

S1. Model Procedure

S1.1 Volume calculation of elliptic paraboloid

Figure S1 illustrates schematic diagrams of the MDL outlet (a) and inlet (b) as elliptical paraboloids, with their respective volumes labeled as V_1 and V_3 . In this section, we derive the expressions for V_1 and V_3 using the definite integral rule. We denote the semi-major and semi-minor axes of the ellipse in Figure S1 a as a_1 and b_1 , and those of the ellipse in Figure S1 b as a_2 and b_2 . As depicted in Figure S1, points A ($w \cdot 2^{-1}$, 0, h) and B (0, m , h) reside on the ellipse V_1 , while points C ($w \cdot 2^{-1}$, 0, h) and D (0, n , h) lie on the ellipse V_3 . By utilizing formulas (1) and (2), we can derive the following expressions:

$$a_1 = a_2 = \frac{w}{2\sqrt{h}}, \quad b_1 = \frac{m}{\sqrt{h}}, \quad b_2 = \frac{n}{\sqrt{h}} \quad (\text{S1})$$

The volume of the elliptical paraboloid can be determined by integrating the cross-sectional area with respect to the height variable, c ($0 \leq c \leq h$). The cross-sectional equation for the ellipsoid paraboloid is expressed as follows:

$$\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = c \quad (\text{S2})$$

Thus,

$$V_1 = \int_0^h \frac{1}{2} \pi a_1 b_1 c dc = \frac{\pi a_1 b_1 h^2}{4} = \frac{\pi w m h}{8} \quad (\text{S3})$$

Similarly,

$$V_3 = \int_0^h \frac{1}{2} \pi a_2 b_2 c dc = \frac{\pi a_2 b_2 h^2}{4} = \frac{\pi w n h}{8} \quad (\text{S4})$$

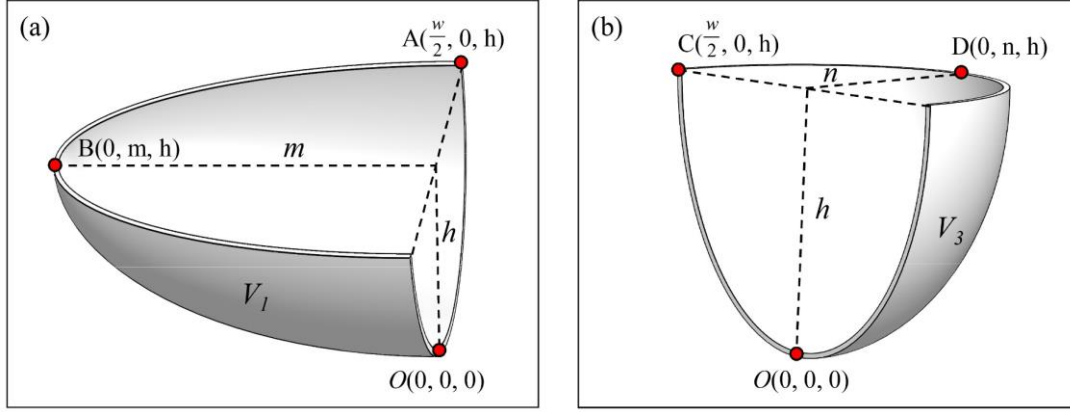


Figure S1. Schematic illustration of the ellipsoidal paraboloid at outlet (a) and inlet (b) of the MDL.

S1.2 Volume calculation of parabolic cylinder

As illustrated in Figure S2 a, the central section of the MDL is represented as a parabolic cylinder to calculate the volume V_2 , with P_s representing its cross-section. Figure S2 b illustrates that P_s takes the shape of a paraboloid, defined by the equation $y=kx^2$. Assuming that point F ($w \cdot 2^{-1}$, h) lies on the parabola, we can express the parameter x as follows:

$$k = \frac{4h}{w^2}, \quad y = \frac{4h}{w^2} x^2, \quad x = \sqrt{y} \frac{w}{2\sqrt{h}} \quad (\text{S5})$$

The area of P_s can be calculated using the following formula:

$$S_{P_s} = 2 \int_0^h \sqrt{y} \frac{w}{2\sqrt{h}} dy = \frac{2wh}{3} \quad (\text{S6})$$

Therefore, the volume of the parabolic cylinder is given by:

$$V_2 = S_{P_s} r = \frac{2}{3} whr \quad (\text{S7})$$

In the volume equations provided for V_1 , V_2 and V_3 of the MDL, the variables involved are w , h , m , n and r . With the exception of the unknown parameter h , all other parameters can be automatically determined based on the boundary data of the

MDL. Subsequently, we can derive the expression for the variable h using point F. As point F ($w \cdot 2^{-1}$, h) lies on the parabola shown in Figure S2 b, we can employ the geometric interpretation of the derivative to obtain the derivative $y'(w \cdot 2^{-1})$, which represents the slope of the tangent line at point F on the parabola. In this context, we denote the average slope around the MDL as a .

Thus,

$$y'(\frac{w}{2}) = 2 \times \frac{4h}{w^2} \times \frac{w}{2} = \tan(\alpha) \quad (S8)$$

$$h = \frac{w \tan(\alpha)}{4} \quad (S9)$$

Finally, considering the four distinct types of MDL, we can focus on the case where $r=0$ and $n=0$, which corresponds to GCL-1. In this scenario, the volume of the GCL-1 can be represented by the following expression:

$$V_{\text{GCL1}} = \frac{\pi w m h}{8} \quad (S10)$$

When $n=0$, the model of MDL corresponds to GCL-2, and its volume can be represented by the following expression:

$$V_{\text{GCL2}} = \frac{\pi w m h}{8} + \frac{2}{3} w h r \quad (S11)$$

When $r=0$, the model of MDL conforms to GUL-1, and its volume can be expressed as:

$$V_{\text{GUL1}} = \frac{\pi w m h}{8} + \frac{\pi w n h}{8} = \frac{\pi w h l}{4} \quad (S12)$$

When the type of MDL corresponds to GUL-2, its volume can be expressed as:

$$V_{\text{GUL2}} = \frac{\pi w m h}{8} + \frac{2}{3} w h r + \frac{\pi w n h}{8} = \frac{\pi w h (l - r)}{4} + \frac{2}{3} w h r \quad (S13)$$

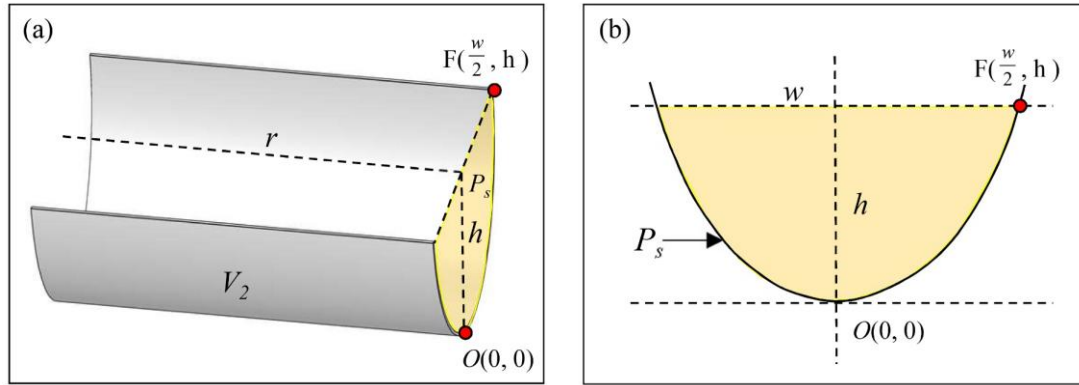


Figure S2. Parabolic cylinder (a) in the middle of the MDL and its schematic of the cross-section (b)