. The sample variogram can then be modelled through a suitable function (e.g. exponential, Gaussian, spherical, fractal, etc.) among the available theoretical models. Skøien et al. (2006) recommend the use of the exponential variogram.

Once the empirical variogram is modelled, the number  $n$  of neighbouring stations on which to base the spatial interpolation is set iteratively by the user on the basis of a first set of preliminary analyses, which aim at identifying the  $n$  value that produces the most accurate predictions in cross-validation (i.e. for predicting TND values in ungauged locations). This means that the local prediction of TND values, i.e. the computation of ordinary linear system in 2, depends on  $n$ -dimensional kriging weights. We assume in our study that the  $n$  kriging weights that are computed for predicting TND in ungauged locations can also be adopted for predicting the flow-duration curve in the same locations as a weighted average of  $n$  standardised empirical curves as,

$$\hat{\psi}(x_0, d) = \sum_{i=1}^n \lambda_i \psi(x_i, d) \quad d \in (0, 1) \quad (8)$$

where  $\lambda_i$  are the Top-kriging weights resulting from TND interpolation,  $\psi(x_i, d)$  indicates the standardised empirical FDC for site  $x_i$ , that is a flow-duration curve in which

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streamflow quantiles are divided either by MAF or by MAP\*,  $\hat{\psi}(x_0, d)$  stands for the standardised FDC predicted for site  $x_0$  and the entire duration domain  $d$ ,  $n$  is the number of neighbouring sites in the vicinity of the site of interest. We will assess this assumption relative to a study area which was extensively analysed in previous studies in the context of regionalisation of FDCs (see e.g. Castellarin et al., 2004a, 2007).

#### 4 Study area and data

The study region includes 18 unregulated catchments, which previous studies describe as a rather heterogeneous group of sites in terms of physiographic and climatic characteristics (see e.g. Castellarin et al., 2007, 2004a). Daily streamflow series were obtained for all basins from the streamgauges belonging to the former National Hydrographic Service of Italy (SIMN) over the time period 1920–2000. The length of the observed series ranges from 5 to 40 yr (average record length: 18 yr). Also, the empirical MAP value relative to each one of the 18 catchments were estimated using data collected from a rather dense raingauge network during the same time-interval of daily streamflow observations.

Empirical FDCs were constructed from the daily streamflow series for the 18 catchments as described in Sect. 3.1. Empirical TND<sub>1</sub> and TND<sub>2</sub> values were computed for each catchment according to standardisations described in Sect. 3.2, and are illustrated in the two maps of Fig. 3. As shown in the left panel of Fig. 3, empirical TND<sub>1</sub> values increase moving from south-east to north-west. This outcome reflects the lower perviousness of the northern catchments, which are then less capable of storing water volumes and consequently are characterised by steeper empirical FDCs. Figure 3 (right) illustrates empirical TND<sub>2</sub> values obtained for the study catchments. Moving from south-east to north-west, one can note for TND<sub>2</sub> similar patterns to those observed for TND<sub>1</sub> values, i.e. TND values tend to increase along the SE–NW direction. On the one hand this general behaviour suggests that in our case study Mean Annual Flow (MAF) is largely controlled by precipitation, on the other hand, karst phenomena

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associated with the presence of fractured limestones result in an increase of  $TND_2$  for the Southern catchments, i.e. sites 3006, 3003 and 3002, for which subsurface flows play a significant role.

Table 1 illustrates the variability over the study region of catchment area  $A$  ( $km^2$ ), mean annual flow  $MAF$  ( $m^3 s^{-1}$ ), mean annual precipitation  $MAP$  (mm),  $MAP^*$  ( $m^3 s^{-1}$ ), empirical  $TND_1$  (–) and  $TND_2$  (–) values, by reporting the minimum, mean and maximum values, together with the 1th, 2nd and 3rd quartiles of each index.

## 5 Analysis and results

### 5.1 Prediction of FDCs in cross-validation

We will refer to the proposed approach as  $TNDTK$  (i.e. Total Negative Deviation Top Kriging) in the remainder of the paper. This section illustrates in detail the application of  $TNDTK$  in cross-validation, describing the accuracy of the procedure when applied in ungauged basins.

#### 5.1.1 Standardisation by $MAF$

The application of  $TNDTK$  to the prediction of FDCs standardised by  $MAF$  requires the preliminary application of Top-kriging to  $TND_1$  values, which we performed by calculating binned sample variogram first, and then by modelling binned empirical data with a 4-parameter exponential theoretical variogram (see details in Skøien et al., 2006). The four parameters were fitted through the Weighted Least Squares (WLS) regression method from Cressie (1985). Top-kriging was then iteratively applied to the study catchments in cross-validation to identify the most suitable number of neighbours  $n$ . Preliminary iterations indicated  $n = 6$  as the optimal number of gauging stations.

We then used the kriging weights obtained for predicting  $TND_1$  in cross-validation at each and every site to estimate dimensionless FDCs. We resampled each curve using  $p = 20$  points equally spaced in the log-normal representation (see Sect. 2.2

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and Fig. 2), choosing  $d_1 = 0.00135$  as lower bound and  $d_{20} = 0.9986$  as the upper one ( $d_1$  and  $d_{20}$  values were selected by referring to the minimum record length in the regional sample, i.e. 5 yr). Predictions were performed through a weighted average, as expressed in Eq. (8), using the optimal Top-kriging cross-validation weighting scheme, i.e.  $\lambda_i$  with  $i = 1, \dots, n$ , where  $n = 6$ .

As anticipated, a comprehensive leave-one-out cross-validation procedure (LOOCV) was performed in order to simulate ungauged conditions at each and every gauged site in the study area and to quantitatively test the reliability and robustness of TNDTK for predicting FDCs in ungauged basins.

The LOOCV that can be summarised by the following steps:

1. empirical and theoretical variograms are computed using the entire dataset of  $TND_1$  values;
2. one of the gauging station, say  $s_j$ , is removed from the set of available stations;
3. a Top-kriging regional model for predicting  $TND_1$  values is developed using the remaining  $N_{\text{site}} - 1$  sites;
4.  $TND_1$  is predicted for site  $s_j$  by referring to  $n = 6$  neighbouring stations (see e.g. Fig. 4);
5. the weighting scheme computed in step 4 is then used to predict a standardised FDC for site  $s_j$  through Eq. (8);
6. steps from 2 to 5 are repeated  $N_{\text{site}} - 1$  times.

The accuracy of the cross-validated standardised FDCs was scrupulously assessed by means of several performance indices and diagrams, which are illustrated in detail in Sect. 5.3. The algorithm described above is tailored for the proposed procedure, TNDTK, but one can implement and apply similar resampling procedures to any regional model for simulating ungauged conditions. The technicalities of each procedure necessarily reflect the particular regional model being considered, but the rationale is

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the same: (i) drop all of the hydrometric information collected at a given streamgauge,  $s_i$ ; (ii) identify the regional model; (iii) use the regional model to predict the FDC at site  $s_i$ ; (iv) repeat steps (i)–(iii) by considering in turn each one of the remaining ( $N_{\text{site}} - 1$ ) sites (see Castellarin et al., 2007, and references therein for further details).

### 5.1.2 Standardisation by MAP\*

Top-kriging was applied also to predict empirical  $\text{TND}_2$  values as well as FDCs standardised by MAP\*. The number of neighbouring stations  $n$ , theoretical variogram, and fitting procedure were the same as for standardisation based on MAF. Also in this case each standardised FDC was resampled on 20 equally-spaced points in the log-normal representation, adopting the interval  $[d_1, d_{20}]$ . We used and LOOCV analogous to the one described above (i.e. standardisation by MAF) in order to identify the weighting scheme to be used for simulating ungauged conditions for all of the study basins.

Furthermore, in order to obtain dimensional prediction, each estimated curve  $\hat{\psi}(x_0, d)$  was than transformed into a dimensional FDC, as

$$\hat{\Psi}(x_0, d) = \hat{\psi}(x_0, d)\text{MAP}^*(x_0) \quad \text{with } d \in [d_1, d_{20}] \quad (9)$$

where  $\text{MAP}^*(x_0)$  indicates the local  $\text{MAP}^*$  value.

## 5.2 Reference regional models of FDCs

The same gauged stations and data considered herein were analysed in previous studies that developed regional models of FDCs (see Castellarin et al., 2004a, 2007). This enabled us to identify for both TNDTK applications two different reference regional models for comparing the performance of the approaches. We report here-below a brief description of such regional models.

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## 5.2.1 Standardisation by MAF

TNDDK predictions of dimensionless FDCs were compared against the dimensionless curves predicted by two reference regional models, which we also applied in cross-validation through a LOOCV procedure:

### 5 KMOD

K model (or KMOD) is a statistical regionalisation model developed by Castellarin et al. (2007) that uses the 4-parameter unit-mean kappa distribution as parent distribution for representing standardised FDCs (see e.g. Hosking and Wallis, 1997). Three parameters, namely the parameter of location and the two shape parameters, were estimated by applying an ordinary least squares (OLS) regression algorithm. The scale parameter is derived as a function of the previous three under the hypothesis that the mean of the distribution is equal to one. Castellarin et al. (2007) regressed the parameters estimates against a suitable set of catchment descriptors through a stepwise-regression procedure in order to enable the estimation of the kappa distribution in ungauged sites. KMOD is therefore a traditional parametric regional model which we adopted as the benchmark regional model for predicting standardised FDCs (see for details Castellarin et al., 2007).

### MEAN

MEAN is a simple approach to regionalisation, which neglects the physiographic and climatic heterogeneities of the study area, and predicts the standardised FDC for any ungauged site in the region as the average of all available standardised FDCs. We adopted MEAN as a baseline model due to its crude assumption and the resulting low-level accuracy.

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## 5.2.2 Standardisation by MAP\*

TNDTK predictions of dimensional FDC were compared with the predictions resulting from two benchmark models, both applied in cross-validation:

### LLK

5 This model, based on an index-flow approach (see Castellarin et al., 2004b), adopts a two-parameter log-logistic (LL) distribution as a suitable distribution for describing the empirical frequency of the annual flow series (i.e. index-flow) and a four-parameter kappa (K) as the parent distribution for dimensionless daily streamflow frequency. Parameters of both distribution were estimated using the routine based on  
10 L-moments developed by Hosking and Wallis (see Hosking and Wallis, 1997), re-estimated through a constrained sequential quadratic programming optimisation procedure aimed at minimising the squared differences between theoretical and empirical nonexceedence probabilities, and then regressed against a suitable set of catchment descriptors through a stepwise-regression procedure. More details can be found in  
15 Castellarin et al. (2007).

### KMOD

Same as KMOD for dimensionless FDCs prediction, but using a multiregression regional model to predict MAF as a function of a suitable set of catchment descriptors in ungauged basins (see e.g. Castellarin et al., 2007 for details).

## 5.3 Performance indices

20 TNDTK performance in cross-validation is analysed for both standardisation methods (MAF and MAP\*) and compared with the results of reference regional models through several performance indices and diagrams. A deep analysis of model performances in terms of relative prediction residuals, i.e. relative errors between modelled and empirical

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values (with sign), is presented through error-duration curves. The curves show relative residuals against duration arranged in gray nested bands containing 50, 80 and 90 % of relative residuals, respectively, while a line illustrates the progression with duration of the median residual (BIAS). Also, we use as performance descriptors scatterdiagrams between cross-validated and empirical streamflow quantiles associated with the same duration. On the basis of the same information, NSE (Nash & Sutcliffe Efficiency) indices for each model are computed, both for natural and in log-transformed streamflows. Such diagrams and indices enhance the overall residual distribution, from low durations (high-flows and floods) to high ones (droughts), at a regional scale.

Concerning the performances of the model at each site, and in particular the assessment of the number of sites for which TNDTK is more reliable than the selected reference regional models, we adopt a comprehensive error index derived from the distance between predicted and empirical FDCs proposed in Ganora et al. (2009):

$$\delta_{\text{mod}} = \sum_{k=1}^p |q_{k,\text{emp}} - \hat{q}_{k,\text{mod}}| \quad (10)$$

where  $p = 20$  resampled points, while  $q_{k,\text{emp}}$  and  $\hat{q}_{k,\text{mod}}$  stand for the empirical and predicted streamflow quantiles (dimensionless or dimensional, depending on the application) ranked at the  $k$ th duration.

## 5.4 Results

### 5.4.1 Standardisation by MAF-dimensionless FDCs

Figure 4 (left) reports empirical  $\text{TND}_1$  values against their predictions in cross-validation. The overall NSE is 0.81. In the same figure one can observe a poor prediction (i.e. significant underprediction) for site 3701, which can be interpreted as a result of the very high empirical TND value obtained for that site (site 3701,  $\text{TND}_1 = 9.8[-]$ ,  $A = 605[\text{km}^2]$ ), the largest in the study region.

TNDTK predictions of standardised FDCs show excellent results from flood (low duration) to low flows (high duration) for the set of global performance indices and diagrams.

The error-duration curves of Fig. 5 clearly shows that TNDTK significantly outperforms KMOD and MEAN: the distribution of relative residuals plotted against duration is characterised by narrower bands (50, 80 and 90 % of the relative errors) for the entire duration interval, even though this behaviour is more marked for low than for high durations. The progression with duration of the median residual (black thick line) in the same figure highlights unbiasedness being close to zero for the entire duration interval. Scatterdiagrams between predicted and observed standardised flows indicate high accuracy of TNDTK, with  $NSE = 0.958$  and  $LNSE \simeq 0.96$ , the latter computed for log-flows. MEAN and KMOD are associated with lower NSE and LNSE values.

Finally, Fig. 6 presents the overall absolute error for each site. In particular in Fig. 6 scatterdiagrams of  $\delta_{mod}$  are illustrated in two panels, where the x-axes reports errors computed for the proposed model (TNDTK) while the y-axes reports in turn errors from reference models. In this representation an equivalence between model performances is represented by the solid bisecting line, hence if one point falls in the top-left above the 1 : 1 line TNDTK provides better predictions then the reference model, otherwise if it falls below the 1 : 1 line. Figure 6 clearly shows that KMOD is less powerful then TNDTK for 14 out of 18 sites, while MEAN performs the poorest, with 16 out of 18 sites characterised by higher  $\delta$  values relative to TNDTK.

#### 5.4.2 Standardisation by MAP\*-dimensional FDCs

Right panel of Fig. 4 highlights satisfactory performance of Top-kriging for predicting  $TND_2$  values in ungauged basins, NSE value is approximately 0.6, and site 3701 still presents an outlying behaviour for the same reason explained before.

Although the cross-validated  $TND_2$  values are less accurate than  $TND_1$  ones, TNDTK performance for predicting dimensional FDCs is good. Comparing TNDTK with LLK models, Fig. 7 shows for LLK narrower bands for  $d < 0.8$ , particularly the band

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illustrating 90 % of residuals, while in the low flow range (i.e.  $0.8 < d < 1$ ) TNDTK shows slightly better performances resulting in narrower error bands. The bottom panels in the same figure report the scatterdiagrams of predicted vs. observed dimensional flows, expressing the goodness and reliability of TNDTK when used for predicting dimensional FDC on the basis of MAP. Even though TNDTK shows an  $NSE = 0.914$  which is lower than the NSE value associated with LLK and equal to KMOD one, TNDTK is associated with the highest LNSE value. TNDTK is associated with the highest value of LNSE (i.e. 0.922) which highlights the very good performance of TNDTK for low-flows. Figure 8 confirms good performance of TNDTK against LLK and KMOD, showing in both cases more accurate predictions for 10 out of 18 catchments. Also, among the 8 catchments for which LLK and KMOD perform better than TNDTK, it is worth nothing that performances are practically the same of TNDTK in 2 cases for LLK and 3 cases for KMOD.

## 6 Discussion and future work

### 6.1 Is Top-kriging suitable for predicting long-term FDCs?

The cross-validation of TNDTK shows that the proposed procedure can be effectively applied in the study region for predicting standardised FDCs (i.e. flow-duration curves divided by an index-flow such as MAF). In particular, the interpolation strategy applied in this study, that is (1) the computation of TND for empirical standardised FDCs, (2) the modelling of spatial correlation of empirical TND values along the stream network, (3) the identification of a linear weighting scheme for averaging empirical dimensionless FDCs on the basis of the correlation model identified at step (2), results in reliable predictions of standardised FDCs. The curves predicted in cross-validation are unbiased for the entire duration range (i.e. from high- to low-flows) and the prediction residuals are as small as, or smaller than, the residuals resulting from the application of traditional regionalisation schemes.

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Analyzing the results in detail, Fig. 6 indicates that TNDTK performed significantly worse than the baseline and benchmark regional models in three cases only. The benchmark model KMOD better predicts the FDC for site 3701 (left panel of Fig. 6). As illustrated in right panel in Fig. 2, site 3701 is associated with the steepest empirical flow duration curve of the study region and therefore the highest empirical TND value (see Table 1 and Figs. 1 and 4). This is a result of the very limited permeability of the catchment, which can be regarded as impervious. The surrounding catchments have higher permeability and, consequently, flatter empirical FDCs that produce a biased interpolation. While information on permeability is explicitly incorporated in the multi-regression models included in KMOD (see e.g. Castellarin et al., 2007), the degree of permeability of the catchment is not considered in the kriging procedure which is mainly driven by spatial proximity. The baseline model MEAN significantly outperforms TNDTK for sites 2502 and 801, and this result can be explained by noticing that both sites are associated with empirical standardised curves that are well represented by the average standardised FDC for the study region (see right panel in Fig. 2), that is the curve associated with the baseline regional model (MEAN) in cross-validation.

This positive outcome derives from the main features of TNDTK, that are associated with several advantages. TNDTK dispenses with the critical phase of delineating hydrologically homogeneous pooling group of sites (see Castellarin et al., 2004a) by exploiting the spatial correlation structure of the streamflow regime (see Archfield and Vogel, 2010). Also, the approach does not require to set up multiregression models for estimating the parameters of a mathematical expression (e.g. a theoretical frequency distribution) controlling the shape of the curve, which are often associated with a large uncertainty and limited robustness (see Castellarin et al., 2007); TNDTK predicts the shape of the curve for an ungauged basin through a non-parametric procedure as a weighted average of empirical standardised FDCs (e. g. Smakhtin et al., 1997; Ganora et al., 2009). The weighting scheme also ensures for the predicted curve a non-increasing relationship between streamflow and duration, which is one of the main properties of flow-duration curves.

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The study also points out that TNDTK can be used for predicting dimensional FDCs in ungauged sites on the basis of a minimal set of hydrological information, that is (a) empirical FDCs for a group of gauges basins and (b) an estimate of Mean Annual Precipitation (MAP) for all gauged basins in the region, as well as for the target ungauged basin. Even though TNDTK does not show a clear supremacy relative to more traditional approaches (see Figs. 7 and 8), it has to be highlighted that its application is rather straightforward and does not require any subjective choice, which, together with the fact that the procedure can be implemented with a limited amount of input data, makes TNDTK a very interesting alternative for predicting dimensional FDCs.

## 6.2 Future analyses

Our study is evidently a preliminary analysis, which tackles the exploration of geostatistical approaches for predicting FDCs. Therefore, the results of our study open up several possible research avenues. In particular, we focus on the prediction of long-term steady-state FDCs, on the basis of Period-of-Record (POR) empirical FDCs. Applicability of TNDTK to the prediction of annual FDCs for typical hydrologic years, as well as for particularly wet or dry years (see e.g. Vogel and Fennessey, 1994; Castellarin et al., 2004b), is an open problem that needs to be specifically and quantitatively addressed.

Evidently, the proposed approach needs to be further investigated in other geographical contexts. In particular, the application of TNDTK for predicting dimensional FDCs on the basis of catchment-scale MAP values deserves some further tests that aim at verifying its suitability for significantly different climatic conditions (e.g. arid regions, alpine catchments, etc.), in which the streamflow regime is not heavily controlled by the rainfall regime, as for the considered case study.

Finally, we propose to summarise empirical flow-duration curves through the index TND, which expresses the total negative deviation of the curve from a reference streamflow value. Although this index proved to be very informative on the similarity of empirical flow-duration curves constructed for the study region, its validity needs a deeper investigation. More importantly, future analyses should focus on the identification of



a global indicator of the similarity between FDCs to be used to analyze and model geographical correlation between the empirical curves themselves, this would enable one to base the definition of the linear weighting scheme on a more comprehensive and descriptive indicator of the streamflow regime, instead of the semivariogram constructed for a point index (i.e. TND).

## 7 Conclusions

This study explores the possibility to extend the application of Top-kriging, which is generally used for spatial interpolation of point streamflow indices (e.g., estimated flood quantiles, low-flow indices, etc.), to the prediction of period-of-record flow-duration curves (FDCs) in ungauged basins. Top-kriging is used in this study to geostatistically interpolate standardised FDCs along the streamnetwork of a broad geographical area in Central-Eastern Italy. We identified the linear weighting typical of any kriging procedure by modeling the spatial correlation structure of an empirical streamflow index, which was shown in the study to be particularly useful in describing the daily streamflow regime of a given catchment. In particular, we defined the index, which we termed Total Negative Deviation (TND), as the overall negative deviation of an empirical FDC relative to a reference streamflow-value used for the standardisation of the empirical curves.

We considered in the study two different reference streamflow values, that is the Mean Annual Flow (MAF) and catchment-scale Mean Annual Precipitation times the drainage area of the catchment (MAP\*). By applying Top-kriging along the streamnetwork to the spatial distribution of empirical TND values, the former standardisation (based on MAF) enabled us to develop a regional model of dimensionless FDCs, while we used the latter (based on MAP\*) to predict dimensional flow-duration curves in ungauged basins. The two regional estimators were cross-validated and compared in terms of prediction performances with other regional models of dimensionless and dimensional flow duration curves that were previously developed for the study area.

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The comparison highlights the superior performances of the proposed procedure (Total Negative Deviation Top-kriging, TNDTK) relative to traditional regional models for predicting standardised FDCs. FDCs predicted with TNDTK are unbiased independently of the considered duration, and prediction residuals are significantly smaller than those associated with traditional regionalization procedures, in particular if high durations are considered (i.e. low-flows). Also, our study points out that applying TNDTK to regionalise FDCs standardised by MAP\* enables one to predict dimensional FDCs in ungauged basins on the basis of a minimal set of hydrological information: (a) empirical FDCs for a group of gauged basins and (b) an estimate of catchment-scale Mean Annual Precipitation (MAP) for all gauged basins in the region, as well as for the target ungauged basin. Moreover, the prediction accuracy of TNDTK is similar to, or higher than, more complex regionalization approaches that use multiregression models that incorporate information on the permeability, morphology, climate, etc. of the catchment, which seems to confirm the value of spatial proximity relative to catchment attributes when hydrological predictions in ungauged basins are concerned (see e.g. Merz and Blöschl, 2005).

Our study is indeed a preliminary analysis, and we are aware that the proposed procedure needs to be further tested in different geographical and climatic contexts before its general validity can be acknowledged. Nevertheless, we believe that this study further highlights the potential of Top-kriging by showing how it can be easily adapted for predicting flow-duration curves. Also, we believe that the TND index identified in this study incorporates a worth of hydrological information and has the potential to be extremely useful in a number of hydrological problems other than the prediction of FDCs, such as catchment classification (see Wagener et al., 2007; Di Prinzio et al., 2011) or regionalization studies (Laaha and Blöschl, 2006; Gaál et al., 2012), future analyses will specifically address this point.

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**Table 1.** Study catchments: variability of drainage area (A), Mean Annual Flow (MAF), Mean Annual Precipitation (MAP), rescaled mean annual precipitation (MAP\*) and empirical TND<sub>1</sub> and TND<sub>2</sub> values; table lists the minima, maxima, means, 1st, 2nd (median) and 3rd quartiles of the sample distributions.

	A [km <sup>2</sup> ]	MAF [m <sup>3</sup> s <sup>-1</sup> ]	MAP [mm]	MAP* [m <sup>3</sup> s <sup>-1</sup> ]	TND <sub>1</sub> [-]	TND <sub>2</sub> [-]
min	61	1.49	918.1	2.17	1.59	1.25
1st Qu.	104	2.63	1079.0	3.60	2.76	4.38
median	164	3.83	1123.0	5.99	3.82	5.78
mean	330	6.51	1118.0	11.69	4.52	6.11
3rd Qu.	562	7.54	1162.0	17.53	5.74	7.55
max	1044	21.29	1298.0	37.07	9.83	13.21

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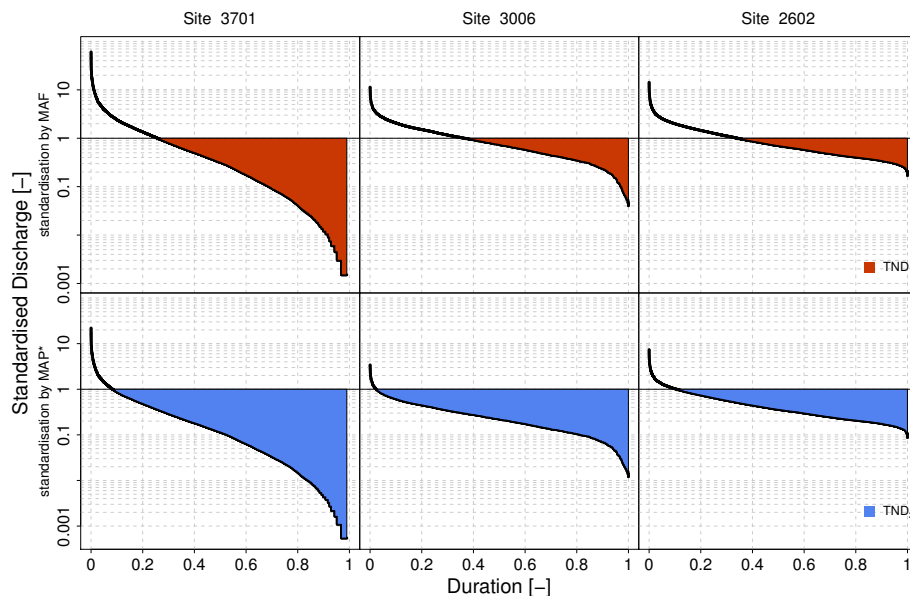
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**Fig. 1.** Total Negative Deviation (TND, filled area) for three catchments with different hydrological behaviours. Top panels:  $TND_1$  (red area) for an empirical FDC (black tick line) standardised by Mean Annual Flow (MAF); bottom panels:  $TND_2$  (blue area) for an empirical FDC (black tick line) standardised by  $MAP^* = MAP \cdot A \cdot CF$ , where MAP is the Mean Annual Precipitation,  $A$  is the drainage area and CF is a unit-conversion factor.

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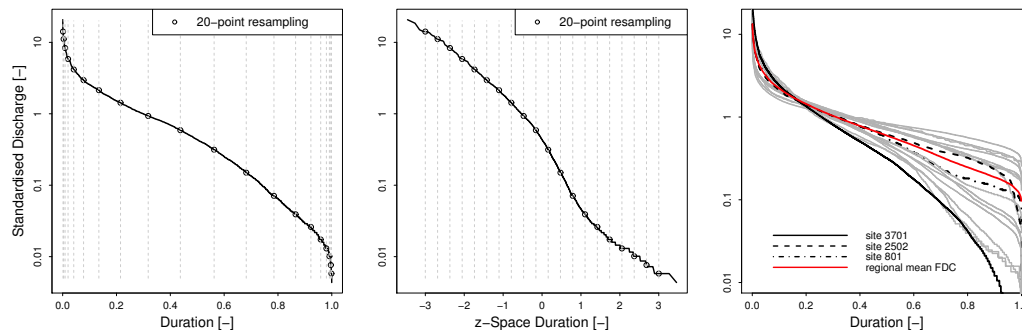
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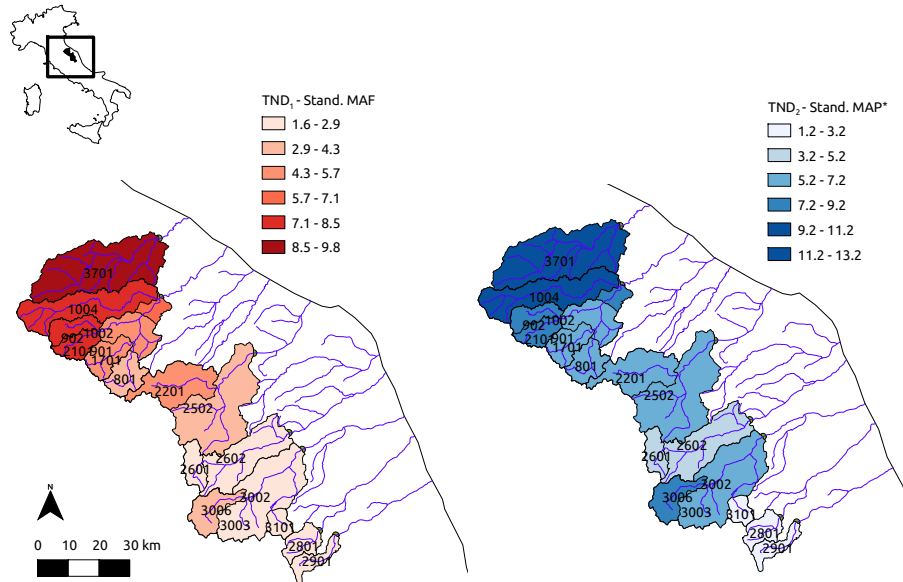
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**Fig. 2.** FDC representations: log-natural scale (left), log-normal scale (center); the panels also show a resampling of the empirical curve (circles) which employs 20 points equally spaced in the standard-normal space; standardised empirical FDCs for the study region (right), FDC for sites 3701, 801, 2502 and regional mean FDC are highlighted.

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**Fig. 3.** Empirical TND<sub>1</sub> and TND<sub>2</sub> values for the study catchments.

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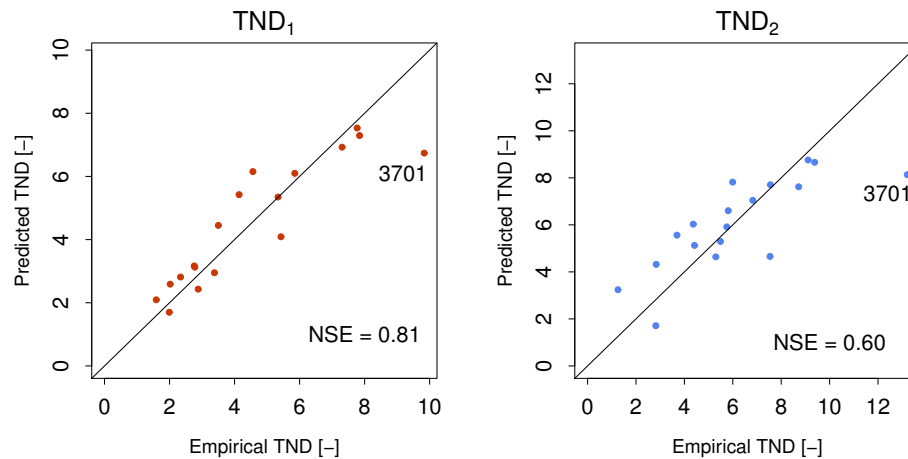
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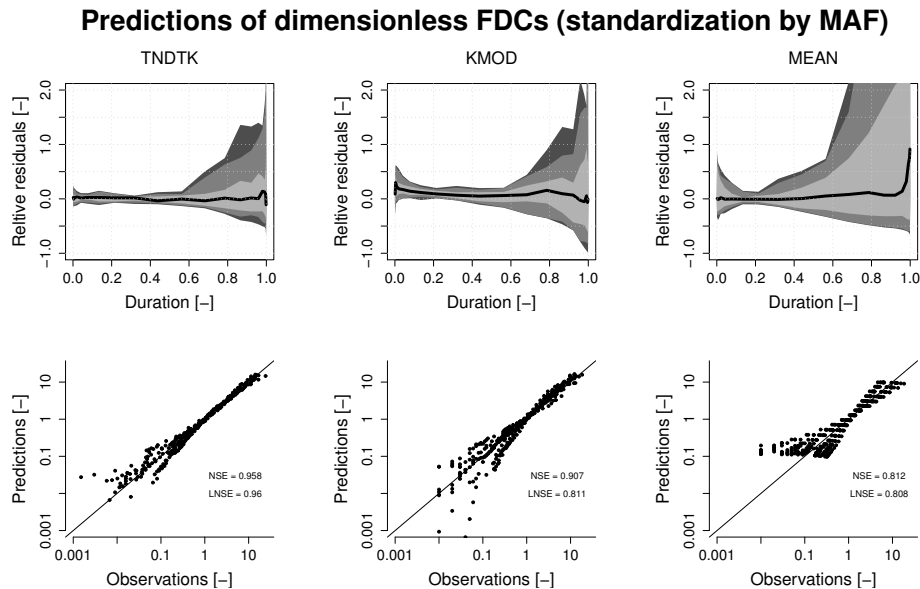
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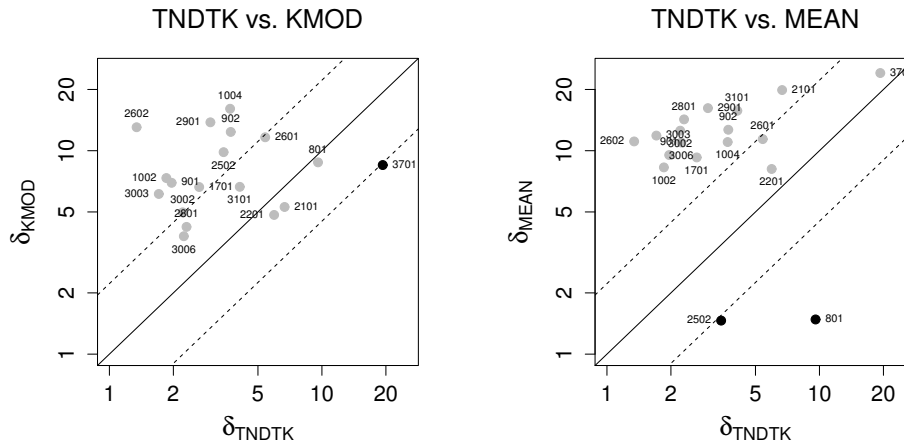


**Fig. 4.** Top-kriging predictions of TND<sub>1</sub> and TND<sub>2</sub> values in cross-validation, predictions for site 3701 are highlighted.



**Fig. 5.** Cross-validation of regional models: MEAN (right), KMOD (center), TNDTK (proposed approach, left); error-duration curves reporting the profile of the median relative error (thick black line) and the bands containing 50 %, 80 % and 90 % of the relative errors (grey nested bands) as a function of duration (top); empirical vs. predicted standardised streamflows (bottom).

### Predictions of dimensionless FDCs (standardization by MAF)



**Fig. 6.** Comparison between TNDTK, MEAN and KMOD models in terms of distances between empirical and predicted FDCs,  $\delta_{mod}$  (where mod stands for TNDTK, MEAN or KMOD); values of  $\delta_{TNDTK}$  are reported against values of  $\delta_{KMOD}$  (left) or  $\delta_{MEAN}$  (right) for each considered basin; the solid line represents the ratio 1 : 1 between the errors, while dashed lines delimit the areas where errors for the TNDTK model are twice the MEAN or KMOD ones and vice versa. Points above the solid line represent curves better estimated by TNDTK; points above the top dashed line represent curves much better estimated by TNDTK (see also Ganora et al., 2009, Fig. 7); sites 3701 and 801 are highlighted.

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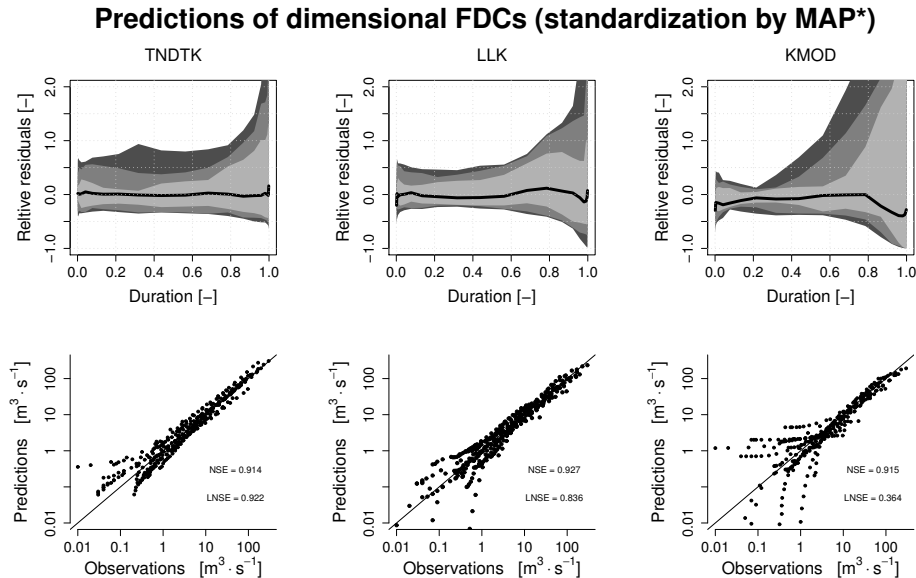
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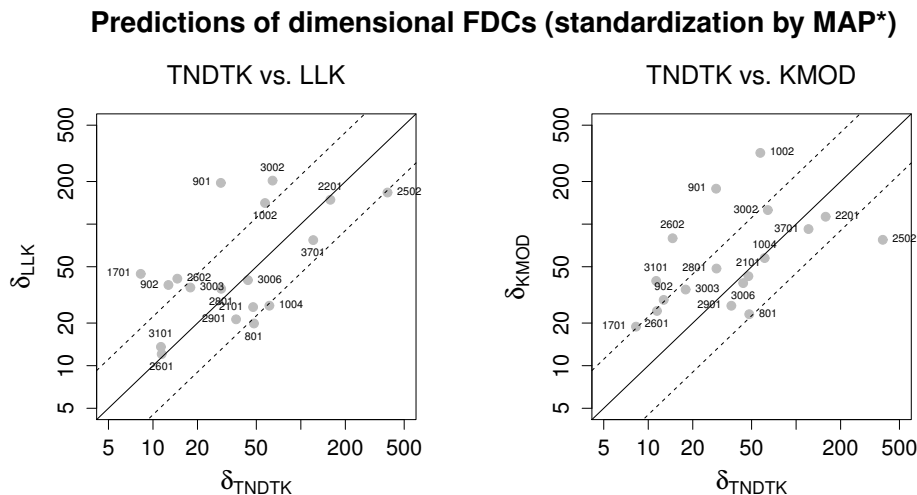
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**Fig. 7.** Cross-validation of regional models: KMOD (right), LLK (center), TNDTK (proposed approach, left); error-duration curves reporting the profile of the median relative error (thick black line) and the bands containing 50 %, 80 % and 90 % of the relative errors (grey nested bands) as a function of duration (top); empirical vs. predicted dimensional streamflows (bottom).



**Fig. 8.** Comparison between TNDTK, KMOD and LLK models in terms of distances between empirical and predicted dimensional FDCs,  $\delta_{\text{mod}}$  (where mod stands for TNDTK, KMOD or LLK); values of  $\delta_{\text{TNDTK}}$  are reported against values of  $\delta_{\text{LLK}}$  (left) or  $\delta_{\text{KMOD}}$  (right) for each considered basin; the solid line represents the ratio 1 : 1 between the errors, while dashed lines delimit the areas where errors for the TNDTK model are twice the LLK or KMOD ones and vice versa. Points above the solid line represent curves better estimated by TNDTK; points above the top dashed line represent curves much better estimated by TNDTK (see also Ganora et al., 2009, Fig. 7).