



**Combination
equations for canopy
evaporation**

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Generalized combination equations for canopy evaporation under dry and wet conditions

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Title Page	
Abstract	Introduction
Conclusions	References
Tables	Figures
⏪	⏩
◀	▶
Back	Close
Full Screen / Esc	
Printer-friendly Version	
Interactive Discussion	



Abstract

The formulation of canopy evaporation is investigated on the basis of the combination equation derived from the Penman equation. All the elementary resistances (surface and boundary layer) within the canopy are taken into account and the exchange surfaces are assumed to be subject to the same vapour pressure deficit at canopy source height. This development leads to generalized combination equations: one for completely dry canopies and the other for partially wet canopies. These equations are rather complex because they involve the partitioning of available energy within the canopy and between the wet and dry surfaces. By making some assumptions and approximations, they can provide simpler equations similar to the common Penman–Monteith model. One of the basic assumptions of this down-grading process is to consider that the available energy intercepted by the different elements making up the canopy is uniformly distributed and proportional to their respective area. Despite the somewhat unrealistic character of this hypothesis, it allows one to retrieve the simple formulations commonly and successfully used up to now. Numerical simulations are carried out by means of a simple one-dimensional model of the vegetation–atmosphere interaction to compare the complete formulations with the simpler ones and to assess the concept of excess resistance.

1 Introduction

The combination equation, which expresses the evaporation from natural surfaces, has certainly been one of the most successful breakthroughs in our understanding of evaporation. It is obtained by combining the energy balance equation with expressions of the convective fluxes of sensible and latent heat. The first equation of this type is the original Penman formula, initially derived to estimate the evaporation from a completely wet surface such as open water (Penman, 1948). It was extended by Monteith (1963)

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



and on foliage area (soil evaporation being neglected). It is interpreted as the effective stomatal resistance of all the leaves acting as resistances in parallel (Shuttleworth, 1976b)

$$\frac{1}{r_s} = \sum_i \frac{1}{r_{s,i}}, \quad (2)$$

$r_{s,i}$ being the stomatal resistance of an individual leaf i . The Penman–Monteith equation is often called “big-leaf model” because the whole canopy is assimilated to a big leaf located at level $d + z_0$ and with stomatal resistance r_s . The transfer processes through the air surrounding the leaves, supposedly negligible, are not taken into account or indirectly through the excess resistance.

The lack of theoretical foundation of Eq. (1) applied to a canopy of leaves was apparent in a controversy which occurred in the seventies about the formulation of evaporation from partially wet canopies (Shuttleworth, 1976a, 1977; Monteith, 1977). The Penman–Monteith equation was considered not to be able to represent the transition between dry and wet canopies, because the definition of canopy resistance according to Penman–Monteith (Eq. 2) implies that, if only a small part of the canopy is wet ($r_{s,i} = 0$), the canopy resistance r_s should be equal to zero, which is unrealistic. In this context, the main objectives of the paper are to investigate the theoretical foundations of the combination equation, applied to a canopy of leaves, and to examine the different ways of aggregating the in-canopy resistances (surface and air) in a general single-source formulation of canopy evaporation. The theoretical analysis is made under dry and wet conditions and the errors made when applying simpler equations of the Penman–Monteith type are numerically assessed. The study is based on principles similar to those developed by Shuttleworth (1978) in his simplified description of the vegetation–atmosphere interaction. The whole canopy (soil surface included) is supposed to be subject to the same vapour pressure deficit D_m at the mean source height z_m ($d + z_0$), as in the original Penman–Monteith model and in most of two-source models (e.g. Shuttleworth and Wallace, 1985; Lhomme et al., 2012). Since the modelling

Combination equations for canopy evaporation

J. P. Lhomme and C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



process accounts for all the boundary layer resistances within the canopy, the question of the exact location of the canopy source height and the corresponding issue of the excess resistance will be indirectly dealt with.

2 Evaporation from a dry canopy

2.1 General formulation

The canopy exchanges sensible and latent heat with the atmosphere through its leaf area and its soil surface. The modelling framework describing this interaction is similar to the one used by Lhomme et al. (2013) to derive the formulation of evaporation from a canopy made of n different components; but, the individual components or elements are represented here by the different leaves of the canopy and the soil surface, as shown in Fig. 1. The elementary evaporation (λE_i) per unit area of exchange surface (each side of a leaf being considered separately) is calculated from an equation of the Penman–Monteith type which involves the corresponding available energy (A_i) and the saturation deficit of the air at canopy source height (D_m)

$$\lambda E_i = \frac{\Delta A_i + \rho c_p D_m / r_{a,i}}{\Delta + \gamma (1 + r_{s,i} / r_{a,i})}. \quad (3)$$

In Eq. (3), $r_{s,i}$ is the leaf stomatal resistance (one side) per unit area of leaf and $r_{a,i}$ is the corresponding leaf boundary-layer resistance for sensible and latent heat. The soil surface is represented with subscript $i = s$, $r_{s,s}$ being the soil surface resistance to evaporation and $r_{a,s}$ the air resistance between the soil surface and the canopy source height (z_m), defined by integrating the reciprocal of the appropriate eddy diffusivity (Choudhury and Monteith, 1988). Canopy leaf area (LAI) being noted L_t , the total exchange surface area per unit area of soil is $S_t = 2L_t + 1$ and total evaporation is obtained by summing the contributions of each individual exchange surface (soil and leaves):

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



$$\lambda E^t = \sum_{i \in S_t} \lambda E_i. \quad (4)$$

The vapour pressure deficit (D_m) in Eq. (3) is calculated from the vapour pressure deficit at reference height (D_a) (Shuttleworth and Wallace, 1985; Lhomme et al., 2013)

$$D_m = D_a + [\Delta A - (\Delta + \gamma) \lambda E^t] r_{a,0} / (\rho c_p), \quad (5)$$

where A is the available energy of the whole canopy and $r_{a,0}$ the aerodynamic resistance between the mean source height (z_m) and the reference height (z_r). Defining

$$R_i = r_{s,i} + \left(1 + \frac{\Delta}{\gamma}\right) r_{a,i}, \quad (6)$$

and introducing Eq. (5) into Eq. (3) and Eq. (3) into Eq. (4) leads to

$$\lambda E^t = \frac{\Delta \left[A + (R_c / r_{a,0}) \sum_{i \in S_t} (A_i r_{a,i} / R_i) \right] + \rho c_p D_a / r_{a,0}}{\Delta + \gamma (1 + R_c / r_{a,0})}, \quad (7)$$

where R_c is expressed as:

$$\frac{1}{R_c} = \sum_{i \in S_t} \frac{1}{R_i}. \quad (8)$$

Equations (7) and (8) represent a kind of generalized combination equation, where all the within-canopy resistances (air and surface) are taken into account. R_c defines a bulk canopy resistance which includes the surface resistances (leaves and soil) and the air resistances within the canopy. The temperature of each exchange surface can be determined from the above equations, as detailed in Appendix B.

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



If the boundary-layer resistances ($r_{a,i}$) within the canopy are neglected, assuming they are small compared to their stomatal counterpart (which is the assumption made in the Penman–Monteith equation), Eqs. (7) and (8) can be easily simplified. Excluding the soil component and putting $r_{a,i} = 0$, the summation in the right-hand term of Eq. (8) defines the canopy stomatal resistance in the sense of Monteith denoted by $r_{s,c}$:

$$\frac{1}{R_c} = \frac{1}{r_{s,c}} = \sum_{i \in 2L_t} \frac{1}{r_{s,i}} \approx \frac{2L_t}{\langle r_{s,l} \rangle}, \quad (9)$$

where $\langle r_{s,l} \rangle$ is the harmonic mean of leaf stomatal resistances (per unit one-sided leaf area). For a hypostomatous canopy $2L_t$ should be replaced by L_t . Hence Eq. (7) becomes:

$$\lambda E^t = \frac{\Delta A + \rho c_p D_a / r_{a,0}}{\Delta + \gamma (1 + r_{s,c} / r_{a,0})}. \quad (10)$$

Equation (10) is the well-known Penman–Monteith equation, which appears now as a particular case of a more general equation (Eq. 7), when all the air resistances within the canopy are set to zero and soil surface is neglected.

The case of a completely wet canopy can also be inferred from Eq. (7). When all the exchange surfaces (leaves and soil surface) are wet, the surface resistances ($r_{s,i}$) are nil and $R_i = (1 + \Delta/\gamma)r_{a,i}$. Noting that $\sum_{i \in 2L_t} A_i = A$ and after some manipulations, Eq. (7) transforms into a Penman type equation

$$\lambda E^t = \frac{\Delta A + \rho c_p D_a / (r_{a,0} + r_{a,c})}{\Delta + \gamma}, \quad (11)$$

where

$$\frac{1}{r_{a,c}} = \sum_{i \in S_t} \frac{1}{r_{a,i}} \approx \frac{2L_t}{\langle r_{a,l} \rangle} + \frac{1}{r_{a,s}}, \quad (12)$$

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



$\langle r_{a,l} \rangle$ being the harmonic mean of leaf boundary-layer resistances and $r_{a,s}$ the air resistance between the soil surface and the canopy source height. There is no surface resistance in the denominator of Eq. (11), as in the original Penman equation, but an additional air resistance ($r_{a,c}$) is added to the common aerodynamic resistance above the canopy ($r_{a,0}$). This additional resistance is the parallel sum of individual air resistances and encapsulates the bulk canopy resistance to heat and water vapour transfer from the wet exchange surfaces (leaves and soil) to the canopy source height.

2.2 Penman–Monteith type formulation

The general combination equation derived above (Eq. 7) does not follow the exact form of the Penman–Monteith equation: an additional term mixing resistances with available energy partitioning is added to total available energy (A). This section investigates under which conditions this general formula of canopy evaporation can be put in the form of a Penman–Monteith equation without neglecting the air resistances within the canopy.

The function A_j giving the partition of available energy within the canopy is supposed to be in the form $A_j = A\Phi_j$, where A is the total available energy and Φ_j is a function depending on canopy structure and on its leaf area distribution. The Beer's law, which is commonly used to express the attenuation of net radiation within the canopy, is a function of this form. Consequently, after some manipulations, it can be shown that canopy evaporation (Eq. 7) writes as

$$\lambda E^t = \frac{\Delta A + \rho c_p D_a / (r_{a,0} + r_{a,c})}{\Delta + \gamma [1 + r_{s,c} / (r_{a,0} + r_{a,c})]}, \quad (13)$$

where the bulk resistances $r_{a,c}$ and $r_{s,c}$ are defined as

$$r_{a,c} = R_c \sum_{i \in S_t} \Phi_i \frac{r_{a,i}}{R_i}, \quad (14)$$

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

$$r_{s,c} = R_c \left[1 - \left(1 + \frac{\Delta}{\gamma} \right) \sum_{i \in S_t} \Phi_i \frac{r_{a,i}}{R_i} \right]. \quad (15)$$

The resistances defined above involve air and surface resistances and the distribution function of available energy within the canopy. In order to get simpler formulations, some approximations are made substituting average values to summations. Introducing the harmonic mean $\langle r_{s,i} \rangle$ of surface resistances per unit area of exchange surface and the corresponding harmonic mean of leaf boundary-layer resistances noted $\langle r_{a,i} \rangle$, Eq. (8) can be written as

$$\frac{1}{R_c} \approx \frac{S_t}{\langle R_i \rangle} = \frac{S_t}{\langle r_{s,i} \rangle + \left(1 + \frac{\Delta}{\gamma} \right) \langle r_{a,i} \rangle}. \quad (16)$$

The summation in Eqs. (14) and (15) can be approximated using means denoted by angle brackets:

$$\sum_{i \in S_t} \Phi_i \frac{r_{a,i}}{R_i} \approx \left\langle \frac{\Phi_i r_{a,i}}{R_i} \right\rangle S_t \approx \frac{S_t}{\langle R_i \rangle} \langle \Phi_i r_{a,i} \rangle. \quad (17)$$

This leads to the following approximate expressions for bulk canopy resistances:

$$r_{a,c} \approx \langle \Phi_i r_{a,i} \rangle, \quad (18)$$

$$r_{s,c} \approx \frac{\langle R_i \rangle}{S_t} - \left(1 + \frac{\Delta}{\gamma} \right) \langle \Phi_i r_{a,i} \rangle. \quad (19)$$

These expressions still depend upon available energy partitioning. It is interesting to note, however, that if available energy is equally distributed within the canopy (soil

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

included), i.e. $\Phi_i = 1/S_t$, the bulk air and surface resistances reduce to simple expressions independent of available energy. Although this assumption is not really realistic and constitutes a priori a strong approximation, it has been used by Shuttleworth (1978) in his “simplified general model”. Using this assumption and separating the soil and leaves components ($S_t = 2L_t + 1$), the bulk canopy resistances can be rewritten in a way similar to Eqs. (9) and (12)

$$\frac{1}{r_{a,c}} \approx \frac{S_t}{\langle r_{a,i} \rangle} = \frac{2L_t}{\langle r_{a,l} \rangle} + \frac{1}{r_{a,s}}, \quad (20)$$

$$\frac{1}{r_{s,c}} \approx \frac{S_t}{\langle R_i \rangle - \left(1 + \frac{\Delta}{\gamma}\right) \langle r_{a,i} \rangle} = \frac{S_t}{\langle r_{s,i} \rangle} = \frac{2L_t}{\langle r_{s,l} \rangle} + \frac{1}{r_{s,s}}, \quad (21)$$

where $\langle r_{a,l} \rangle$ and $\langle r_{s,l} \rangle$ are the harmonic means of leaf boundary-layer resistances and stomatal resistances respectively. If the canopy is hypostomatous and if the average stomatal resistance $\langle r_{s,l} \rangle$ applies to the lower side of the leaves, $2L_t$ should be replaced by L_t in Eq. (21).

Equation (13) appears now as a typical Penman–Monteith equation with its bulk resistances defined in the conventional way. The canopy surface resistance ($r_{s,c}$) accounts for all surface resistances, including leaves and soil. The “extra” resistance ($r_{a,c}$), added to the common aerodynamic resistance above the canopy ($r_{a,0}$), accounts for the air resistances opposed to heat and water vapour transfer within the canopy and can be perceived as similar to the excess resistance (B^{-1}/u^*) introduced by Thom (1972) in the formulation of canopy evaporation.

3 Evaporation from a partially wet canopy

The partially wet canopy is taken here in the sense of “double canopy limit” described by Shuttleworth (1976b, 1978), all the individual elements being considered either totally

Combination
equations for canopy
evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



dry or totally wet. It is opposed to the “single canopy limit”, where the distribution of surface water resembles that of stomata, as when droplets of fog and mist impact the leaves. The “double canopy” is the most realistic case applicable to canopies which are drying out or in the process of wetting up by rainfall.

5 3.1 General formulation

The whole canopy is divided into two parts assumed to be independent: one is dry (with exchange surface S_d) and the other wet (with exchange surface S_w) and $S_t = S_d + S_w$. The assumption of independence is certainly questionable, but, as expressed by Shuttleworth (1978, p. 8): “such an assumption is certainly essential if theoretical progress is to be made in this field”. Consequently, Eq. (4) can be rewritten in the following way:

$$\lambda E^t = \lambda E^d + \lambda E^w = \sum_{i \in S_d} \lambda E_i + \sum_{i \in S_w} \lambda E_i. \quad (22)$$

After substituting the expression of D_m (Eq. 5) into Eq. (3), elementary evaporation can be rewritten as:

$$\lambda E_i = \left[\frac{\Delta}{\gamma} (r_{a,i} A_i + r_{a,0} A) + \left(\frac{\rho c_p}{\gamma} \right) D_a - \left(1 + \frac{\Delta}{\gamma} \right) r_{a,0} \lambda E^t \right] / R_i, \quad (23)$$

where R_i is given by Eq. (6). Bulk canopy resistances for the dry and wet parts of the canopy will be respectively defined as:

$$\frac{1}{R_{c,d}} = \sum_{i \in S_d} \frac{1}{R_i} \quad \text{and} \quad \frac{1}{R_{c,w}} = \sum_{i \in S_w} \frac{1}{R_i}. \quad (24)$$

With these definitions, the evaporation from the dry part of the canopy writes as:

$$\lambda E^d = \frac{\Delta}{\gamma} \left(\frac{r_{a,0}}{R_{c,d}} A + \sum_{i \in S_d} \frac{r_{a,i} A_i}{R_i} \right) + \frac{\rho c_p}{\gamma} \frac{D_a}{R_{c,d}} - \left(1 + \frac{\Delta}{\gamma} \right) \frac{r_{a,0}}{R_{c,d}} \lambda E^t, \quad (25)$$

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



and the contribution of the wet part is:

$$\lambda E^w = \frac{\Delta}{\Delta + \gamma} \left(\frac{r_{a,0}}{R_{c,w}} A + \sum_{i \in S_w} A_i \right) + \frac{\rho C_p D_a}{\Delta + \gamma R_{c,w}} - \frac{r_{a,0}}{R_{c,w}} \lambda E^t. \quad (26)$$

After some rearrangement, putting $A_w = \sum_{i \in S_w} A_i$ and defining a bulk canopy resistance for a partially wet canopy as

$$\frac{1}{R_{c,pw}} = \frac{1}{R_{c,d}} + \frac{\gamma}{\Delta + \gamma} \frac{1}{R_{c,w}}, \quad (27)$$

Eq. (22) becomes:

$$\lambda E^t = \frac{\Delta \left\{ A + \frac{R_{c,pw}}{r_{a,0}} \left[\left(\frac{\gamma}{\Delta + \gamma} \right) A_w + \sum_{i \in S_d} \frac{r_{a,i} A_i}{R_i} \right] \right\} + \rho C_p \frac{D_a}{r_{a,0}}}{\Delta + \gamma (1 + R_{c,pw} / r_{a,0})}. \quad (28)$$

The contribution of each part of the canopy (wet and dry) to total evaporation is obtained by replacing λE^t by its expression in Eqs. (25) and (26). As could be expected, the limit of Eq. (28) when the canopy becomes completely dry is Eq. (7) and it is Eq. (11) when it becomes entirely wet. Consequently, Eq. (28) constitutes a kind of generalized combination equation applicable in all conditions (dry, wet or partially wet canopy). It is also a different and simpler writing of the single-source limit of the General model developed by Shuttleworth (1976b, 1978). It is worthwhile noting that neglecting the air resistances within the canopy (i.e., $r_{a,i} = 0$) would lead to an inconsistency, as is the case for the Penman–Monteith equation applied in partially wet conditions. The bulk resistance $R_{c,pw}$ would become zero and Eq. (28) would turn into a simple Penman equation (λE_p), which is not realistic.

3.2 Penman–Monteith type formulation

This section examines under which conditions the general evaporation formula for partially wet canopies (Eq. 28) can be put in a form similar to the Penman–Monteith

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



equation. The same assumptions as those made by Shuttleworth (1978) to derive the “Double Canopy Limit of the Simplified General model” are used here. Soil surface is neglected and a proportion W of the canopy is supposed to be wet, which means that $S_w = WS_t$ and $S_d = (1 - W)S_t$ with $S_t = 2L_t$. Available energy is supposed to be equally distributed amongst the exchange surfaces as discussed above. Consequently the available energy of each part (wet and dry) is assumed to be proportional to its area: $A_w = AW$ and $A_d = A(1 - W)$. Additionally, the canopy is supposed to be amphitomatous. Eq. (24) can be approximated by:

$$\frac{1}{R_{c,w}} \approx \frac{2WL_t}{\langle r_{a,l} \rangle}, \quad (29)$$

$$\frac{1}{R_{c,d}} \approx \frac{2(1 - W)L_t}{\langle r_{s,l} \rangle + \left(1 + \frac{\Delta}{\gamma}\right) \langle r_{a,l} \rangle}. \quad (30)$$

For the sake of convenience, the mean (harmonic) leaf boundary-layer resistance $\langle r_{a,l} \rangle$ (assumed to be the same for the dry and wet parts of the canopy) and the mean (harmonic) stomatal resistance $\langle r_{s,l} \rangle$ of the dry part are assumed to be equal to that of the whole canopy. So, Eq. (27) can be rewritten as:

$$\frac{1}{R_{c,pw}} \approx \frac{2L_t}{\langle r_{a,l} \rangle} \frac{\gamma}{\Delta + \gamma} \left[\frac{W \langle r_{s,l} \rangle + \left(1 + \frac{\Delta}{\gamma}\right) \langle r_{a,l} \rangle}{\langle r_{s,l} \rangle + \left(1 + \frac{\Delta}{\gamma}\right) \langle r_{a,l} \rangle} \right] \quad (31)$$

The assumption on equally distributed available energy ($A_i = A/(2L_t)$) leads to:

$$\sum_{i \in S_d} \frac{r_{a,i} A_i}{R_i} \approx [2(1 - W)L_t] \frac{A}{2L_t} \frac{\langle r_{a,l} \rangle}{\langle R_i \rangle} = \frac{A(1 - W) \langle r_{a,l} \rangle}{\langle r_{s,l} \rangle + \left(1 + \frac{\Delta}{\gamma}\right) \langle r_{a,l} \rangle}. \quad (32)$$

Combination equations for canopy evaporation

J. P. Lhomme and C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

1 to n from the top of the canopy to the soil surface. The different components or unit exchange surfaces (i) of the system are represented here by the different layers of vegetation making up the canopy plus the soil surface. The parameterizations used for the microclimatic profiles, air and surface resistances are given in the appendix. The vertical profile of leaf area is supposed to be uniform (Eq. C4), which implies that the leaf area of each layer is $\Delta L_i = (L_t/z_h)\Delta z_i$ and the corresponding available energy (see Eq. C2) is expressed as

$$A_i = cR_{n,a} \exp(-cL_i)\Delta L_i, \quad (35)$$

where L_i is the cumulative leaf area above layer i and $R_{n,a}$ the net radiation of the whole canopy. Component air and stomatal resistances (amphistomatous case) are expressed as

$$r_{a,i} = \frac{r_{a,l}(z_i)}{2\Delta L_i} \quad \text{and} \quad r_{s,i} = \frac{r_{s,l}(z_i)}{2\Delta L_i}, \quad (36)$$

where $r_{a,l}(z_i)$ and $r_{s,l}(z_i)$ are respectively the leaf boundary layer resistance and the leaf stomatal resistance (per unit area of leaf) at each height within the canopy, given by Eqs. (C5) and (C8) respectively. The soil surface resistance $r_{s,s}$ has a fixed value depending on soil surface moisture and the corresponding air resistance $r_{a,s}$ (between the soil surface and the canopy source height) is given by Eq. (C7). Calculations are made for an amphistomatous canopy with $z_h = 1.2$ m and $L_t = 4$ under the following weather conditions at a reference height $z_r = 3$ m: incoming solar radiation $R_{s,a} = 700$ W m⁻², air temperature $T_a = 25$ °C, vapour pressure deficit $D_a = 10$ hPa, wind speed $u_a = 2$ ms⁻¹. The canopy is divided into 20 layers plus the soil surface.

4.2 Numerical results

In Fig. 2, the generalized combination equation in dry conditions expressed by Eq. (7) (called E_{D1}) is compared with two simplified equations: first Eq. (13) (called E_{D2}), where

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



the bulk air and surface resistances of the canopy are calculated by Eq. (20) and (21) respectively; it is derived assuming available energy to be equally distributed amongst the exchange surfaces; second, the common Penman–Monteith equation (E_{D3}) expressed by Eq. (10), where the canopy stomatal resistance $r_{s,c}$ is calculated as the harmonic mean of leaf stomatal resistances divided by twice the canopy leaf area: $\langle r_{s,l} \rangle / 2L_t$ (Eq. 9). The comparison is made as a function of the canopy water stress represented by the minimal stomatal resistance ($r_{s,l,n}$) in Eq. (C8). When the soil surface is dry ($r_{s,s} = 1000 \text{ s m}^{-1}$), the two simplified equations approximate very well the complete formulation, E_{D2} being practically mingled with E_{D1} (Fig. 2b). This clearly justifies the use of the Penman–Monteith equation in such conditions. However, when the soil surface becomes wetter ($r_{s,s} = 100 \text{ s m}^{-1}$) (Fig. 1a), there is a large discrepancy between the formulations: the common Penman–Monteith equation (E_{D3}) clearly underestimates canopy evaporation, as could be anticipated, and E_{D2} overestimates it. In Fig. 3, the generalized combination equation established in partially wet conditions (Eq. 28) (called E_{W1}) is compared with its simpler form (called E_{W2}) (Eqs. 33 and 34) based upon a series of simplifying assumptions. This comparison is made as a function of the fractional surface wetness W , assuming the wetting process begins by the top layers, as occurs during rainy events. The discrepancy is maximal ($\approx 100 \text{ W m}^{-2}$) when the canopy is half wet and decreases when the canopy becomes drier or wetter. The discrepancy is also greater when the canopy is water stressed (large $r_{s,l,n}$).

As already noticed, the “extra” resistance ($r_{a,c}$) (Eq. 20), added to the aerodynamic resistance ($r_{a,0}$) in the Penman–Monteith form of the combination equations (Eqs. 13 and 33), plays the same role as the excess resistance ($r_{a,ex} = B^{-1}/u^*$) introduced by Thom (1972) and mentioned in the introduction. The dimensionless parameter B^{-1} can be estimated by equating $r_{a,c}$ to $r_{a,ex}$: $kB^{-1} = \ln(z_0/z_{0h}) = ku^*r_{a,c}$. In Fig. 4, $r_{a,c}$ and kB^{-1} are plotted vs. wind speed at reference height for different LAI. In Fig. 1a, the extra resistance $r_{a,c}$ is also compared with the rough approximation based on $B^{-1} = 4$, which is a typical value for permeable vegetation (Thom, 1972). The extra resistance $r_{a,c}$ is

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

[Title Page](#)
[Abstract](#)
[Introduction](#)
[Conclusions](#)
[References](#)
[Tables](#)
[Figures](#)
[⏪](#)
[⏩](#)
[◀](#)
[▶](#)
[Back](#)
[Close](#)
[Full Screen / Esc](#)
[Printer-friendly Version](#)
[Interactive Discussion](#)

be made. In this down-grading process, one of the basic assumptions is to consider that the available energy is equally distributed amongst the exchange surfaces. This hypothesis appears to be rather unrealistic, both in dry and wet conditions, but it leads to simple formulations of the Penman–Monteith type (Eqs. 13 and 33, respectively), which have been successfully used up to now. The numerical simulations, based on a simple one-dimensional model, confirm that the Penman Monteith equation performs well in dry conditions, when the soil surface does not evaporate. In partially wet conditions, a discrepancy with the comprehensive formulation exists, but it tends to be nil when the canopy becomes completely wet.

Appendix A

List of symbols

See Table A1.

Appendix B

Expressing the temperature of exchange surfaces ($T_{c,i}$)

The basic equations for the transfer of sensible heat are:

$$H_i = \frac{\rho c_p (T_{c,i} - T_m)}{r_{a,i}} \quad \text{with} \quad H_i = A_i - \lambda E_i, \quad (\text{B1})$$

$$H^t = \frac{\rho c_p (T_m - T_a)}{r_{a,0}} \quad \text{with} \quad H^t = A - \lambda E^t. \quad (\text{B2})$$

Surface temperature is inferred from Eqs. (B1) and (B2):

$$T_{c,i} = T_a + \frac{(A_i - \lambda E_i) r_{a,i}}{\rho c_p} + \frac{(A - \lambda E^t) r_{a,0}}{\rho c_p}. \quad (\text{B3})$$

Elementary flux λE_i is given by Eq. (3) with D_m expressed by Eq. (5). Substituting and rearranging gives the following expression of $T_{c,i}$ as a function of λE^t (Eq. 7):

$$T_{c,i} - T_a = \frac{1}{\rho c_p} \left\{ [(A - \lambda E^t) r_{a,0} + A_i r_{a,i}] \left(1 - \frac{\Delta r_{a,i}}{\gamma R_i} \right) \right\} + \frac{r_{a,i}}{R_i} \left(\frac{\lambda E^t r_{a,0}}{\rho c_p} - \frac{D_a}{\gamma} \right). \quad (\text{B4})$$

Appendix C

Parameterizations used in the simulation process

Solar radiation R_s and net radiation R_n are assumed to decrease within the canopy as exponential functions of the cumulative leaf area index $L(z)$ (Beer's law) counted from the top of the canopy

$$R_s(z) = R_{s,a} \exp[-cL(z)], \quad (\text{C1})$$

$$R_n(z) = R_{n,a} \exp[-cL(z)]. \quad (\text{C2})$$

The attenuation coefficient is assumed to be the same for both profiles: $c = 0.60$. Net radiation above the canopy $R_{n,a}$ is calculated as 60 % of global radiation $R_{s,a}$ and soil heat flux G , as half the net radiation reaching the soil surface. The profile of wind speed within the canopy is given by

$$u(z) = u(z_h) \exp[-\beta L(z)], \quad (\text{C3})$$

where $u(z_h)$ is the wind speed at canopy height z_h (inferred from wind speed u_a at reference height z_r using a simple logarithmic profile) and $\beta = 0.5$ (Inoue, 1963). The profile of leaf area is taken as a uniform function of height

$$L(z) = L_t \left(1 - \frac{z}{z_h}\right). \quad (C4)$$

Leaf boundary-layer resistance (per unit one-sided leaf area) is calculated as a function of wind speed and leaf width w (0.01 m) as (Choudhury and Monteith, 1988)

$$r_{a,l}(z) = \alpha [w/u(z)]^{1/2}, \quad (C5)$$

with $\alpha = 200$ in SI units. For the sake of convenience, the aerodynamic resistance above the canopy is expressed as a simple function of wind speed without stability correction

$$r_{a,0} = \frac{1}{ku^*} \ln \left[\frac{z_r - d}{z_0} \right], \quad (C6)$$

where $u^* = ku_a / \ln[(z_r - d)/z_0]$ with $d = 0.63z_h$ and $z_0 = 0.13z_h$. The air resistance between the soil surface and the canopy source height is given by (Choudhury and Monteith, 1988)

$$r_{a,s} = \frac{z_h \exp(\omega)}{\omega K(z_h)} \left\{ \exp[-\omega z_{0,s}/z_h] - \exp[-\omega(d + z_0)/z_h] \right\}, \quad (C7)$$

where $K(z_h) = k^2 u_a (z_h - d) / \ln[(z_r - d)/z_0]$ is the value of eddy diffusivity at canopy height, $\omega = 2.5$ (dimensionless) and $z_{0,s} = 0.01$ m. The profile of leaf stomatal resistance (per unit one-sided leaf area) is made a function of solar radiation within the canopy following a Jarvis-type formulation:

$$r_{s,l}(z) = \frac{r_{s,l,n}}{1 - \exp[-\nu R_s(z)]}, \quad (C8)$$

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



where $r_{s,l,n}$ is a minimal stomatal resistance, which depends on available soil water, and $\nu = 0.009$ with R_s expressed in Wm^{-2} (Lhomme et al., 2001).

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Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

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Table A1. List of symbols.

A	Available energy of the whole canopy (W m^{-2})
A_i	Available energy of the unit area of exchange surface i (W m^{-2})
$R_{s,a}$	Incoming solar radiation (W m^{-2})
$R_{n,a}$	Net radiation of the whole canopy (W m^{-2})
G	Soil heat flux (W m^{-2})
H^t	Sensible heat flux from the whole canopy (W m^{-2})
H_i	Sensible heat flux from the unit area of exchange surface i (W m^{-2})
λE^t	Latent heat flux from the whole canopy (W m^{-2})
λE^d	Latent heat flux from the dry part of the canopy (W m^{-2})
λE^w	Latent heat flux from the wet part of the canopy (W m^{-2})
λE_i	Latent heat flux from the unit area of exchange surface i (W m^{-2})
D_a	Vapour pressure deficit at reference height ($e(T_a) - e_a$) (Pa)
D_m	Vapour pressure deficit at canopy source height ($e(T_m) - e_m$) (Pa)
e_a	Vapour pressure at reference height (Pa)
e_m	Vapour pressure at canopy source height (Pa)
$e^*(T)$	Saturated vapour pressure at temperature T (Pa)
T_a	Air temperature at reference height ($^{\circ}\text{C}$)
T_m	Air temperature at canopy source height ($^{\circ}\text{C}$)
$T_{c,i}$	Surface temperature of the unit area of exchange surface i ($^{\circ}\text{C}$)
u_a	Wind speed at reference height (m s^{-1})
u^*	Friction velocity (m s^{-1})
k	von Karman's constant (0.41)
C_p	Specific heat of air at constant pressure ($\text{J kg}^{-1}\text{K}^{-1}$)
ρ	Air density (kg m^{-3})
γ	Psychrometric constant (Pa K^{-1})
Δ	Slope of the saturated vapour pressure curve at air temperature (Pa K^{-1})
Canopy physical characteristics:	
d	Zero plane displacement height (m)
L_t	Leaf area index of the whole canopy ($\text{m}^2 \text{m}^{-2}$)
S_t	Canopy exchange surface area per unit area of soil ($\text{m}^2 \text{m}^{-2}$)
ΔL_i	Leaf area of the vegetation layer i with width Δz_i ($\text{m}^2 \text{m}^{-2}$)
$r_{a,0}$	Aerodynamic resistance between the source height and the reference height (s m^{-1})
$r_{a,i}$	Boundary-layer resistance for sensible heat and water vapour of the unit area of exchange surface i (s m^{-1})
$r_{s,i}$	Surface resistance per unit area of exchange surface (s m^{-1})
$r_{a,l}$	Boundary-layer resistance for sensible heat and water vapour of the unit area of leaf (one side) (s m^{-1})
$r_{s,l}$	Leaf stomatal resistance per unit area of leaf (one side) (s m^{-1})
$r_{s,l,n}$	Minimal leaf stomatal resistance (Eq. C8) (s m^{-1})
$r_{a,s}$	Air resistance between the soil surface and the canopy source height (s m^{-1})
$r_{s,s}$	Soil surface resistance to evaporation per unit area of soil (s m^{-1})
$r_{a,c}$	Bulk air resistance of the canopy (s m^{-1})
$r_{s,c}$	Bulk surface resistance of the canopy (s m^{-1})
Z_r	Reference height (m)
Z_h	Mean canopy height (m)
Z_m	Mean canopy source height ($= d + z_0$) (m)
Z_0	Canopy roughness length for momentum (m)
$Z_{0,h}$	Canopy roughness length for sensible and latent heat (m)

**Combination
equations for canopy
evaporation**

J. P. Lhomme and
C. Montes

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

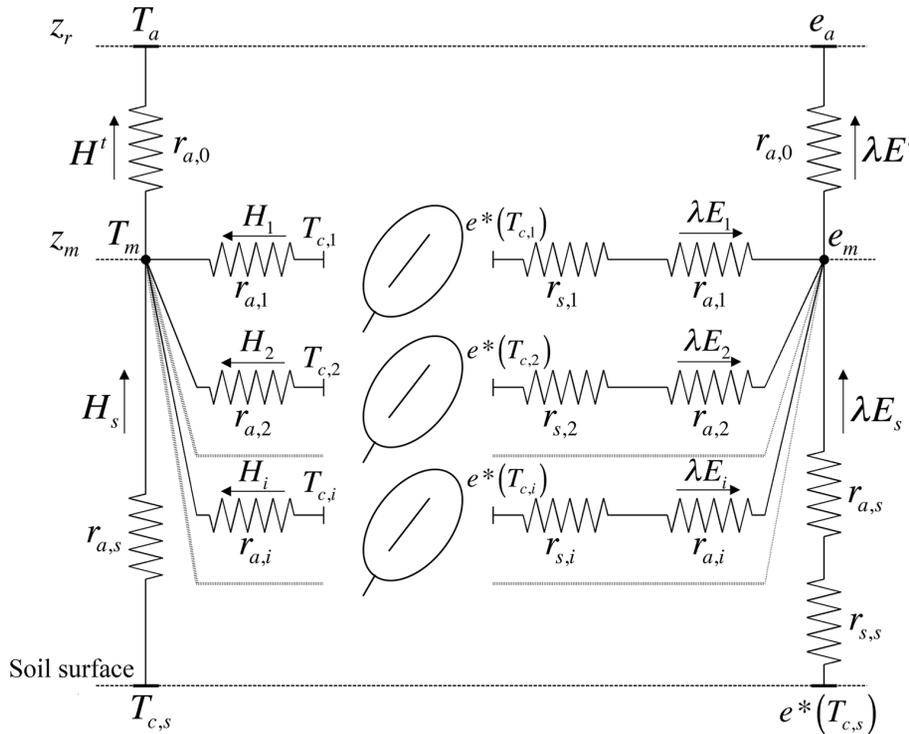


Fig. 1. Resistance network and potentials for a canopy represented by its elementary exchange surfaces (see list of symbols). All the component fluxes (sensible heat H_i and latent heat λE_i) converge at canopy source height (z_m). $T_{c,s}$ is soil surface temperature.

Title Page	
Abstract	Introduction
Conclusions	References
Tables	Figures
◀	▶
◀	▶
Back	Close
Full Screen / Esc	
Printer-friendly Version	
Interactive Discussion	



Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

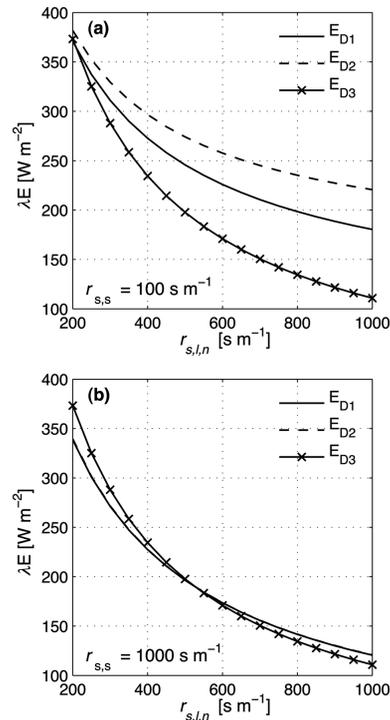


Fig. 2. Latent heat flux (λE) from a dry canopy as a function of the minimal stomatal resistance ($r_{s,l,n}$) (Eq. C8) representing the canopy water stress. Comparison of three formulations for two different values of soil surface resistance: **(a)** $r_{s,s} = 100 \text{ s m}^{-1}$; **(b)** $r_{s,s} = 1000 \text{ s m}^{-1}$. E_{D1} is the most complete formulation given by Eq. (7); E_{D2} is a simplified formulation represented by Eqs. (13), (20) and (21), where soil surface is taken into account; E_{D3} is the common Penman–Monteith equation, where soil surface is ignored (Eqs. 9 and 10).

[Title Page](#)
[Abstract](#)
[Introduction](#)
[Conclusions](#)
[References](#)
[Tables](#)
[Figures](#)
[⏪](#)
[⏩](#)
[◀](#)
[▶](#)
[Back](#)
[Close](#)
[Full Screen / Esc](#)
[Printer-friendly Version](#)
[Interactive Discussion](#)

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

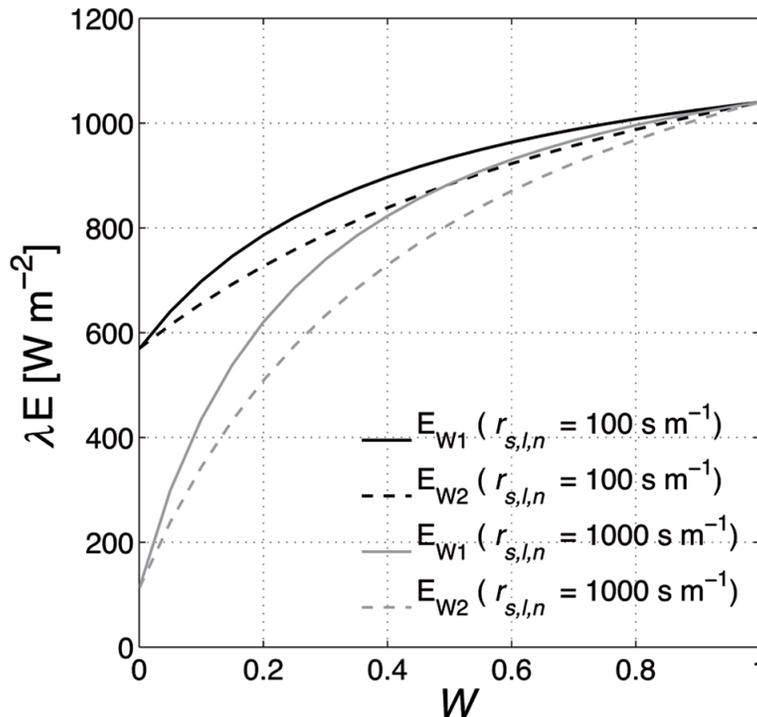


Fig. 3. Latent heat flux (λE) from a partially wet canopy as a function of its fractional surface wetness (W). Comparison of two formulations for two different values of the minimal stomatal resistance $r_{s,l,n}$ representing the canopy water stress: E_{W1} is the general formulation given by Eq. (28); E_{W2} is the simplified formulation given by Eqs. (33) and (34). Soil surface resistance $r_{s,s}$ is set to 500 s m^{-1} .

[Title Page](#)
[Abstract](#)
[Introduction](#)
[Conclusions](#)
[References](#)
[Tables](#)
[Figures](#)
[⏪](#)
[⏩](#)
[◀](#)
[▶](#)
[Back](#)
[Close](#)
[Full Screen / Esc](#)
[Printer-friendly Version](#)
[Interactive Discussion](#)

Combination equations for canopy evaporation

J. P. Lhomme and
C. Montes

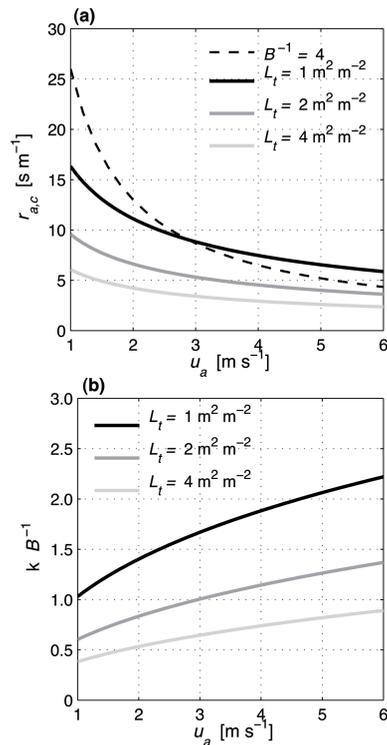


Fig. 4. (a) Additional aerodynamic resistance ($r_{a,c}$) given by Eq. (20) is plotted as a function of wind speed at reference height (u_a) for different LAI and compared with the rough estimate based on $B^{-1} = 4$; (b) the bulk parameter kB^{-1} (inferred from the value of $r_{a,c}$) is plotted as a function of u_a for different LAI (L_t).