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Hydroeconomic optimization of water management in the Yellow River Basin: dealing with scarcity

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Table S1. Key equations used in the YRB-HEM.

Equations	**Variable**	**Parameter**	**Description**
Objective function			
$Max\ Z = \sum_t \sum_r HP_{r,t} + \sum_y \sum_i AG_{i,y} + \sum_y \sum_c UR_{c,y} - \sum_y \sum_i Cost_{PMP}^{i,y} - \sum_t \sum_g Cost_{Pump}^{g,t} - \sum_t \sum_d Cost_{water}^{d,t}$	Z		Objective function: total net benefit (CNY million)
	$HP_{r,t}$		Hydropower revenue for reservoir r in time period t (CNY million/month)
	$AG_{i,y}$		Irrigated crop production revenue on irrigation area i in year y (CNY million/year)
	$UR_{c,y}$		Urban water use benefit in city c in year y (CNY million/year)
	$Cost_{PMP}^{i,y}$		Crop production cost in irrigation area i in year y estimated using PMP approach (CNY million/year)
	$Cost_{Pump}^{g,t}$		Pumping cost from groundwater aquifer g in time period t (CNY million/month)
	$Cost_{water}^{d,t}$		Water resource fees paid by water demand node d in time period t (CNY million/month)
Benefit			
$HP_{r,t} = P_{r,t} \cdot p_{HP_r} \cdot \varphi_h$		φ_h	Unit conversion factor
	$P_{r,t}$		Solved hydropower output (MW)
$P_{r,t} = K_r \cdot QT_t^r \cdot (H_t^r - T_r)$	QT_t^r		Solved monthly average flow through hydropower turbine (MCM/month)
	H_t^r		Solved reservoir water table (m.a.s.l.)
		T_r	Tailwater elevation (m.a.s.l.)
		p_{HP_r}	Price of electricity (CNY/kWh)
		K_r	Hydropower output coefficient, the product of hydropower generation efficiency, water density and gravitational acceleration ($kg \cdot m^{-2} \cdot s^{-2}$)

$AG_{i,y} = \sum_{cp} RY^i_{cp,y} \cdot Ymax^i_{cp} \cdot A^i_{cp,y} \cdot p_{AG\,cp} \cdot \varphi_a$	$RY^i_{cp,y}$		Solved relative yield for crop cp at irrigation district i in year y
		$Ymax^i_{cp}$	Maximum yield (mt/ha)
	$A^i_{cp,y}$		Harvested area (ha/year)
		$p_{AG\,cp}$	Crop price (CNY/mt)
		φ_a	Unit conversion factor
$UR_{c,y} = \sum_{c}\sum_{k} CS^{k,y}_c + \sum_{c}\sum_{k} (p^{k,y}_c - cost^{k,y}_c) Q^{k,y}_c$	$CS^{k,y}_c$		Consumer surplus for city c and industrial or domestic water use activity k (CNY million/year)
$p^{k,y}_c = \alpha^k_c - \beta^k_c \cdot Q^{k,y}_c$	$p^{k,y}_c$		Urban water tariff (CNY·m^{-3}·year^{-1})
		$cost^{k,y}_c$	Cost from suppliers providing water to users (CNY·m^{-3}·year^{-1})
	$Q^{k,y}_c$		Industrial or urban domestic water use (MCM/year)
$CS^{k,y}_c = \int_{p^{k,y}_c}^{p_{max}} \left(\alpha^k_c - \beta^k_c \cdot Q^{k,y}_c\right) dQ = \frac{1}{2} Q^{k,y}_c (\alpha^k_c - p^{k,y}_c)$		α^k_c, β^k_c	Intercept and the slope of the demand function
Cost			
$Cost^{g,t}_{Pump} = \frac{\rho G}{\eta} \cdot H^g_t \cdot Q^g_t \cdot p_{GW} \cdot \varphi_p$		ρ	Water density (kg/m^3)
		G	Gravitational acceleration (9.81 m/s^2)
		η	Pumping efficiency
	H^g_t		Solved pumping depth at groundwater aquifer g (m/month), constrained by minimum allowable water-table level
	Q^g_t		Solved groundwater withdrawal (MCM/month)
		p_{GW}	Pumping cost (CNY/kWh)
		φ_p	Unit conversion factor
$Cost^{i,y}_{PMP} = \sum_{cp} (\mu^i_{cp} + 0.5\varphi^i_{cp} \cdot A^i_{cp,y}) A^i_{cp,y}$		$\mu^i_{cp}, \varphi^i_{cp}$	Intercept and slope for crop cp in irrigation district i
$Cost^{d,t}_{water} = \sum_{nu} cost^d_{sur} \cdot {Q_{sur}}^{nu,d}_t + \sum_{g} cost^d_{gw} \cdot Q^{g,d}_t$		$cost^d_{sur}$	Surface water resources fees (CNY/m^3)
		$cost^d_{gw}$	Groundwater water resources fees (CNY/m^3)

	${Q_{sur}}_t^{nu,d}$ $Q_t^{g,d}$		Solved total surface water used (MCM/month) Solved total groundwater used (MCM/month)
Constraints			
Flow Balance			
$S_t^n - S_{t-1}^n = \sum_n Q_t^{nu,n} - \sum_n Q_t^{n,nd}$	S_t^n $Q_t^{nu,n}$ $Q_t^{n,nd}$		Storage of node n at the end of period t (MCM/month) Inflow from upstream node nu to node n (MCM/month) Outflow from node n to downstream node nd (MCM/month)
Water Delivery			
$Q_t^d = \sum_{nu} {Q_{sur}}_t^{nu,d} + \sum_g Q_t^{g,d}$	Q_t^d		Total water delivery to node *n* (MCM/month)
Irrigation			
Water consumption $Q_t^i \cdot e_i = \sum_{cp} IWD_{cp,t}^i \cdot A_{cp,y}^i \cdot \varphi_i$ Return flow ${Q_{RF}}_t^{i,nd} = Q_t^i \cdot I_r^i$	Q_t^i $IWD_{cp,t}^i$ ${Q_{RF}}_t^{i,nd}$	e_i φ_i I_r^i	Irrigation water delivery (MCM/month) Irrigation efficiency Irrigation water demand requirement (mm/month) Unit conversion factor Irrigation return flow (MCM/month) Coefficient of irrigation return flow
Water demand $IWD_{cp,t}^i = \begin{cases} ETc_{cp,t}^i - Pe_{cp,t}^i, & ETc_{cp,t}^i > Pe_{cp,t}^i \\ 0, & ETc_{cp,t}^i \leq Pe_{cp,t}^i \end{cases}$	$ETc_{cp,t}^i$	$Pe_{cp,t}^i$	Potential crop evapotranspiration (mm/month) Effective precipitation (mm/month)
Crop evapotranspiration FAO: $ET_c = k_c \cdot ET_0 \cdot \varphi_c$	ET_0	k_c φ_c	Crop coefficient Reference-crop evapotranspiration (mm/day) Unit conversion factor

Priestly-Taylor: $$ET_0 = \alpha \frac{\Delta}{\Delta + \gamma}(R_n - G)$$		α Δ γ R_n G	Priestly-Taylor coefficient (default is 0.23 for surface covered with short reference crop) Gradient of saturated vapor pressure (kPa/°C) Psychrometric constant (kPa/°C) Net shortwave solar radiation (MJ·m^{-2}·day^{-1}) Ground heat flux (MJ·m^{-2}·day^{-1})
Actual evapotranspiration $$\theta \cdot ETc_{cp,t}^{i} \leq ETa_{cp,t}^{i} \leq ETc_{cp,t}^{i}$$	$ETa_{cp,t}^{i}$	 θ	Actual crop evapotranspiration (mm/month) Minimum evapotranspiration factor
Relative yield $$RY_{cp,y}^{i} = \prod_{m=1}^{M} \left(\frac{ETa_{cp,t}^{i}}{ETc_{cp,t}^{i}}\right)^{\lambda_{cp}^{m}}$$		λ_{cp}^{m}	Sensitivity factor at growing stage m of crop cp. The growing stage m represents the order of months within the growing period of crop cp in irrigation area i, which is harvested in year y.
Reservoir			
Evaporation $$ER_t^r = Emax_t^r \cdot A_t^r \cdot \varphi_r$$	ER_t^r A_t^r	 $Emax_t^r$ φ_r	Solved reservoir evaporation (MCM/month) Open water evapotranspiration rate at reservoir r (mm/month) Solved reservoir surface area (km^2/month) Unit conversion factor
Release $$\sum_r Q_t^{r,nd} = QT_t^r + QS_t^r$$	QS_t^r		Reservoir spill (MCM/month)
Elevation vs. storage $$H_t^r = {h_1}^r + {h_2}^r \cdot (\frac{S_t^r + S_{t-1}^n}{2})^{{h_3}^r}$$ Area vs. storage $$A_t^r = {a_1}^r + {a_2}^r \cdot (\frac{S_t^r + S_{t-1}^r}{2})^{{a_3}^r}$$		${h_1}^r, {h_2}^r, {h_3}^r$ ${a_1}^r, {a_2}^r, {a_3}^r$	Reservoir water table curve coefficients (elevation vs. storage) Reservoir area curve coefficients (area vs. storage)

Range control $$S_{dead}^{r} \le S_{t}^{r} \le S_{max}^{r,t}$$ $$P_{min}^{r} \le P_{r,t} \le P_{max}^{r}$$ $$Q_{t}^{r,nd} \ge Q_{control,t}^{r,nd}$$ $$H_{t}^{r} \ge H_{control,t}^{r}$$		$S_{dead}^{r}, S_{max}^{r,t}$ P_{min}^{r}, P_{max}^{r} $Q_{control,t}^{r,nd}$ $H_{control,t}^{r}$	Dead storage (MCM) and allowable storage upper limit (MCM) for reservoir r Firm power and installed hydropower capacity at powerplant r (MW) Minimum reservoir release for ice flood control and sediment flushing requirements (MCM/month) Minimum reservoir water table for ice flood and sediment flushing requirements (m.a.s.l.)
City			
Water use control $$D_{min}^{k,y} \le Q_{c}^{k,y} \le D_{max}^{k,y}$$		$D_{max}^{k,y}, D_{min}^{k,y}$	Maximum and minimum industrial or municipal water demand (MCM/month)
Ecosystem			
Replenishment control $$Q_{min}^{e,t} \le Q_{t}^{e} \le Q_{max}^{e,t}$$	Q_{t}^{e}	 $Q_{min}^{e,t}$ $Q_{max}^{e,t}$	Delivered ecosystem flow to wetland area e (MCM/month) Ecosystem flow lower limit (MCM/month) Ecosystem flow upper limit (MCM/month)

Notes:

1. Node types: q-surface inflow; g-groundwater; r-reservoir; i-irrigation area; c-city; e-ecosystem; j-junction; s-sink; d-demand nodes; n-storage, conveyance and junction.
2. Year index value y can be determined using the time period value t (month sequence, t = 1, …, 240 in this study): y = int(ord(t - 1) / 12) + 1, where “int” is the floor function that rounds down a fractional number to its nearest integer, and “ord” means ordinal number.
3. Units: MCM-million cubic meters; m.a.s.l.-meters above sea level.

Table S2-1 to **S2-6** provide details of each type of nodes, and related location information.

Table S2-1. Irrigation nodes, names, and provinces.

IRND	Irrigated Area	Province
I1	D02_QH_HuangShui	Qinghai
I2	D03_NX_WeiNing	Ningxia
I3	D03_NX_Qingtongxia	
I4	D03_NM_Hetao	Inner Mongolia
I5	D03_NM_Nanan	
I6	D03_NM_TMC	
I7	D05_SX_Fenhe	Shanxi
I8	D05_SN_Weihe	Shaanxi
I9	D06_HN_Xiaolangdi	Henan
I10	D06_HN_Yinqin	
I11	D07_HN_Henan	
I12	D07_SD_Shandong	Shandong
I13	D02_GS	Gansu
I14	D03_GS	
I15	D04_NM	Inner Mongolia
I16	D04_SN	Shaanxi
I17	D04_SX	Shanxi
I18	D05_GS	Gansu
I19	D05_HN	Henan
I20	D05_NX	Ningxia
I21	D06_HN	Henan
I22	D06_SX	Shanxi

Table S2-2. Secondary water resources regionalization of the YRB. Regionalization was delineated by official authorities, considering both natural characteristics and administrative boundaries. Endorheic basin areas are excluded from this study.

Code	Region
D01	Above Longyangxia
D02	Longyangxia to Lanzhou
D03	Lanzhou to Hekouzhen
D04	Hekouzhen to Longmen
D05	Longmen to Sanmenxia
D06	Sanmenxia to HuaCNYkou
D07	Below HuaCNYkou

Table S2-3. City nodes, names, and provinces.

CND	City	Province
XN	Xining	Qinghai
LZ	Lanzhou	Gansu
BY	Baiyin	
YC	Yinchuan	Ningxia
ZW	Zhongwei	
WZ	Wuzhong	
SZS	Shizuishan	
HHHT	Hohhot	Inner Mongolia
BYNE	Bayan Nur	
BT	Baotou	
EEDS	Ordos	
TY	TaiCNY	Shanxi
JINZ	Jinzhong	
DT	Datong	
SZ	Shuozhou	
LL	Lvliang	
YUNC	Yuncheng	
LF	Linfen	
XA	Xi'an	Shaanxi
XY	Xianyang	
YL	Yulin	
YA	Yan'an	
WNA	Weinan	
ZZ	Zhengzhou	Henan
LY	Luoyang	
JZ	Jiaozuo	
XX	Xinxiang	
KF	Kaifeng	
PY	Puyang	
JN	Jinan	Shandong
LC	Liaocheng	
TA	Taian	
BZ	Binzhou	
DY	Dongying	

Table S2-4. Ecosystem nodes, sites, and provinces.

END	Site	Province
XHH	Xinghai Lake	Ningxia
WLSH	Wuliangsu Lake	Inner Mongolia
DaiH	Daihai Lake	
BYD	Baiyangdian Lake	Hebei
YRD	Yellow River Delta	Shandong

Table S2-5. Inflow nodes, names of tributaries, and locations.

QND	Tributary	Location	Notes
TP	Tibetan Plateau	Upper reaches	Mainstream inflow to Longyangxia Reservoir
TH	Taohe		Tributary
DTH	Datonghe		
HS	Huangshui		
DHH	Daheihe		
KYH	Kuyehe	Middle reaches	
WDH	Wudinghe		
FH	Fenhe		
JH	Jinghe		
WH	Weihe		
YLH	Yiluohe	Lower reaches	
QH	Qinhe		
DWH	Dawenhe		

Table S2-6. Reservoir nodes, names and provinces.

RND	Reservoir	Province
LYX	Longyangxia	Qinghai
LiJX	Lijiaxia	
LiuJX	Liujiaxia	Gansu
YGX	Yanguoxia	
BPX	Bapanxia	
DX	Daxia	
QTX	Qingtongxia	Ningxia
WJZ	Wanjiazhai	Shanxi
SMX	Sanmenxia	Henan
XLD	Xiaolangdi	

Table S3. Crop calendar with sensitivity factor for all irrigated crops in the YRB.

Crop	Months											
	1	2	3	4	5	6	7	8	9	10	11	12
Spring wheat			0.04	0.13	0.57	0.35	0.37					
Winter wheat	0.12	0.10	0.18	0.32	0.24					0.14	0.11	0.12
Rice	0.29				0.36	1.05	0.95	0.60	0.58	0.60	0.58	0.60
Spring maize				0.11	0.75	1.40	0.26	0.22	0.09			
Summer maize					0.02	0.92	1.39	0.33	0.16	0.01		
Sorghum					0.08	0.35	0.35	0.25	0.11			
Other cereals				0.11	0.58	0.21	0.21	0.31				
Potato					0.52	0.67	0.43	0.23	0.04			
Bean					0.56	0.93	0.51	0.14				
Lentil				0.53	0.89	0.34	0.26	0.11				
Other pulses			0.13	0.90	0.61	0.14						
Soybean					0.12	0.85	0.29	0.52	0.25	0.01		
Groundnut					0.13	0.54	0.54	0.28	0.08			
Sunflower					0.27	0.36	0.68	0.84	0.21			
Rapeseed	0.71	0.54								0.27	0.27	0.56
Cotton					0.16	0.25	0.25	0.24	0.17	0.11		
Vegetable	0.12	0.11	0.56							0.20	0.25	0.14
Tomato	0.45								0.13	0.21	0.24	0.34
Melon				0.28	0.79	0.64	0.46	0.35	0.02			
Rest				0.74	0.76	0.56	0.15					

Table S4. Model performance for major hydrological gauge stations, reservoirs' end-of-year storage values, and agricultural water use against observed data, assessed with Percent Bias (PBIAS) and Normalized Root Mean Square Error (NRMSE).

Category	**Name**	**PBIAS (%)**	**NRMSE (%)**	**Period**
Hydrological station	Lanzhou	-5.48	15.99	1996-2015
	Xiaheyan	-9.61	18.50	
	Tongguan	10.22	21.34	
	HuaCNYkou	-0.99	18.69	
Reservoir storage	Longyangxia Reservoir	3.80	25.79	1997-2015
	Xiaolangdi Reservoir	0.40	25.96	2005-2015
Agricultural water use	Total irrigation withdrawal	7.66	10.58	1996-2015

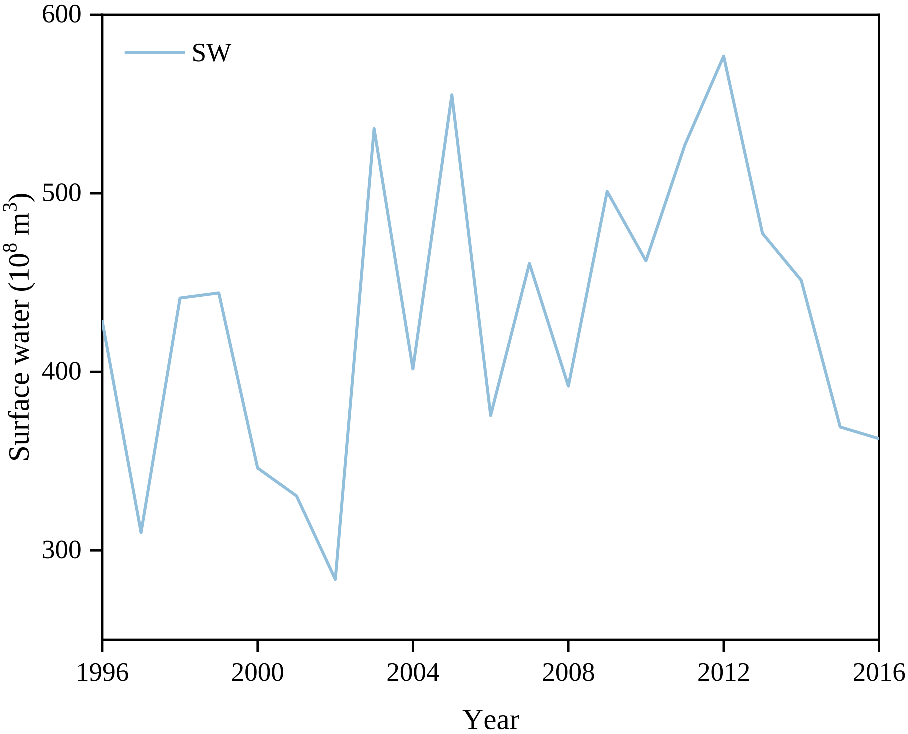

Fig. S1. Total surface water (SW) inflow during 1996 to 2016 in the YRB.

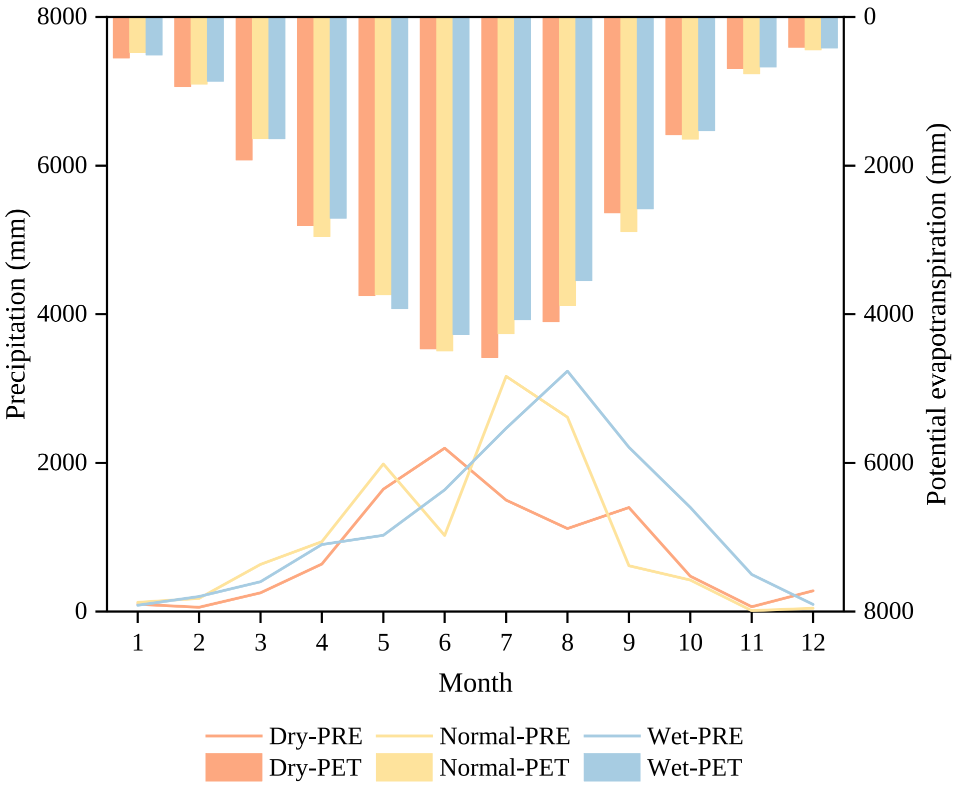

Fig. S2. Monthly precipitation (PRE) and reference evapotranspiration (PET) by hydrological year type, averaged over the entire YRB.

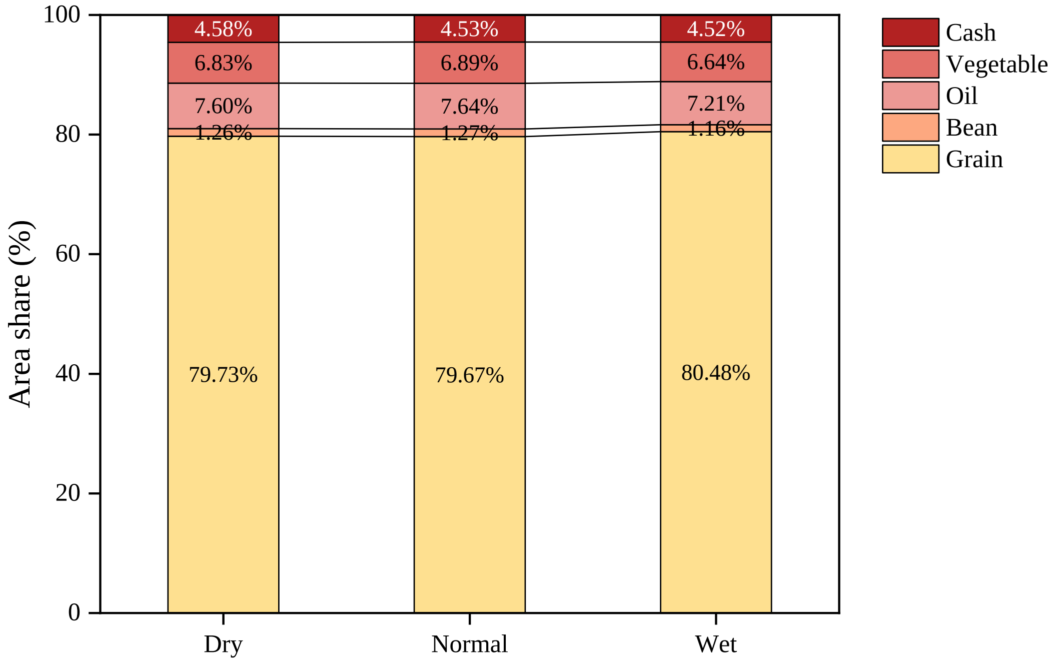

Fig. S3. Cropping patterns in the YRB by hydrological year type.

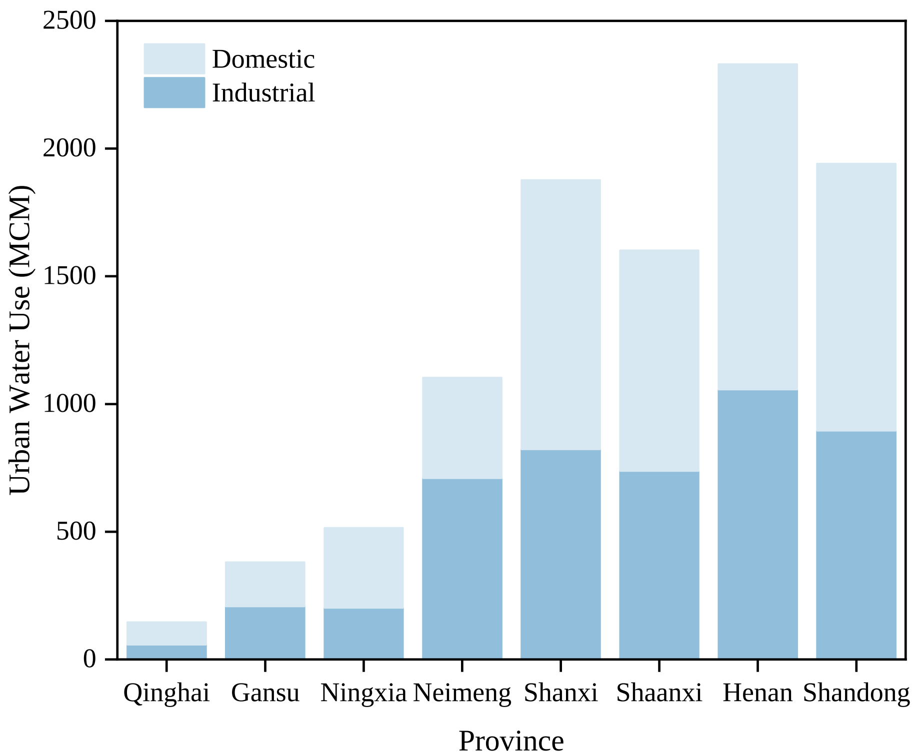

Fig. S4. Annual urban water uses by sector and province in the YRB.

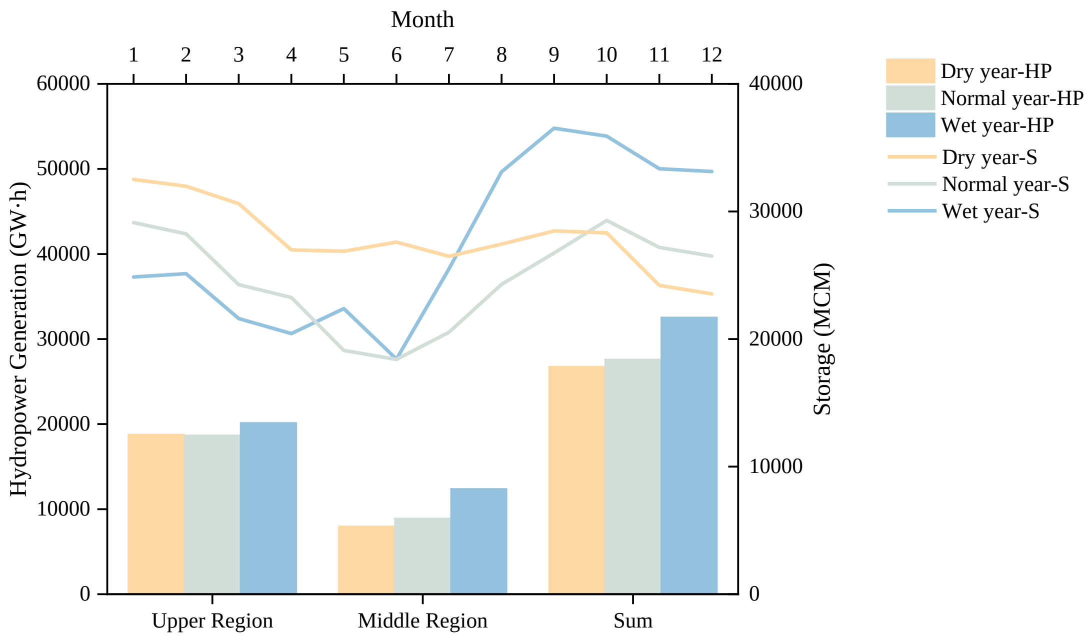

Fig. S5. Annual hydropower generation (HP) and monthly water storage (S) in the YRB, by hydrological year type.