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Rating curve estimation using local stages, upstream discharge data and a simplified hydraulic model

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Abstract

This article proposes a methodology for synthesising the rating curve in one or more cross-sections of a watercourse provided with stage data, when a reliable rating curve and stage data are also available in the upstream cross-section; the synthesised rating curves are consistent with each other. The proposed methodology uses a variable parameter Muskingum-Cunge model whose parameters take express account of travel times and attenuation of the flood wave, and are expressed in such a way that allows for an integration in the time-space domain even when a topographic survey of the river is not available. Furthermore, the methodology proposed implicitly provides a ready-calibrated simulation model whose ease of application suggests that it could also be useful in real time stage forecasting. The paper includes a description of a numerical application to a reach of the Po River (Italy).

Introduction

Knowledge of the rating curve in one or more cross-sections of a watercourse is essential to make hydrological balances or to calibrate both flood propagation models or rainfall-runoff models. However, estimation of the rating curve presupposes a complex and extensive measuring survey in which the chief difficulty lies in the discharge measurements which, irrespective of the technique adopted, must be repeated frequently. In fact, the rating curve may vary over time as a result of changes which occur, for example, because of seasonal changes in vegetation, the passage of significant flood waves, or human intervention; the updating of the rating curve also carries a high cost in terms of the measurements themselves and the maintenance of the service.

Discharge measurements are usually carried out in low and medium flow conditions since they may prove dangerous and/or impossible during flood events; accordingly, the extension of the rating curve to the highest discharge values is usually performed either by extrapolation (by fitting a curve to the measuring points) or on the basis of a series of uniform flow simulations. However, it would be more appropriate to carry out this extension when, for example, the rating curve is known in one cross-section and the recorded level measurements in others, using a gradually varied unsteady flow hydraulic model, based on knowledge of the geometry of the river, in which roughness is the only parameter that can be calibrated. Unfortunately, the topographical survey is frequently missing or out of date due to the high costs connected with performing it.

This article presents a method which obviates the need to know the geometry of the watercourse and allows the rating curve to be estimated in one or more cross-sections when, specifically, (a) the stages are provided by a water level recorder (this is not rare nowadays, given the relative ease with which instruments of this type can be installed and operated) and (b) the rating curve is also available in the upstream cross-section of the reach in question.

The proposed method uses a variable parameter Muskingum-Cunge (VPMC) model (Cunge, 1969) parameterised in a slightly different way with respect to the standard formulation. This model, being hydraulic in nature, allows for its parameters to be estimated on the basis of physical considerations to take account of the space-time dynamic of the propagation event, even in situations such as that examined in this paper, in which a reliable survey of the geometry of the watercourse is not available.

Definition of the problem

Let us take a river reach of length L for which the following statements hold true:

(a) the flow is sub-critical;
(b) the geometry and roughness are not known;
(c) there are no concentrated tributaries or distributed inflows;
(d) in several cross-sections, and particularly the outermost sections, recorded stage values $h(t)$, usually referring to a local gauge zero, are available;
(e) the reach under examination is not affected by upstream/downstream conditions (weirs, sluices, etc.) that might influence flow conditions;
(f) the flood wave propagation is of the quasi-kinematic type;
(g) the upstream cross-section has a reliable rating curve over the calibration period.

It is worth noting that point (e) ensures the existence in each cross-section of a one to one relationship between discharge and stages; furthermore, point (f) implies the presence of a (very) narrow rating loop.

The purpose of this study is to identify the rating curve in cross-sections equipped with water level recorders by using a VPMC model whose parameters take express account of travel times and attenuation of the flood wave. The methodology proposed here is similar to that presented by Franchini and Lamberti (1994) whose sole purpose, however, since it referred to a stretch without reliable rating curves, was to create a hydraulic model for the computation of the levels in the downstream cross-section once the upstream section levels were known. It differs, however, from the methodology proposed by Birkhead and James (1998); indeed, whilst referring to a problem similar to the one described above, they suggest a methodology based on the *hydrological* formulation of the Muskingum model which allows the indirect parameterisation of the rating curve of given equation by imposing compliance with the function of the volume enclosed between the two outermost cross-sections of the sub-reach considered, which is assumed to be prismatic.

A variable parameter Muskingum-Cunge formulation

The Muskingum model in its standard hydrological formulation referring to a stretch without lateral inflow, is frequently written:

$$Q_{t+\Delta t} = C_1 I_{t+\Delta t} + C_2 I_t + C_3 Q_t \tag{1}$$

with :

$$C_1 = \frac{-KX + 0.5\Delta t}{K - KX + 0.5\Delta t};\quad C_2 \frac{KX + 0.5\Delta t}{K - KX + 0.5\Delta t};\quad C_3 = \frac{K - KX - 0.5\Delta t}{K - KX + 0.5\Delta t} \tag{2a}$$

$$C_1 + C_2 + C_3 = 1 \tag{2b}$$

where I is the upstream discharge, Q is the downstream discharge, K and X are the Muskingum model parameters, and Δt is the simulation time step. Equation (2b) ensures the conservation of mass.

Referring to the time-space computational grid in the x–t plane, the Muskingum routing equation can be written for the discharge at position $x = (i + 1)\Delta x$ and the time $t = (j + 1)\Delta t$ as:

$$[Q]_{i+1}^{j+1} = C_1[Q]_i^{j+1} + C_2[Q]_i^j + C_3[Q]_{i+1}^j \tag{3}$$

where C_1, C_2 and C_3 are as defined in Eqn. (2a).

Cunge (1969) showed that Eqn. (3) is an approximate solution to the continuous kinematic wave when K and X are taken as constant. He further demonstrated that Eqn. (3) can be considered also an approximate solution of a modified diffusion equation when:

$$K = \frac{\Delta x}{c_k};\quad X = \frac{1}{2}\left(1 - \frac{Q}{Bc_k S_w \Delta x}\right) \tag{4}$$

where B is the width of the water surface, $c_k = dQ/dA$ the kinematic celerity, A the wetted area, S_w is the slope of the water profile.

The Muskingum-Cunge formulation (i.e. Eqns. (3), (2a) and (4)), allows for the updating of the parameters K and X for each time-space computational step and for this reason will be referred to as the Variable Parameter Muskingum-Cunge (VPMC), as already mentioned.

As demonstrated in NERC (1975), this model tends to show less wave attenuation than any direct numerical approximation of the continuous kinematic wave. In other words, the VPMC may be considered appropriate also for describing the flood waves characterised by narrow rating loops, such as those considered in the present study. However, in the present study, a modified formulation of the three coefficients C_1, C_2 and C_3 is preferred:

$$C_1 = \frac{-1 + r_1 + r_2}{1 + r_1 + r_2};\quad C_2 = \frac{1 + r_1 - r_2}{1 + r_1 + r_2};\quad C_3 = \frac{1 - r_1 + r_2}{1 + r_1 + r_2};$$
$$r_1 = c_k \frac{\Delta t}{\Delta x};\qquad r_2 = 2\frac{\lambda_0}{\Delta x};\qquad \lambda_0 = \frac{Q}{2S_w c_k B} \tag{5}$$

This formulation intends to highlight the parameters c_k (kinematic celerity) and λ_o (characteristic length) (Miller and Cunge, 1975) which have a well-defined physical meaning and are strictly related to the hydraulic condition of the flow. In particular, λ_o may be interpreted as the length of the stream reach where the water level drops by ⅓ of the uniform flow depth y; in fact (see Fig. 1):

$$\Delta y = \lambda_0 S_0 = \lambda_0 S_w = \frac{Q}{2c_k B} \cong \frac{VA}{2(1.5V)B} = \frac{y_0}{3} \tag{6}$$

where V is the mean water velocity and S_o the bottom slope.

For the sake of numerical stability, the three coefficients C_1, C_2, C_3 must be less than one in absolute value. Furthermore, to prevent numerical oscillations and thus avoid the classic 'Muskingum effect' (increase in discharge

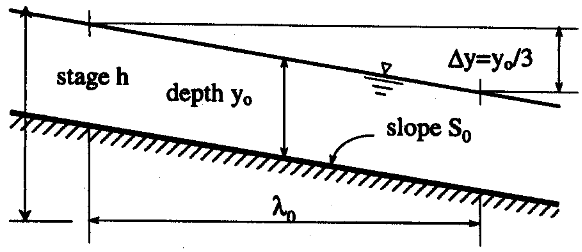

Fig. 1. *Geometrical representation of the meaning of λ_0 in a prismatic channel in uniform flow conditions. S_0: slope of bottom matching the water profile slope S_w.*

upstream which translates into a decrease in the discharge downstream), the time step Δt and the corresponding space step Δx must be selected in relation to the values of the parameters c_k and λ_0 so that the three ratios are greater than zero. In addition, in order to avoid excessive 'numerical diffusion' the ratio r_1 must be close to one. Thus, the following constraints must be respected:

$$|(c_k\Delta t - \Delta x)| < 2\lambda_0\,(a);\ (c_k\Delta t + \Delta x) > 2\lambda_0\,(b);$$
$$r_1 = c_k\Delta t\,/\,\Delta x \cong 1\,(c) \qquad (7)$$

Lastly, with regard to the parameters c_k and λ_0, these can be estimated as a function of the flow discharge calculated at the point (i,j) on the computational grid (explicit method).

Rating curve estimation methodology

In this section, it should be borne in mind that in quasi-kinematic conditions the discharge, Q, flowing in unsteady conditions is almost equal to that flowing in steady conditions, water levels being equal; thus, in any cross-sections, the discharges, Q, flowing in unsteady conditions can be related directly to the water level through the rating curve.

For a single stretch delimited by two water level recorders, it is assumed that: (a) the flood propagation can be represented exactly by the VPMC model and (b) its parameters c_k and λ_0, relating to the n Δx steps into which the stretch is divided, and the rating curve $Q(h)$ in the downstream cross-section are known. Thus, for a given flood wave in the upstream section, the discharges simulated in the downstream section (Q_{sim}) and the corresponding stages observed (h_{obs}) would be arranged on the above-mentioned rating curve.

If, on the other hand, the parameters c_k and λ_0 and the rating curve at the downstream section are not known, we could proceed as follows: let the parameters c_k and λ_0 be expressed as a function of the discharge Q using laws valid for the entire stretch; these relationships, referred to below as 'stretch-laws', can be 'parameterised' so that the discharge values Q_{sim} at the downstream cross-section, computed using the VPMC, are arranged on a one to one curve in relation to the corresponding stages observed; by its very nature, this curve constitutes an estimation of the rating curve in the downstream cross-section.

The reasoning set out above can be repeated, proceeding cascade-fashion from upstream to downstream, for any number of stretches delimited by cross-sections provided with recorders and can be used to parameterise both the rating curves of these cross-sections and the stretch-laws.

THE STRETCH-LAWS GOVERNING CELERITY AND CHARACTERISTIC LENGTH

The expressions for the stretch-laws adopted are as follows:

$$c_{k,i} \equiv c_{k,i}(Q,\boldsymbol{\alpha}) = \alpha_{i,1} + (\alpha_{i,2} - \alpha_{i,1})\left(\frac{Q-Q_l}{Q_r-Q_l}\right)^{\alpha_{i,3}};$$
$$\boldsymbol{\alpha} \equiv \alpha_{i,m}(m=1,2,3) \qquad (8)$$

$$\lambda_{0,i} \equiv \lambda_{0,i}(Q,\beta) = \beta_{i,1} + (\beta_{i,2} - \beta_{i,1})\left(\frac{Q-Q_l}{Q_r-Q_l}\right)^{\beta_{i,3}};$$
$$\boldsymbol{\beta} \equiv \beta_{i,m}(m=1,2,3) \qquad (9)$$

where the subscript i denotes the ith stretch, and $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the coefficient-vectors. Specifically, in Eqn. (8) $\alpha_{i,1}$ is the celerity value in the ith stretch corresponding to a small reference-discharge Q_l value (l: left), and $\alpha_{i,2}$ is the value for a high reference-discharge Q_r value (r: right); the coefficient $\alpha_{i,3}$ is the exponent in the power-law relationship. Similar observations hold good for the three coefficients present in Eqn. (9). Q_l and Q_r are determined *a priori* on the basis of the knowledge, *which may even be approximate*, of the range of the possible discharge values affecting the stretch in question. Note that, for obvious numerical reasons, Q_l must always be less than the smallest observed value of Q.

Numerical simulations, not shown here for reasons of space, show clearly that Eqns. (8) and (9) accurately represent the two parameters c_k and λ_0 for several flow conditions in channels with regular cross-sections. This representativeness diminishes as the geometrical irregularities of the cross-sections increase, especially in the presence of sudden widenings such as those of the flood plains. In spite of this limitation, the results which can be achieved, as shown by the numerical example, are of good quality.

As has been pointed out, in the application of the VPMC model the generic stretch is discretised in segments of length Δx in accordance with the relation (7c) and the Δt selected. The celerity and characteristic length of the stretch, as described by Eqns. (8) and (9), are thus updated in each integration time-space step according to the current values of Q.

THE OBJECTIVE FUNCTION AND CALIBRATION PROCEDURE

In the quasi-kinematic situation, it is possible to handle one stretch at a time, proceeding from upstream to downstream. Thus consider, by way of an example, the first upstream stretch, whose laws $c_{k,1}(Q,\alpha)$ and $\lambda_{0,1}(Q,\beta)$ are characterised by a total of six coefficients. The VPMC model, for any set of values of these coefficients and for a given upstream flood wave, provides at the downstream cross-section the discharge values $Q_{sim}(t,\alpha,\beta)$ (written this way to highlight their dependence on the value assigned to the coefficient-vectors α and β). The points $[Q_{sim}(t,\alpha,\beta), h_{obs}(t)]$ are then fitted by a rating curve $Q(h,\gamma)$, whose coefficient-vector γ is *estimated* using the least squares method (a possible equation for the rating curve may be, for instance, $Q = \gamma_1(h - \gamma_2)\gamma_3$; however, the general expression $Q(h,\gamma)$ is here preferred to highlight that the procedure is completely independent of the equation selected for the rating curve).

Once the vector γ has been estimated using the least-squares method, consider the errors $\varepsilon(t)$ at the downstream section:

$$\varepsilon(t) = Q_{sim}(t,\alpha,\beta) - Q(h_{obs}(t),\gamma) \qquad (10)$$

The objective function is thus expressed as:

$$OF = \sqrt{\sum_{l-1}^{N} \varepsilon^2(t = \ell\Delta t)/N} \qquad (11)$$

where ℓ spans stage values corresponding to N time intervals on the hydrograph. The OF function in itself represents the minimum value of the sum of the squares of the errors $\varepsilon(t)$, *given a particular set of values of the coefficient-vectors* α and β. It follows that the values of the coefficient-vectors α and β sought are those which *minimise* the *minimum value* of the sum of the squares of the errors $\varepsilon(t)$ as defined above. Once the *coefficient-vectors* α and β are found, the corresponding vector γ allows for the description of the rating curve sought.

This procedure, which can be performed automatically, produces the estimation of α, β and γ; however, the optimisation algorithm has to operate and control the elements of the first two vectors alone, while the vector γ is estimated at each optimisation step through the least square method applied to the points $[Q_{sim}(t,\alpha,\beta), h_{obs}(t)]$. In this study the optimisation algorithm chosen is SCE-UA (Duan *et al.*, 1992, 1993), since recent studies (Luce and Cundy, 1994, Franchini *et al.*, 1998) have shown that it offers a strong probability of success (identification of the global minimum) and is undoubtedly one of the most robust, effective and efficient algorithms to have been proposed in recent years.

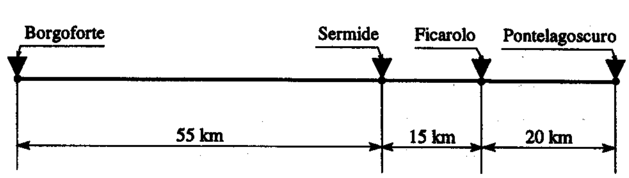

Fig. 2. *Schematic representation of the reach of the Po River (Italy).*

Numerical application: synthesising the rating curve in several cross-sections of the river Po

THE AVAILABLE DATA

Figure 2 is a schematic representation of a reach of the river Po highlighting the cross-sections at Borgoforte, Sermide, Ficarolo and Pontelagoscuro. Specifically, the Borgoforte section has a reliable rating curve for discharges of up to approximately 7000 m^3/s (see Fig. 3, observation period 1981–1995). The recorded level values $h(t)$, available for the four stations, cover the whole of 1994 at half-hourly time intervals. The period 11th August—31st December was used for the calibration since it was characterised by the widest ranges of $h(t)$ at all four stations. The preceding period was used for validation. The reference discharges Q_l and Q_r were set at 250 and 7000 m^3/s respectively and were deemed valid over the entire reach analysed [note that while the value 250 m^3/s is less than the smallest observed discharge, as required by Eqns. 8 and 9, the value 7000 assumed for Q_r is not bigger than the highest value observed (see Fig. 3).] Indeed, as already mentioned, the proposed procedure requires only an approximate estimation of the range of the possible discharge values affecting the stretch in question.

Between the Borgoforte and Pontelagoscuro sections, there are three main tributaries (Secchia, Panaro and

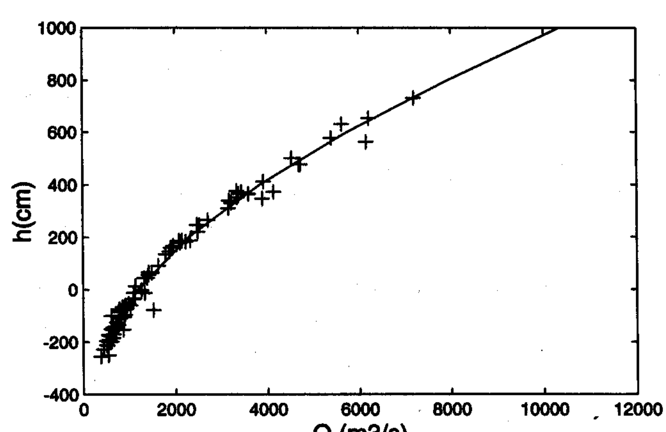

Fig. 3. *Rating curve in the Borgoforte section (solid line) estimated using the least squares method in relation to the measuring points for the period 1981–1995 (+).*

Mincio) and several structures capable of introducing or withdrawing discharge quantities which are negligible except at specific times. For the year 1994, and this is the reason for its selection, all these contributions (positive and negative) were very small with respect to discharges flowing in the Po river and so they were disregarded. Thus, this set of data was used to describe the methodology proposed for application to river reaches without lateral inflow.

CALIBRATION

The calibration of the stretch-laws and rating curves is described from upstream to downstream. The procedure is, however, one and the same in all cases and therefore the relevant details are provided only for the first Borgoforte-Sermide stretch.

First stretch—synthesising the rating curve for the Sermide station

The SCE-UA algorithm requires the delimitation of the search range for each of the elements of the two coefficient-vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, which represent the variables on which it operates to minimise the objective function of Eqn. (11) *OF*. With regard to the stretch-law $c_{k,1}(Q,\boldsymbol{\alpha})$, an analysis of the travel times of the stages in the stretch delimited by the two cross-sections at Borgoforte and Sermide suggested using the ranges (0.3–1.8 m/s) and (1.3–3.5 m/s) for the celerity in low and high flow conditions respectively. The range (0.1–2.0) was adopted for the exponent according to the values observed in simulations that allowed for the selection of the type of equation for the coefficient-vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

With regard to the stretch-law $\lambda_{0,1}(Q, \boldsymbol{\beta})$, the following should be noted. The parameter λ_0 according to its definition in Eqn. (5) and using the definition of kinematic celerity, can be written as:

$$\lambda_0 = \frac{Q}{2S_w}\frac{dh}{dQ} \qquad (12)$$

An initial estimation of the law $\lambda_{0,1}(Q,\boldsymbol{\beta})$ can therefore be made using Eqn. (12) on the basis of the known rating curve in the upstream section (Borgoforte) and a water profile slope 'typical' of the stretch (for example, inferred from the recorded level data at the ends of the stretch), thus obtaining, by fitting, the following values for the three elements of the coefficient-vector $\boldsymbol{\beta}$: $\beta_{1,1}$ = 10 km, $\beta_{1,2}$ = 25 km and $\beta_{1,3}$ = 1.1; these are referred to below as 'central values'.

In general, these values should serve as a reference around which to define the search range mentioned above. However, in this specific case, a preliminary sensitivity analysis pointed up the impossibility of improving these initial estimations by calibration; in fact, large variations in the elements of the coefficient-vector $\boldsymbol{\beta}$ did not have any effect on the value of the *OF*. As a consequence, only the elements of the coefficient-vector $\boldsymbol{\alpha}$ (Eqn. 8) were calibrated, while the coefficients of the law $\lambda_{0,1}(Q,\boldsymbol{\beta})$ were set at the three central values indicated above.

Lastly, to apply the VPMC model correctly, the conditions given by constraints. (7) have to be respected. Thus, with reference to a celerity value of 1.5 m/s, the following steps were chosen: Δt = 5 h and Δx equal to the half-distance between the two stations (Δx = 27.5 km). These values ensure a ratio of $r_1 \cong 1$ and, in general, they conform to the constraints (7a) and (7b). During the calibration process, any combinations of the celerity law coefficients that do not match these constraints or combinations which would produce non-monotonic increasing rating curves in the downstream section, were penalised with very high *OF* values. The time step Δt = 5 h, while quite large, does not affect the description of the stage hydrographs recorded given their slow evolution, which moreover is consistent with the assumed kinematic conditions.

The calibration procedure required 540 *OF* calls and produced the estimation of the stretch-law $c_{k,1}(Q_t,\boldsymbol{\alpha})$ ($\alpha_{1,1}$=*1.2* m/s, $\alpha_{1,2}$ = 2.2 m/s and $\alpha_{1,3}$ = 1.9); Fig. 4 shows, directly in a graphical way, the rating curve in the Sermide section estimated by the procedure proposed. Given the very nature of the procedure, this rating curve is, for the calibration period, ***congruent*** with the one ***assumed to be valid*** at Borgoforte. Figure 4 also shows the discharge-stage measuring points, which were actually recorded during the period 1986–1995, ***but which have so far not been used.*** An ***a posteriori*** comparison with these points underlines the validity of the whole methodology whose purpose is to synthesise the rating curve in cross-sections provided with water level measurements but totally devoid of discharge measurements. However, the synthesised rating curve shows a small but systematic

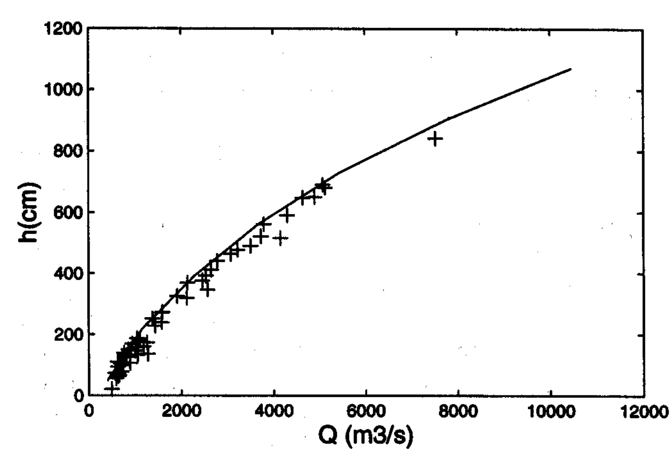

Fig. 4. First stretch – Sermide station: calibration period. Synthesised rating curve (solid line), obtained with the proposed method, and measuring points (+) for the period 1986–1995.

under-estimation of Q with respect to the measuring points; this apparent contradiction can be explained by the fact that at the Sermide station, there is not a perfect and systematic match between the gauge zero of the stages recorded continuously, and the gauge zero of the measurements taken over a time period of about 15 years.

Lastly, even though the main purpose of this methodology is the synthesis of rating curves, it implicitly provides a ready-calibrated simulation model whose ease of application suggests that it could also be useful in real time stage forecasting. Indeed, Fig. 5 reproduces the stages simulated using the VPMC model, calibrated following the procedure illustrated above, together with the stages observed: the average error does not exceed 7 cm (less than 1% of the total range).

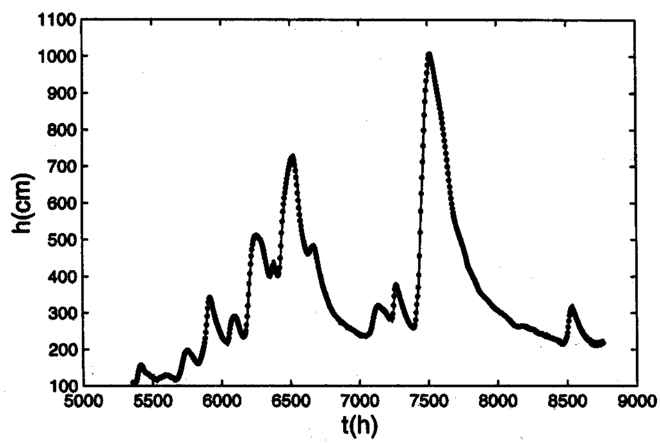

Fig. 5. *First stretch – Sermide station: calibration period. Observed (solid line) and simulated (dotted line) stages. The time is scanned in hours and counted from the start of the year.*

Second and third stretch—synthesising the rating curve for the Ficarolo and Pontelagoscuro stations

The Ficarolo station is situated 15 km downstream of Sermide, while Pontelagoscuro is situated 20 km downstream of Ficarolo. For both of them, coherent values of Δx and Δt, according to the conditions given by constraints (7), were chosen and the stretch-laws were calibrated taking account of the results of the sensitivity analysis described above, i.e. only the celerity law was calibrated, whereas with regard to the stretch-law of the characteristic length the one identified for the reach between Borgoforte and Sermide was used since the available information on the characteristics of the watercourse in the stretch in question would not have justified its variation.

For the second stretch, the celerity law $c_{0,2}(Q,\alpha)$ identified by calibration is: $\alpha_{2,1}$ = 1.23 m/s, $\alpha_{2,2}$ = 2.01 m/s and $\alpha_{2,3}$ = 1.96; it is virtually identical to that for the first stretch and this confirms that the hydraulic characteristics are roughly the same in the two stretches. Figure 6 shows

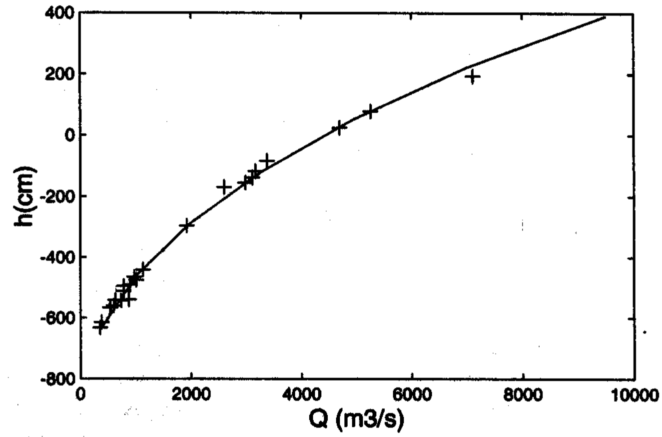

Fig. 6. *Second stretch – Ficarolo station: calibration period. Synthesised rating curve, obtained with the proposed method, and measuring points for the period 1990–1994.*

the rating curve obtained at Ficarolo and the discharge-stage measurments observed during the period 1990–1994; as with the Sermide station, these were not used in any way in the proposed procedure. In this case too, it was established that the stages simulated using the VPMC model produced a mean error, in relation to the values observed, of not more than 7 cm (less than 1% of the total range).

For the third stretch, the celerity law $c_{k,3}(Q,\alpha)$ identified is: $\alpha_{3,1}$= 1.49 m/s, $\alpha_{3,2}$= 3.27 m/s and $\alpha_{3,3}$= 1.81; Fig. 7 shows the rating curve at Pontelagoscuro together with the measuring points for the period 1979–1990. As with the two previous stations, the stages simulated give an average error, in relation to the observations, of not more than 10 cm (less than 1% of the total range).

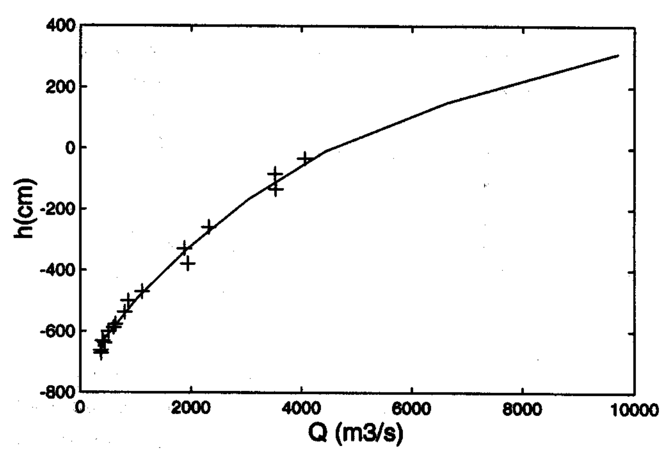

Fig. 7. *Third stretch – Pontelagoscuro station: calibration period. Synthesised rating curve, obtained with the proposed method, and measuring points for the period 1979–1994.*

All the previous examples show that the proposed procedure allows the rating curve to be extended to the maximum levels observed during the calibration period and, therefore, well beyond the available measurements (see, for example, Pontelagoscuro); this extension, given that it is based on the application of a gradually varied unsteady flow hydraulic model, albeit simplified, produces results which should be seen as more reliable than those produced by a simple statistical extrapolation of the measuring points.

Finally, the automatic calibration requires, for each stretch, short computation times of not more than 20 minutes (PC with 166 Mhz Pentium processor).

VALIDATION

Validation was performed using, for each stretch, the laws $c_{k,i}(Q,\alpha)$ and $\lambda_{0,i}(Q,\beta)$ as described above, the estimated rating curves $Q(h,\gamma)$ at the three stations and a different data period (first few months of 1994). Figure 8 shows, with reference to the Sermide station, the stages calculated and the corresponding observed values: the mean error is ≅ 16 cm. Similar results are obtained for the other two stations; Ficarolo station: mean error ≅ 7cm; Pontelagoscuro station: mean error ≅ 9cm). The analysis of Fig. 8, while it confirms the basic extendibility of the results obtained to other flood events, highlights some differences between observed and simulated, especially in the first few months of the year. However, these differences are consistent with the uncertainty present at the discharge—stage measurements (cf. Figs. 3 and 4); these may be attributed to recorder measuring errors and to the variability of the flow conditions in time (in fact, the stage-discharge measurements were made over a period of several years).

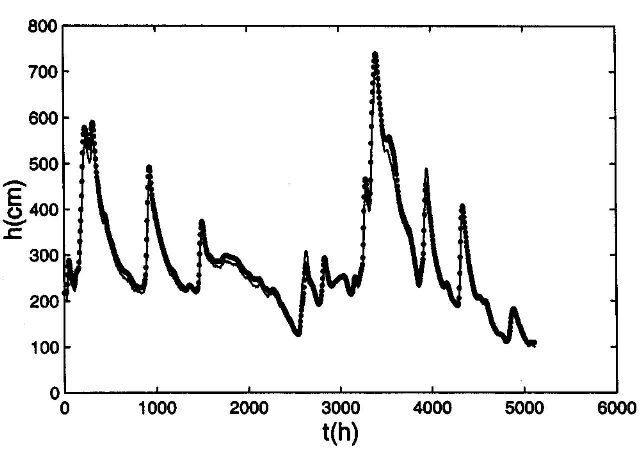

Fig. 8. *First stretch – Sermide station: validation period. Observed (soild line) and simulated (dotted line) stages. The time is scanned in hours and counted from the start of the year.*

Conclusions

The methodology described allows the synthesis, or perhaps just the validation and extension to higher discharge values, of the rating curve in one or more cross-sections provided with water level recorders; specifically, its application requires that, in addition to a level recorder, a reliable rating curve is also available in the upstream section of the reach considered.

The proposed methodology, which is based on the hypothesis that the flood propagation occurs in quasi-kinematic conditions (i.e. narrow rating loop in any cross sections), can be summarised as follows: in the upstream cross-section, the observed levels are converted to discharge values Q using the known rating curve. This discharge Q is propagated downstream using the VPMC model whose parameters are expressed as power laws of Q itself, considered valid on each stretch of river delimited by cross-sections provided with level recorders. This propagation, including an attenuation effect, in turn compatible with the "quasi-kinematic" wave assumption, can be considered as "correct" when the discharge values Q, calculated in the downstream section of the stretch in question, and the corresponding observed levels are positioned on a one to one curve. This curve represents the rating curve sought and it is congruent with the upstream curve which is assumed to be known; it is estimated together with the laws of the two stretch parameters in the VPMC model. Thus, the propagation model calibrated and the rating curve obtained can also be used to simulate or forecast in real time the levels in the downstream section once the levels in the upstream section are known.

The numerical application described refers to the river Po in the reach between Borgoforte (upstream cross-section with known rating curve) and Pontelagoscuro. Proceeding from upstream to downstream, in accordance with the quasi-kinematic characteristics of the flood events observed, the rating curves at Sermide, Ficarolo and Pontelagoscuro were estimated. A comparison between these curves and the stage-discharge measurements, which are actually available in these sections but were not used in the calculations, confirms the validity of the procedure proposed.

Lastly, this procedure should be applied with caution in the case of a reach where the tributaries produce large contributions and/or where the downstream conditions might be such as to affect the upstream river reach.

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